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Breakdown of the classical description of a local system

Anders S. Sørensen

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The Niels Bohr Institute, University of Copenhagen

* Department of Applied Science, University of California, Davis

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A well known story - with a twist

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Non-classical effects

John Doe *et al*, Journal of Something, Vol. Whatever, p. something (200x)

In this article we demonstrate a genuine non-classical effect....

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When is an effect truly non-classical?

Why important?

Quantum/classical transition

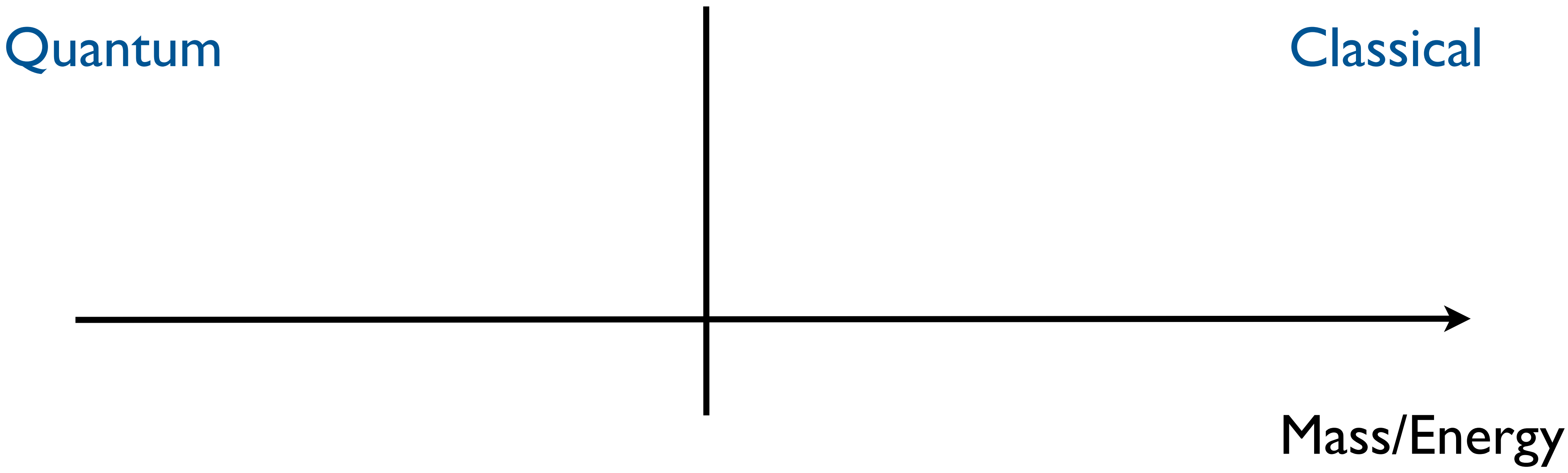
Quantum

Classical

Mass/Energy

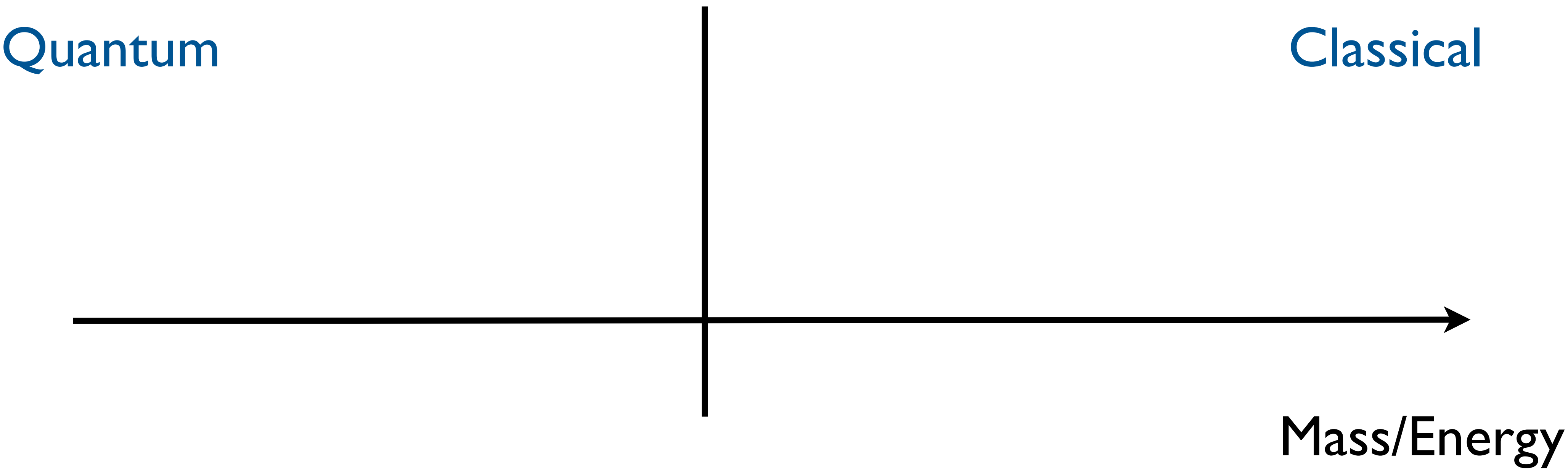
Why important?

Quantum/classical transition



Why important?

Quantum/classical transition   Is there a separation?



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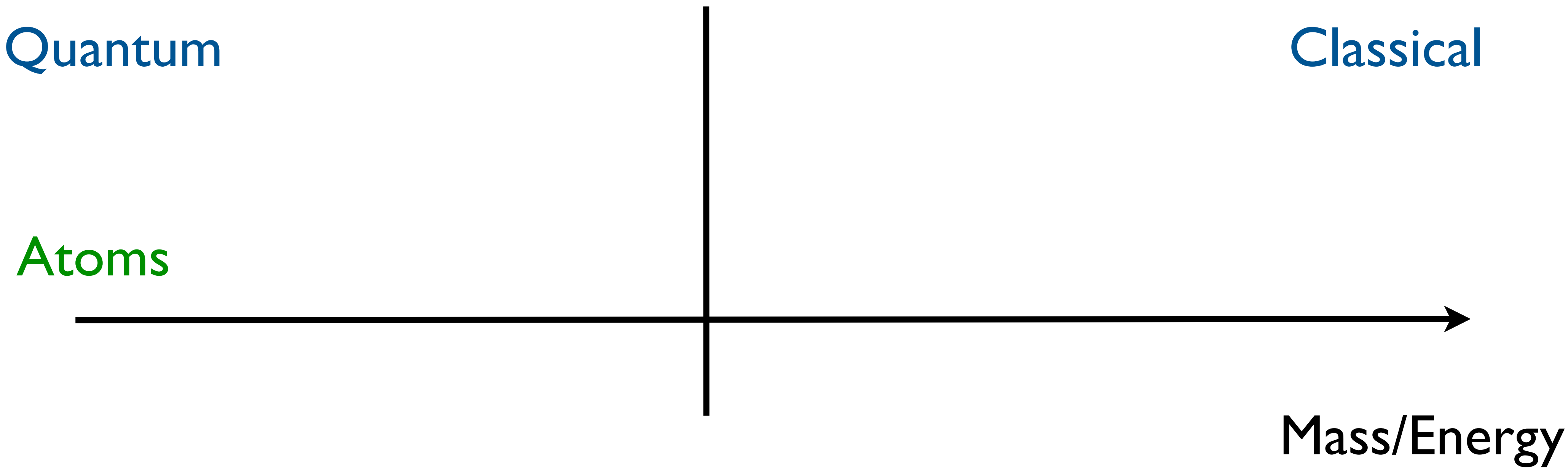

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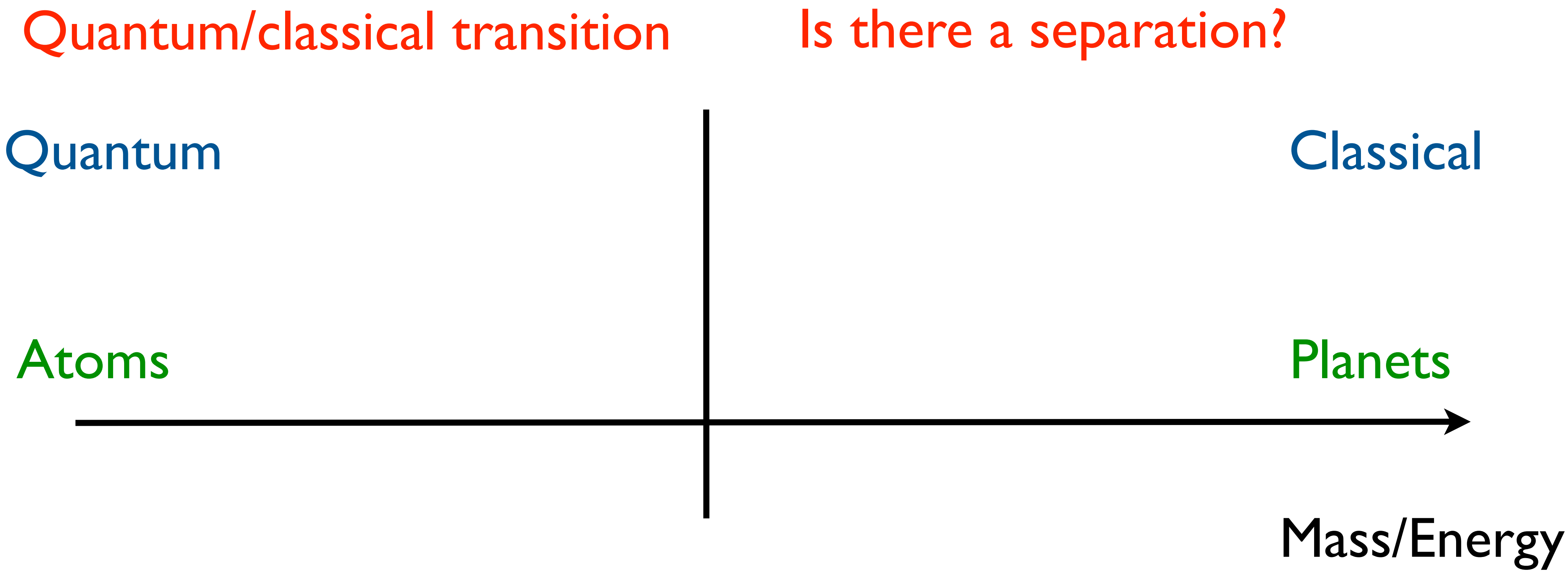

