Fidelity induced transparency and the quantum illusionist game

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- 一 Interference at a beam splitter
  - *Transparency and bath engineering*
- 二 The birth of (Gaussian) entanglement
  - *A necessary and sufficient condition in terms of fidelity*
- 二 The quantum illusionist game
  - *An experiment revealing hidden correlations*

The birth of correlations in bilinear interactions

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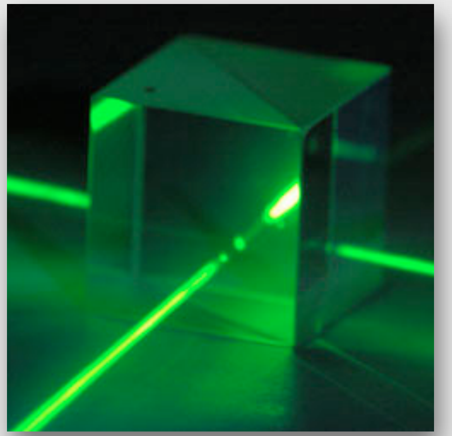
Any pair of bosonic modes:

- prepared in independent Gaussian states
- interact through an exchange Hamiltonian

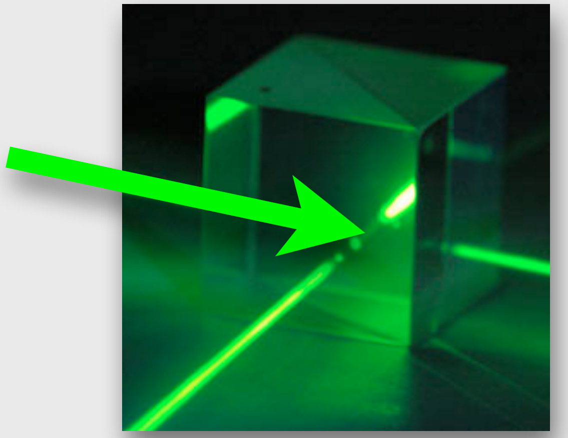
BS and interference

Beam splitter



BS and interference

Beam splitter



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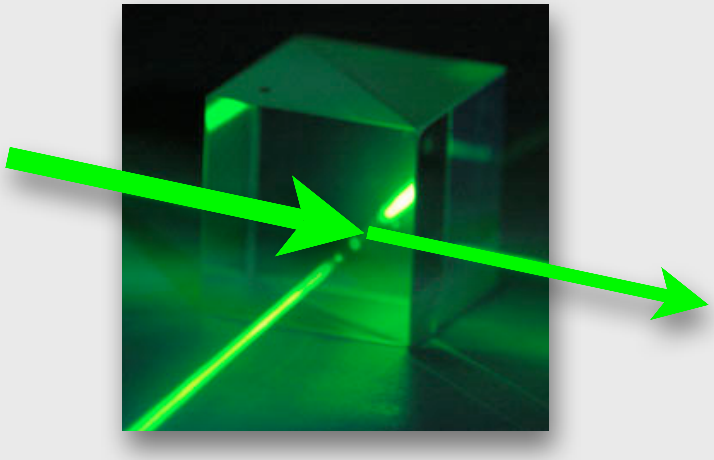

BS and interference

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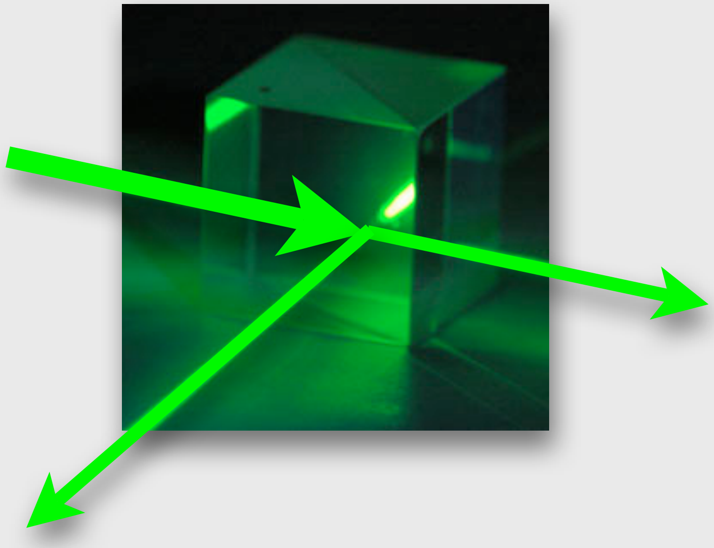
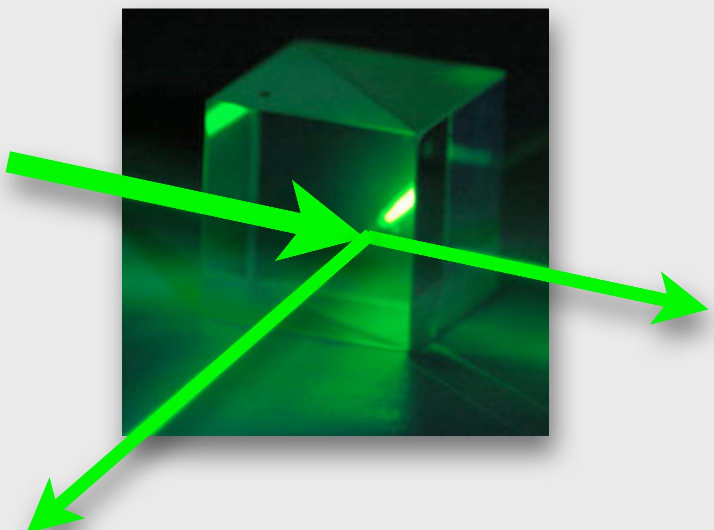
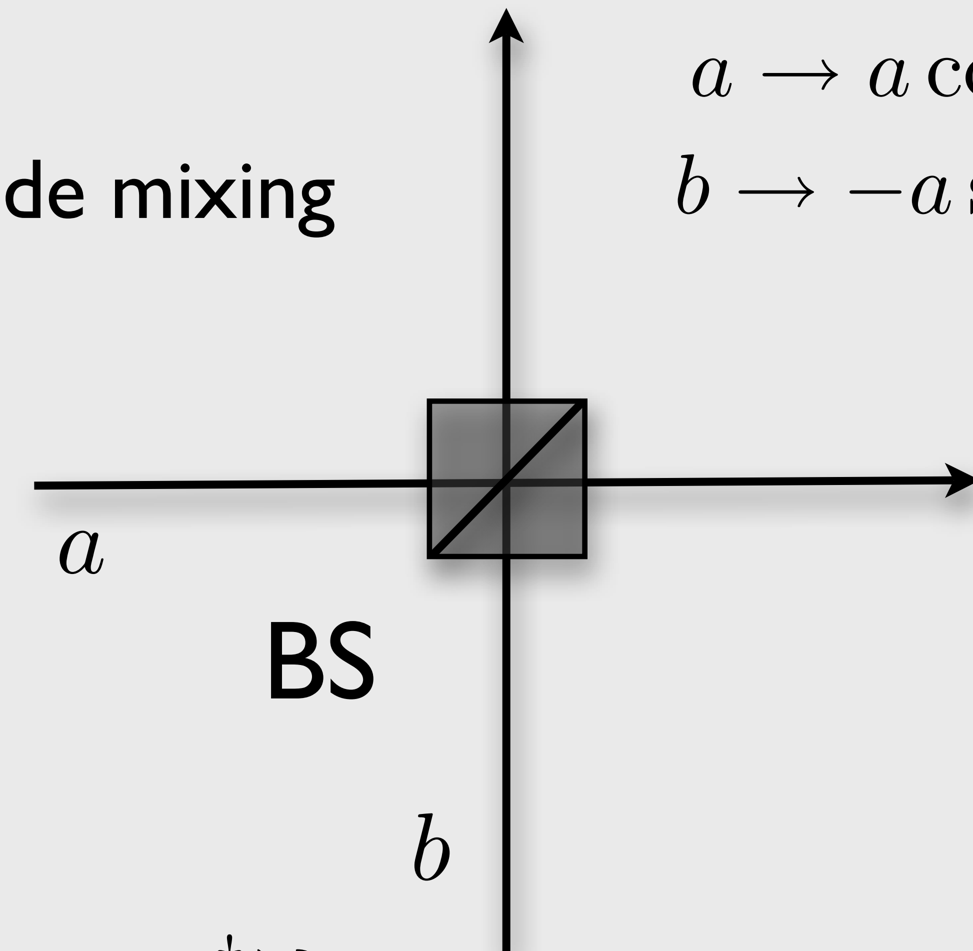


BS and interference

Beam splitter



Mode mixing



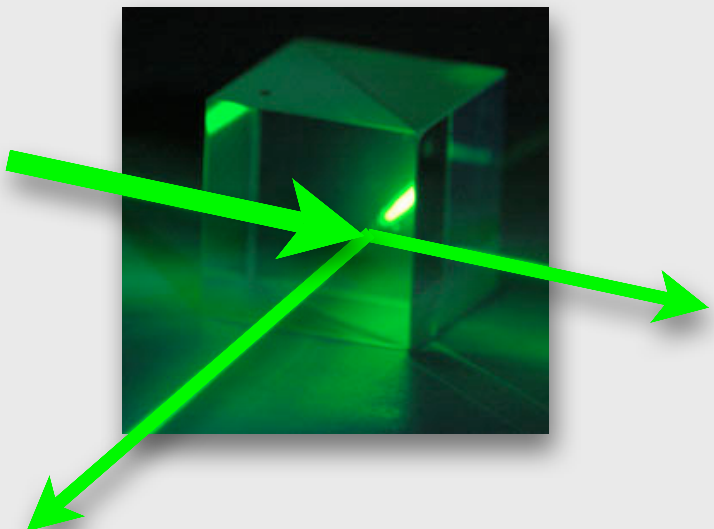
$$a \to a\cos\phi + b\sin\phi$$
$$b \to -a\sin\phi + b\cos\phi$$

$$U(\phi) = \exp\{\phi(a^\dagger b - a b^\dagger)\}$$

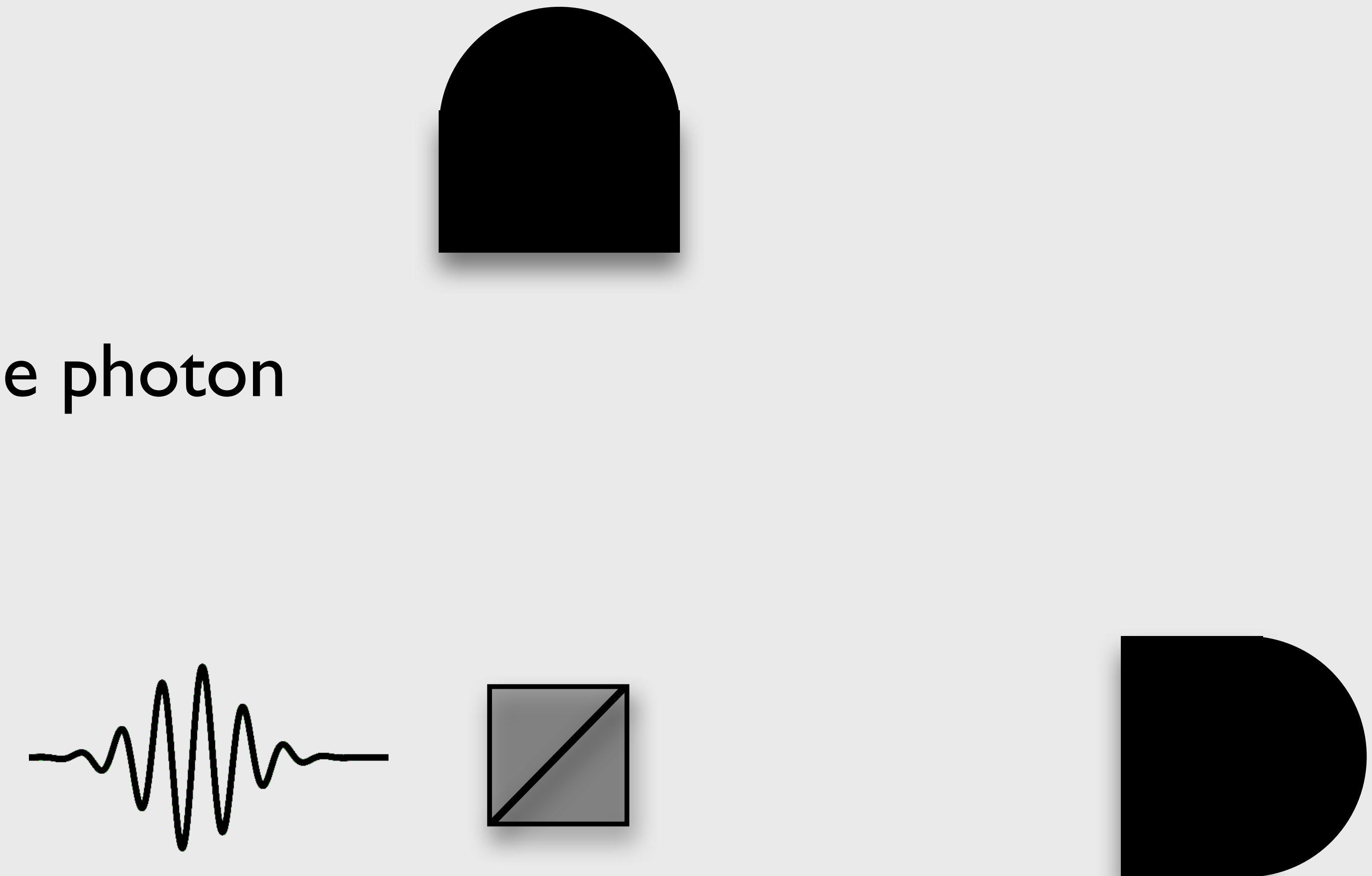
$\tau = \cos^2\phi$ transmissivity of the BS

BS and interference

Beam splitter



Single photon

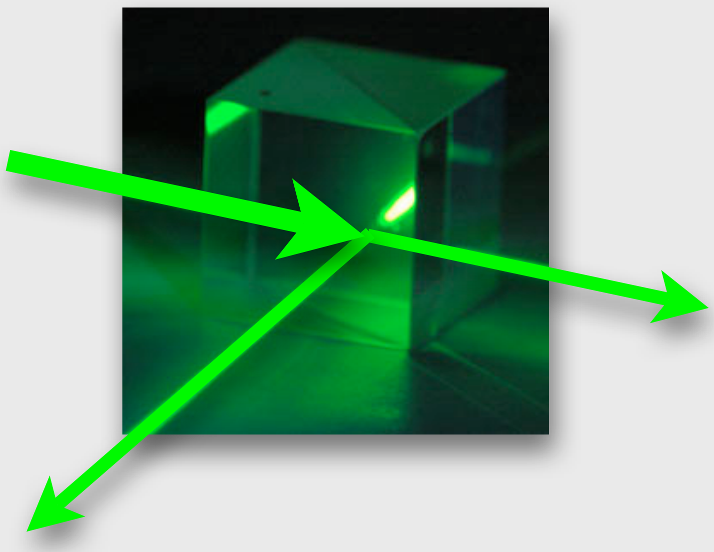


balanced BS: τ = 1/2

$$U(\phi)|0\rangle|1\rangle = \frac{1}{\sqrt{2}}\Big(|1\rangle|0\rangle + |0\rangle|1\rangle\Big)$$

BS and interference

Beam splitter



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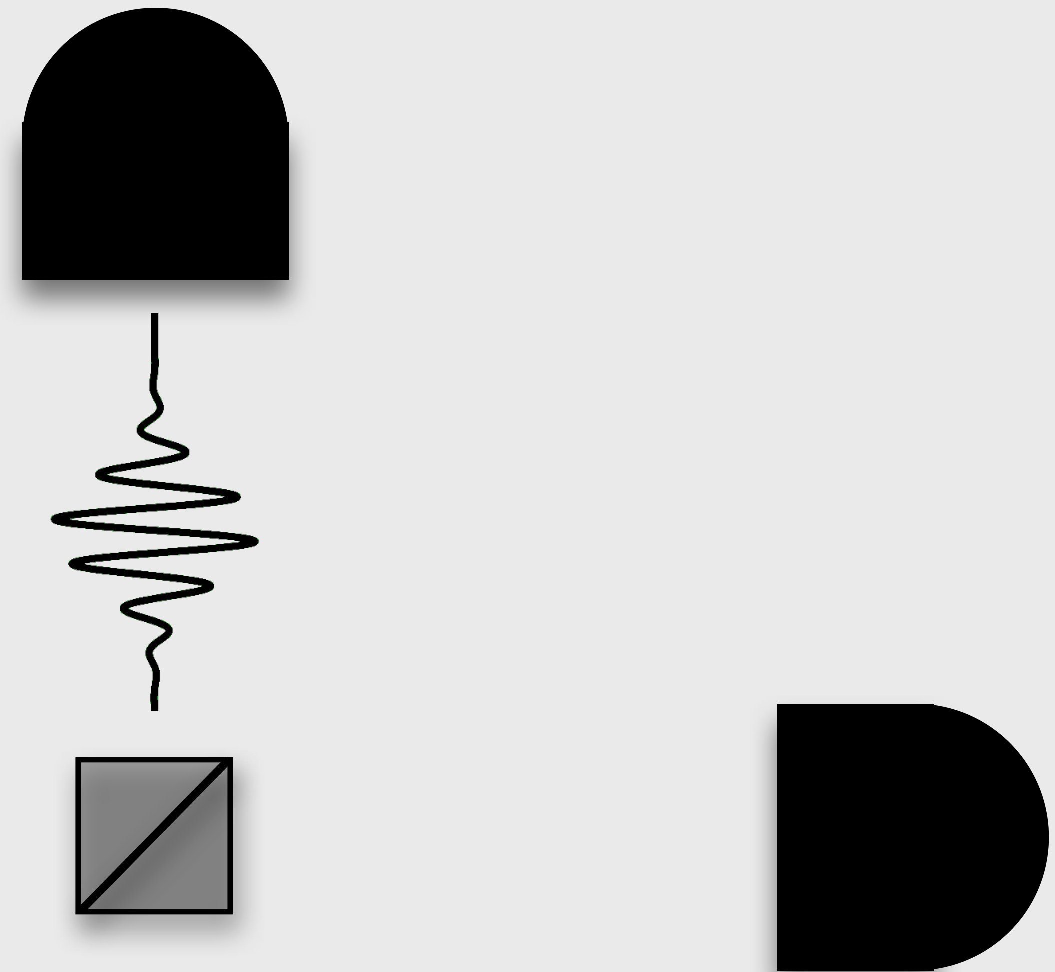

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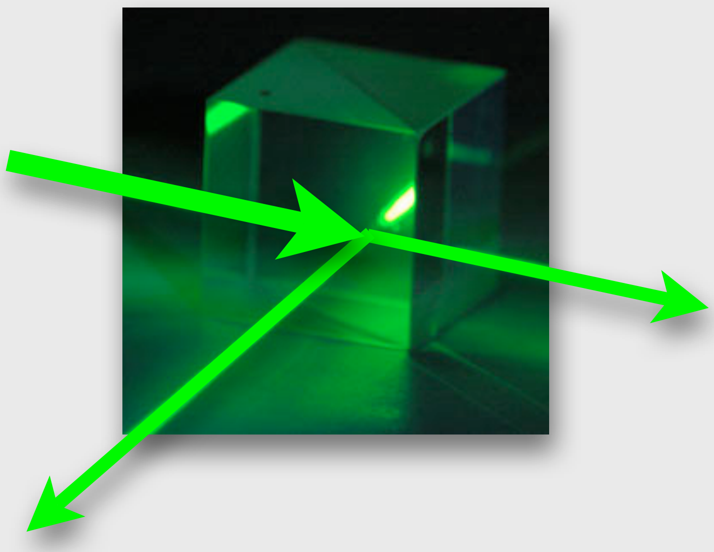
BS and interference

