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PART 1 : Tutorial on Quantum Nonlinear Optics

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Summer School on Quantum and Nonlinear Optics
(QNLO 2015 Sørup Herregaard)

OVERVIEW OF TUTORIAL and RESEARCH TALK:

PART 1 : Tutorial on Quantum Nonlinear Optics

EM field quantization; Wigner distribution;
Homodyne detection; Quantum tomography;
Temporal modes; Beam splitter; Basics of NLO;
Parametric amplification; Squeezing;
Quantum frequency conversion;

PART 2 : Discussion on Quantum Nonlinear Optics

PART 3 : Photon Temporal Modes: a Complete Framework for Quantum Information Science

Pulse-code multiplexing; TMs as qubits and qudits;
Quantum pulse gate; Completing the tool kit for
photons as an information resource;

Unifying Theme: Temporal Modes of Photons

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GENERAL REFERENCES for PART 1

1. R Loudon, "The Quantum Theory of Light"

2. L Mandel, E Wolf, "Optical Coherence and Quantum Optics"

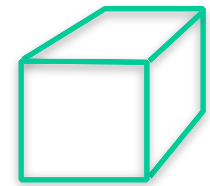
3. M Raymer, "Measuring the quantum mechanical wave function," Contemporary Physics 38, 343 (1997).

4. A Lvovksy and M Raymer, Rev. Mod. Phys., 81, 299 (2009), "Continuous-variable optical quantum state tomography,"

5. B Smith and M Raymer, New J. Phys. 9, 414 (2007). (advanced treatment of wave-packet quantization)

1. QUANTIZATION OF THE OPTICAL FIELD

E obeys classical Maxwell's equations:
So the modes obey Helmholtz equation: $\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\underline{u}_j(\underline{r}) = 0$



An imaginary box with side lengths L has allowed MONOCHROMATIC modes with frequencies $\omega_j = j\,\pi / L \; (j = 1,2,3\ldots)$

modes: $\underline{u}_j(\underline{r}) = V^{-1/2}\underline{\varepsilon}_j \exp(i\,\underline{k}_j \cdot \underline{r})\,; \quad V = L^3,\; \underline{\varepsilon}_j = polarization$

expand: $\underline{\hat{E}}^{(+)}(\underline{r},t) = i\sum_j \sqrt{\frac{\hbar\omega_j}{2\varepsilon_0}}\;\hat{a}_j\;\underline{u}_j(\underline{r})\exp(-i\omega_j t)$ $\quad(\omega_j > 0)$

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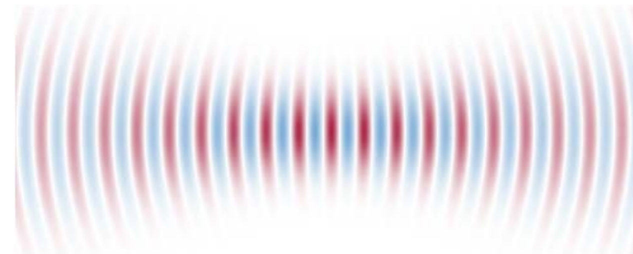
photon annihilation and creation operators: $\hat{a}_j,\; \hat{a}_k^{\dagger}$

commutator: $[\hat{a}_j, \hat{a}_k^{\dagger}] = \delta_{jk}$

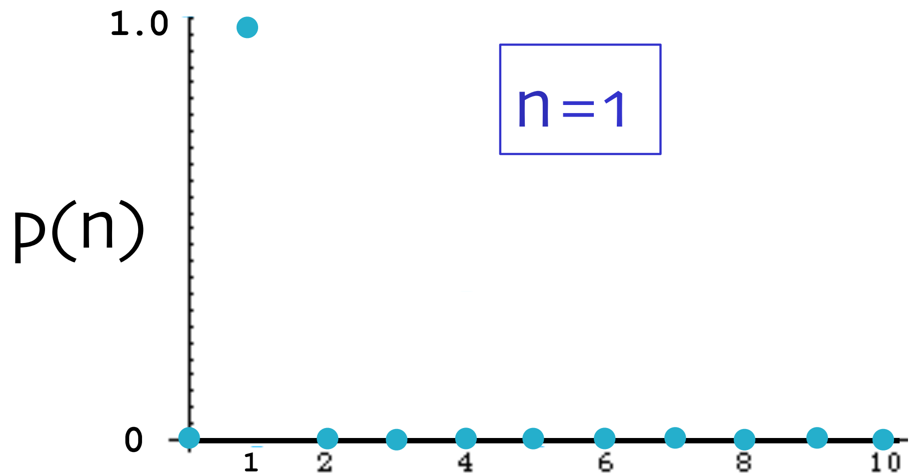
QUANTIZATION OF THE OPTICAL FIELD

MONOCHROMATIC "PHOTON": A single-frequency excitation (state) of the quantum EM field.



for a particular mode: $\underline{u}_0(z)$

one-photon state: $\left|1_\omega\right\rangle = \hat{a}_\omega^{\dagger}\left|vac\right\rangle$



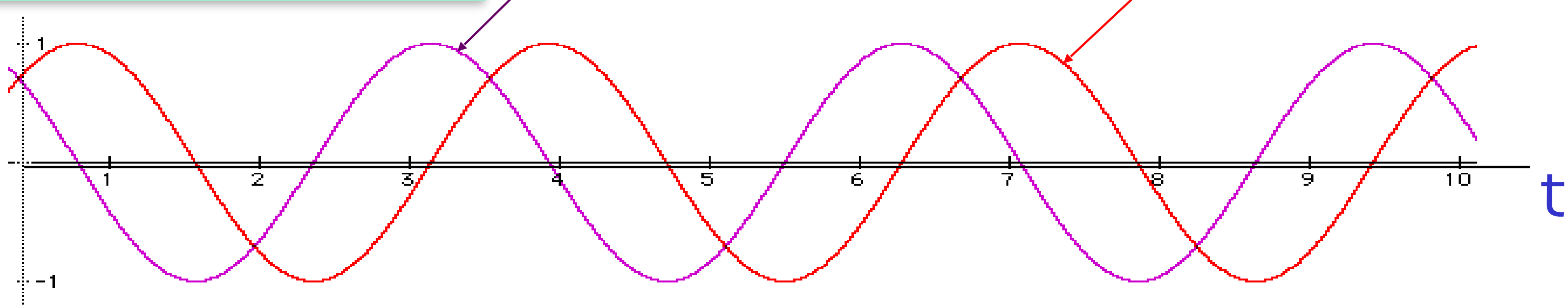
n-photon state: $\left|n_\omega\right\rangle = (\hat{a}_\omega^{\dagger})^n\left|vac\right\rangle$