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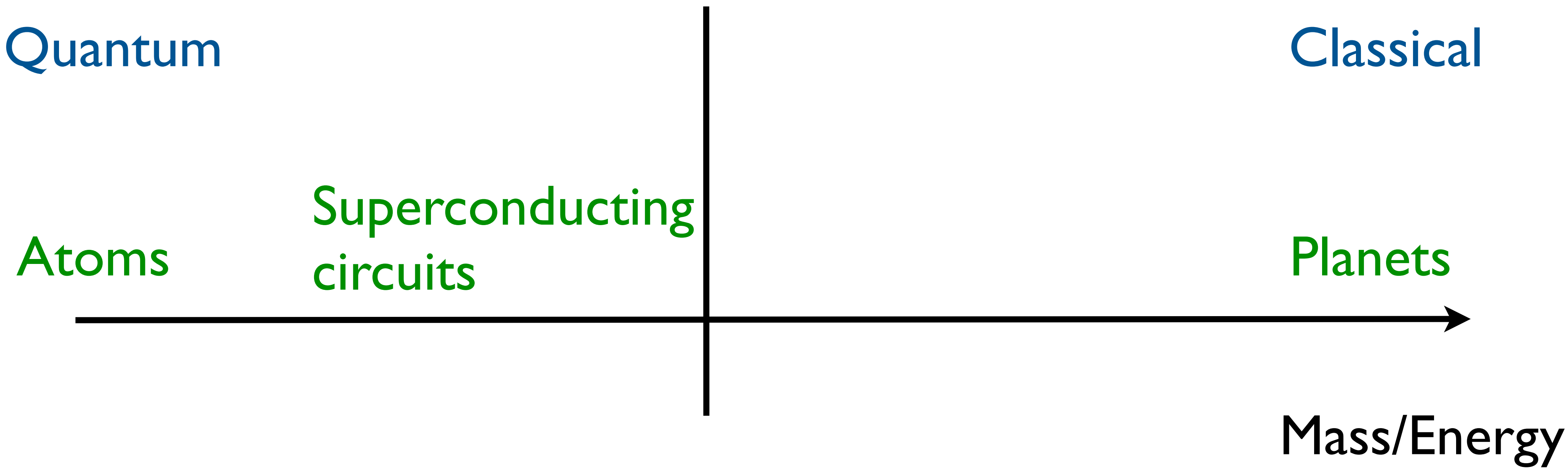

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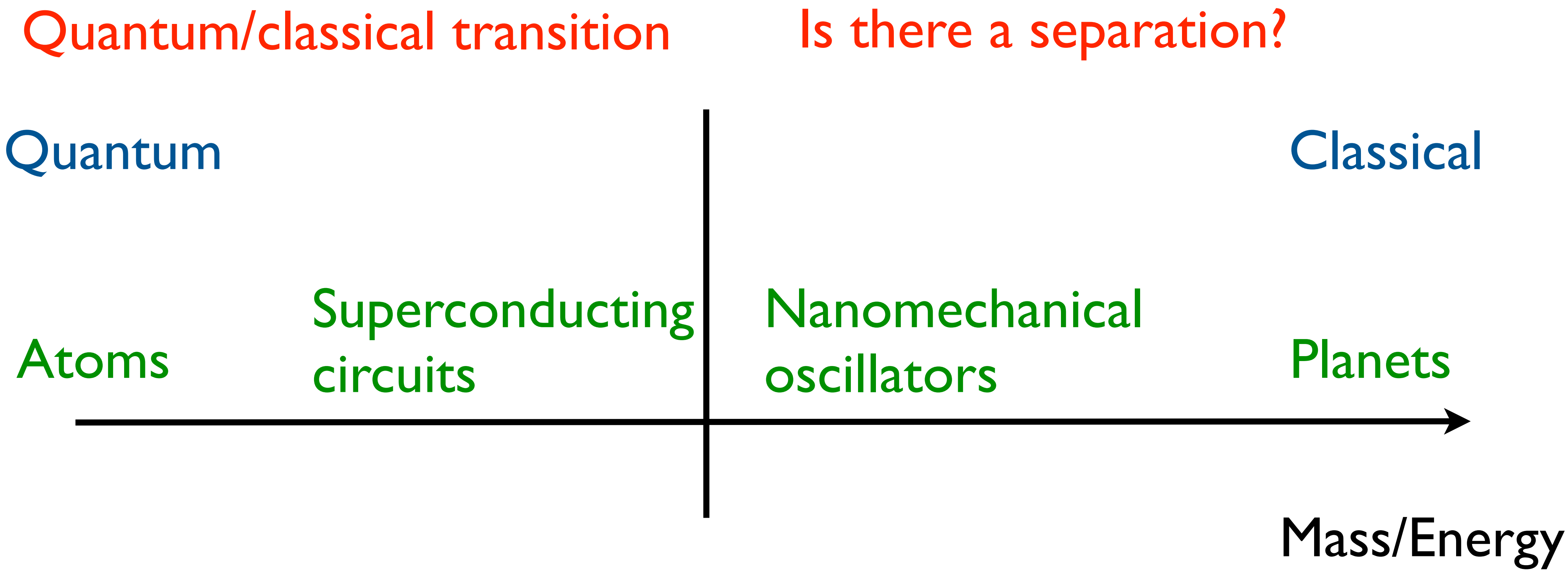

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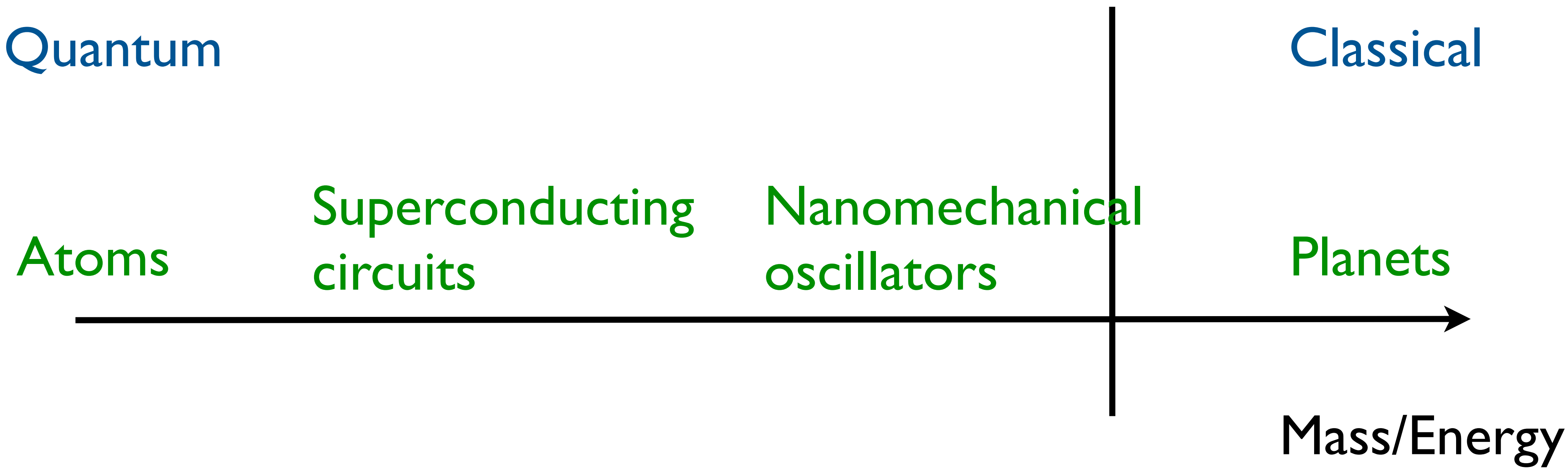