Beam splitter



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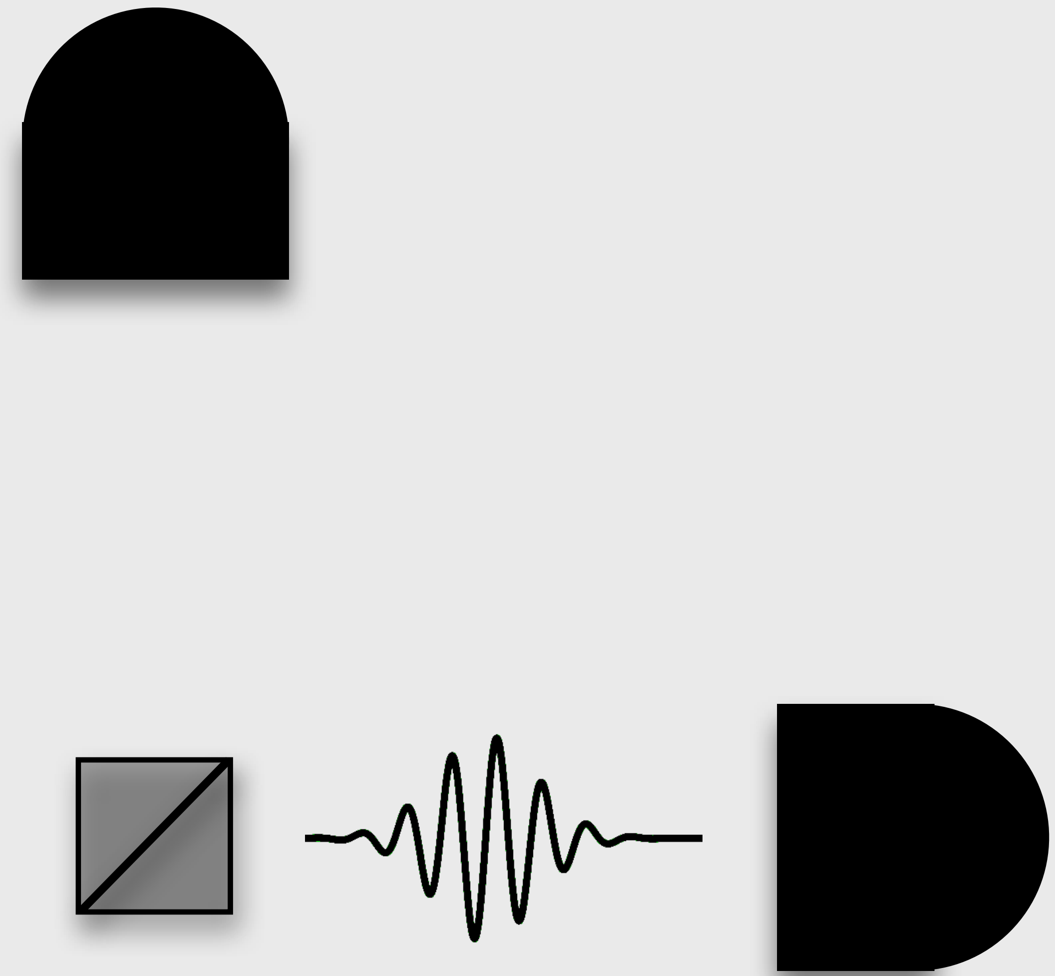

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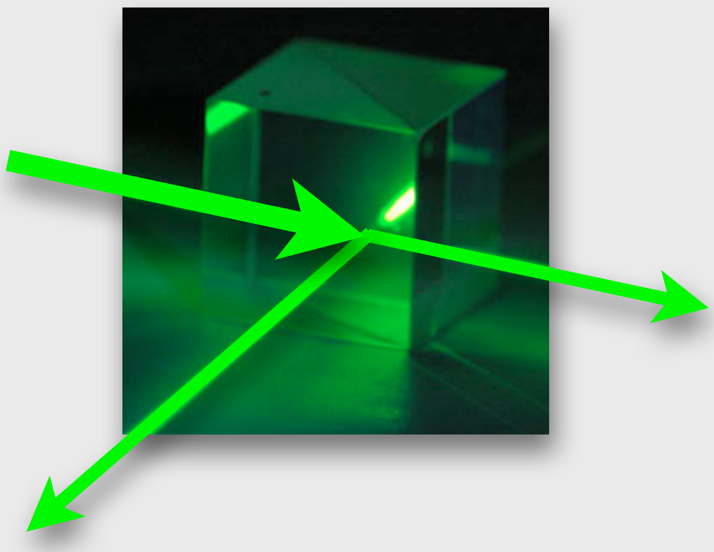
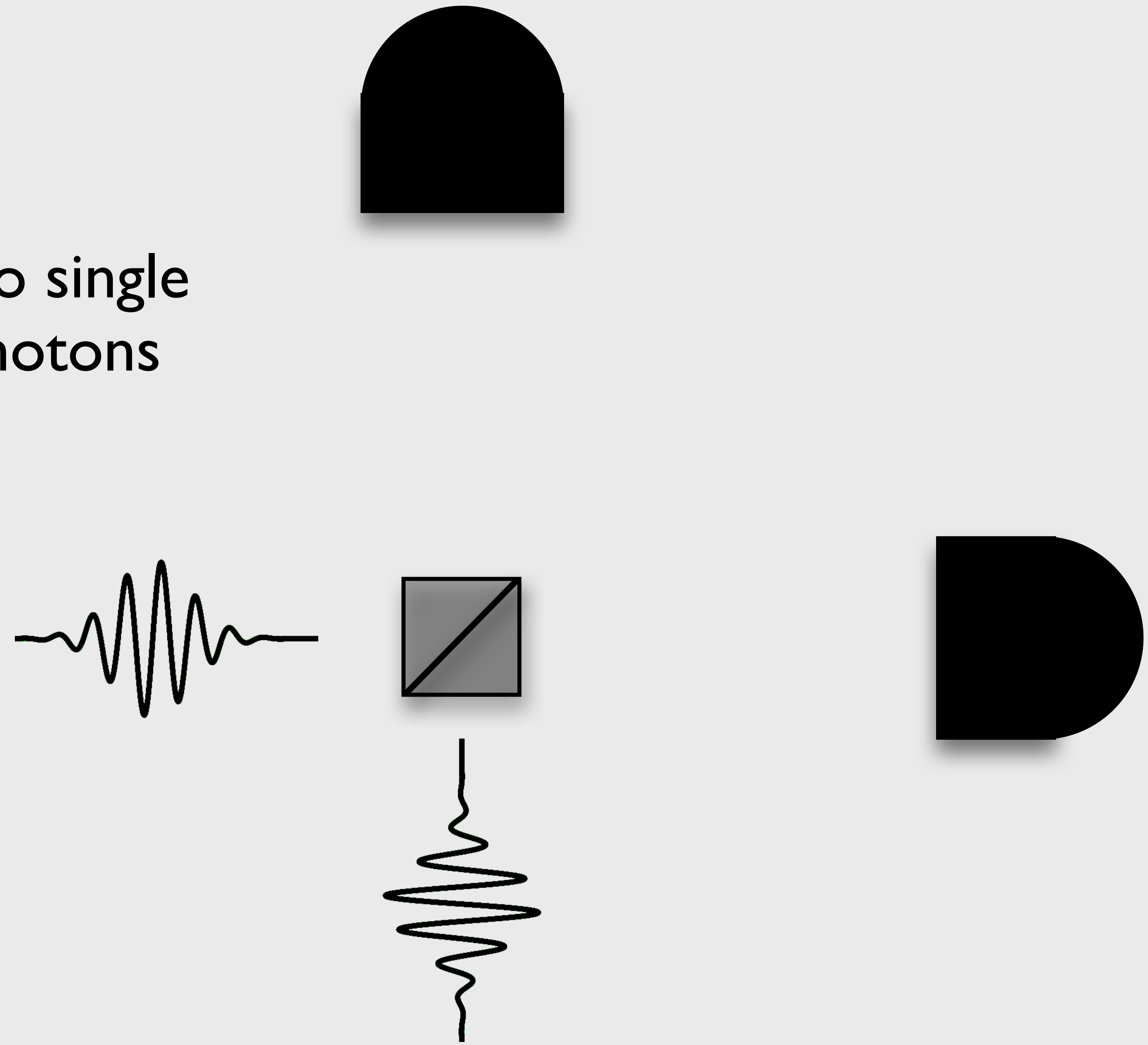
$$U(\phi)|0\rangle|1\rangle = \frac{1}{\sqrt{2}}\Big(|1\rangle|0\rangle + |0\rangle|1\rangle\Big)$$

BS and interference

Beam splitter



Two single photons



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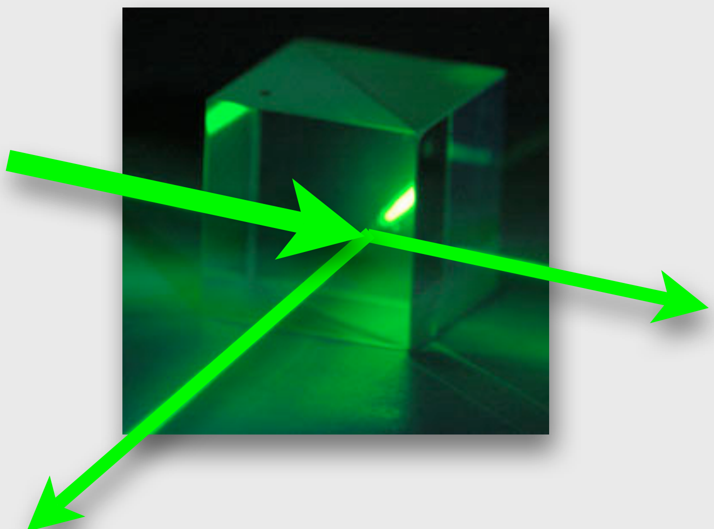
BS and interference

Beam splitter



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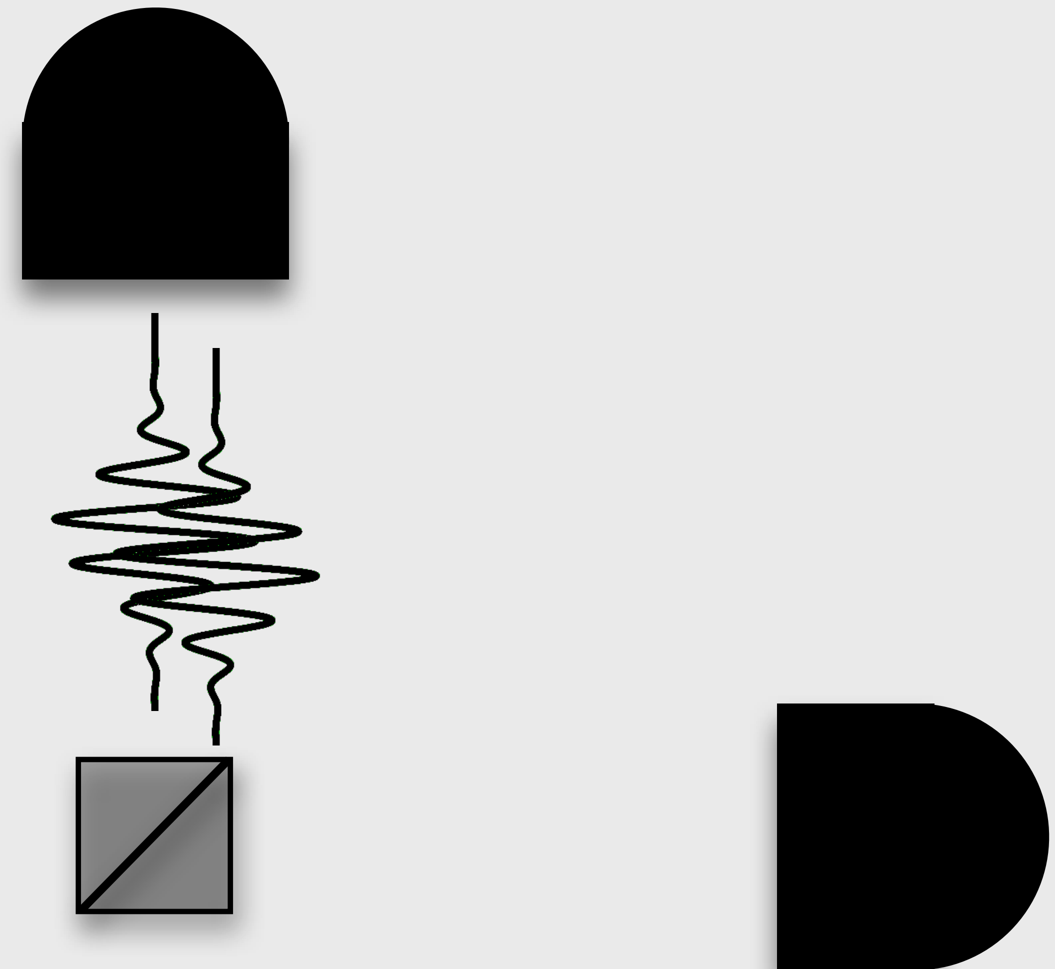

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BS and interference

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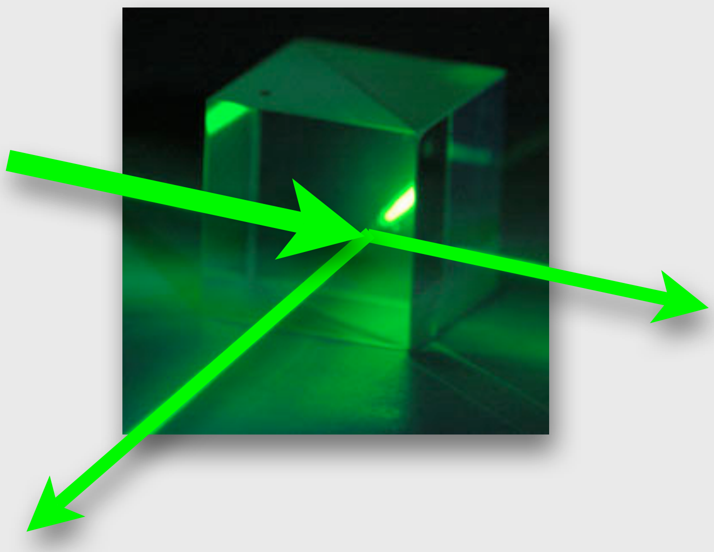
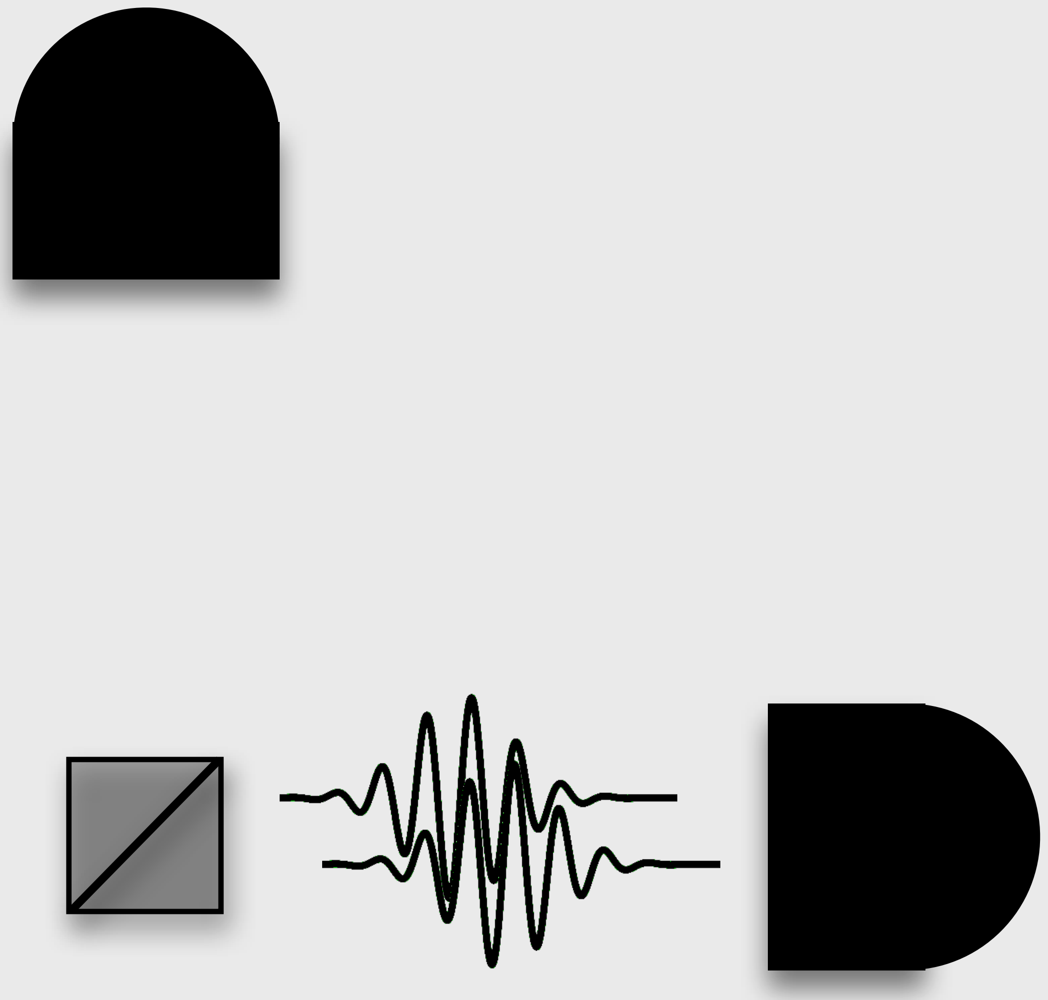


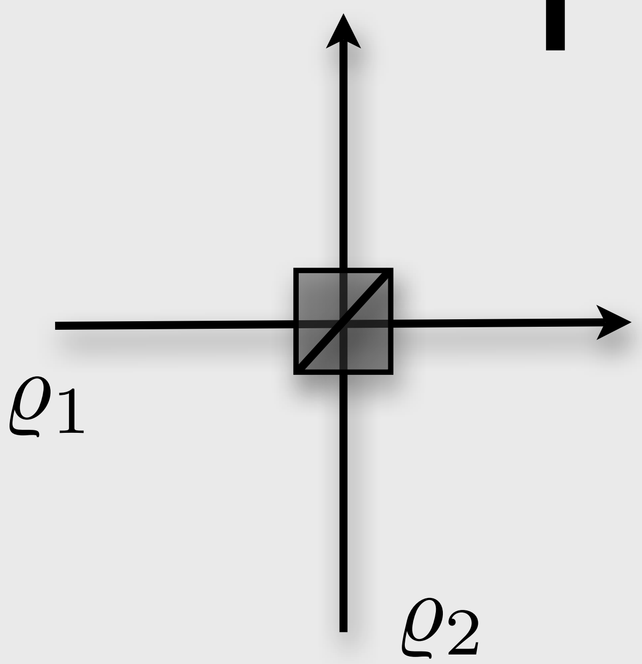
Two single photons



balanced BS: τ = 1/2

$$U(\phi)|1\rangle|1\rangle = \frac{1}{\sqrt{2}}\Big(|2\rangle|0\rangle + |0\rangle|2\rangle\Big)$$

Multiphoton states



$$\varrho = \sum_{n,m=0}^{\infty} \varrho_{nm} \, |n\rangle\langle m| \qquad \varrho_{nm} = \langle n|\varrho|m\rangle$$

$$U(\phi)|n_1\rangle \otimes |n_2\rangle = \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{n_2} A^{n_1 n_2}_{k_1 k_2} \, |k_1 + k_2\rangle \otimes |n_1 + n_2 - k_1 - k_2\rangle$$

Campos et al (1989)

$$A^{n_1 n_2}_{k_1 k_2} = \sqrt{\frac{(k_1+k_2)!(n_1+n_2-k_1-k_2)!}{n_1! n_2!}} (-)^{k_2} \begin{pmatrix} n_1 \\ k_1 \end{pmatrix} \begin{pmatrix} n_2 \\ k_2 \end{pmatrix} \sin\phi^{n_1-k_1+k_2} \cos\phi^{n_2-k_2+k_1}$$

Any simple picture in some specific cases?