http://www.rp-photonics.com/gaussian_beams.html

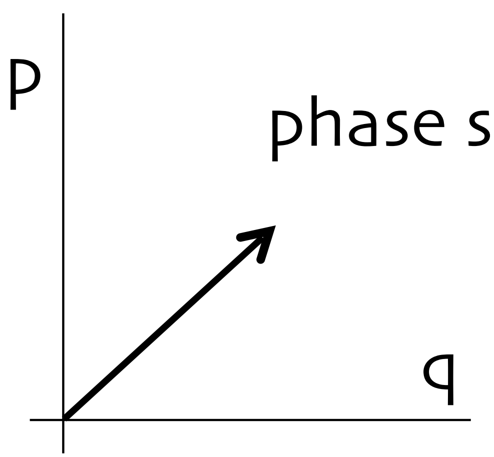
Field-Quadrature Operators

A monochromatic plane-wave mode: $\underline{\hat{E}}^{(+)} = i\sqrt{\dfrac{\hbar\omega_j}{2\varepsilon_0}}\;\hat{a}\;\dfrac{\exp(ik_0 z)}{\sqrt{V}}\exp(-i\omega_0 t)$

quadrature operators: $\hat{q} = (\hat{a} + \hat{a}^\dagger)/2^{1/2}$ $\hat{p} = (\hat{a} - \hat{a}^\dagger)/i2^{1/2}$



$$\hat{E}^{(+)}(z,t) \propto \underline{\hat{q}}\cos(\omega_0 t - k_0 z) + \underline{\hat{p}}\sin(\omega_0 t - k_0 z)$$



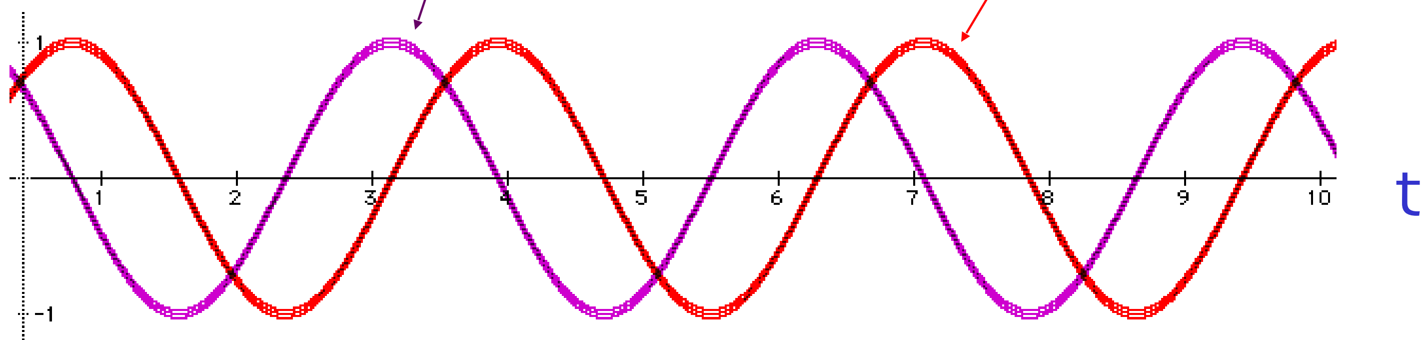
Uncertainty relation: $[\hat{q}, \hat{p}] = i$

$$std(q)\; std(p) \geq 1/2$$

Coherent State - ideal laser output

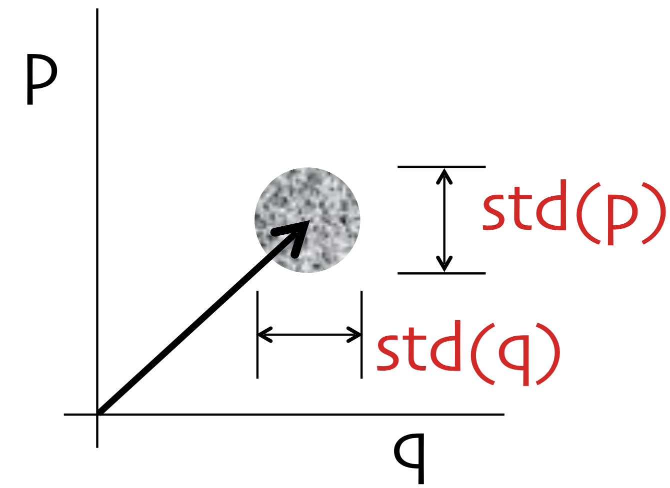
$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

quadrature operators: $\hat{q} = (\hat{a} + \hat{a}^\dagger)/2^{1/2}$ $\hat{p} = (\hat{a} - \hat{a}^\dagger)/i2^{1/2}$