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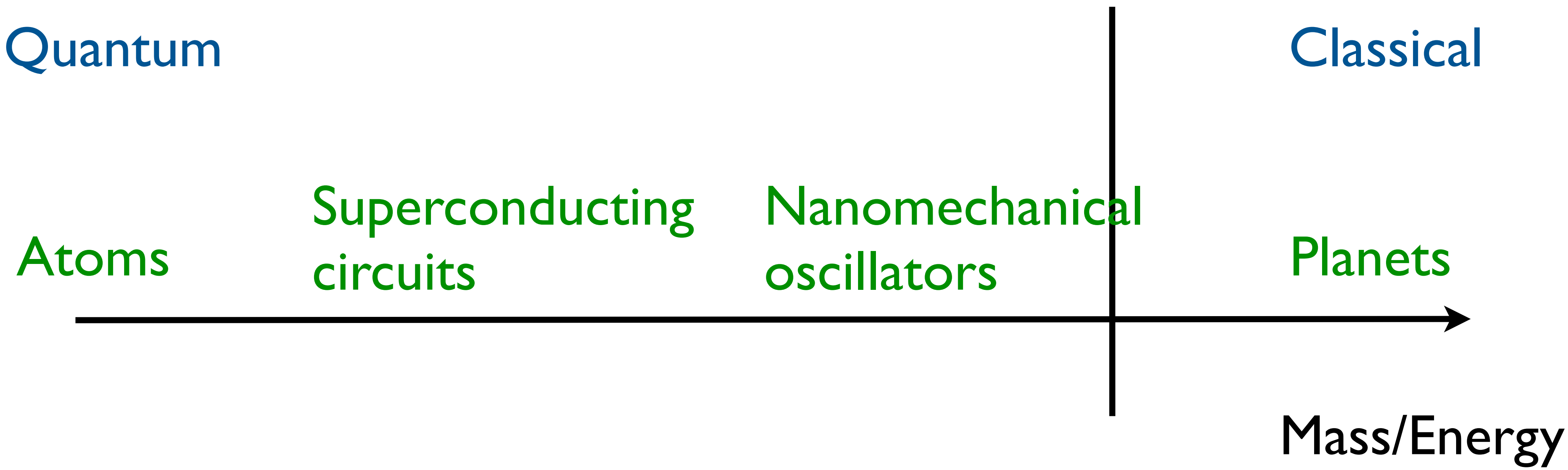

We need criteria to test that something is non-classical

What is not

- Discrete spectra
- Spontaneous emission
- Squeezing
- Continuous variable quantum teleportation

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What is

Negative Wigner functions

Types of non-classicality

1. Agrees with quantum mechanics

2. The quantum description is different

3. Non-classical according to quantum mechanics

4. Violates *any* classical description

5. Bell inequalities

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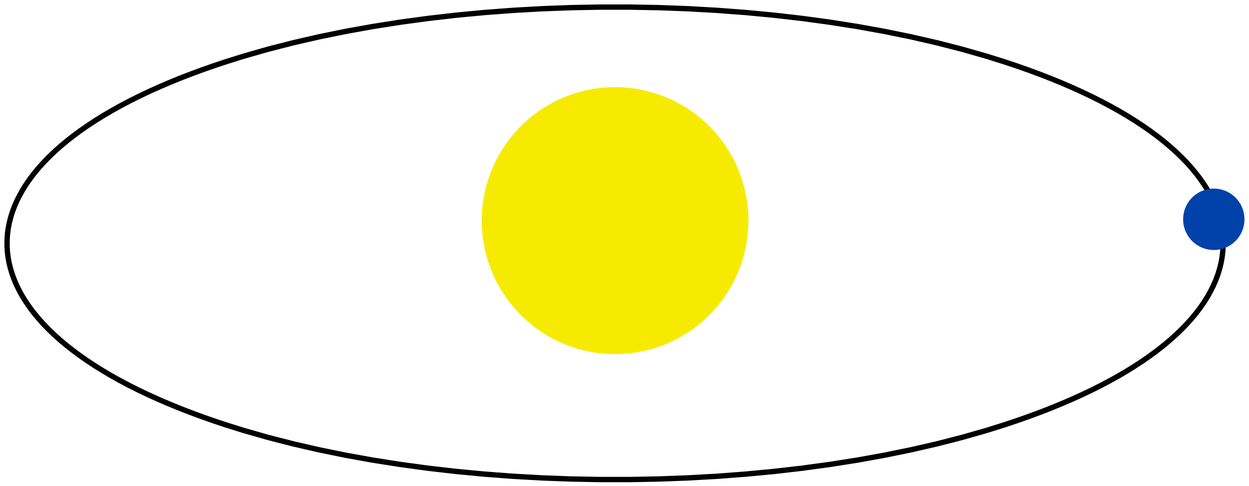
Agrees with quantum theory

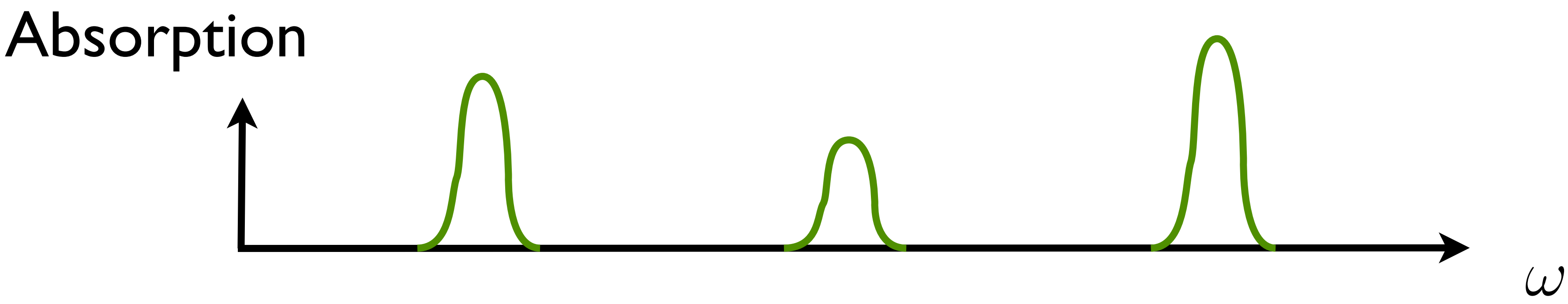
True for planetary motion



$$\left\langle \frac{\partial p}{\partial t} \right\rangle = -\langle \nabla V \rangle$$

Agrees with quantum theory

Discrete spectra



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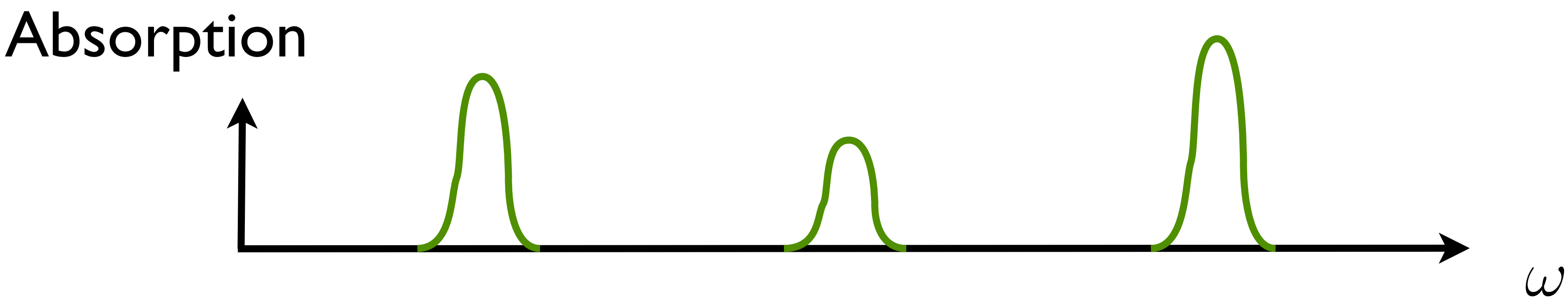

Absorption of classical harmonic oscillator

$$\text{Abs} \;\propto\; \frac{\omega^2\gamma}{(\omega_0 - \omega^2)^2 + \omega^2\gamma^2}$$

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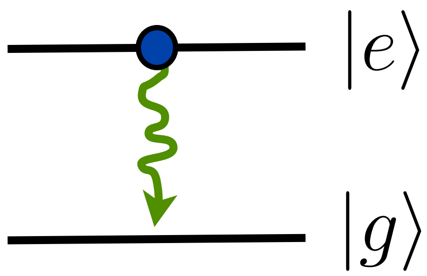
Ex: Spontaneous emission



Dipole moment vanish $\langle \hat{\vec{d}} \rangle = 0$

No electric field $\vec{E}(\vec{r}) = G(\vec{r})\langle \hat{\vec{d}} \rangle = 0$

=> No radiation

The quantum description is different

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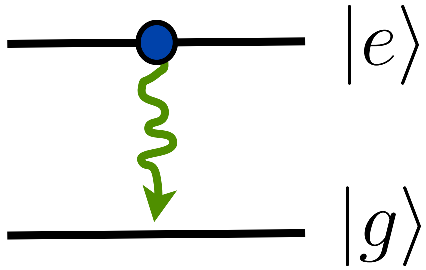

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Quantize: $\hat{\vec{E}}(\vec{r}) = G(\vec{r}) d\sigma_-$

$$\hat{\vec{E}}^\dagger \hat{\vec{E}}(\vec{r}) = G(\vec{r})^2 d^2 \sigma_+ \sigma_- \sim |e\rangle\langle e|$$

The quantum description is different

Harmonic oscillator with random phase

Dipole moment vanish $\langle d \rangle \sim d_0 \langle \mathrm{e}^{\mathrm{i}\phi} \rangle = 0$

Square of dipole does not $\langle d^*(t+\tau) d(t) \rangle \sim d_0^2 \mathrm{e}^{\mathrm{i}\omega\tau} \neq 0$

Radiation as before $\left\langle \vec{E}^\dagger \vec{E} \right\rangle = G(\vec{r})^2 d_0^2$

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Rabi oscillation: phase lost during excitation

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Ideal test

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Complications:

Requires two systems

Known Bell inequalities for continuous variables require complicated states

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Also theory hard

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Normal ordered products
Commutators etc.