Gaussian states

$$\varrho = \varrho(\alpha, \xi, N) = D(\alpha)\, S(\xi)\, \nu_{th}(N)\, S^{\dagger}(\xi)\, D^{\dagger}(\alpha)$$

$$\nu_{th}(N) = \frac{N^{a^{\dagger}a}}{(1+N)^{a^{\dagger}a}} \qquad N = 0 \to \nu_{th}(N) = |0\rangle\langle 0|$$

$$S(\xi) = \exp\{\frac{1}{2}\left(\xi a^{\dagger 2} - \bar{\xi} a^2\right)\}$$

$$D(\alpha) = \exp\{\alpha a^{\dagger} - \bar{\alpha} a\}$$

Gaussian states

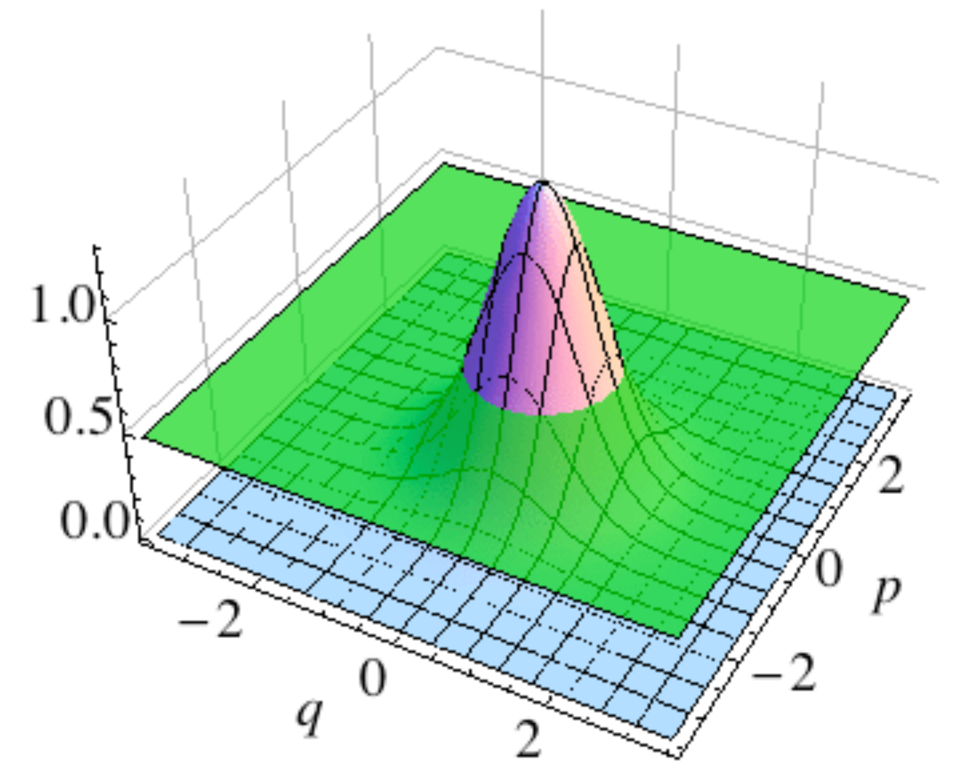
$$\varrho = \varrho(\alpha, \xi, N) = D(\alpha)\, S(\xi)\, \nu_{th}(N)\, S^\dagger(\xi)\, D^\dagger(\alpha)$$

Quadrature operators: $q = \frac{1}{\sqrt{2}}\,(a + a^\dagger) \quad p = \frac{1}{i\sqrt{2}}\,(a^\dagger - a)$

Gaussian states have Gaussian Wigner functions:

$$W(q, p)$$

Wigner function (vacuum state)



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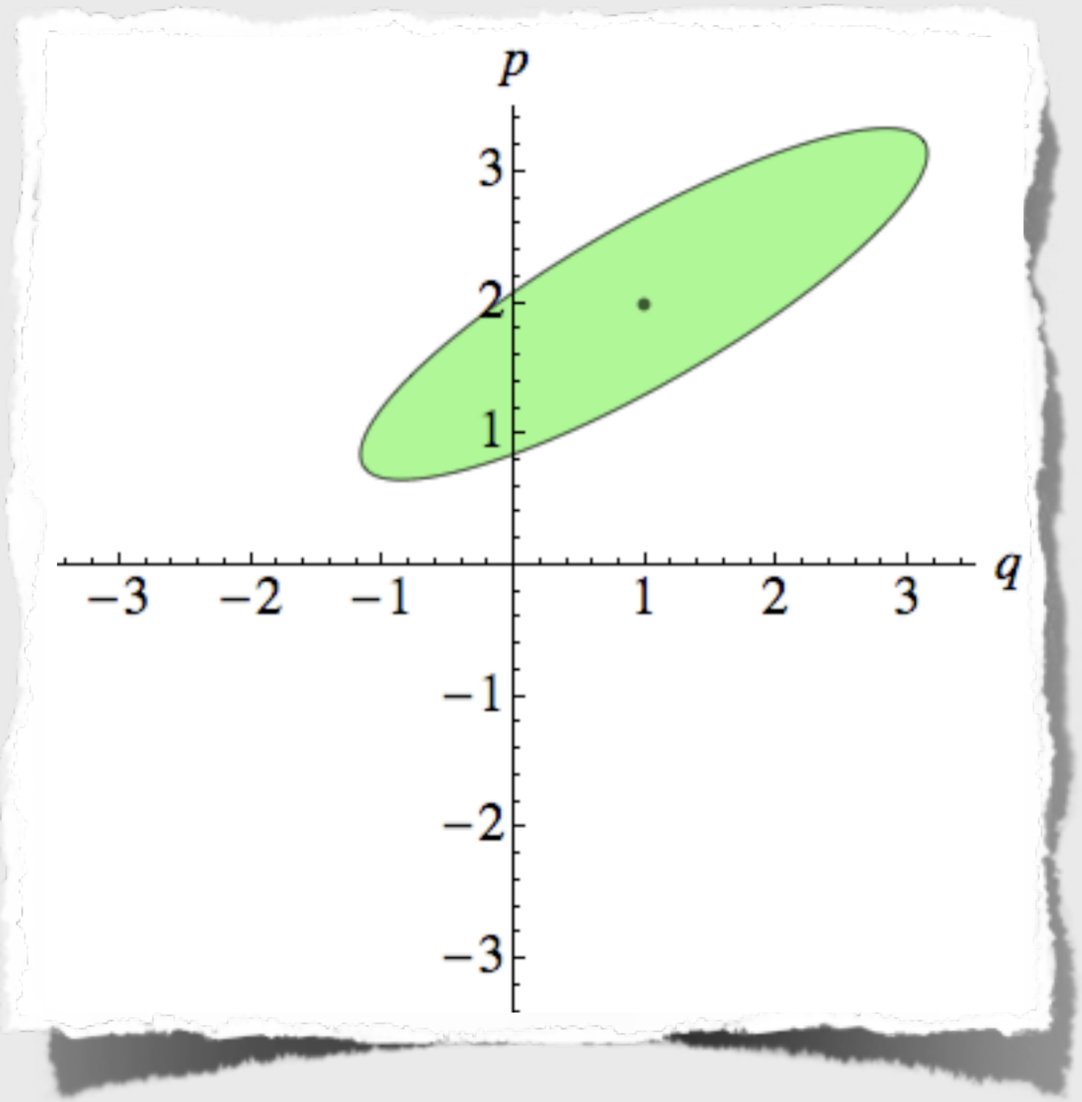
vacuum $|0\rangle\langle 0|$

thermal $\nu_{th}(N)$

squeezing $S(\xi)\nu_{th}(N)S^\dagger(\xi)$

displacement

$D(\alpha)\, S(\xi)\, \nu_{th}(N)\, S^\dagger(\xi)\, D^\dagger(\alpha)$



Gaussian states

$$\varrho = \varrho(\alpha, \xi, N) = D(\alpha)\, S(\xi)\, \nu_{th}(N)\, S^\dagger(\xi)\, D^\dagger(\alpha)$$

Quadrature operators: $R^T = (R_1, R_2) = (q, p)$

Mean values vector: $\langle R^T \rangle = \sqrt{2}(\mathrm{Re}\, \alpha, \mathrm{Im}\, \alpha)$

Covariance matrix (CM): $[\boldsymbol{\sigma}]_{hk} = \frac{1}{2}\langle R_h R_k + R_k R_h \rangle - \langle R_h \rangle \langle r_k \rangle$

$$[\boldsymbol{\sigma}]_{kk} = (2\mu)^{-1} \left[\cosh(2r) - (-1)^k \cos(\psi) \sinh(2r)\right]$$

$$[\boldsymbol{\sigma}]_{12} = [\boldsymbol{\sigma}]_{21} = -(2\mu)^{-1} \sin(\psi) \sinh(2r)$$

$\xi = re^{i\psi}$ $\quad \mu = \mathrm{Tr}[\varrho^2] = (1 + 2N)^{-1}$ purity

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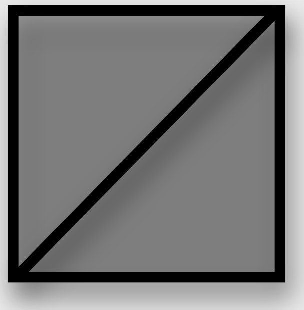
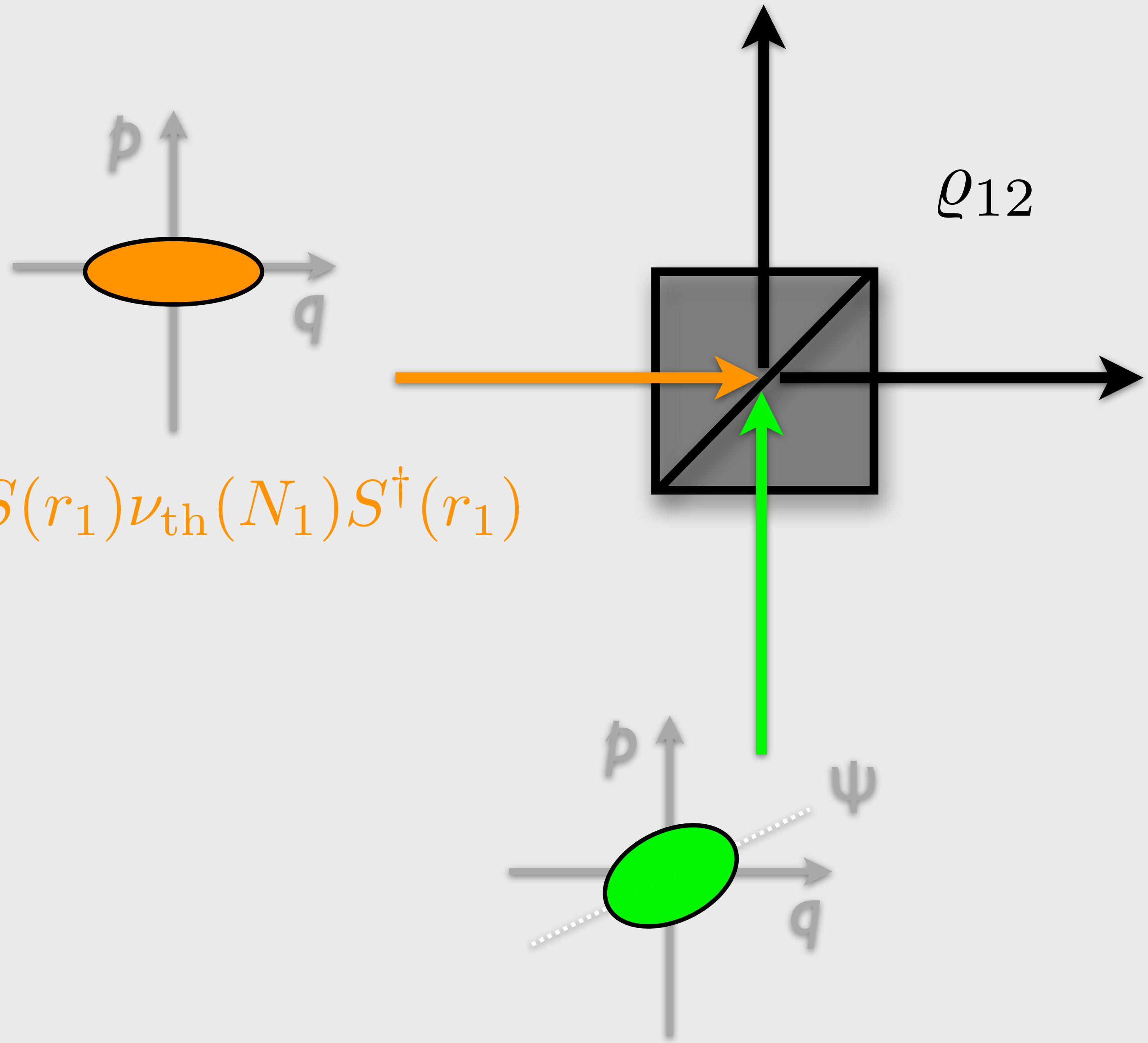
Interference of Gaussian states



Interference of Gaussian states

$$\varrho_1 = S(r_1)\nu_{\text{th}}(N_1)S^\dagger(r_1)$$

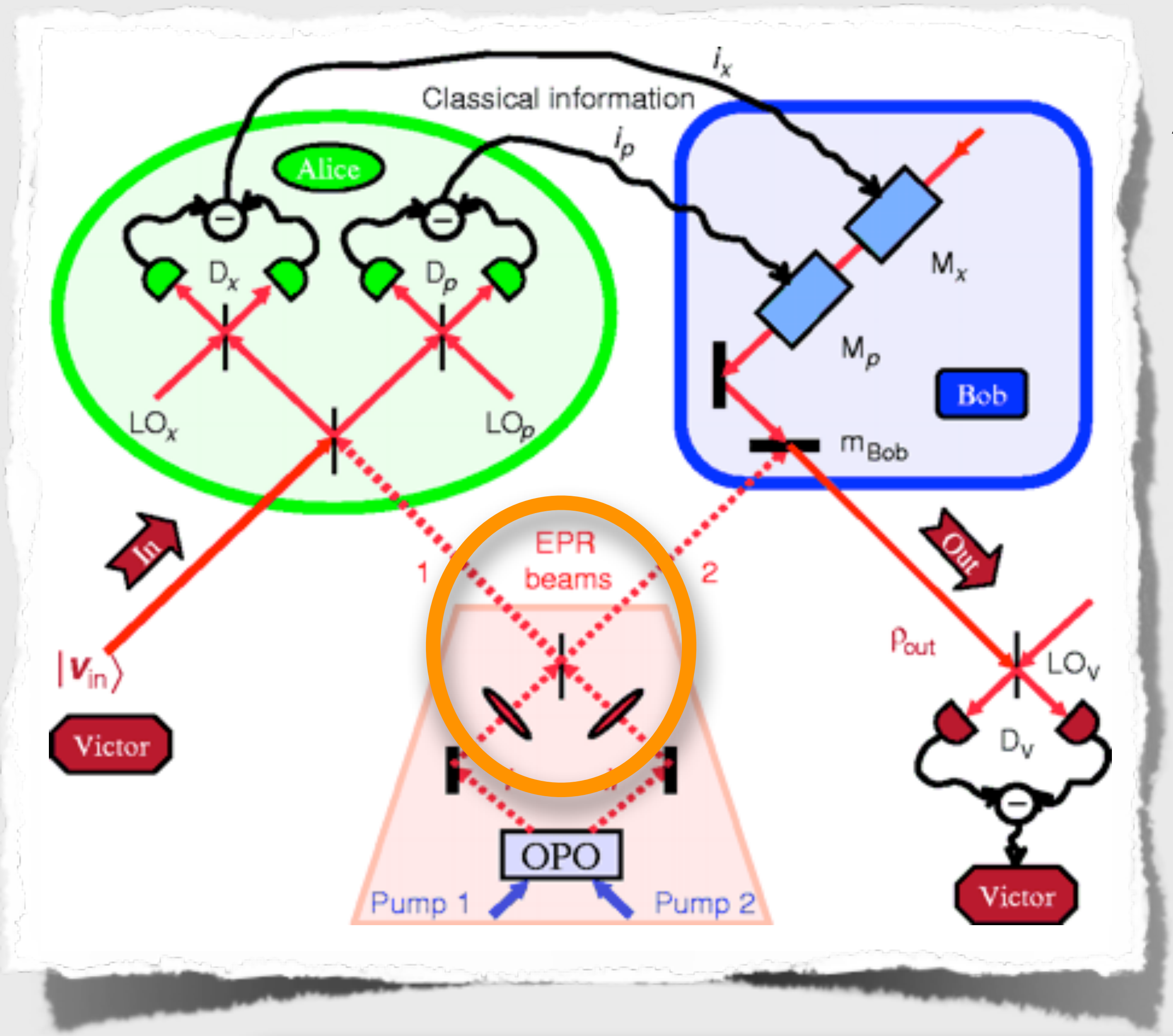
Interference of Gaussian states



$$\varrho_1 = S(r_1)\nu_{\text{th}}(N_1)S^\dagger(r_1)$$

$$\varrho_2 = S(r_2 e^{i\psi})\nu_{\text{th}}(N_2)S^\dagger(r_2 e^{i\psi})$$

Interference of Gaussian states



"*Unconditional Quantum Teleportation*", A. Furusawa *et al.*, Science **282,** 706 (1998)

In the first teleportation experiment involving continuous variables, the entangled resource was generated by the interference of two squeezed, and thus Gaussian, states with orthogonal squeezing phases:

$$\varrho_1 = S(r)|0\rangle\langle 0|S^\dagger(r)$$

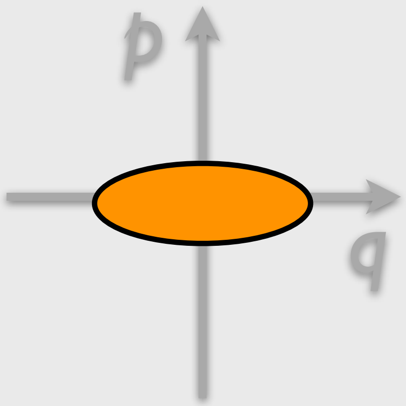
$$\varrho_2 = S(-r)|0\rangle\langle 0|S^\dagger(-r)$$

$$|\Psi_{twb}\rangle\rangle = \sqrt{1-\mathrm{Th}^2 r}\;\sum_n \mathrm{Th}^n r\,|n\rangle \otimes |n\rangle$$

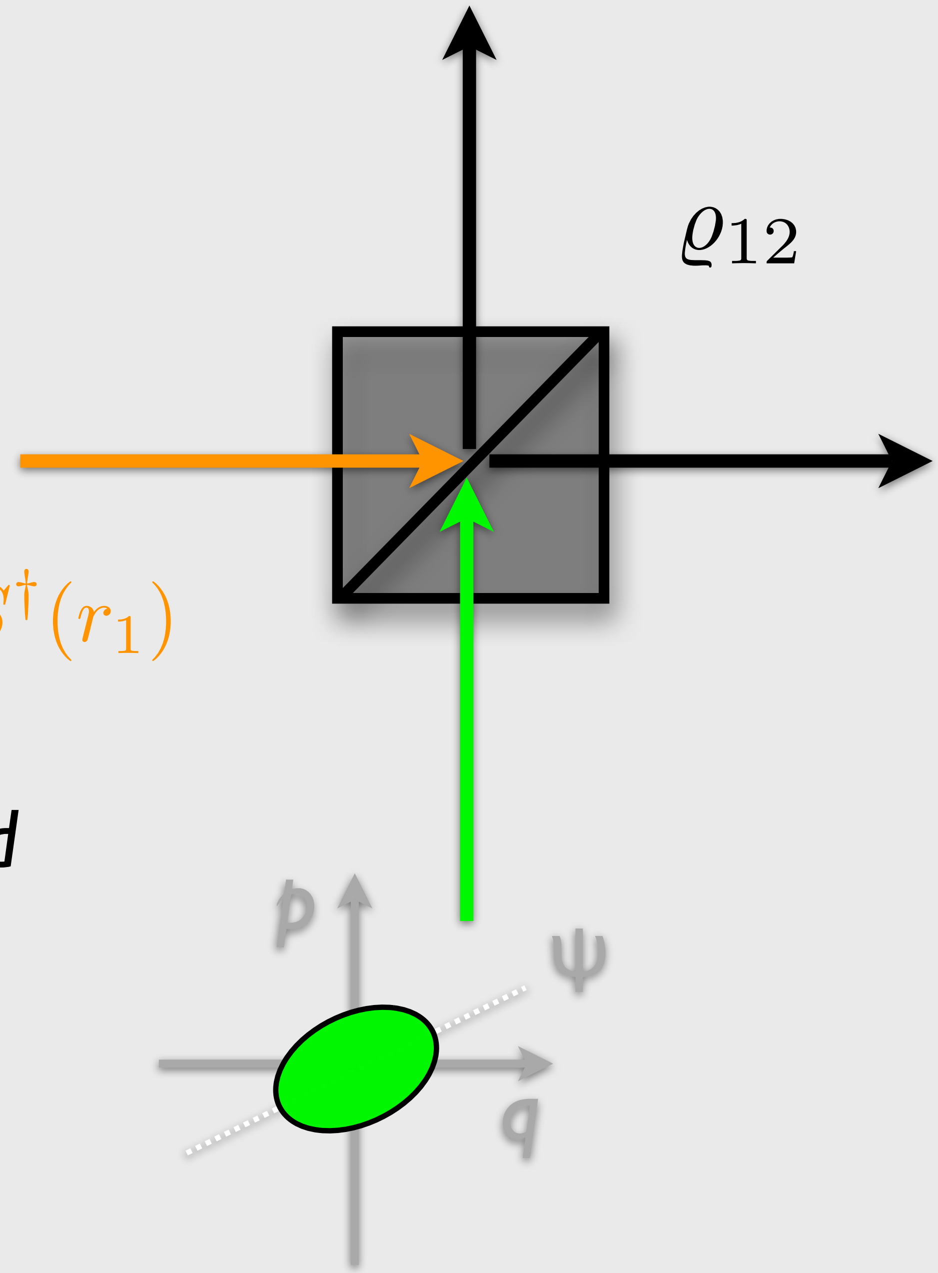
twin-beam state: maximally entangled state