Equal Uncertainties:

$$std(q) = std(p) = 1/\sqrt{2}$$

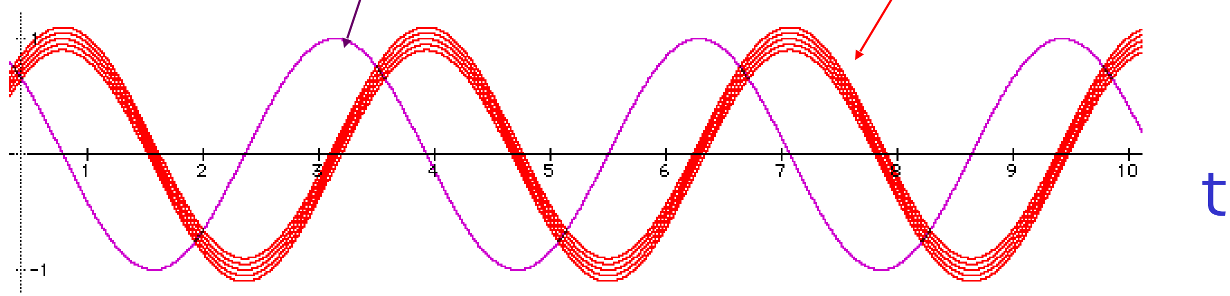


Squeezed Coherent State

$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

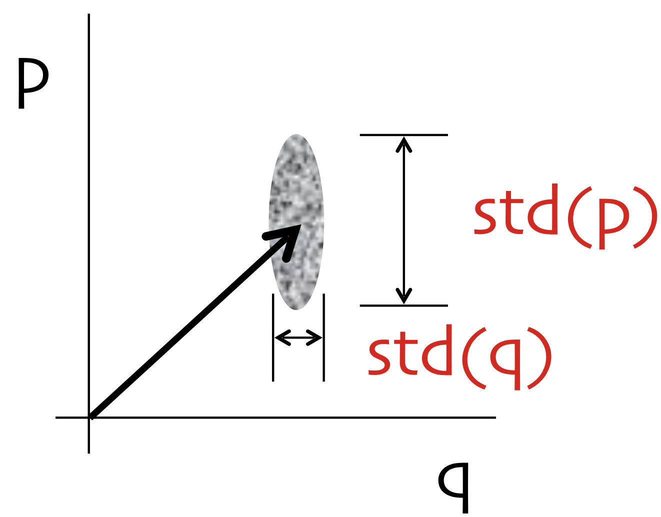
quadrature-squeezed light:

$\hat{q} = (\hat{a} + \hat{a}^\dagger)/2^{1/2}$ $\hat{p} = (\hat{a} - \hat{a}^\dagger)/i2^{1/2}$



q fluctuation reduced

p fluctuation increased

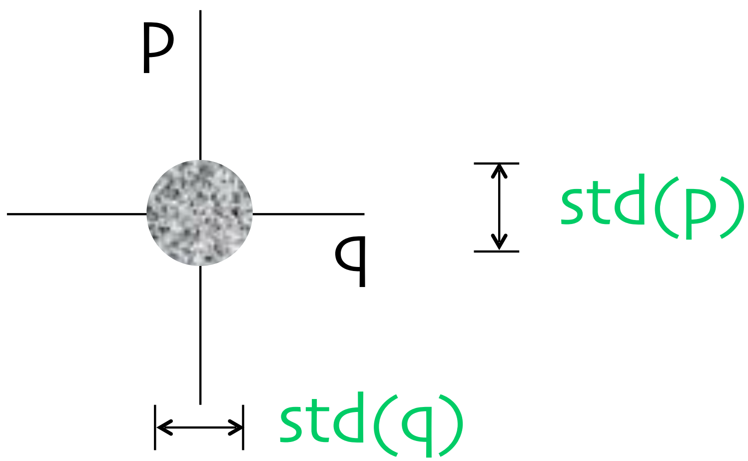


Quadrature-Squeezed Vacuum State

$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

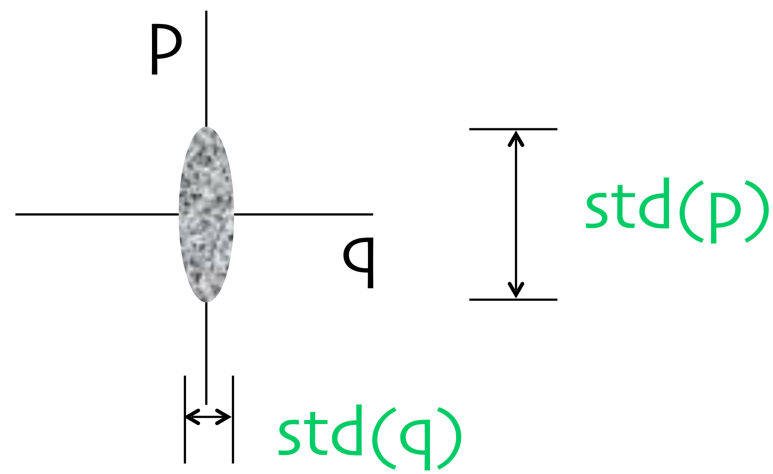
vacuum state:

$$\psi(q) = \exp\left[-q^2/2\right]$$



squeezed-vacuum state:

$$\psi(q) = \exp\left[-q^2/2e^{-2s}\right]$$

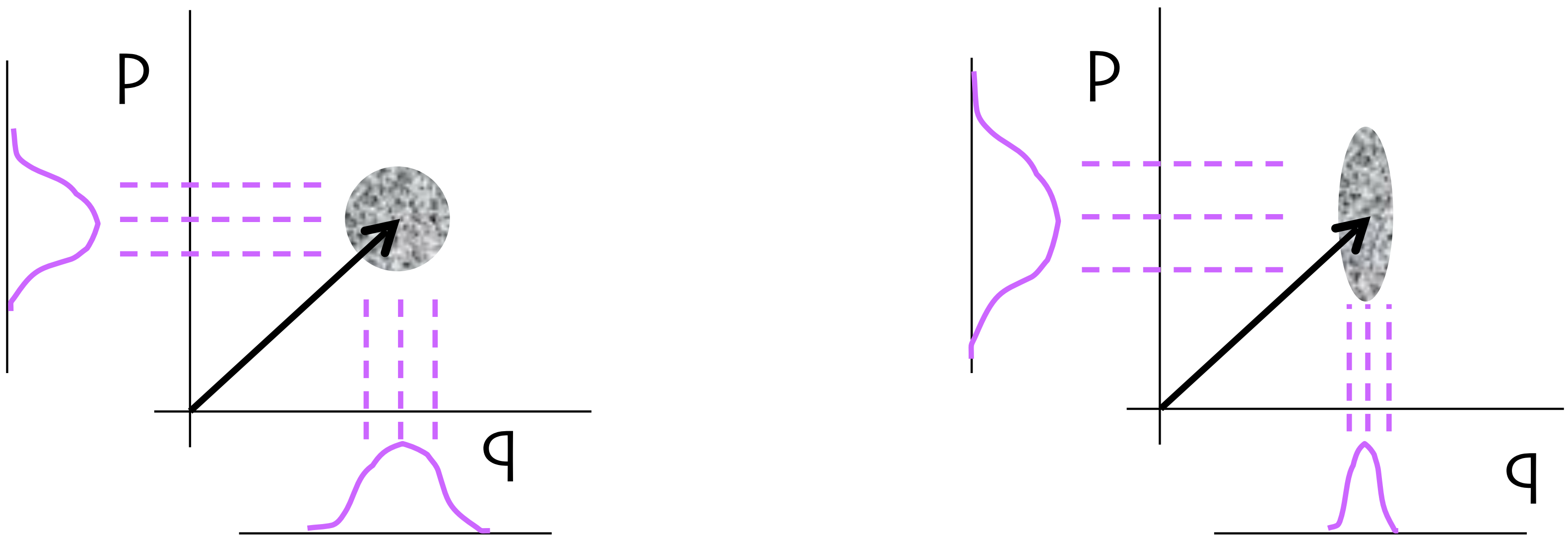


2. WIGNER DISTRIBUTION

represent the state of a single mode in (q, p) phase space.

$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

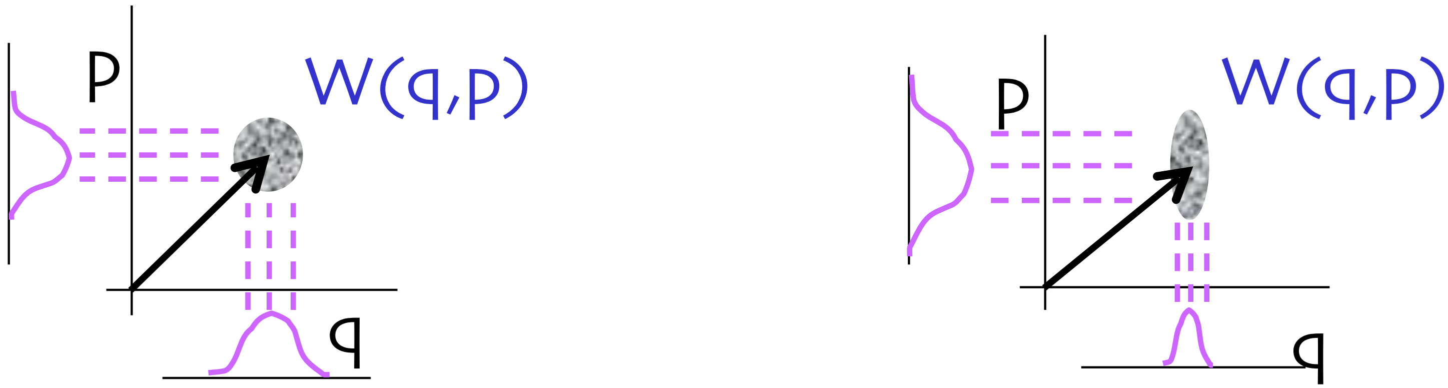
projected distributions: Pr(q), Pr(p)



Underlying Joint Distribution?

$$W(q,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle \psi(q+q'/2)\psi^*(q-q'/2)\rangle \exp(-ipq')\,dq'$$

WIGNER DISTRIBUTION



$$W(q,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle \psi(q+q'/2)\psi^*(q-q'/2)\rangle \exp(-ipq')\,dq'$$

projected distributions:

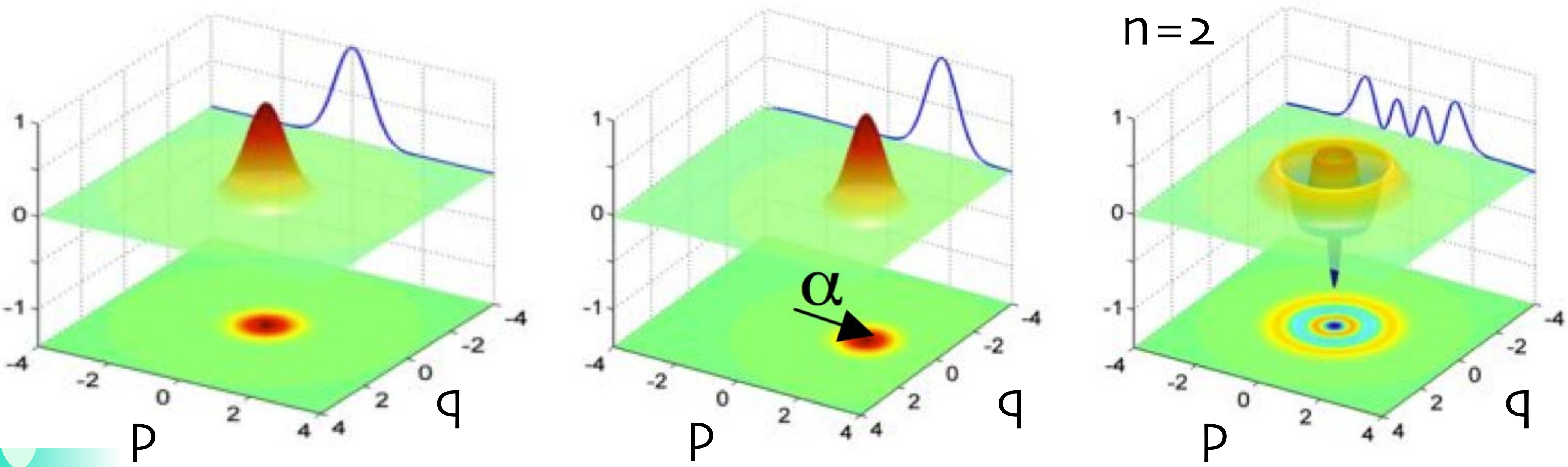
$$Pr(q) = \int_{-\infty}^{\infty} W(q,p)\ dp \quad , \quad Pr(p) = \int_{-\infty}^{\infty} W(q,p)\ dq$$

W(q,p) acts like a joint probability distribution.