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Squeezing: non-classical if fluctuations reduced below vacuum fluctuations

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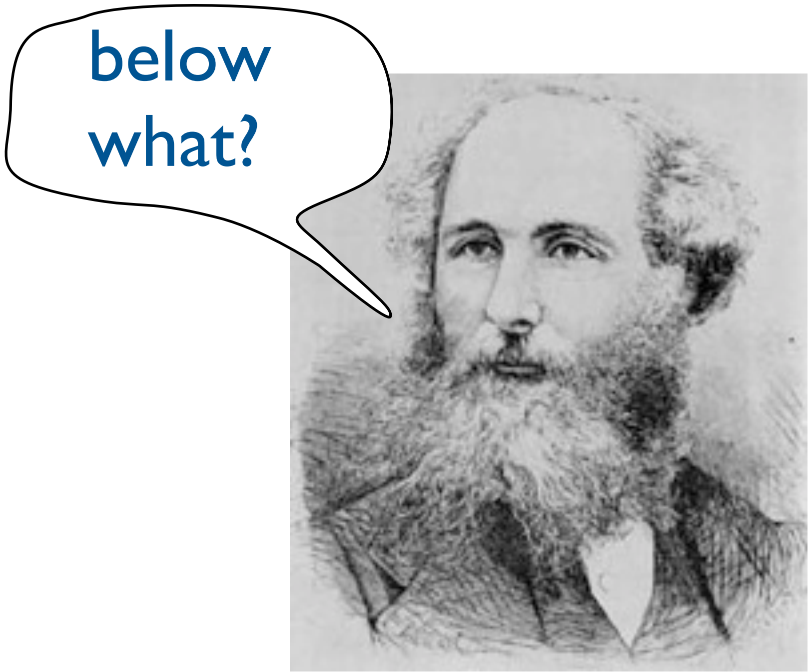

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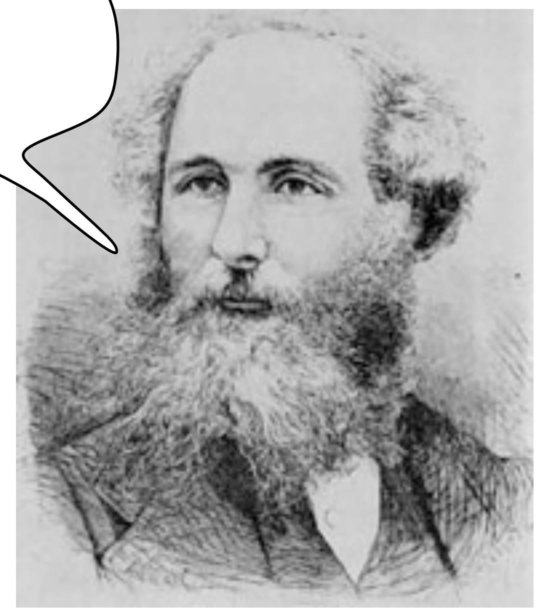
Squeezing: non-classical if fluctuations reduced below vacuum fluctuations

Well quantum mechanics tells us that every mode has fluctuations even vacuum

below what?



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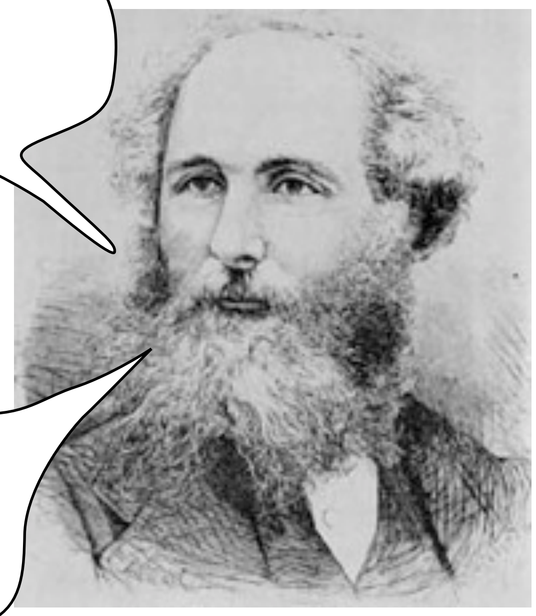

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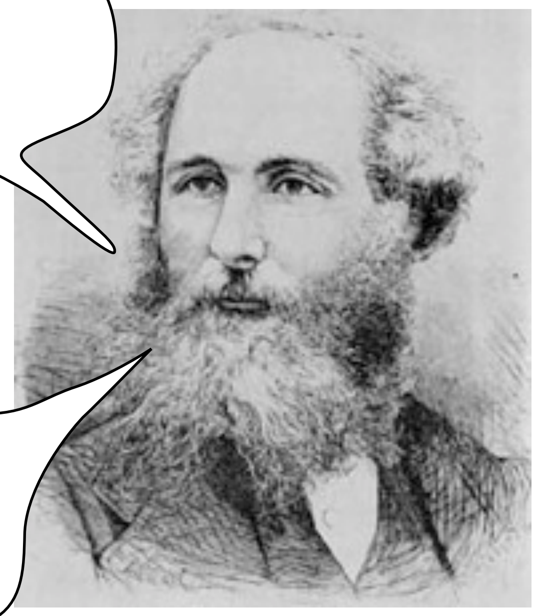

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Well so you say

Squeezing

Classical theory?

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Wigner function $W(x,p) \geq 0$ => Probability distribution

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Pick x,p according to $W(x,p)$ and evolve

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Same arguments to apply continuous variable quantum teleportation,......

Not bad science. Different objective.

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Genuine non-classical

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What is the most general description of a system?

Classical description

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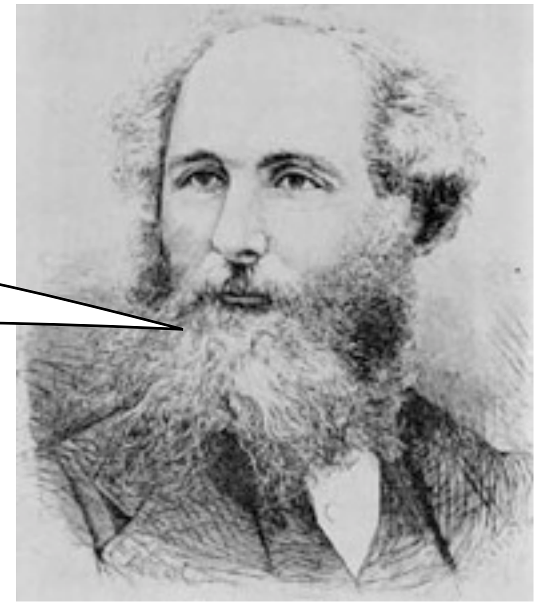
Well it has a certain position and momentum



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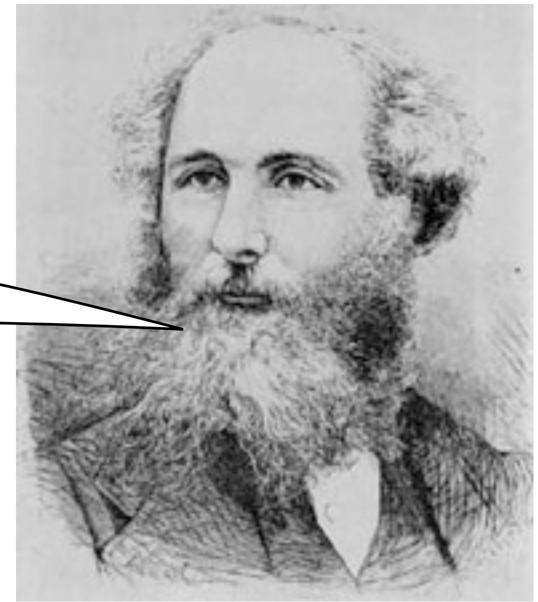