Interference of Gaussian states

$$\begin{pmatrix} \boldsymbol{\sigma}_1 & \boxed{\mathbf{0}} \\ \mathbf{0} & \boldsymbol{\sigma}_2 \end{pmatrix}$$



$\varrho_1 = S(r_1)\nu_{\mathrm{th}}(N_1)S^\dagger(r_1)$



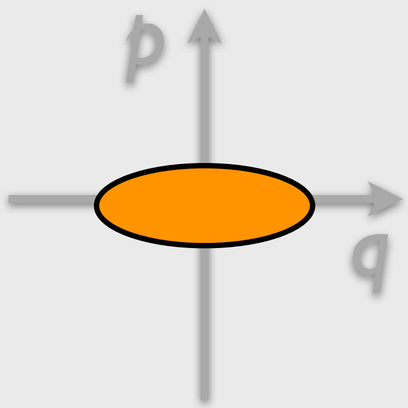
ϱ_{12}

Inputs: two uncorrelated Gaussian states

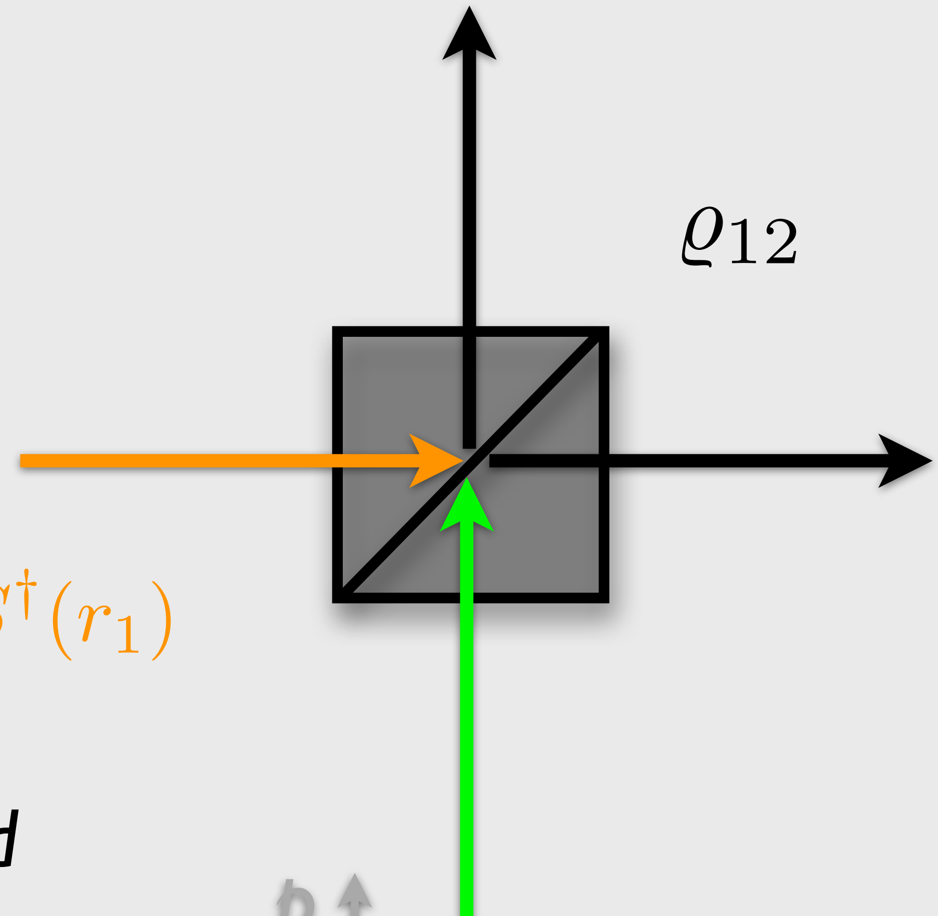
$\varrho_1 \otimes \varrho_2$

$\varrho_2 = S(r_2 e^{i\psi})\nu_{\mathrm{th}}(N_2)S^\dagger(r_2 e^{i\psi})$

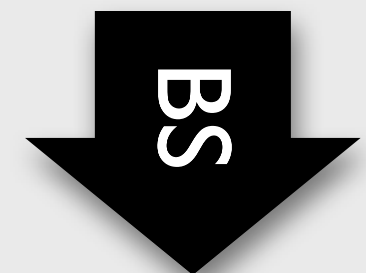
Interference of Gaussian states



$\varrho_1 = S(r_1)\nu_{\rm th}(N_1)S^\dagger(r_1)$



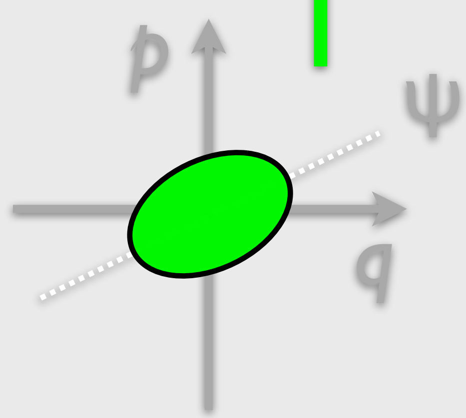
$$\begin{pmatrix} \boldsymbol{\sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_2 \end{pmatrix}$$



$$\begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_2 \end{pmatrix}$$

Inputs: two uncorrelated Gaussian states

$\varrho_1 \otimes \varrho_2$



$$\begin{aligned} \boldsymbol{\Sigma}_1 &= \tau\boldsymbol{\sigma}_1 + (1-\tau)\boldsymbol{\sigma}_2, \\ \boldsymbol{\Sigma}_2 &= \tau\boldsymbol{\sigma}_2 + (1-\tau)\boldsymbol{\sigma}_1, \\ \boldsymbol{\Sigma}_{12} &= \tau(1-\tau)(\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1) \end{aligned}$$

$\varrho_2 = S(r_2 e^{i\psi})\nu_{\rm th}(N_2)S^\dagger(r_2 e^{i\psi})$

Interference of Gaussian states

$$\begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_2 \end{pmatrix}$$

$$\boldsymbol{\Sigma}_1 = \tau\boldsymbol{\sigma}_1 + (1-\tau)\boldsymbol{\sigma}_2,$$
$$\boldsymbol{\Sigma}_2 = \tau\boldsymbol{\sigma}_2 + (1-\tau)\boldsymbol{\sigma}_1,$$
$$\boldsymbol{\Sigma}_{12} = \tau(1-\tau)(\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1)$$

If the two input modes are *excited in the same Gaussian state*, i.e., they have the same CM, $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_2 = \boldsymbol{\sigma}$, then:

$$\begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} \boldsymbol{\sigma} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma} \end{pmatrix} \Leftrightarrow \varrho_{12} = \varrho \otimes \varrho$$

the output is factorized and equal to the input (transparency).

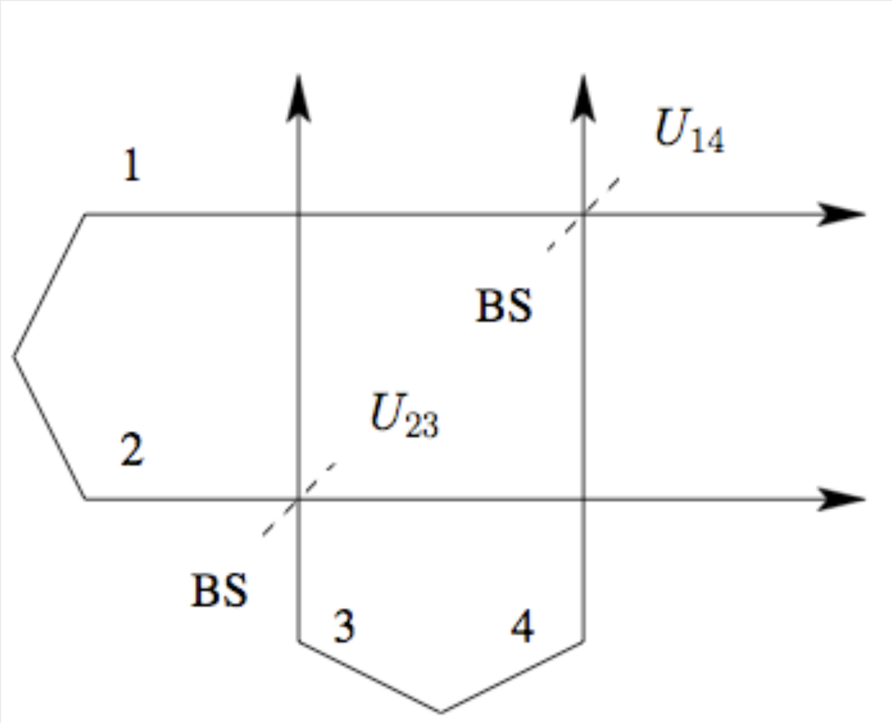
S. Olivares et al Phys. Rev. A **80**, 032329 (2009)
S.C. Springer et al Phys. Rev. A **79**, 062303 (2009)

Interference of Gaussian states

(multimode transparency)

$$U_{\mathrm{BS},N}(\phi)\varrho_{\boldsymbol{A}} \otimes \varrho_{\boldsymbol{B}}\, U^{\dagger}_{\mathrm{BS},N}(\phi) = \varrho_{\boldsymbol{A}} \otimes \varrho_{\boldsymbol{B}},$$

if and only if $\varrho_{\boldsymbol{A}}$ *and* $\varrho_{\boldsymbol{B}}$ *are excited in the same state.*



R. Bloomer, M. Pysher and O. Pfister (NJP 2011)

Interference of Gaussian states

(bath engineering to control decoherence)

$$\varrho_0 \rightsquigarrow \varrho_t(\Gamma, N)$$

$$\dot{\varrho} = \frac{1}{2}\Gamma(1+N)L[a]\varrho + \frac{1}{2}\Gamma L[a^\dagger]\varrho$$

$$L[O]\varrho = 2O\varrho O^\dagger - O^\dagger O \varrho - \varrho O^\dagger O$$

$$H_{SB} = \sum_j g_j (a b_j^\dagger + a^\dagger b_j) = a B^\dagger + a^\dagger B$$

The effective temperature of the bath sets the maximum purity of a signal that may be transmitted without decoherence

Interference of Gaussian states

$$\begin{pmatrix} \mathbf{\Sigma}_1 & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{12} & \mathbf{\Sigma}_2 \end{pmatrix}$$

$$\begin{aligned} \mathbf{\Sigma}_1 &= \tau\boldsymbol{\sigma}_1 + (1-\tau)\boldsymbol{\sigma}_2, \\ \mathbf{\Sigma}_2 &= \tau\boldsymbol{\sigma}_2 + (1-\tau)\boldsymbol{\sigma}_1, \\ \mathbf{\Sigma}_{12} &= \tau(1-\tau)(\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1) \end{aligned}$$

After the evolution (interference) the two modes are (classically or quantum) correlated.

(Gaussian) Discord is always different from zero.

- *What about the relation between the "similarity" of the inputs and the birth of entanglement?*

- *What is the actual role of squeezing?*

Gaussian entanglement

We recall that a *bipartite state* is entangled if and only if the *partially transposed* density matrix is no longer semi-positive defined:

$$\varrho_{12} = \sum_k p_k \varrho_{1k} \otimes \varrho_{2k} \longleftrightarrow \varrho_{12}^{\tau} < 0$$

In the case of *bipartite Gaussian states* this criterion can be rewritten in term of the CM. The state is entangled if and only if the *minimum symplectic eigenvalue* of CM associated with the partially transposed density matrix is less than 1/2; in our case:

$$\begin{pmatrix} \mathbf{\Sigma}_1 & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{12} & \mathbf{\Sigma}_2 \end{pmatrix}$$

$$I_1 = \det[\mathbf{\Sigma}_1] \; I_2 = \det[\mathbf{\Sigma}_2]$$
$$I_3 = \det[\mathbf{\Sigma}_{12}] \; I_4 = \det[\mathbf{\Sigma}]$$
$$\Delta = I_1 + I_2 - 2I_3$$

$$\tilde{\lambda} = \frac{1}{\sqrt{2}}\sqrt{\Delta - \sqrt{\Delta^2 - 4I_4}} < \frac{1}{2}$$

R. Simon, Phys. Rev. Lett. **84**, 2726 (2000)

The birth of entanglement

In general, we can state the following (we assume τ = 1/2):

Theorem 1 *The bipartite state* $\varrho_{12} = U_{\mathrm{BS}}\varrho_1 \otimes \varrho_2 U_{\mathrm{BS}}^{\dagger}$, *resulting from the evolution of two single-mode Gaussian states with zero first moments,* $\varrho_1(r_1, N_1)$ *and* $\varrho_2(r_2 e^{i\psi}, N_2)$, *through a balanced BS, is entangled if and only if the fidelity* $F(\varrho_1, \varrho_2)$ *between the inputs falls below a threshold value* $F_{\mathrm{e}}(\mu_1, \mu_2)$, *which depends only on their purities* $\mu_k = Tr[\varrho_k^2] = (1 + 2N_k)^{-1}$, $k = 1, 2$.

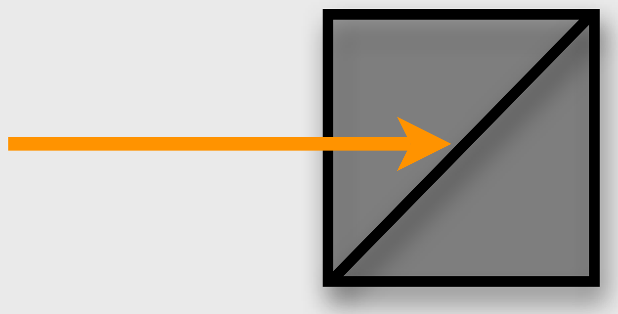
$$F_{\mathrm{e}}(\mu_1, \mu_2) = \frac{2\mu_1\mu_2}{\sqrt{2(1+\mu_1^2\mu_2^2)} - \sqrt{(1-\mu_1^2)(1-\mu_2^2)}}$$

$$F(\varrho_1, \varrho_2) = \left(\mathrm{Tr}\left[\sqrt{\sqrt{\varrho_1}\,\varrho_2\sqrt{\varrho_1}}\right]\right)^2$$

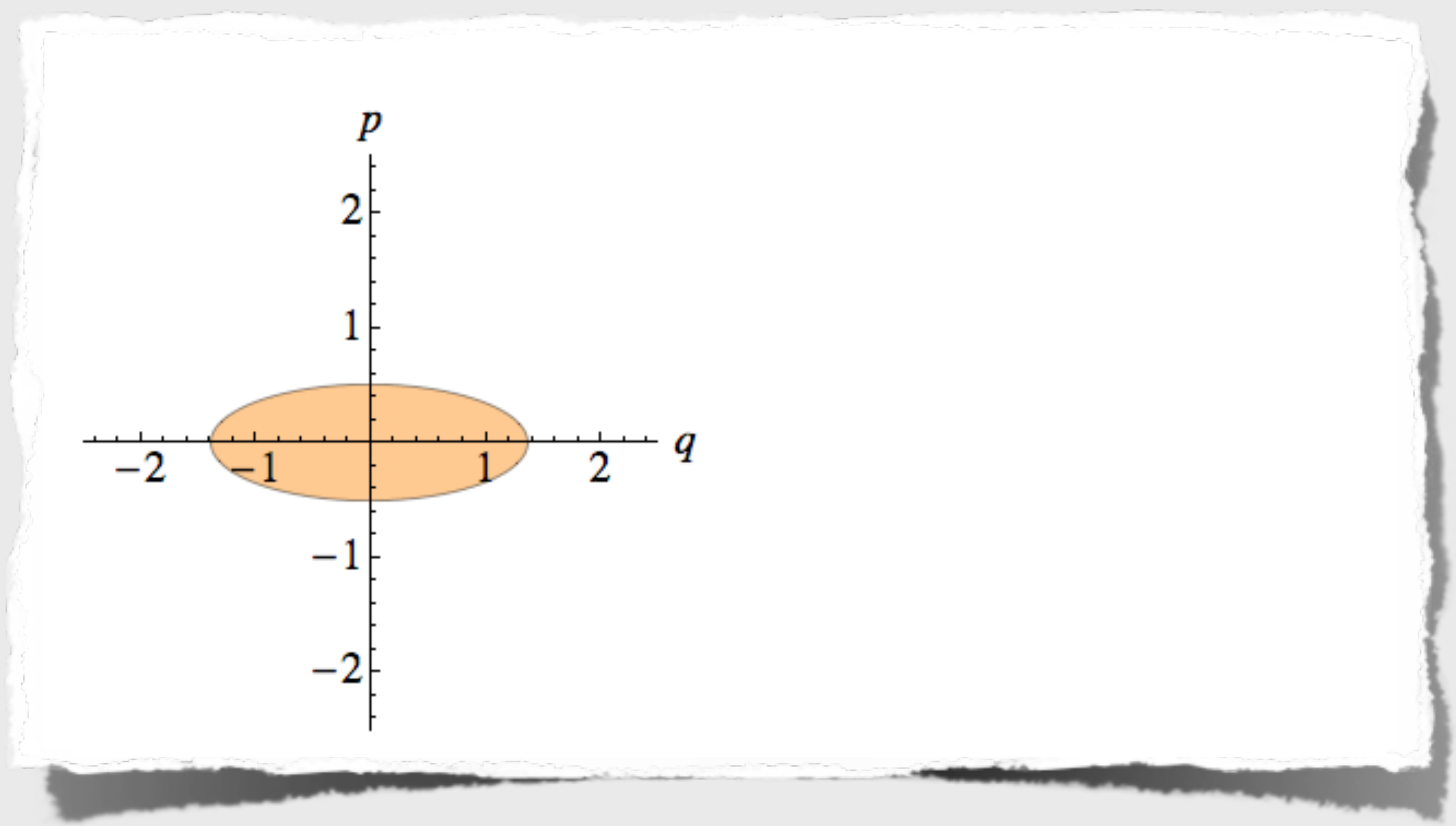
S. Olivares and M. G.A. Paris, PRL 2011

The birth of entanglement

(balanced beam splitter)