But it can be negative.

Some Wigner Distributions

vacuum | coherent state | number state n=2

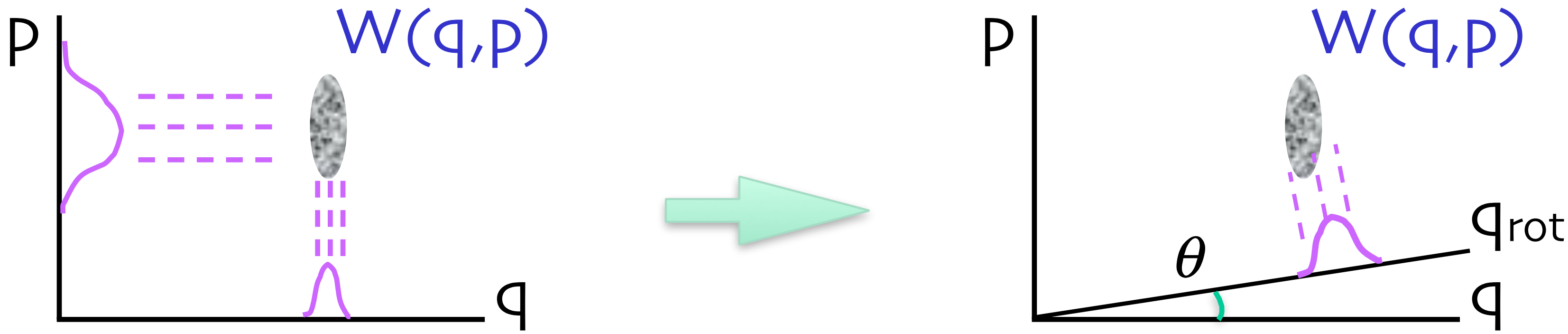


solid curves show projected distributions,
which are measurable

from S. Haroche lectures

http://www.quantumlah.org/highlight/120301_haroche_lectures.php

3. QUANTUM-STATE TOMOGRAPHY: MEASURING THE WIGNER DISTRIBUTION



1. measure a set of projected distributions:

$$Pr(q_{rot},\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(q,p)\,\delta(q_{rot} - q\cos\theta - p\sin\theta)\,dq\,dp$$

2. invert using a tomography kernel K:

$$W(q,p) = \int_{-\infty}^{\infty}\int_{0}^{\pi} Pr(q_{rot},\theta)\,K(q_{rot},\theta;q,p)\,dq_{rot}\,d\theta$$

3. invert to obtain density matrix:

$$\langle \psi(q+q'/2)\psi^*(q-q'/2)\rangle = \int_{-\infty}^{\infty} W(q,p)\exp(i\,p\,q')\,dp$$

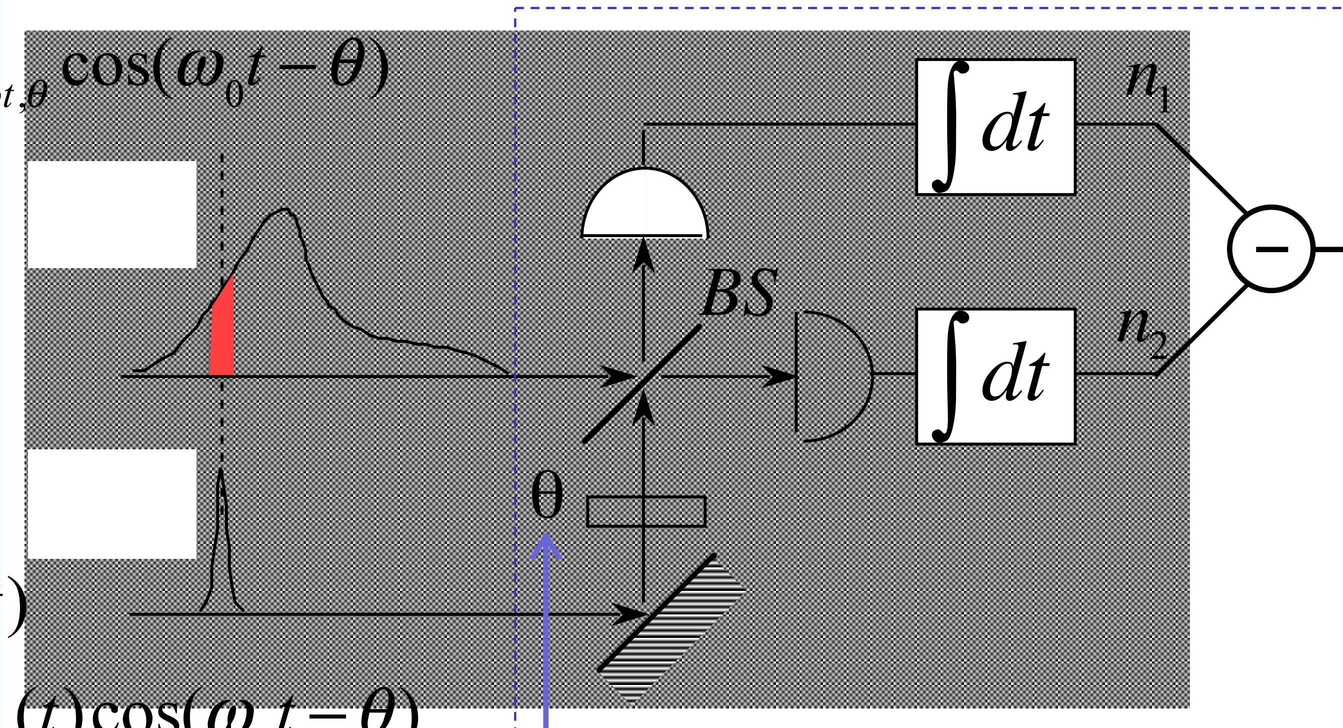
How to measure Pr(q)?