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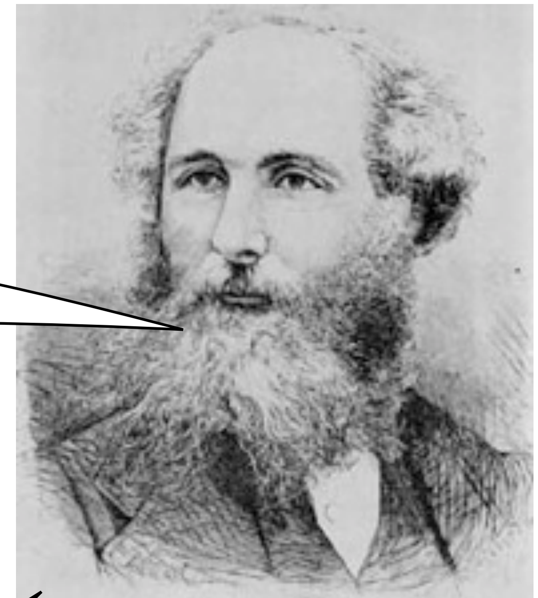

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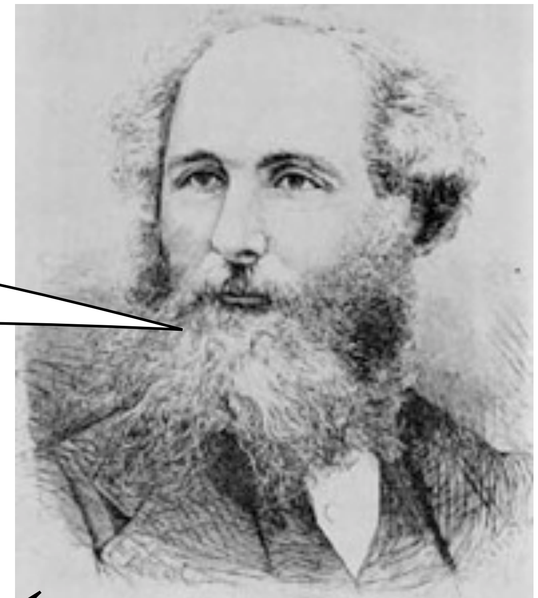

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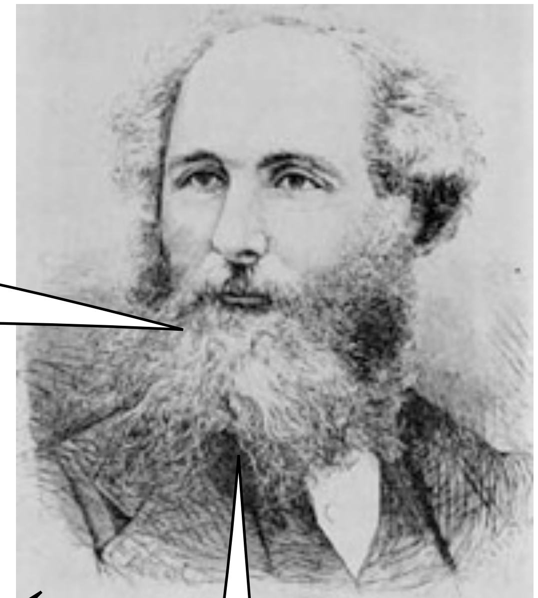
Well prove it



Wigner functions

Grey background => quantum input (don't tell Maxwell)

Single photon state => negative Wigner function
=> not a probability distribution

Have been done*:

State reconstruction
Maximum likelihood

Inverse Radon

* Large fraction of audience *et al*

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Can we do something simple?

* Large fraction of audience *et al*

Test*

$$\langle M^2(x,p)\rangle = \int dxdpW(x,p)M^2(x,p) \geq 0$$

*Bednorz and Belzig, Phys. Rev. A **83**, 52113 (2011)
See also: E. Shchukin, T. Richter, and W. Vogel, J of Optics B: Q. and Semi. Optics 6, S597 (2004).
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I agree, so let us measure x and p and see that it fits

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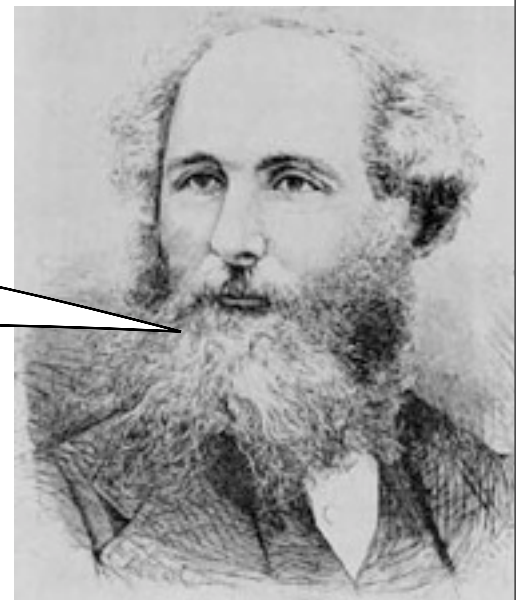

Unfortunately I cannot measure both x and p but I can measure combination and infer $W(x,p)$

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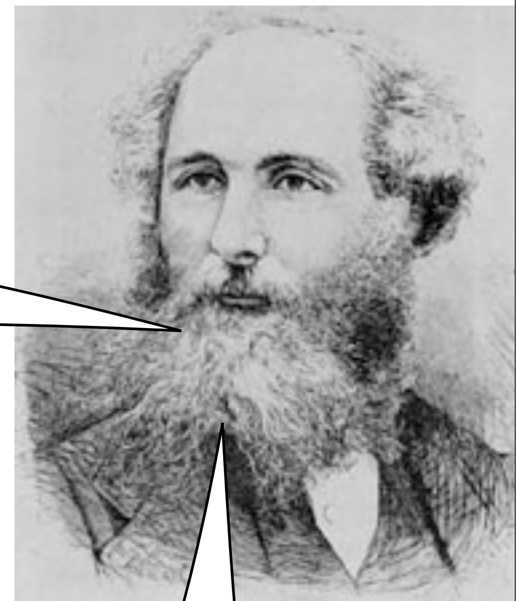

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Ok, let's see

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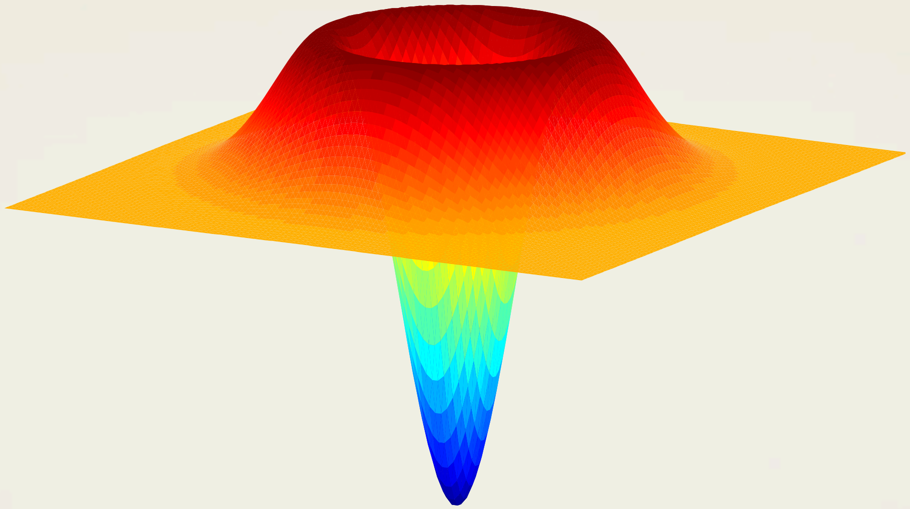
Picking the right function

Single photon state $$W(x,p) = \frac{1}{\pi}(1-2r^2)\mathrm{e}^{-r^2}$$

$$r^2 = x^2 + p^2$$



Rotational symmetry $$M(x,p) = 1 + \sum_{n=1}^{N/2} C_{2n} r^{2n}$$

Pick M so that strong weight on center: $\langle M^2 \rangle < 0$

Measurable Test

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Easy case l=1 $\langle r^2\rangle = \langle x^2\rangle + \langle p^2\rangle$

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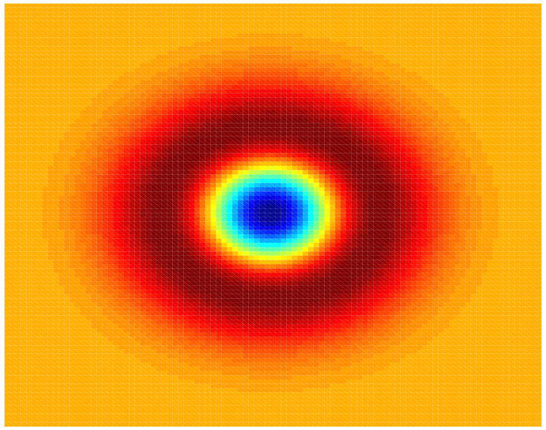