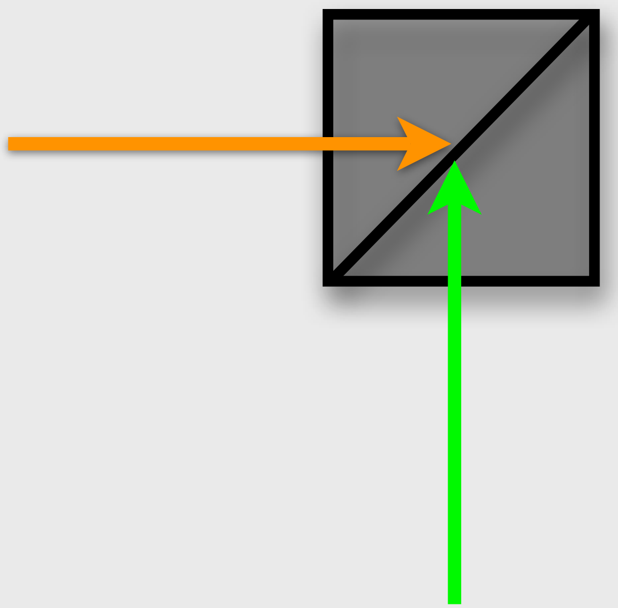


$N_1 = 0.2$
$r_1 = 0.5$



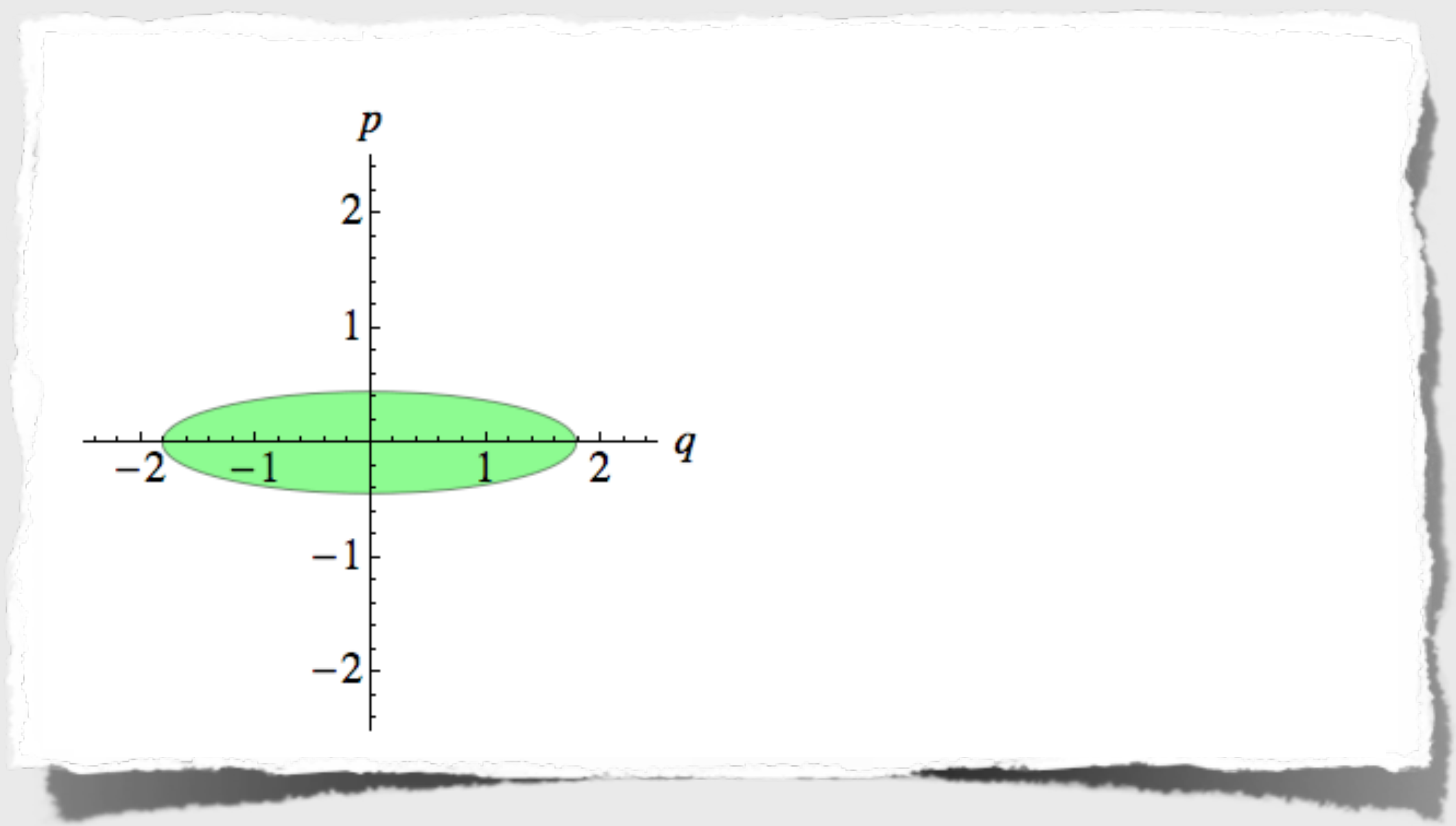
The birth of entanglement

(balanced beam splitter)



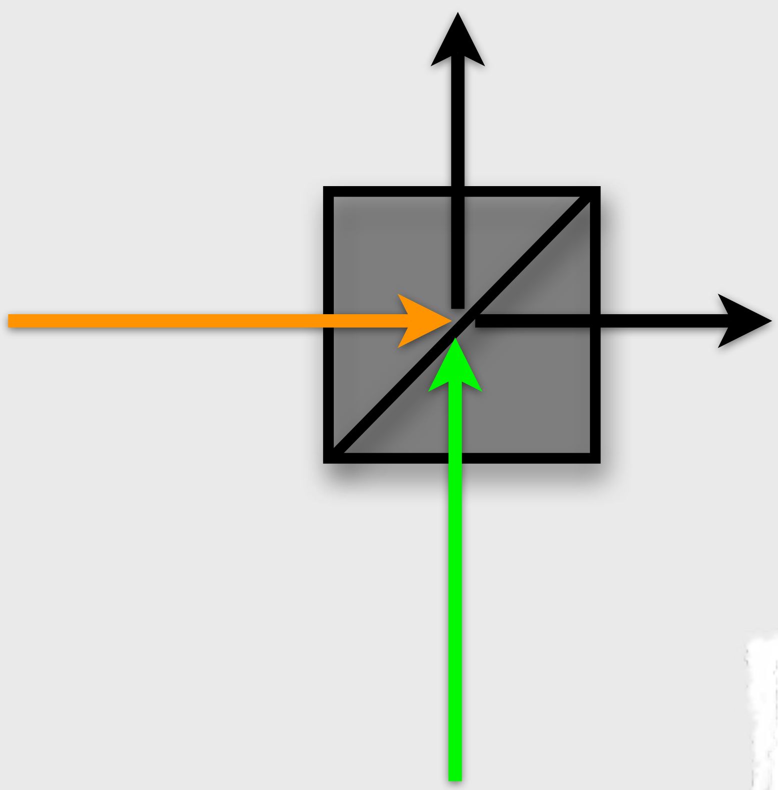
$N_1 = 0.2$
$r_1 = 0.5$

$N_2 = 0.3$
$r_2 = 0.7$



The birth of entanglement

(balanced beam splitter)



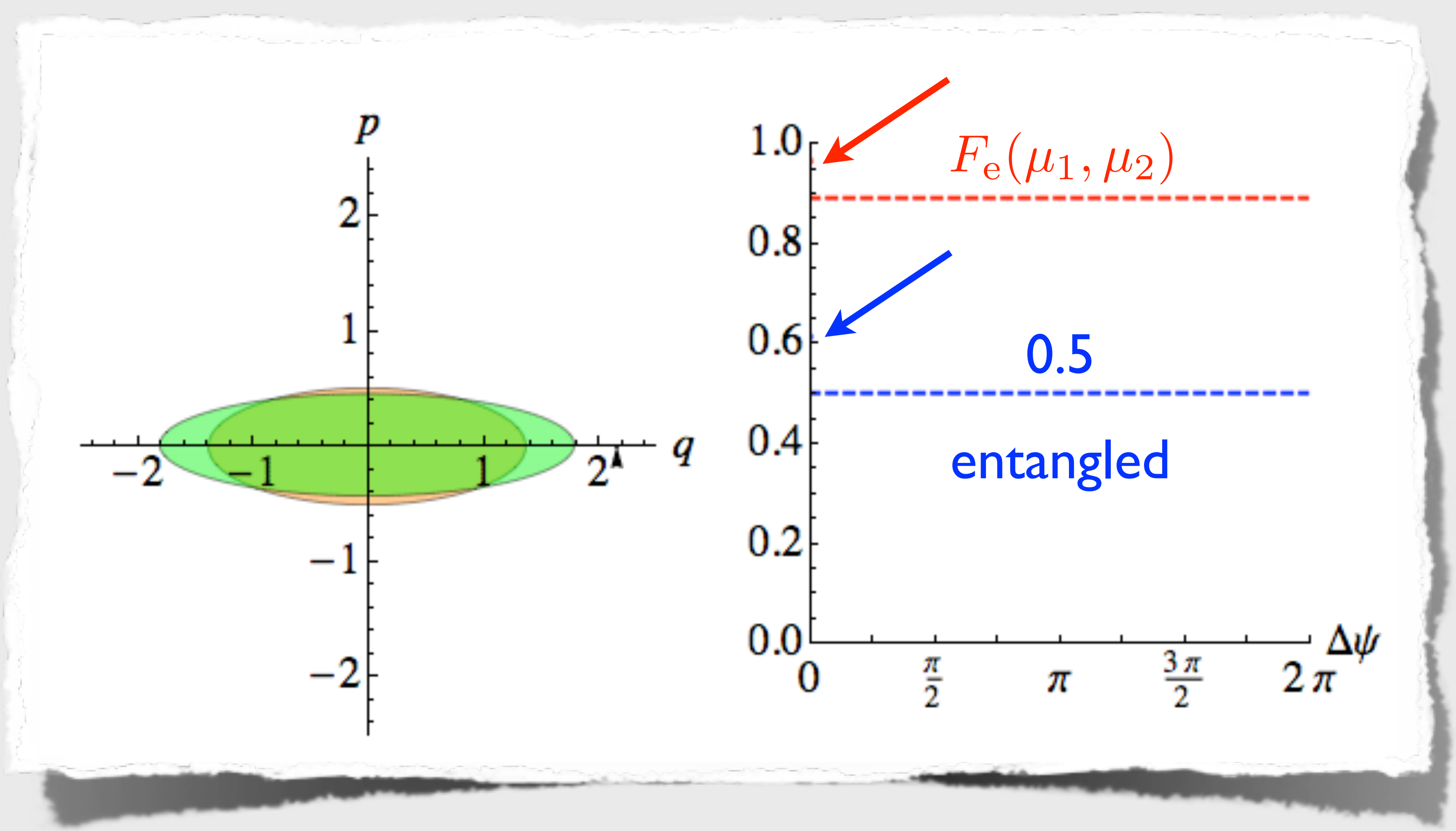
$$\tilde{\lambda} = \frac{1}{2}\frac{\left[\gamma - \sqrt{\gamma^2 - (2\mu_1\mu_2)^2}\right]^{\frac{1}{2}}}{\sqrt{2}\mu_1\mu_2}$$

$$\gamma = 2\mu_1\mu_2\cosh[2(r_1 + r_2)]$$

$$F_{\mathrm{e}}(\mu_1, \mu_2) = \frac{2\mu_1\mu_2}{\sqrt{2(1+\mu_1^2\mu_2^2)} - \sqrt{(1-\mu_1^2)(1-\mu_2^2)}}$$

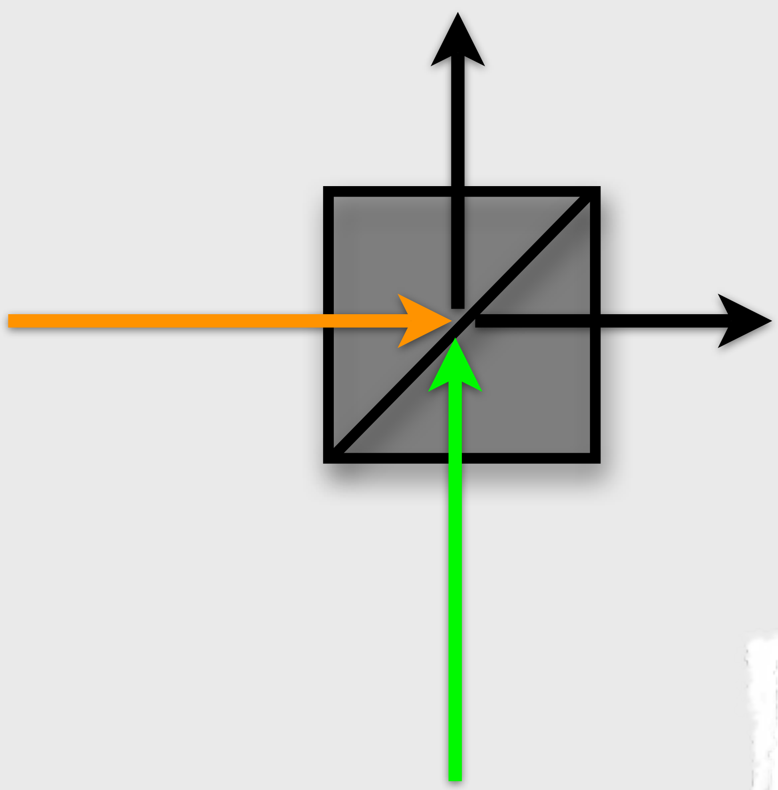
$N_1 = 0.2$
$r_1 = 0.5$

$N_2 = 0.3$
$r_2 = 0.7$



The birth of entanglement

(balanced beam splitter)



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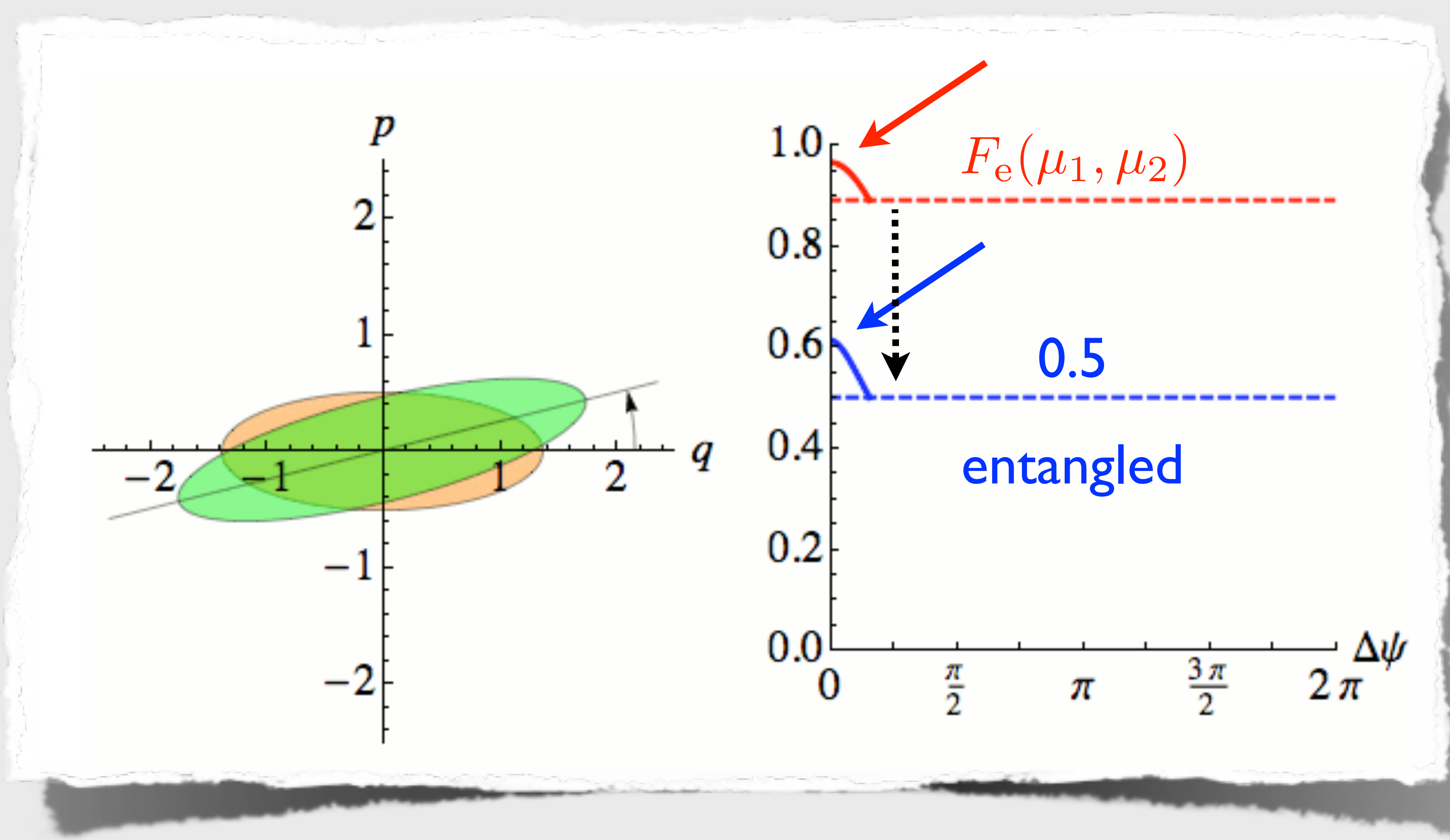

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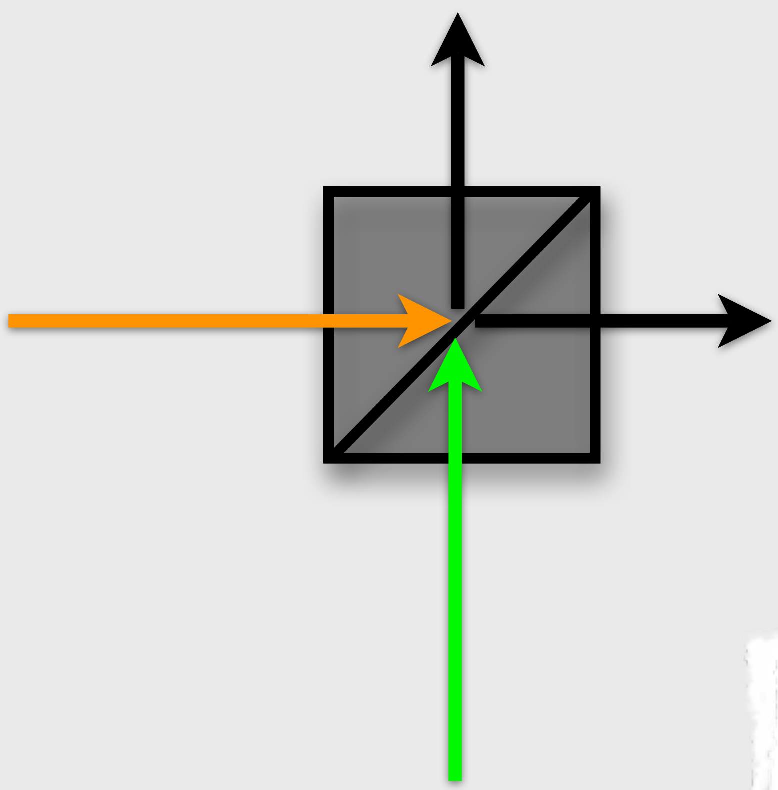
(balanced beam splitter)



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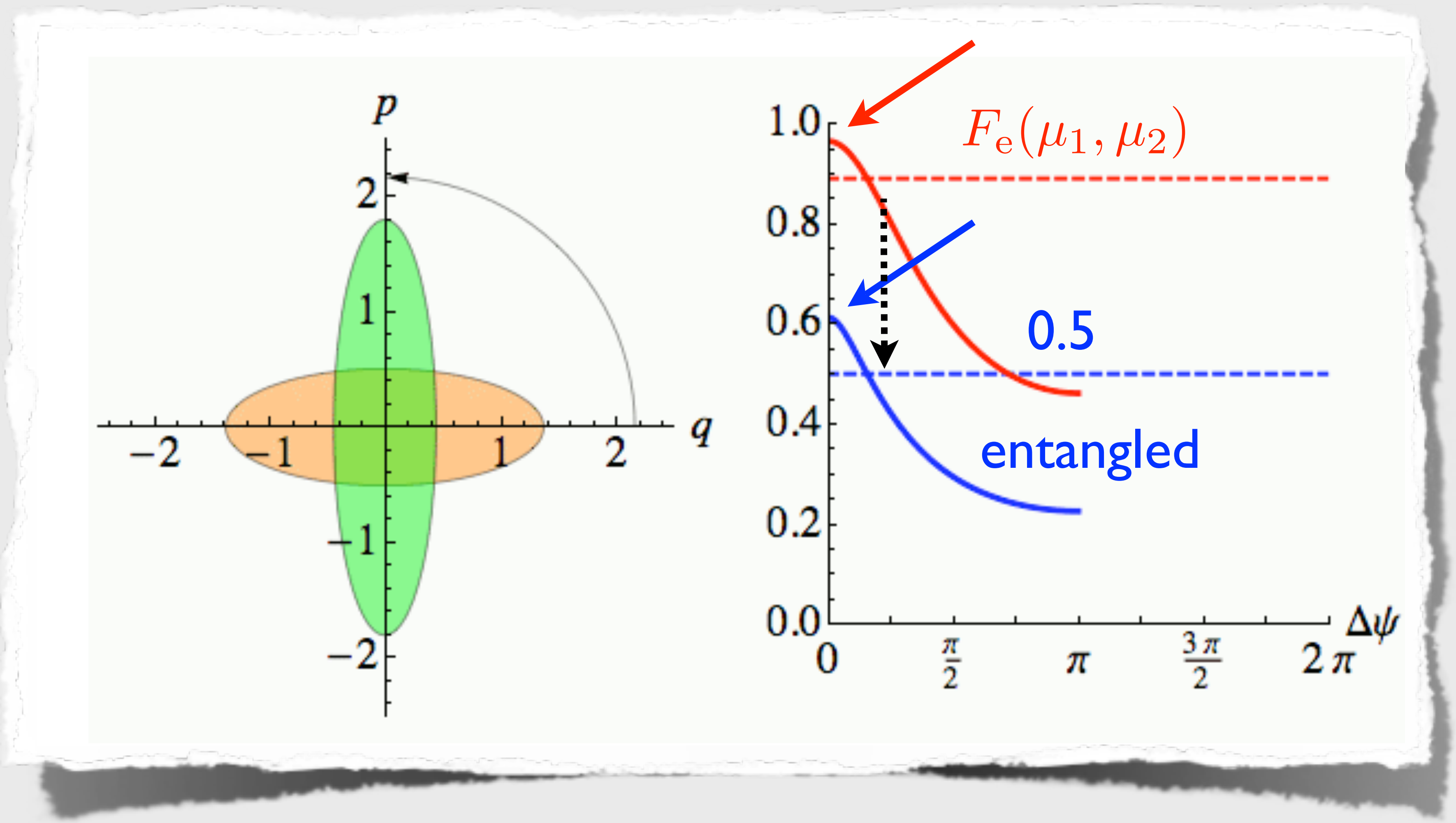
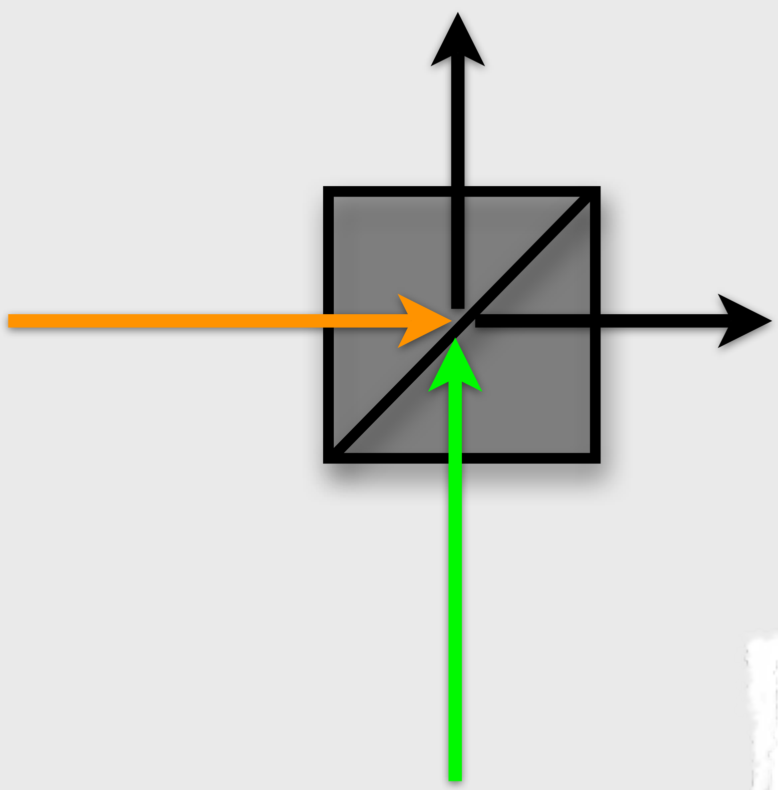