Measuring Quadrature Distributions using BALANCED HOMODYNE DETECTION

D. T Smithey, M. Beck, M. G. Raymer and A. Faridani, Phys. Rev. Lett. **70**, 1244 (1993).

$E_S(t)$ signal

$\propto q_{rot,\theta} \cos(\omega_0 t - \theta)$



$\Delta N = n_1 - n_2$

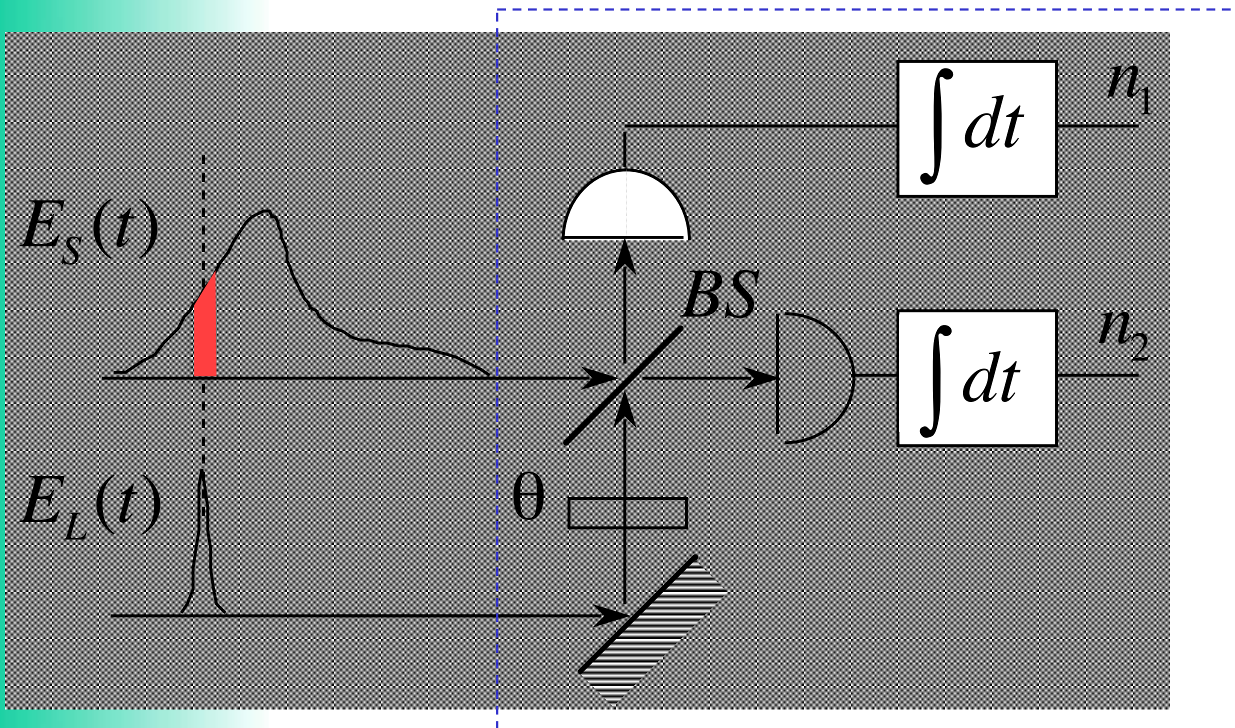
$\propto q_{rot,\theta}$

$E_L(t)$

$\propto A_L(t) \cos(\omega_0 t - \theta)$

- local oscillator
- temporal slice selection

Measuring the Wigner Distributions using Balanced Homodyne Detection



→ Set of projected quadrature distributions

↓

Phase Space Tomography

↓

W(q,p)

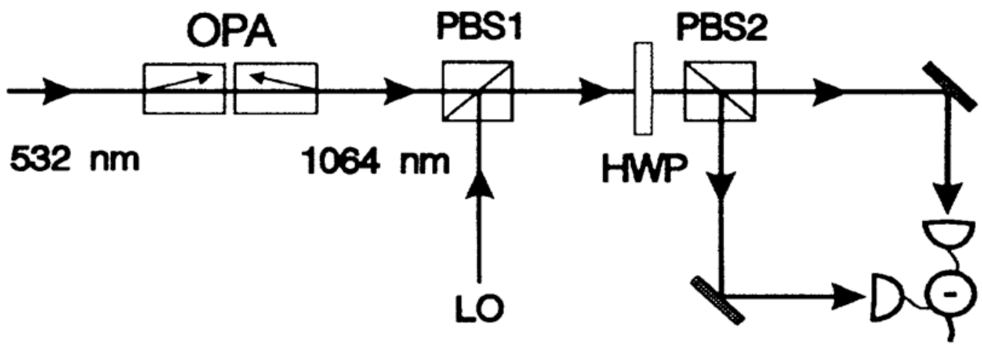
↓

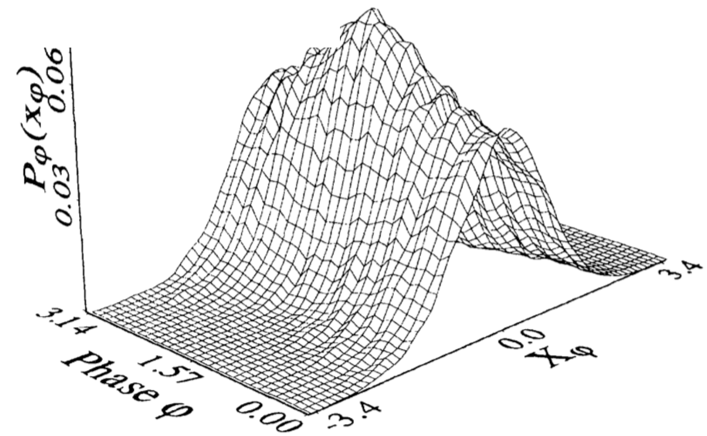
density matrix

→ if pure $Trace(\rho^2)=1$ →

quadrature wave function $\psi(q)$

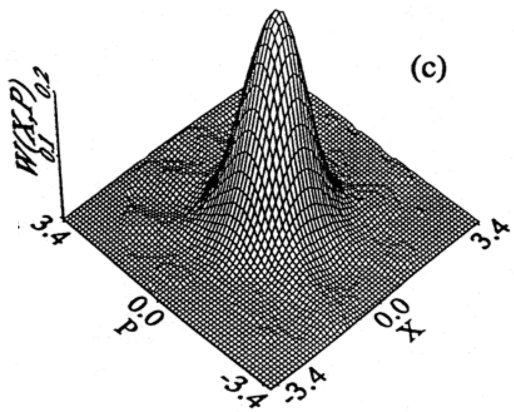
D. T Smithey, M. Beck, M. G. Raymer and A. Faridani, Phys. Rev. Lett. 70, 1244 (1993).