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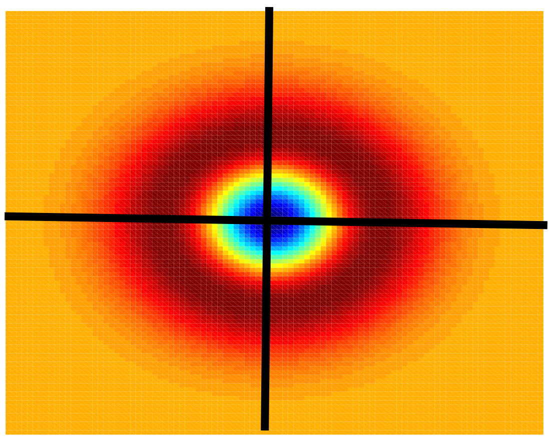
Need to know $\langle r^{2l}\rangle$

Easy case l=1 $\langle r^2\rangle = \langle x^2\rangle + \langle p^2\rangle$ => measure x and p



Measuring higher orders

l=2 $$\langle r^4 \rangle = \langle (x^2 + p^2)^2 \rangle = \langle x^4 \rangle + \langle p^4 \rangle + 2\langle x^2 p^2 \rangle$$

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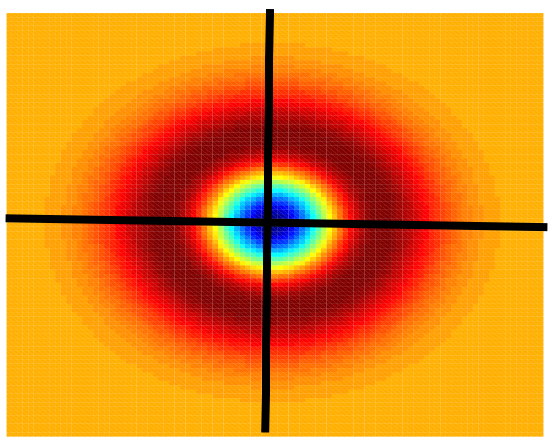

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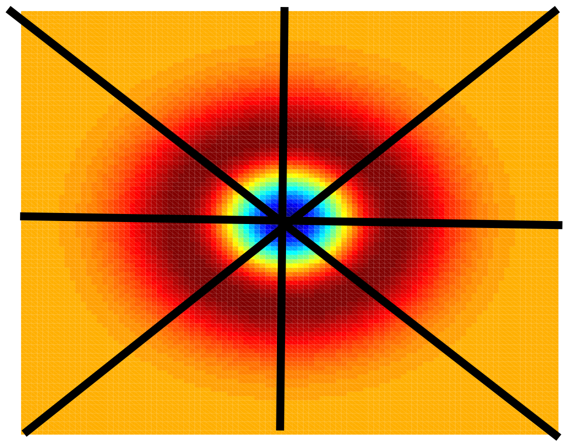
Measure "diagonal" quadratures

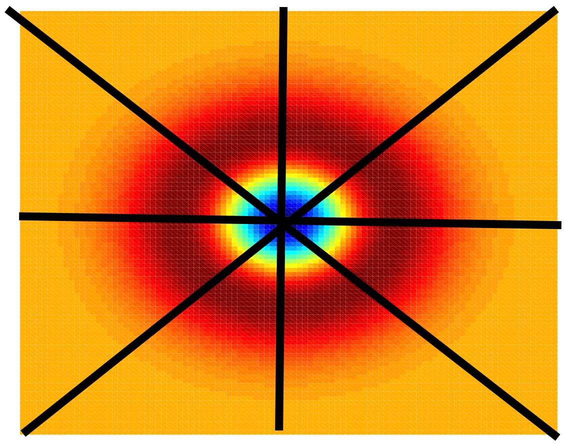


Measuring higher orders

l=2 $\langle r^4 \rangle = \langle (x^2+p^2)^2 \rangle = \langle x^4 \rangle + \langle p^4 \rangle + 2\langle x^2 p^2 \rangle$

Measure "diagonal" quadratures

$$\left\langle \left(\frac{x+p}{\sqrt{2}} \right)^4 \right\rangle + \left\langle \left(\frac{x-p}{\sqrt{2}} \right)^4 \right\rangle = \frac{1}{2}(\langle x^4 \rangle + \langle p^4 \rangle) + 3\langle x^2 p^2 \rangle$$



General test

Measure $2l$ quadratures: $\left\langle \left(x^2 + p^2\right)^l \right\rangle = \binom{2l}{l}^{-1} \frac{2^{2l}}{2l} \sum_{m=1}^{2l} \langle Q^{2l}_{\pi m/2l} \rangle$

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General test

$$\langle M^2 \rangle = \ldots C_{2k} \ldots C_{2n} \ldots \sum_{m=1}^{2l} \langle Q^{2l}_{\pi m/2l} \rangle \geq 0$$

For *any* Cs

General test

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General test

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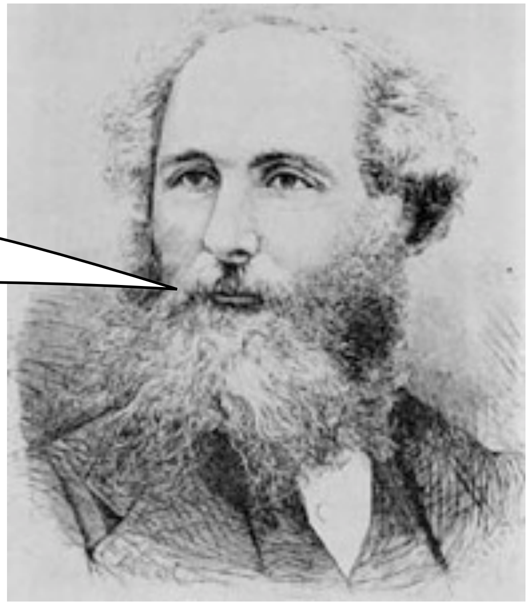
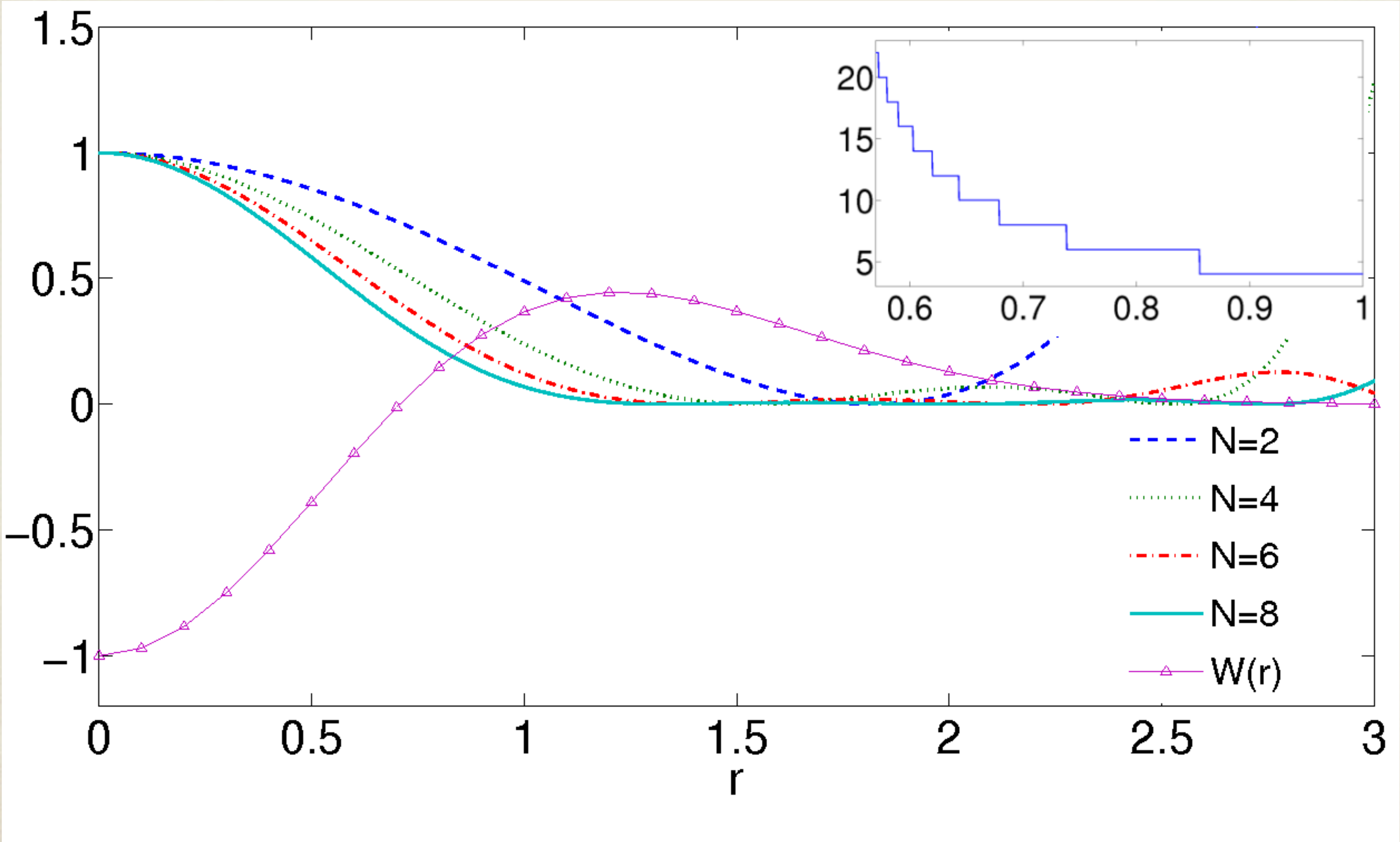
For *any* Cs