The birth of entanglement

(balanced beam splitter)



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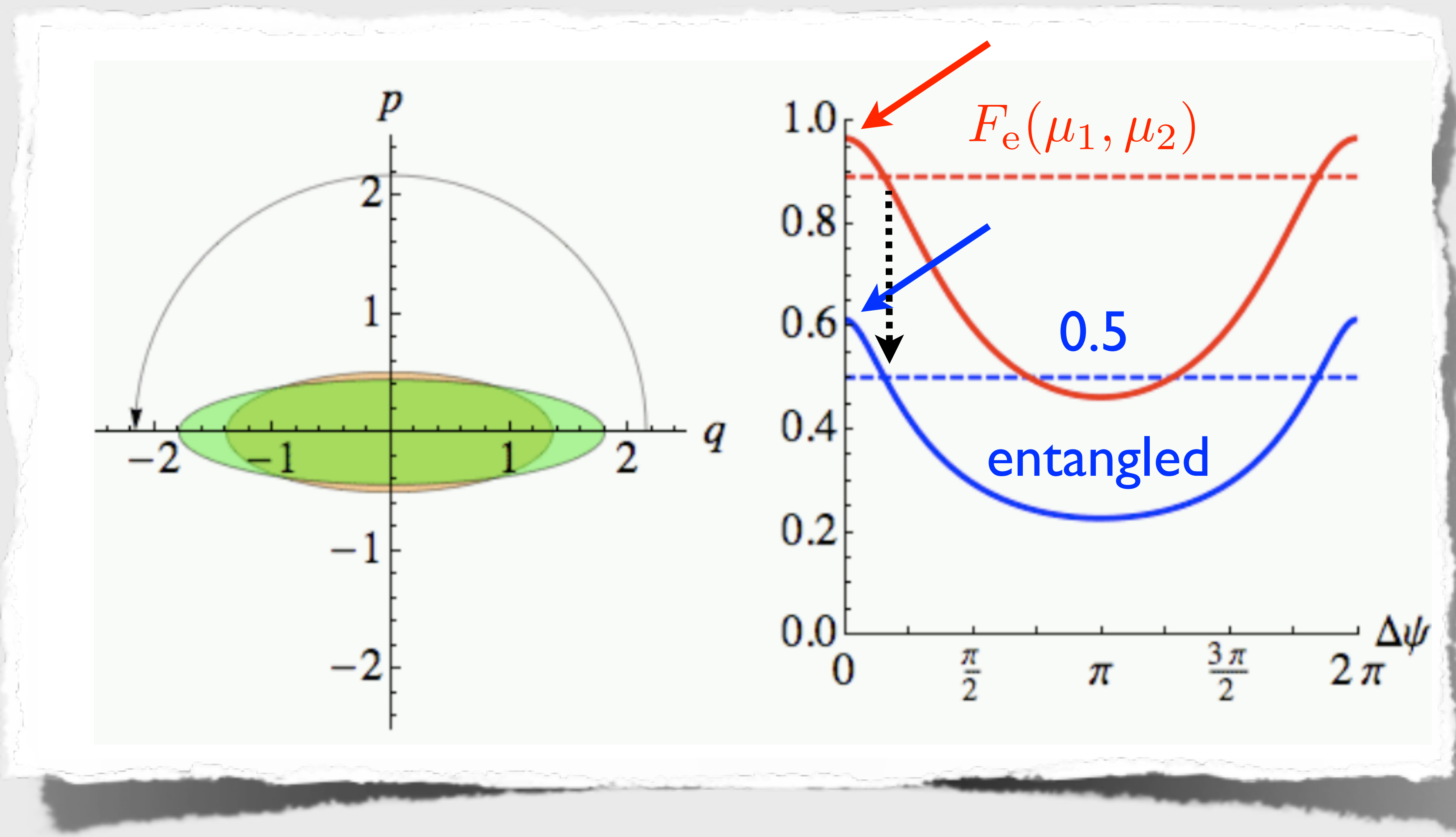
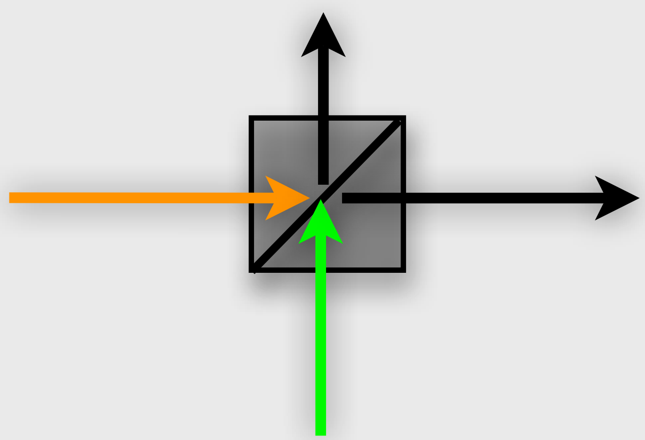


The birth of entanglement

Take two input thermal (classical) states:

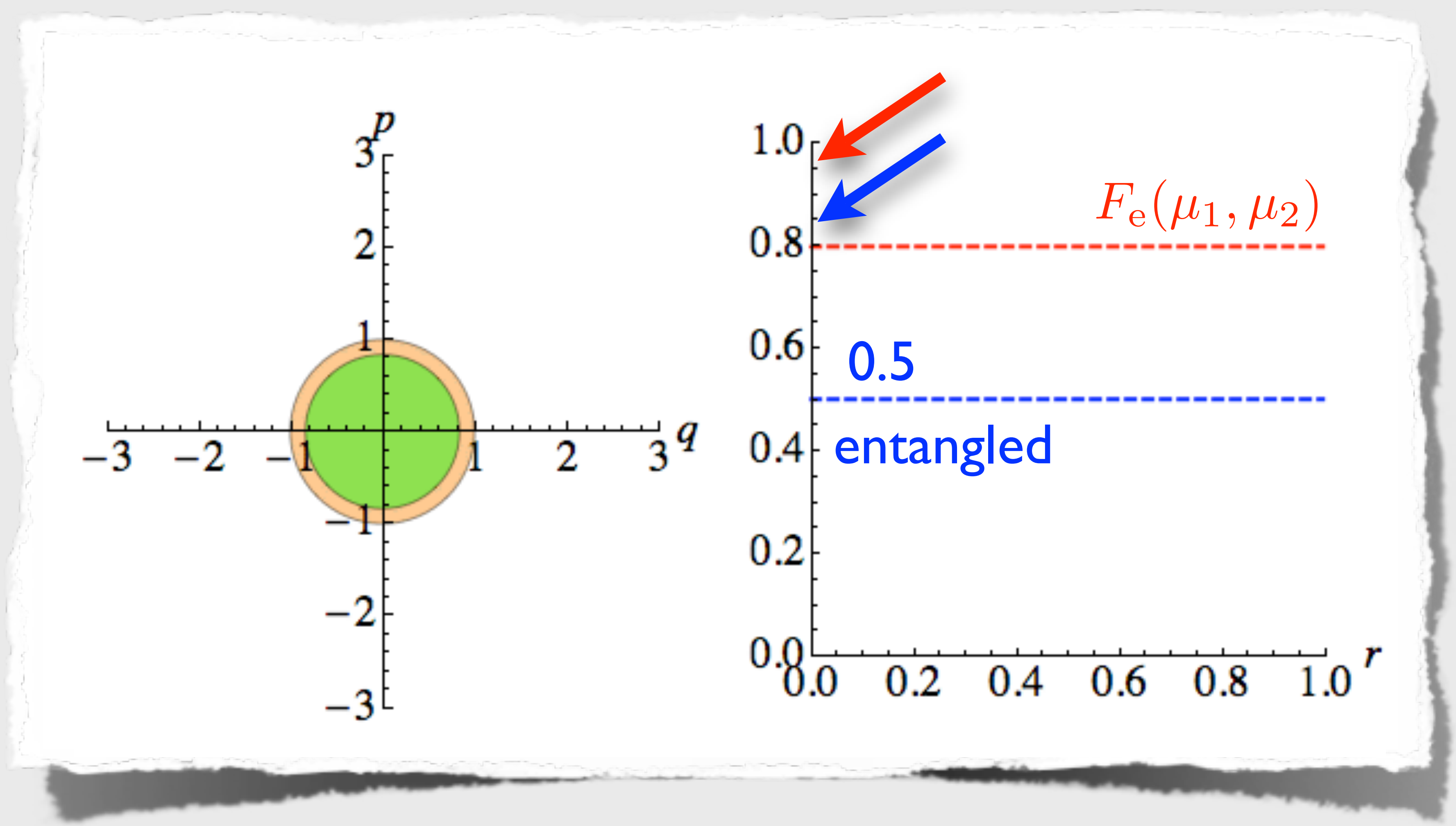
$$F(\varrho_1, \varrho_2) = \frac{2\mu_1\mu_2}{(1+\mu_1\mu_2) - \sqrt{(1-\mu_1^2)(1-\mu_2^2)}} > F_{\mathrm{e}}(\varrho_1, \varrho_2)$$



$N_1 = 0.5$

r

$N_2 = 0.2$



The birth of entanglement

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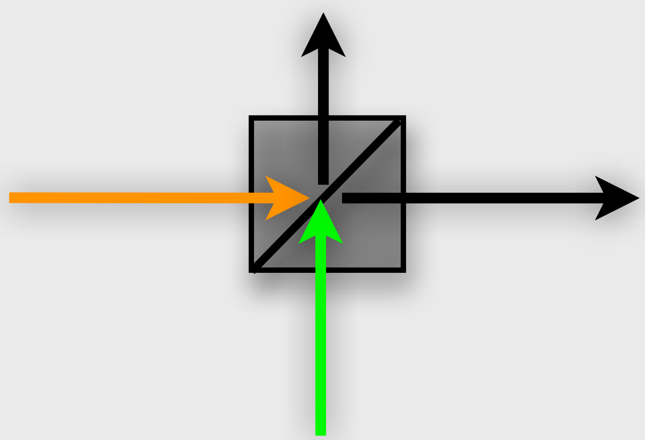

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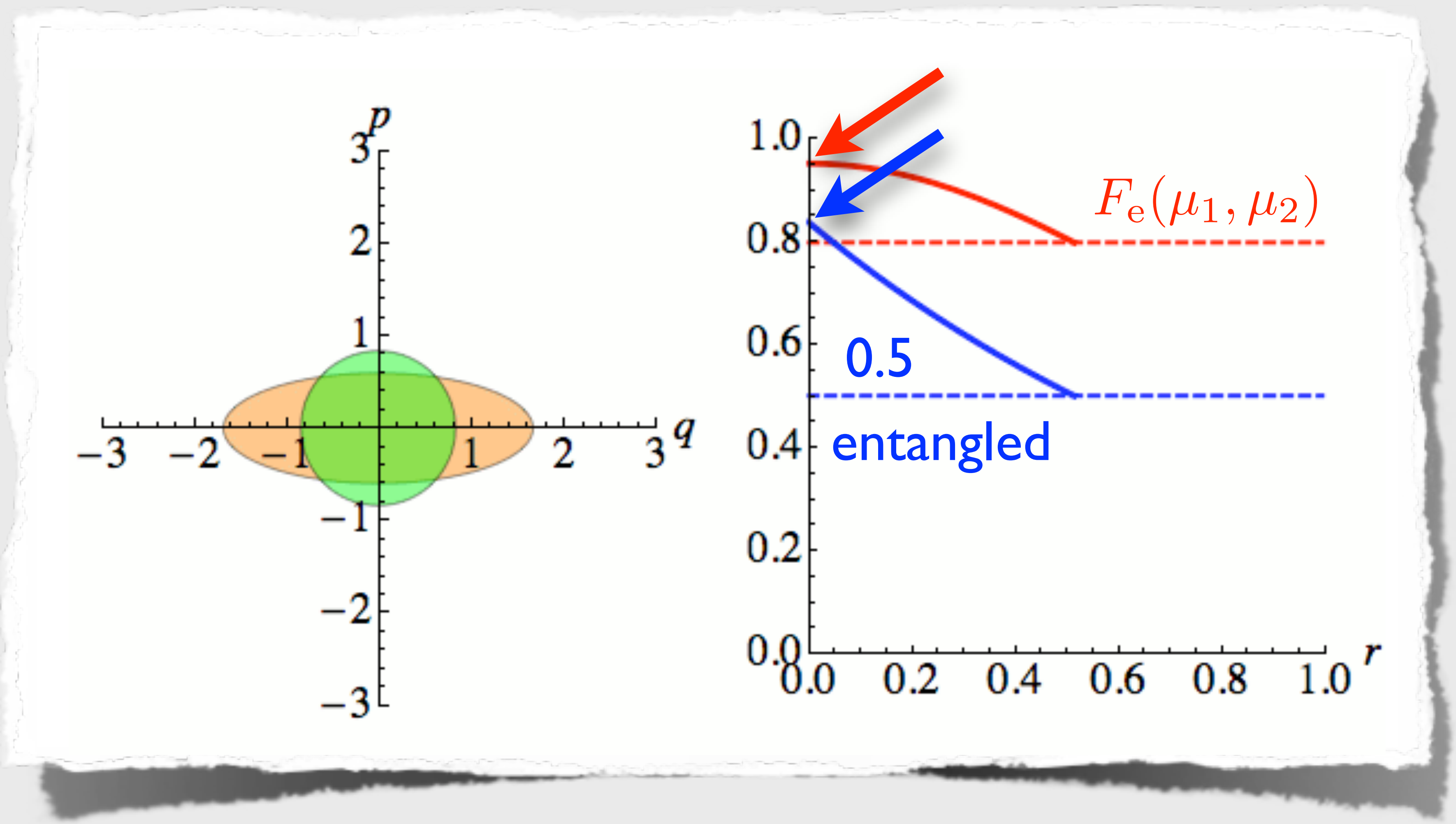

The birth of entanglement

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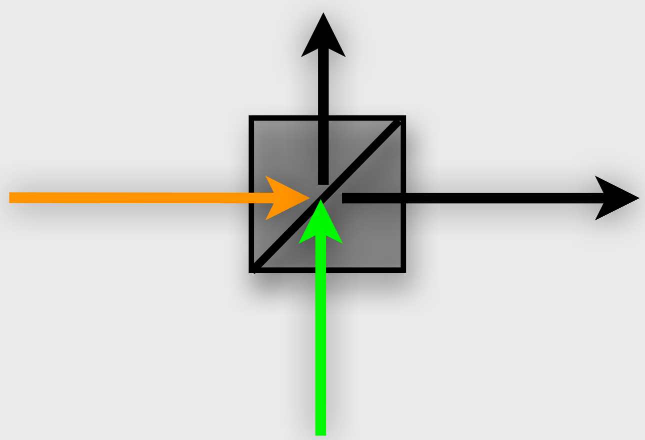

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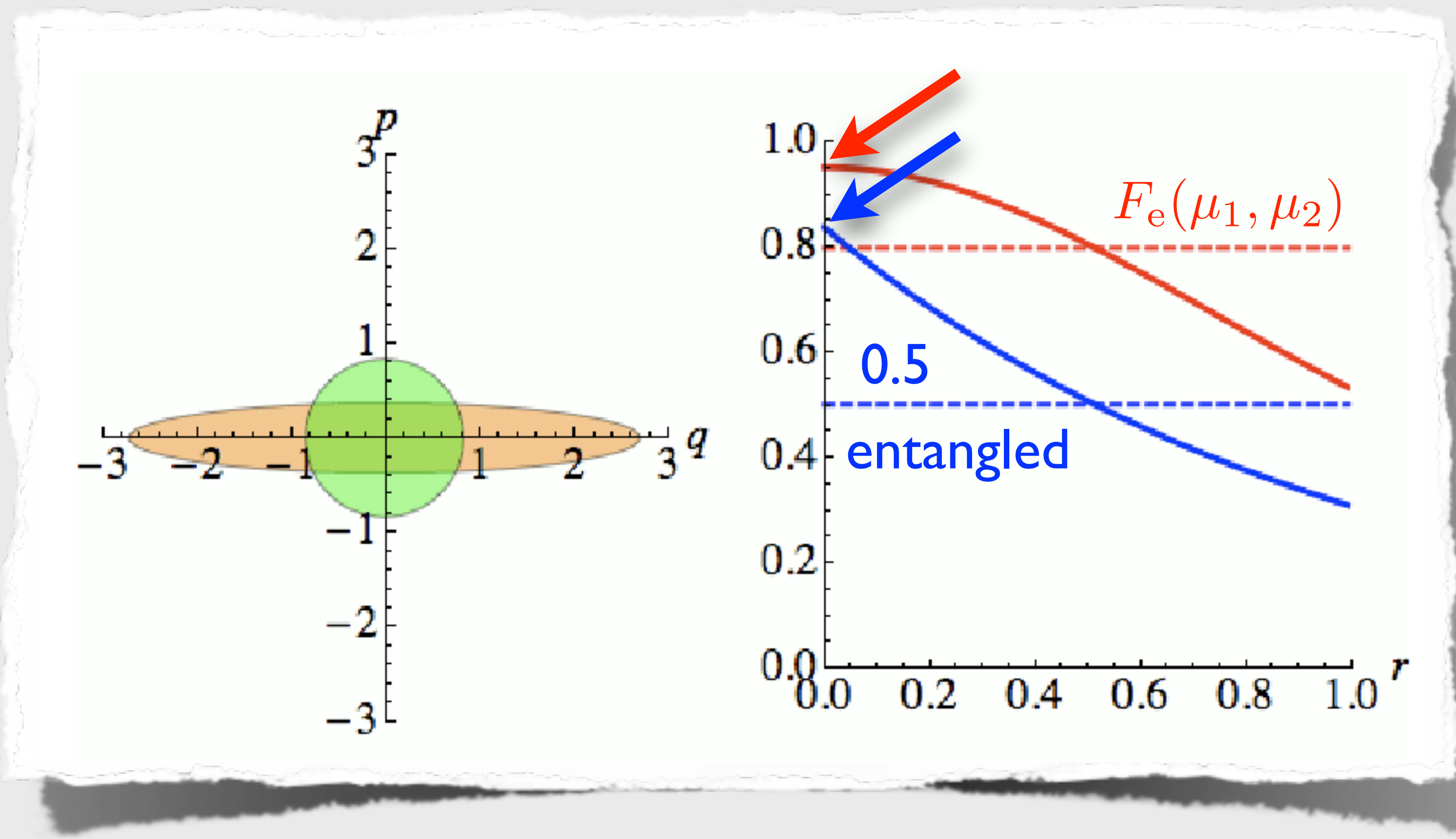


The birth of entanglement

(non zero displacement) $\boldsymbol{X}_{12} = \langle \boldsymbol{R}_1 - \boldsymbol{R}_2 \rangle$

Corollary 1 *If* $\overline{\boldsymbol{X}}_k^T = Tr[(q_k, p_k)\,\varrho_k] \neq 0$, *where where* $q_k = (a_k + a_k^\dagger)/\sqrt{2}$ *and* $p = (a_k^\dagger - a_k)/(i\sqrt{2})$ *are the quadrature operators of mode* $k = 1, 2$, *then the bipartite state* $\varrho_{12} = U_{\mathrm{BS}} \varrho_1 \otimes \varrho_2 U_{\mathrm{BS}}^\dagger$ *is entangled if and only if:*

$$F(\varrho_1, \varrho_2) < \Gamma(\overline{\boldsymbol{X}}_1, \overline{\boldsymbol{X}}_2)\, F_{\mathrm{e}}(\mu_1, \mu_2), \tag{3}$$

where $F_{\mathrm{e}}(\mu_1, \mu_2)$ *is still given in Eq. (2) and:*

$$\Gamma(\overline{\boldsymbol{X}}_1, \overline{\boldsymbol{X}}_2) = \exp\left[-\tfrac{1}{2}\,\overline{\boldsymbol{X}}_{12}^T (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)^{-1} \overline{\boldsymbol{X}}_{12}\right], \tag{4}$$

where $\overline{\boldsymbol{X}}_{12} = (\overline{\boldsymbol{X}}_1 - \overline{\boldsymbol{X}}_2)$.