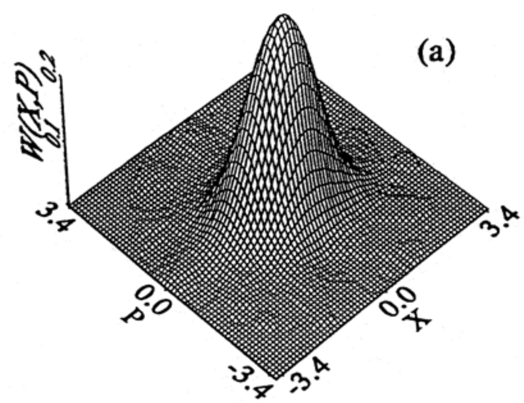


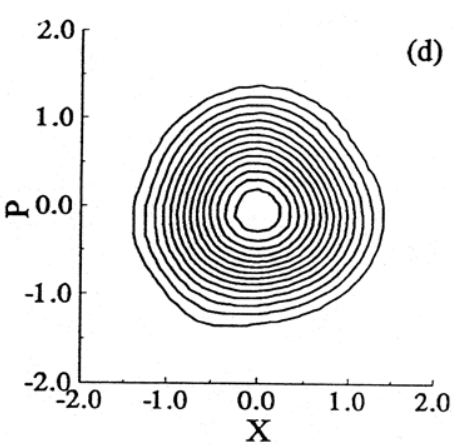
vacuum

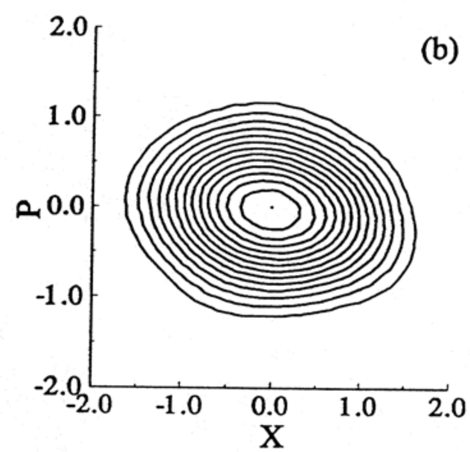
squeezed vacuum



W(X,P)

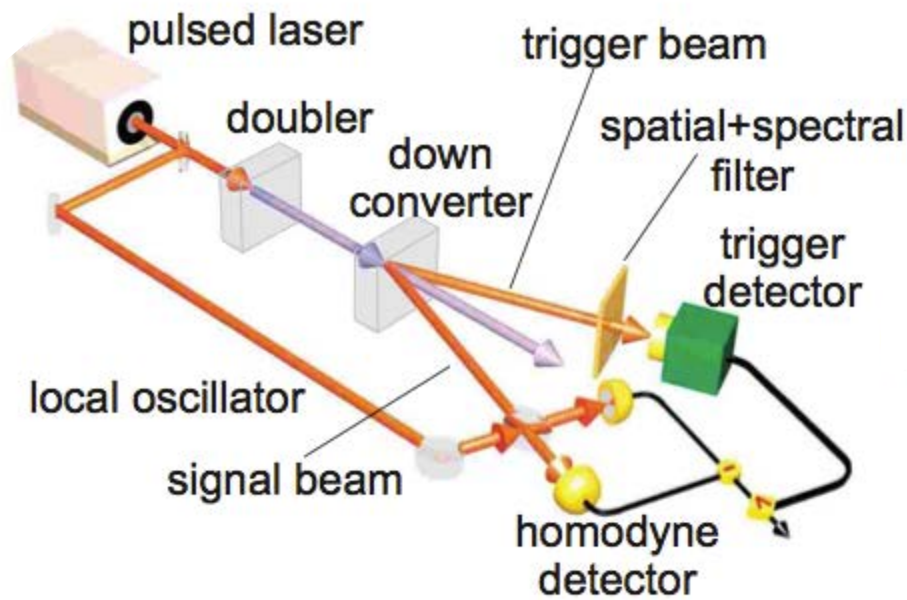


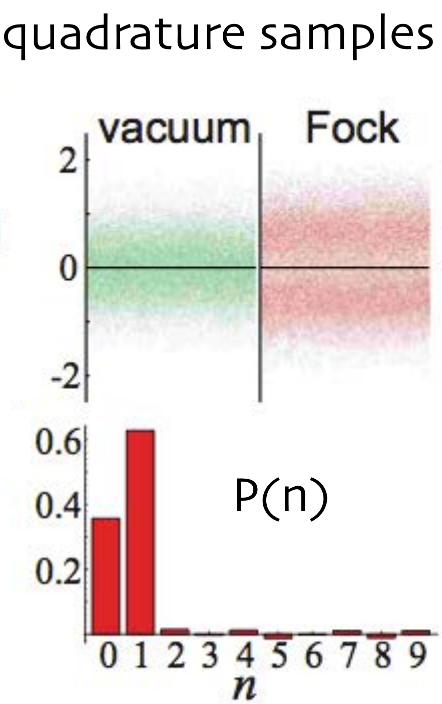




Single-Photon Fock State Tomography

Lvovsky, et al, Phys. Rev. Lett. 87, 050402 (2001)