I agree, so let
us try it out



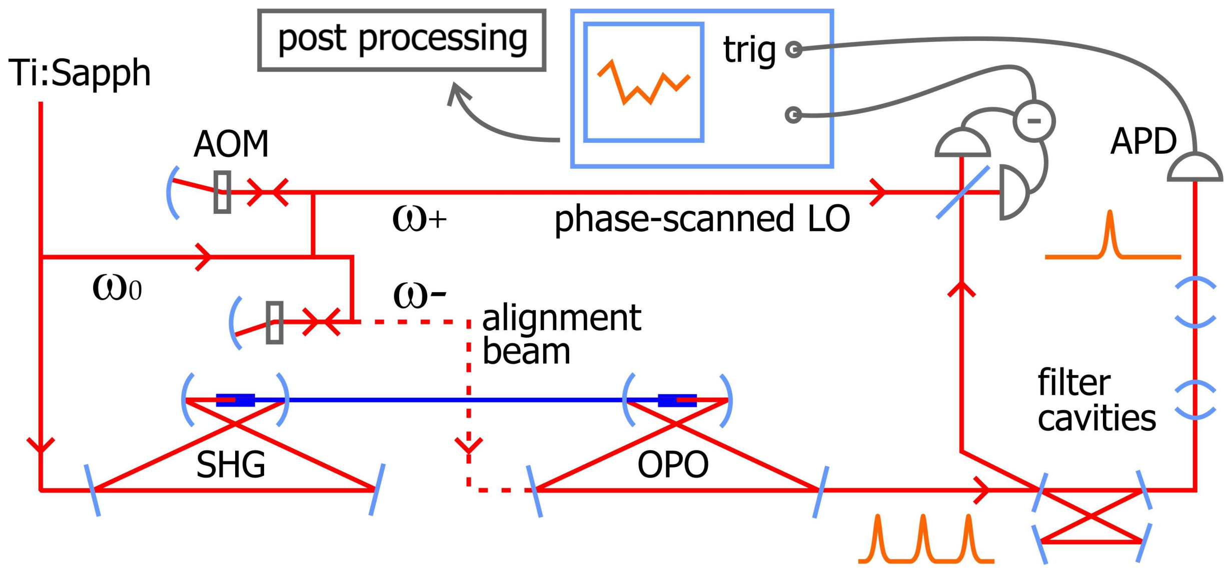
Quantum expectation

Optimize Cs => negative for $N \geq 4$ (requires 8 quadratures)



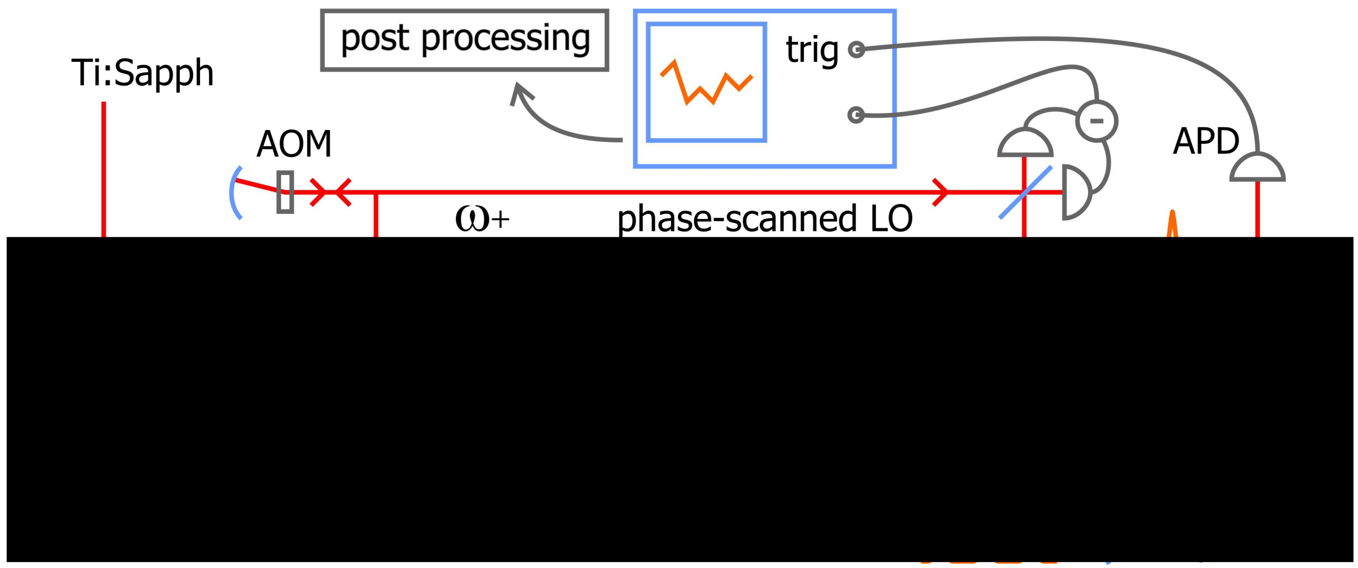
Experiment

“Standard” photon subtraction experiments



Experiment

"Standard" photon subtraction experiments



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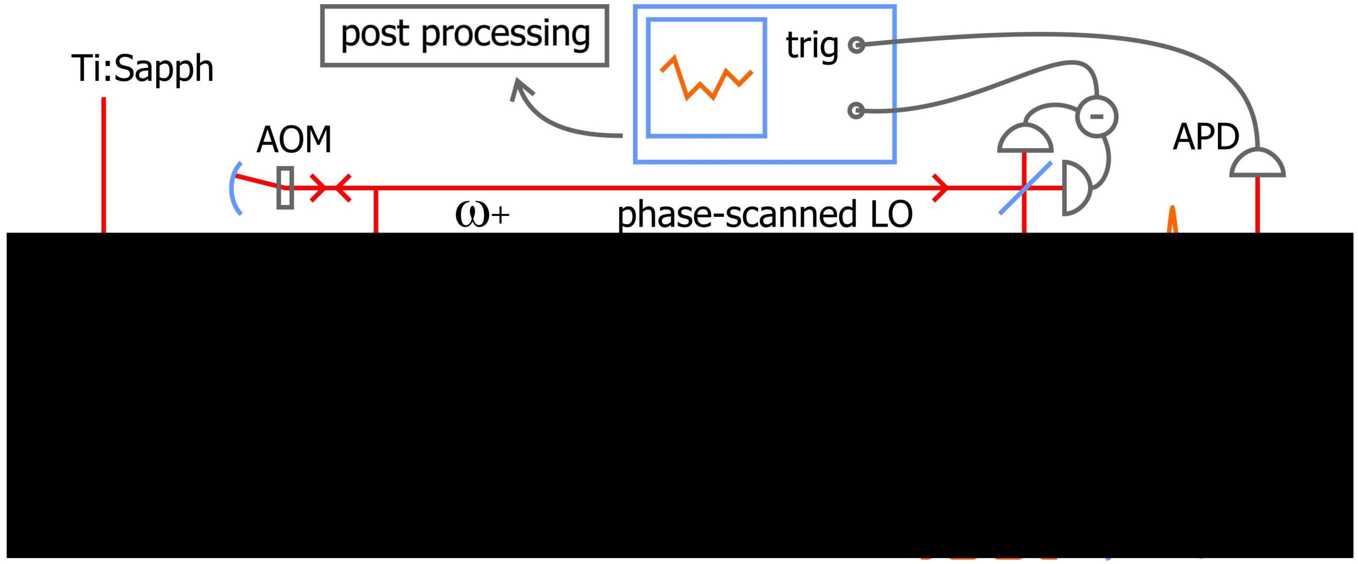
Experiment

"Standard" photon subtraction experiments



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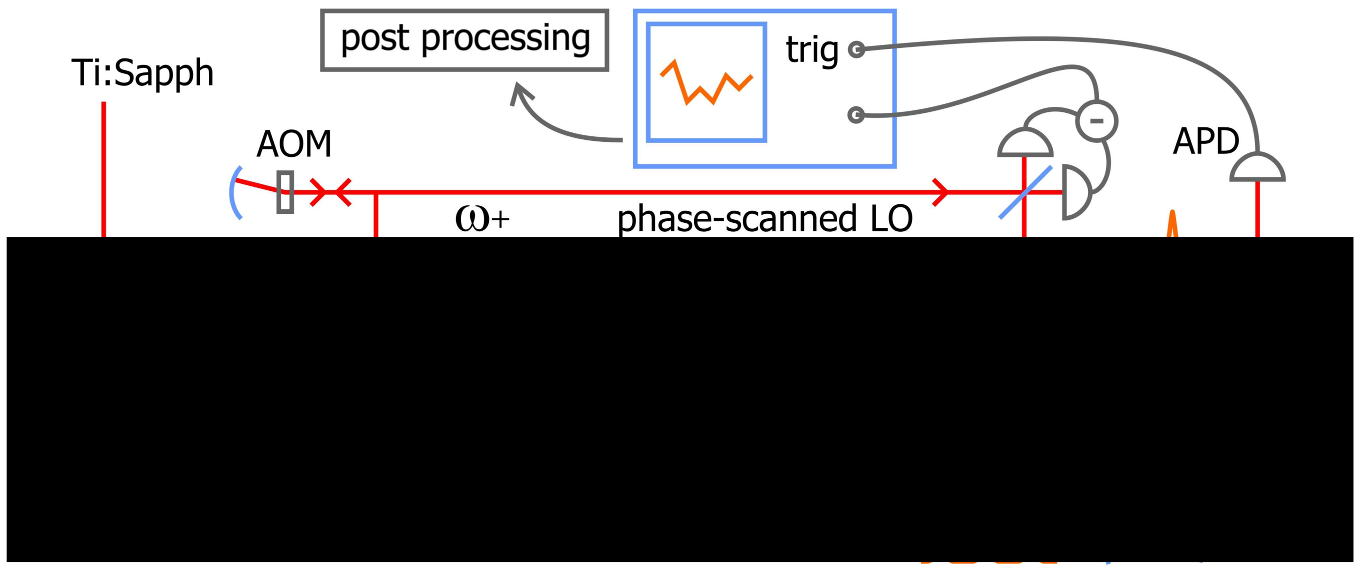
Homodyne detection with varying phase