(reduction of fidelity due to displacement is not relevant)

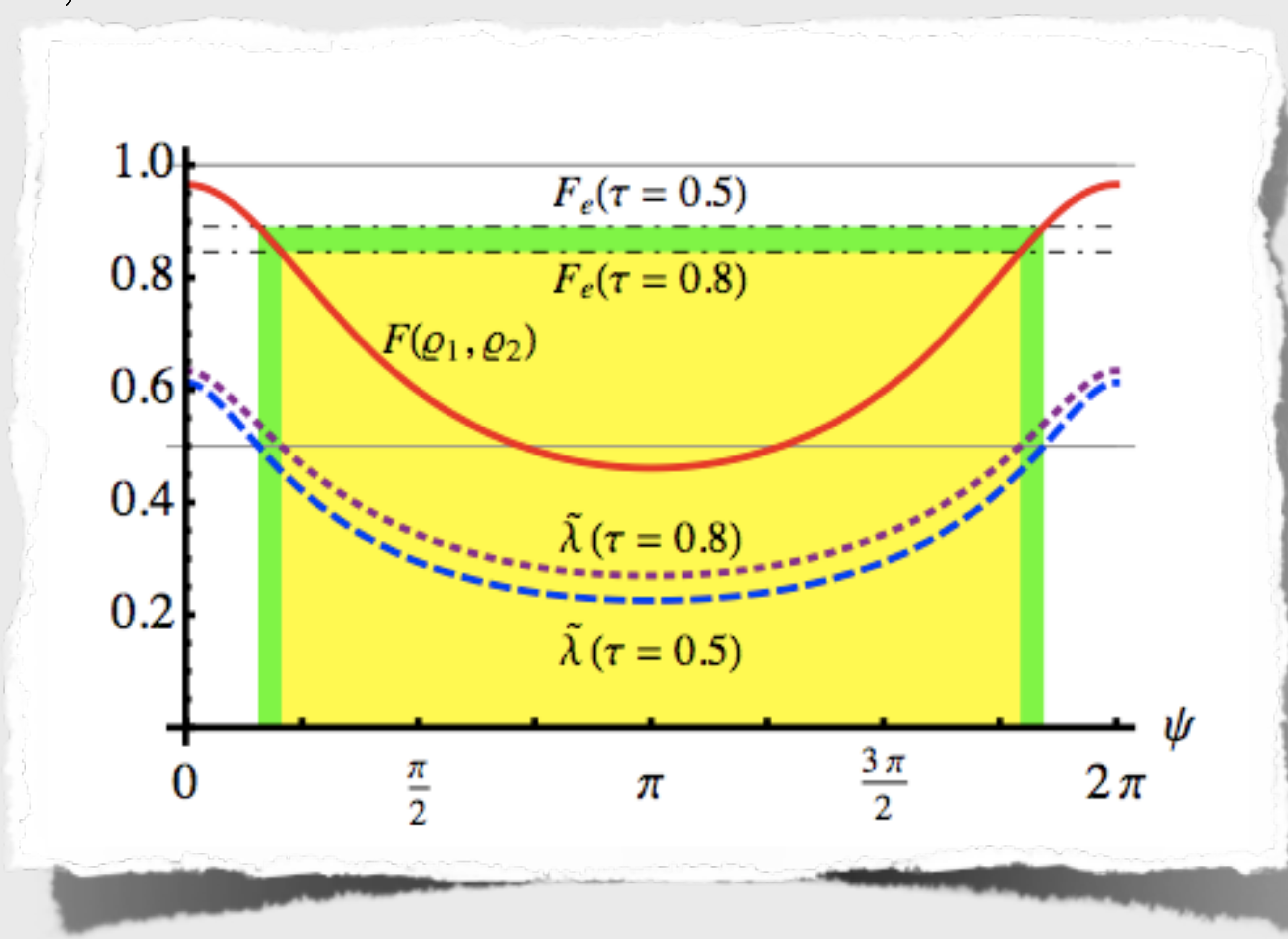
The birth of entanglement

(unbalanced beam splitter)

In general, if the transmissivity of the BS is τ :

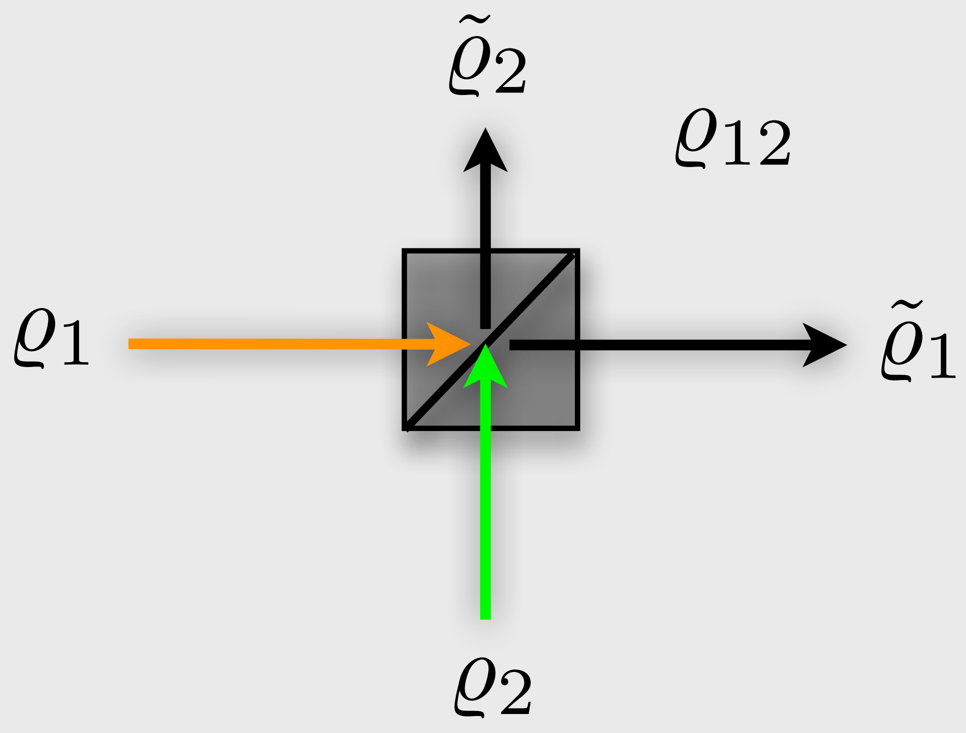
$$F_{\mathrm{e}} = \frac{4\mu_1\mu_2\sqrt{\tau(1-\tau)}}{\sqrt{g_- + 4\tau(1-\tau)g_+} - \sqrt{4\tau(1-\tau)g_-}}$$

where $g_\pm \equiv g_\pm(\mu_1,\mu_2) = \prod_{k=1,2}(1 \pm \mu_k^2)$.



The birth of entanglement

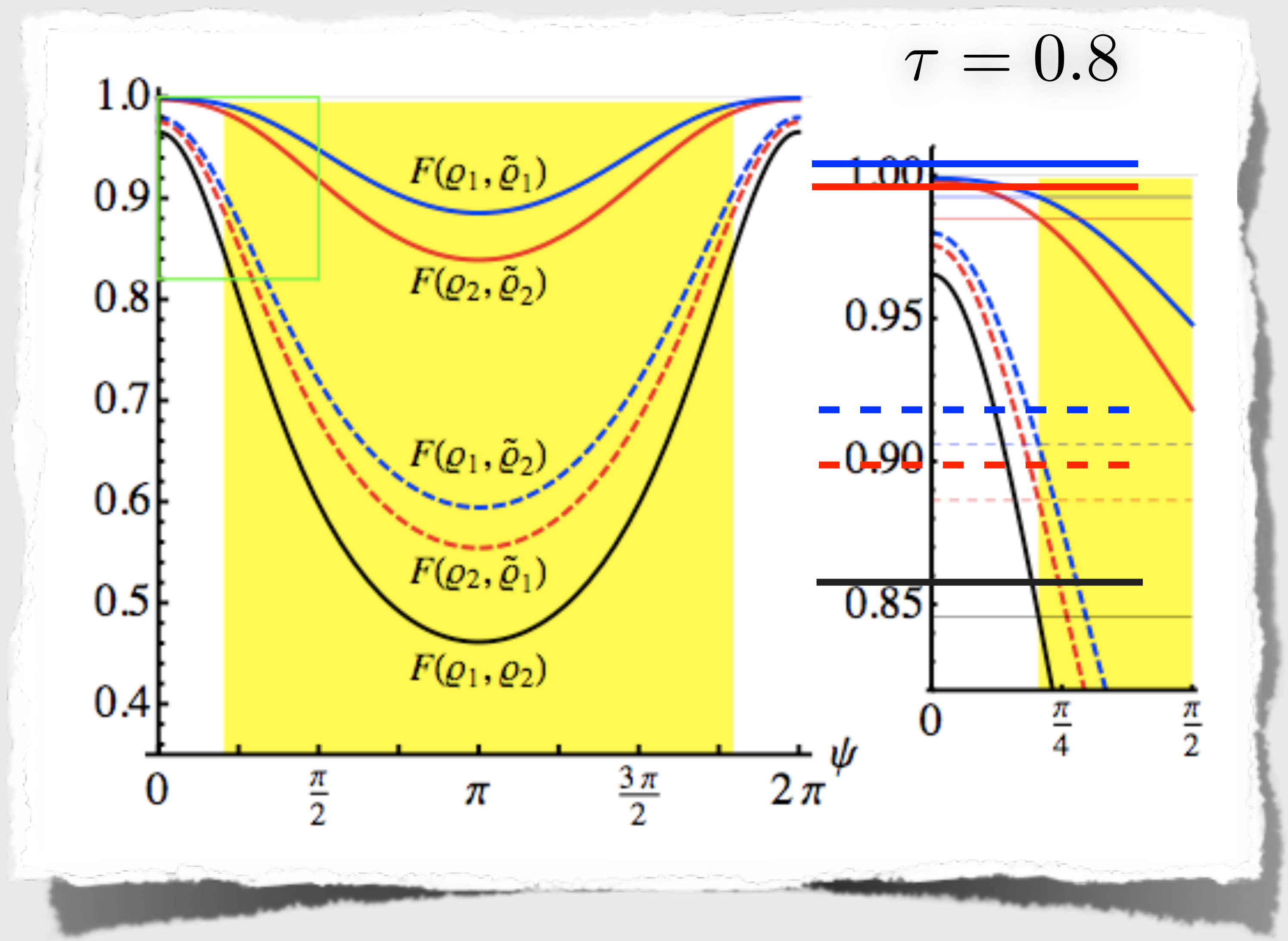
(input/output fidelities)



$$\tilde{\varrho}_k = \mathrm{Tr}_{\neq k}\left[\varrho_{12}\right]$$

BS as a quantum channel:
thresholds on the
input-output fidelities

The birth of correlations between the output modes corresponds to a distortion of the single-mode states and thus to a reduction of the input-output fidelity: the less is the fidelity, the more are the correlations.



Summary of the first part

一 Interference at a beam splitter

- *(multimode) Transparency and bath engineering*

二 The birth of (Gaussian) entanglement

- *Necessary and sufficient condition in terms of fidelity*
- *Role of squeezing*
- *Input output fidelities (loss of information)*

Hidden correlations and the optical illusionist game

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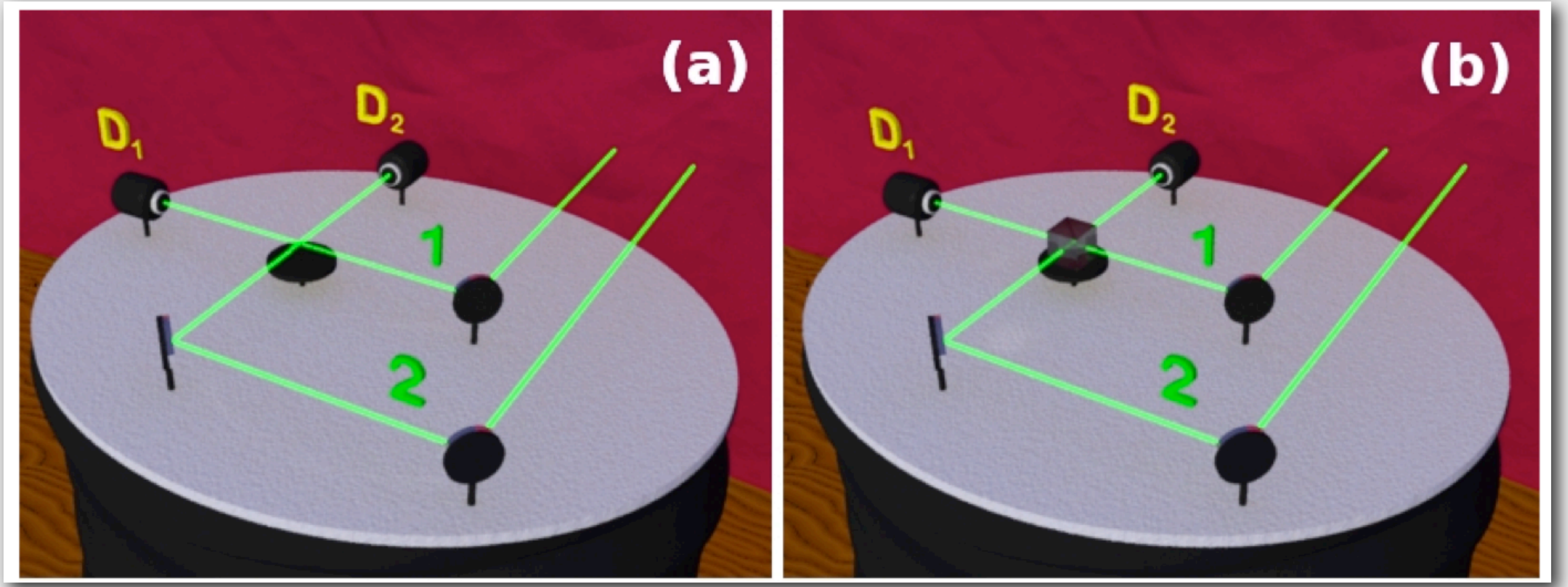
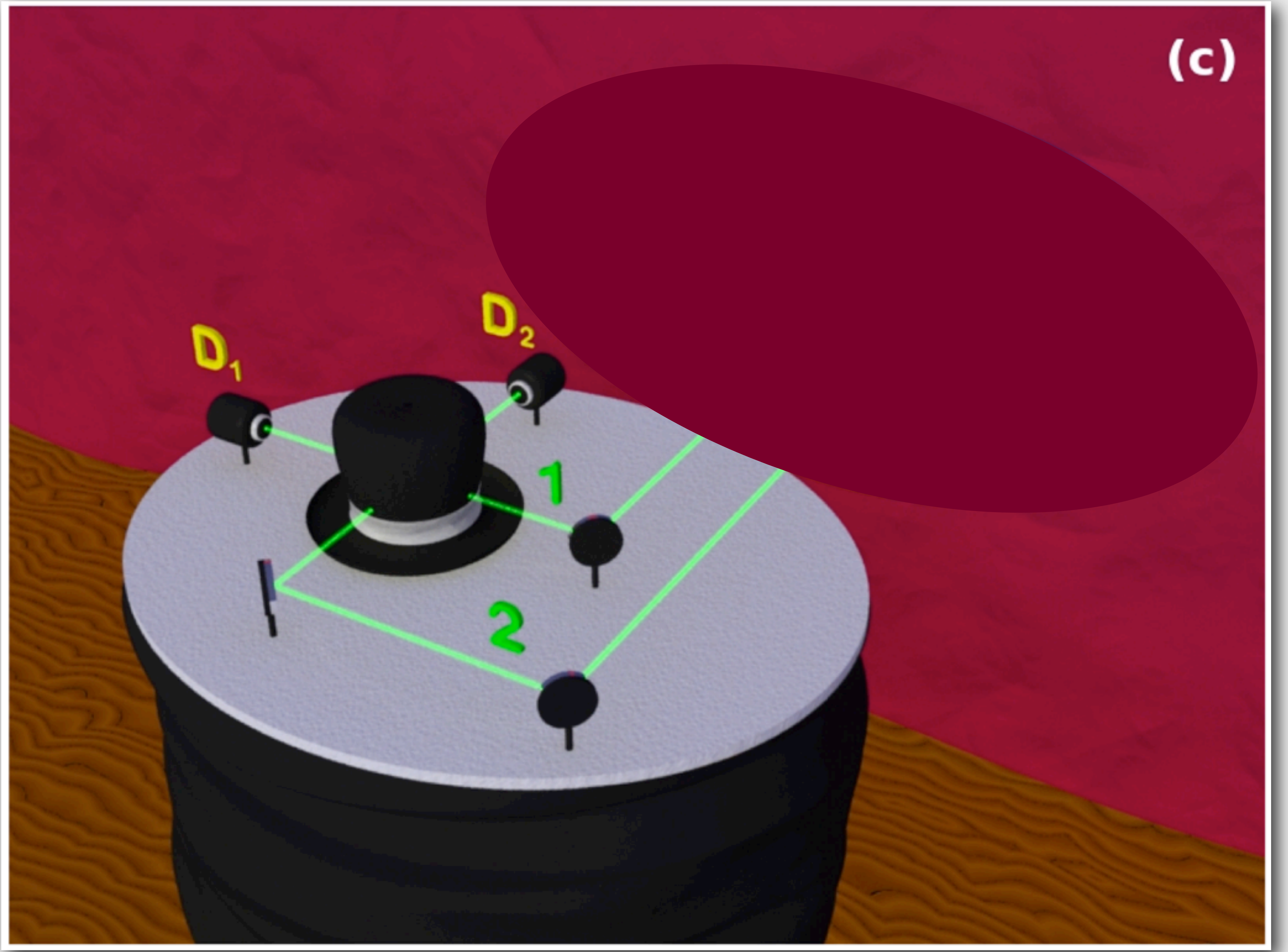
A. Meda, G. Brida, M. Genovese, I. P. Degiovanni
INRIM Torino, Italy

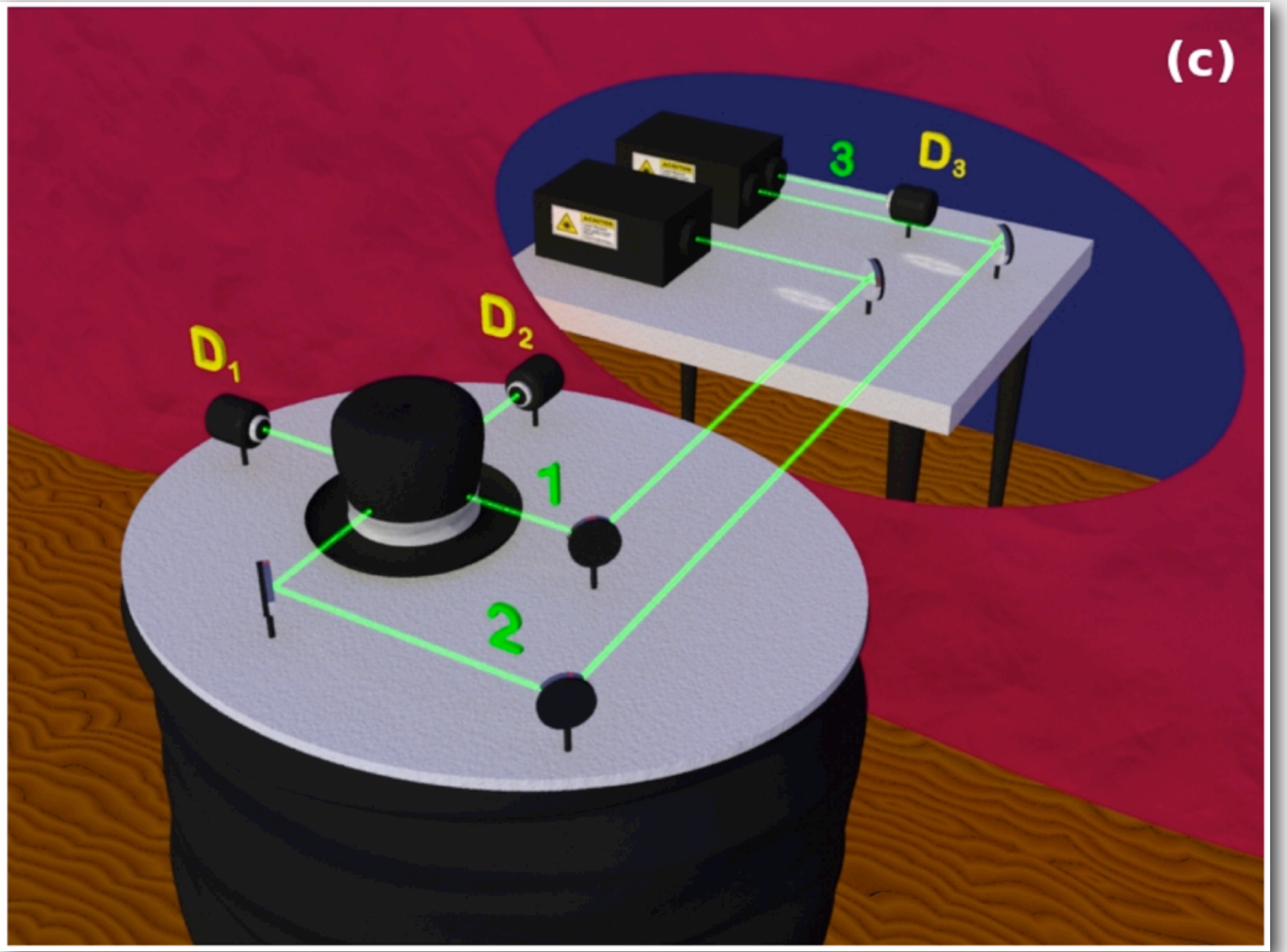
Fidelity induced transparency



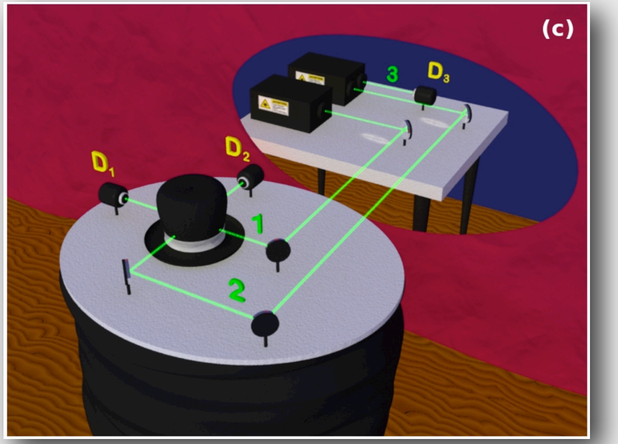
The optical illusionist game



The optical illusionist game



The optical illusionist game



$$\boldsymbol{\Sigma}_{123} = \begin{pmatrix} \boldsymbol{\sigma} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma} & \boldsymbol{\delta}_{23} \\ \mathbf{0} & \boldsymbol{\delta}_{23}^T & \boldsymbol{\sigma}_3 \end{pmatrix}$$

$$\boldsymbol{\Sigma}_{123}^{(\mathrm{out})} = \begin{pmatrix} \boldsymbol{\sigma} & \mathbf{0} & \sqrt{1-\tau}\,\boldsymbol{\delta}_{23} \\ \mathbf{0} & \boldsymbol{\sigma} & \sqrt{\tau}\,\boldsymbol{\delta}_{23} \\ \sqrt{1-\tau}\,\boldsymbol{\delta}_{23} & \sqrt{\tau}\,\boldsymbol{\delta}_{23} & \boldsymbol{\sigma}_3 \end{pmatrix}$$

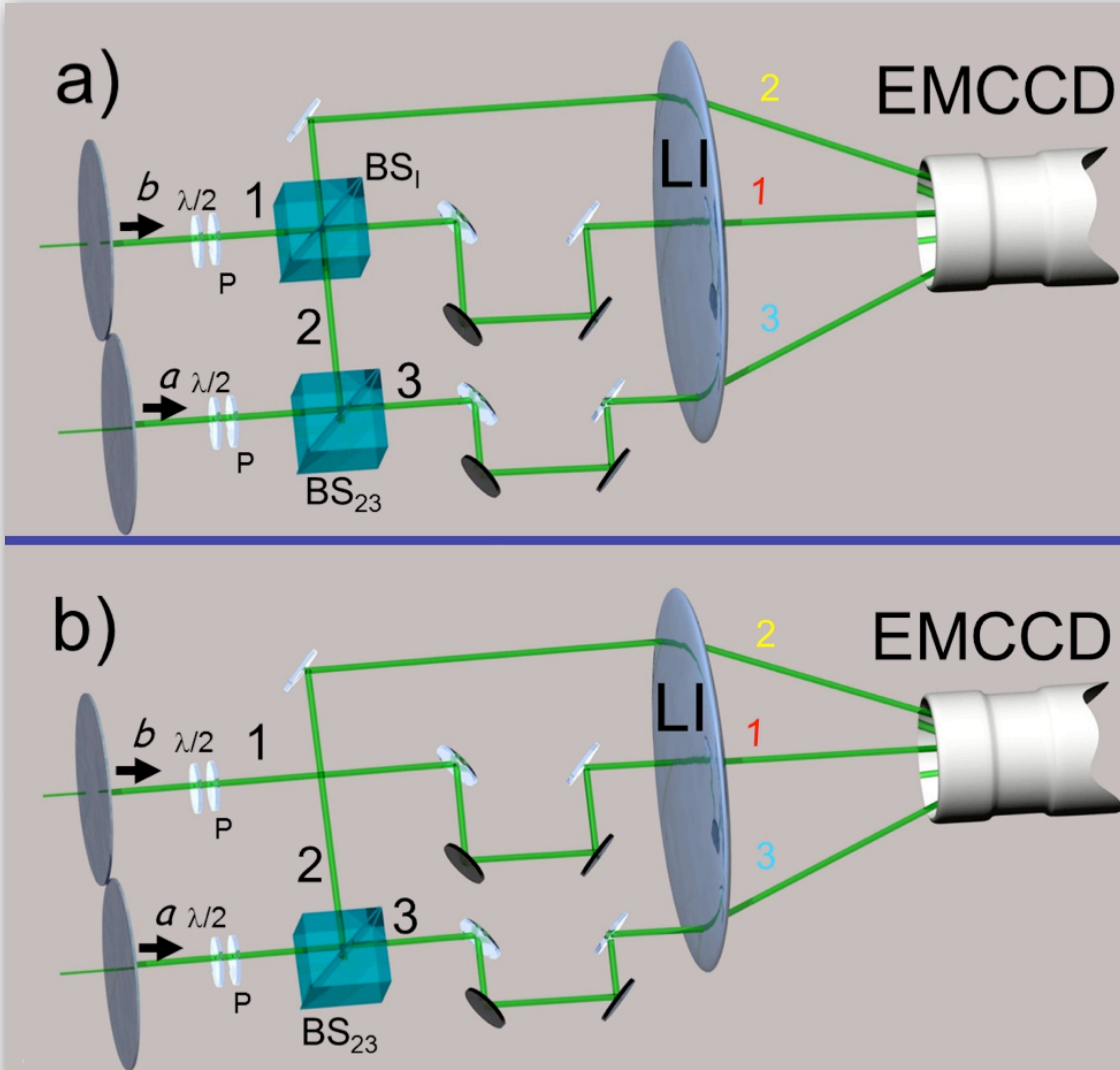
No correlation arises between the interacting modes 1 & 2

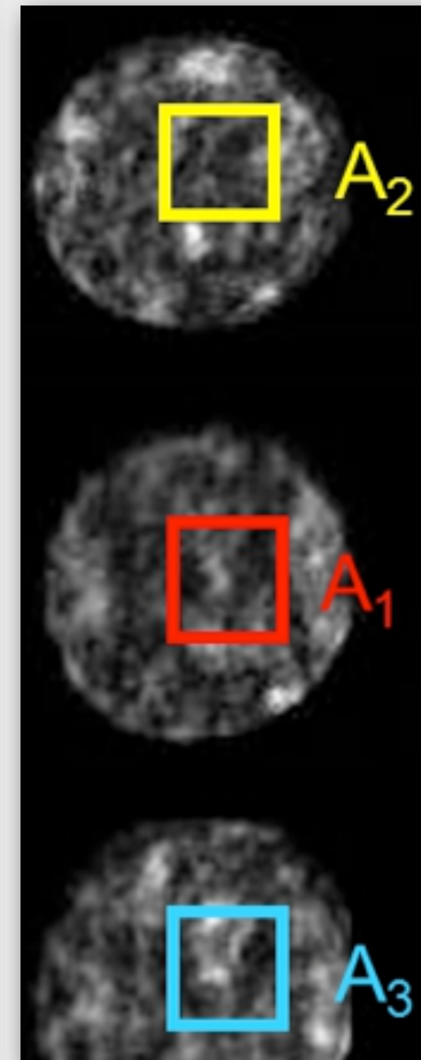
The illusionist exploits "hidden" correlations to detect the BS

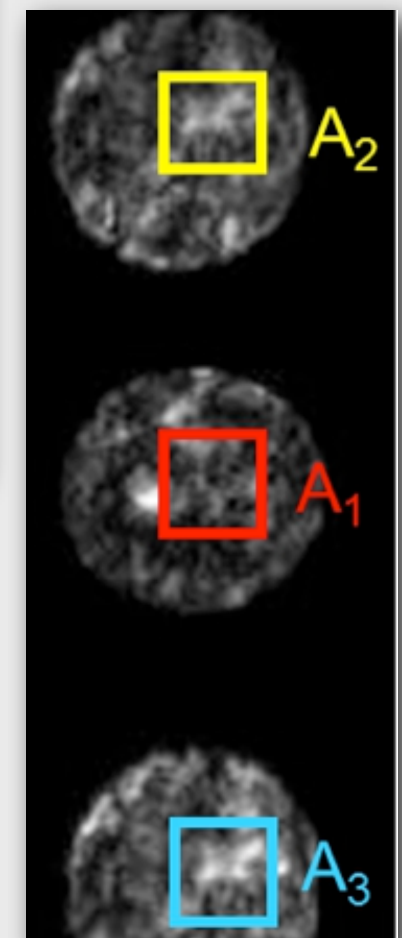
Setup and results

- The same results hold also in the presence of multimode Gaussian states.
- Tensor product nature of the multimode state.
- Pairwise interaction.
- Each mode interferes with one mode in the other beam.

Setup and results

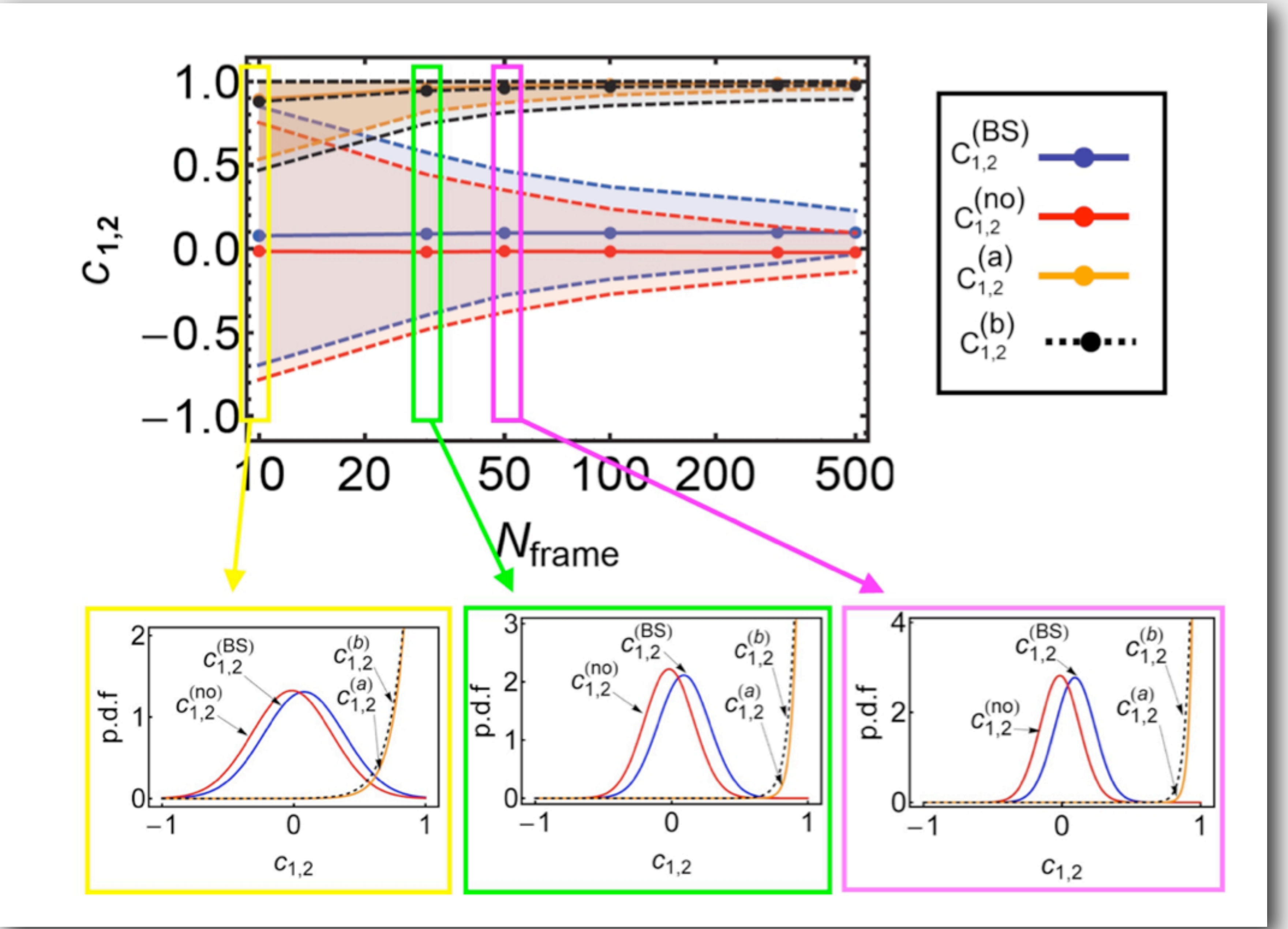






Two (speckled) spatial multimode and single temporal mode pseudo-thermal beams generated by scattering two 1 ns laser pulses @532 nm - 12.4 Hz rep rate, on two independent rotating ground glasses

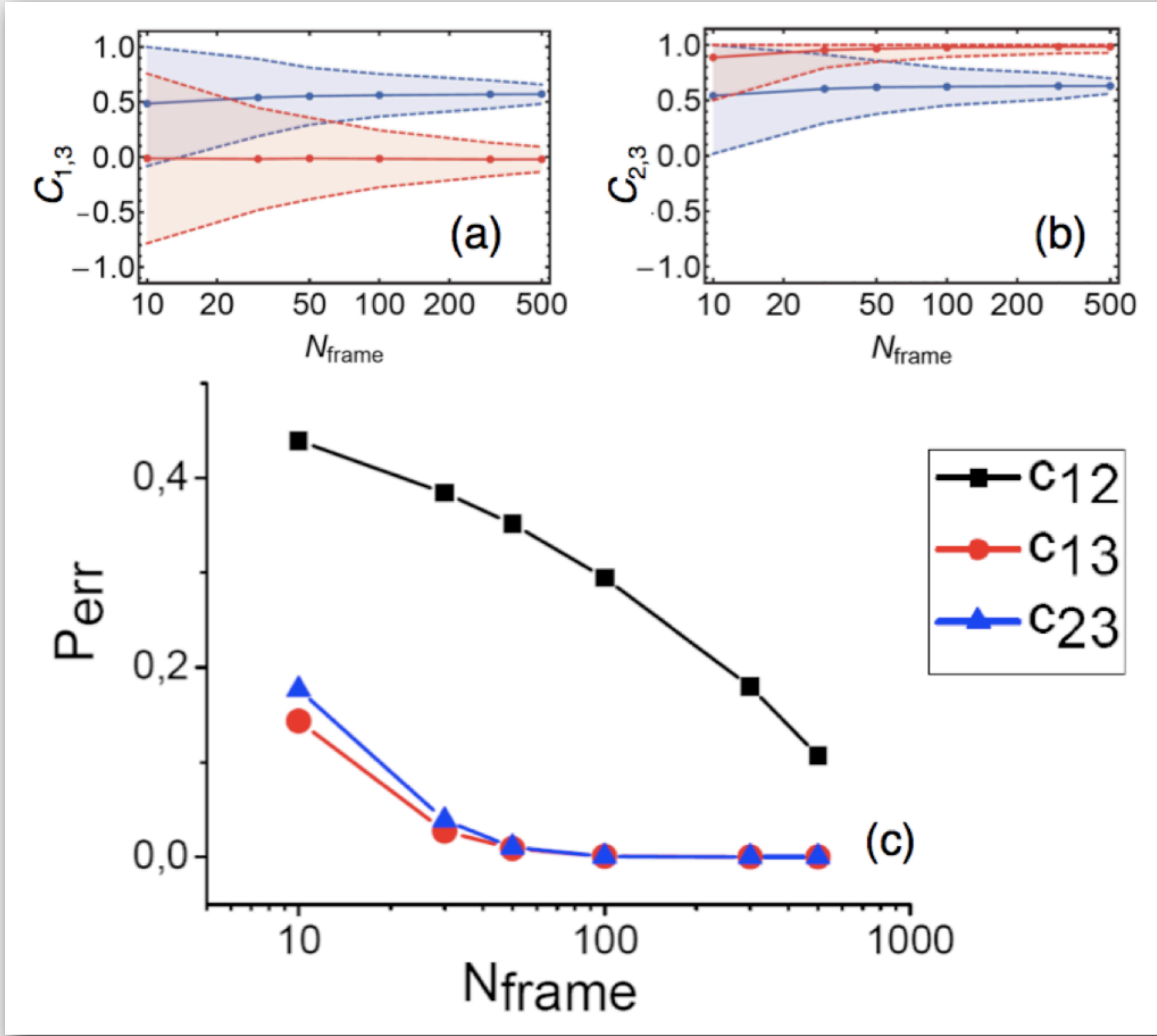
Setup and results



Detecting the BS is quite difficult for the public (only modes 1&2 available)

$$c_{h,k} = \frac{\langle I_k I_h \rangle_{\rm fr} - \langle I_h \rangle_{\rm fr} \langle I_k \rangle_{\rm fr}}{\Delta_{\rm fr}(I_h) \Delta_{\rm fr}(I_k)}$$

Setup and results



Detecting the BS is quite easy for the illusionist (accessing also mode 3)

Conclusions

一 Gaussian states in a beam splitter

- *A necessary and sufficient condition for entanglement in terms of fidelity*
- *A condition for transparency*

二 The quantum illusionist game

- *An experiment revealing hidden correlations*

- S. Olivares, M. G. A. Paris, Phys. Rev. A **80**, 032329 (2009)
- S. Olivares, M. G. A. Paris, Phys. Rev. Lett. **107**, 170505 (2011)
- G. Brida, I. P. Degiovanni, M. Genovese, A. Meda, S. Olivares, M. G. A. Paris, arXiv 1204.5499