=> Also works classically

Experiment

"Standard" photon subtraction experiments



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Homodyne detection with varying phase

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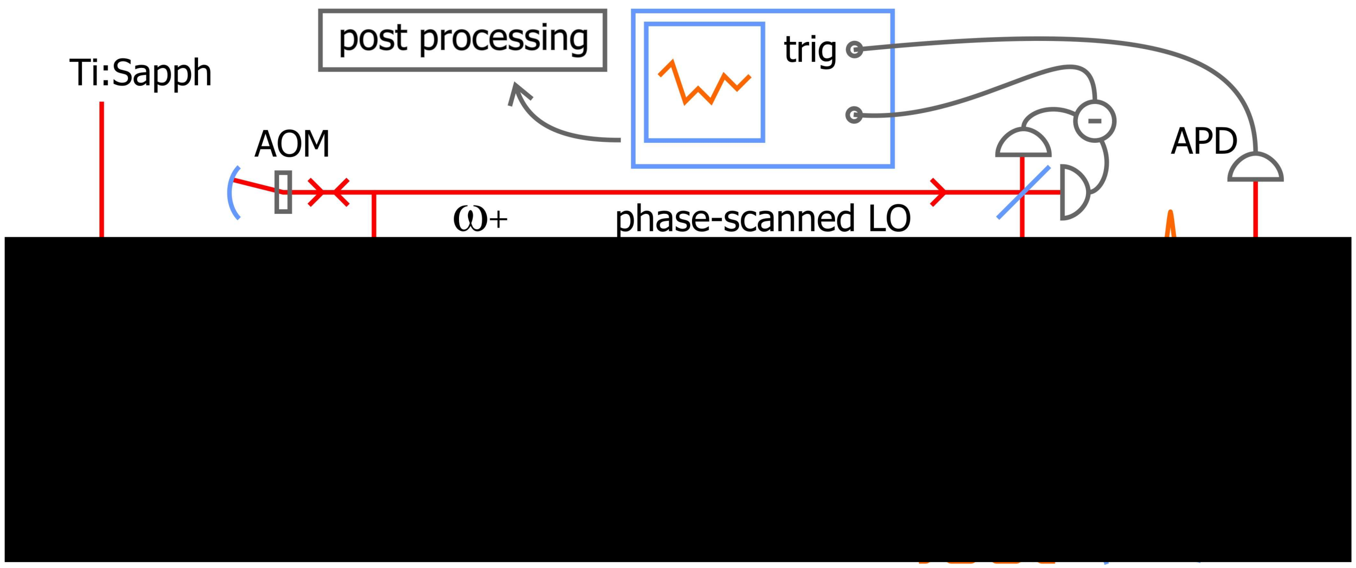
Phase not locked => All quadratures the same

Experiment

"Standard" photon subtraction experiments



[redacted]

Homodyne detection with varying phase

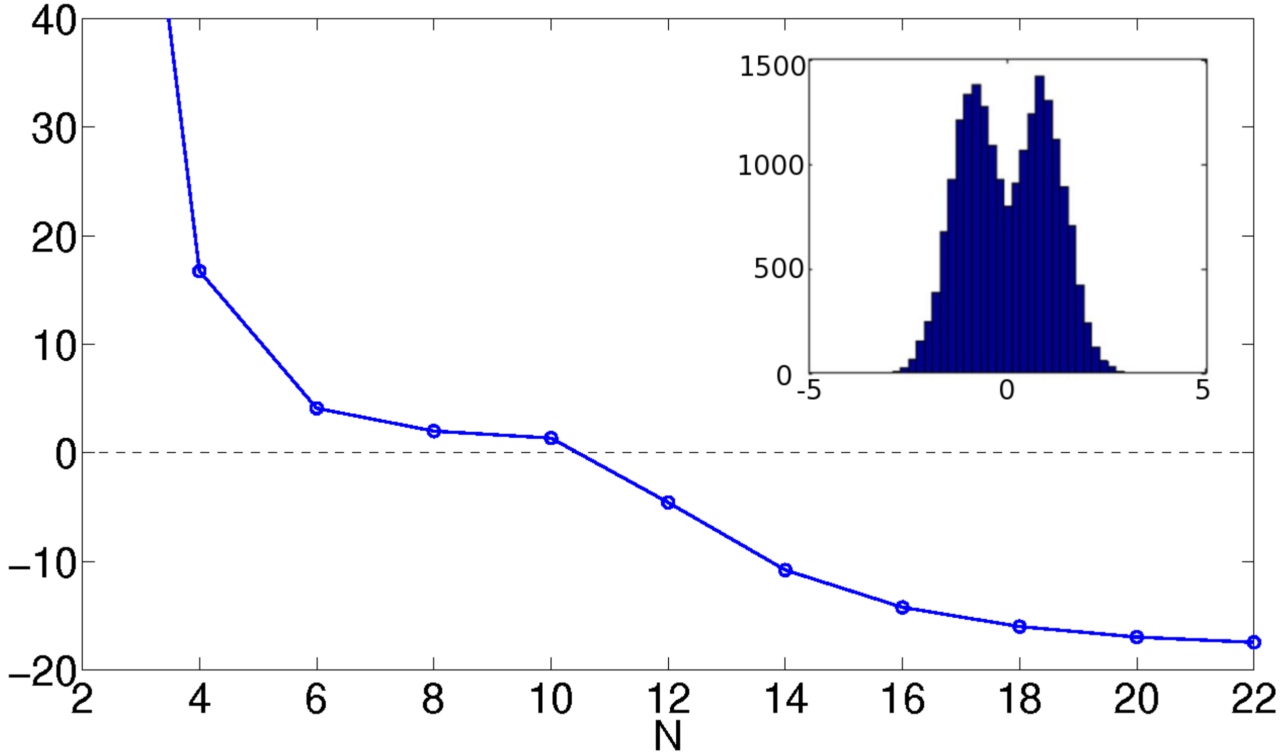
=> Also works classically

Phase not locked => All quadratures the same

Cannot introduce violation

Results

$$\frac{\langle M^2 \rangle}{\sigma_{M^2}}$$



Violation by nearly 20 standard deviations.

Conclusion

Non-classical: no classical description
(don't assume quantum mechanics)

Simple strict non-classicality test

Can be violated on a single system using homodyne detection

Light field: one cannot assign a probability distribution to the position and moment - not even nature can know x and p simultaneously

Conclusion

Non-classical: no classical description
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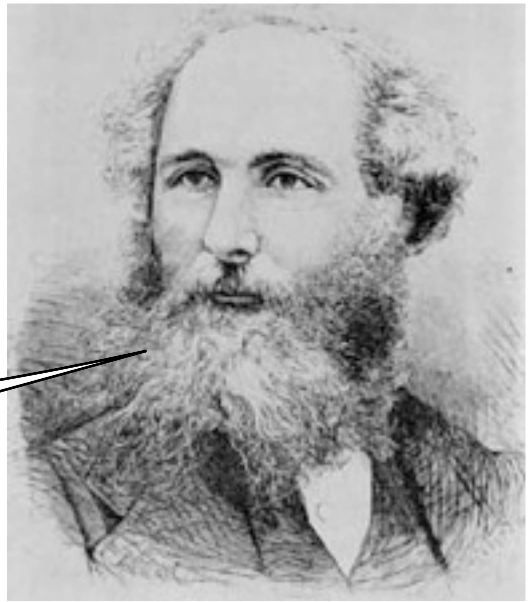
Simple strict non-classicality test

Can be violated on a single system using homodyne detection

Light field: one cannot assign a probability distribution to the position and moment - not even nature can know x and p simultaneously

I didn’t see that coming. I guess I will have to study this quantum thing.



Outlook

Similar test should be applied to other macroscopic systems

Superconducting systems

Nanomechanical systems => this test works directly

Extension to Bell inequalities?

Acknowledgements

Thanks to:

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Apologies to Maxwell