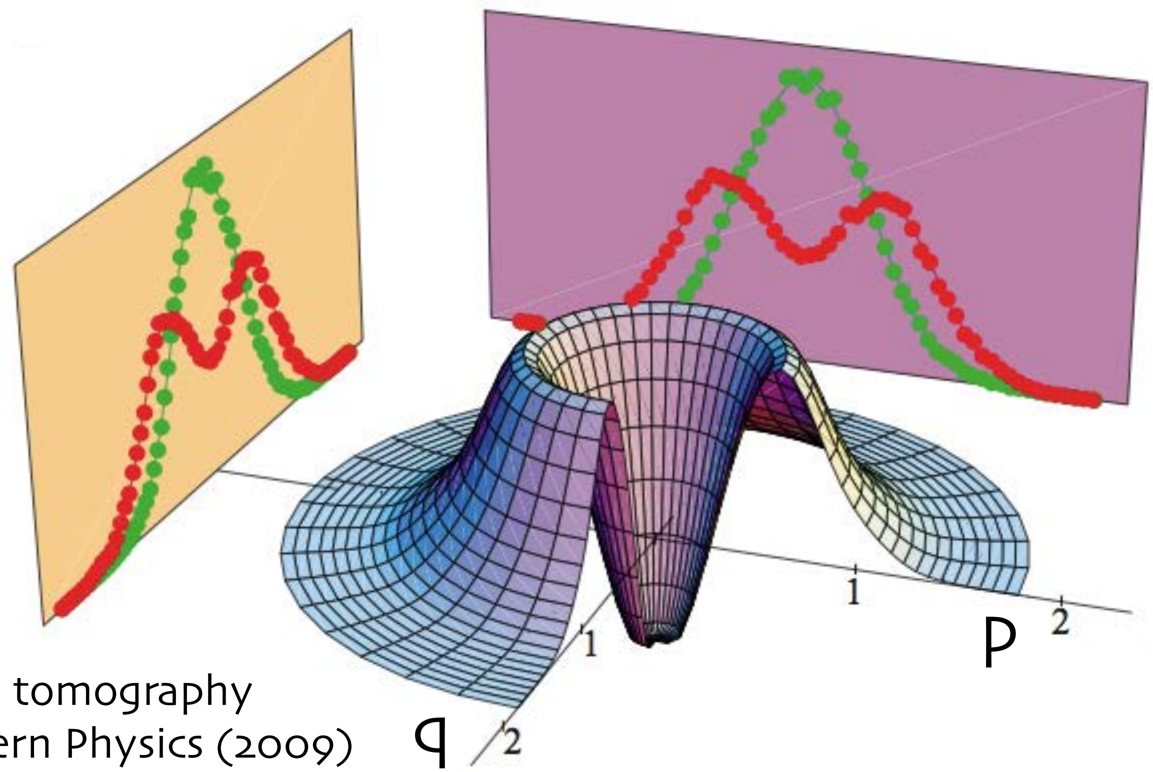


reconstructed Wigner function

projected Pr(q):

vacuum

Fock state

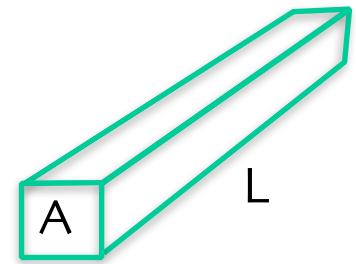


Continuous-variable optical quantum-state tomography
AI Lvovsky, MG Raymer - Reviews of Modern Physics (2009)

4. FREQUENCY CONTINUUM MODES

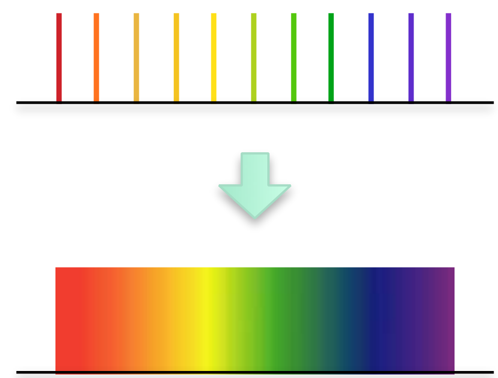
In free space, frequency is continuous

- Replace discrete mode sum by frequency continuum integral.
- Consider 1D propagation, as in a waveguide with area A.



$$\underline{\hat{E}}^{(+)}(\underline{r},t) = i\sum_j \sqrt{\frac{\hbar\omega_j}{2\varepsilon_0}}\, \hat{a}_j\, \frac{\underline{\varepsilon}_j \exp(i\underline{k}_j \cdot \underline{r})}{\sqrt{V}} \exp(-i\omega_j t)$$

$$\to i\sum_j \sqrt{\frac{\hbar\omega_j}{2\varepsilon_0}}\, \hat{a}_j\, \frac{\underline{\varepsilon}_j \exp(i k_j z)}{\sqrt{AL}} \exp(-i\omega_j t)$$



- Use $k = \omega / c$ *and* $L \to \infty$

$$\underline{\hat{E}}^{(+)}(z,t) = \frac{i}{2\pi}\int_0^\infty d\omega \sqrt{\frac{\hbar\omega}{2\varepsilon_0 A c}}\, \hat{a}(\omega)\, \underline{\varepsilon}(\omega) \exp[-i\omega(t - z/c)]$$

where $[\hat{a}(\omega), \hat{a}^\dagger(\omega')] = 2\pi\delta(\omega - \omega')$

5. QUANTIZATION OF EM FIELD IN TERMS OF TEMPORAL MODES

MOTIVATIONS:

1. quantum mechanics deals with discrete degrees of freedom.
2. how do I define a single mode from within a continuum?
3. a homodyne detector measures a ‘temporal slice.’
4. a good pulsed laser creates an isolated, transform-limited (coherent) pulse.
5. we know from Fourier Analysis that wave packets are made by adding monochromatic waves

WHAT ARE TEMPORAL MODES (TMs)?

TMs are <u>Non-Monochromatic</u> optical wave packets

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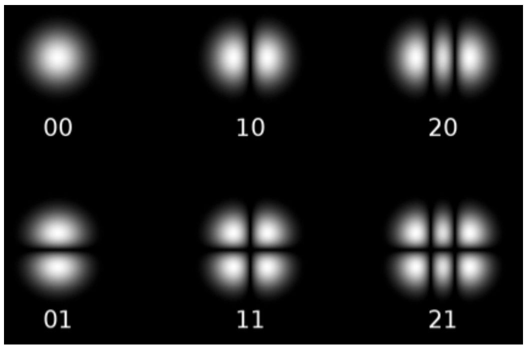
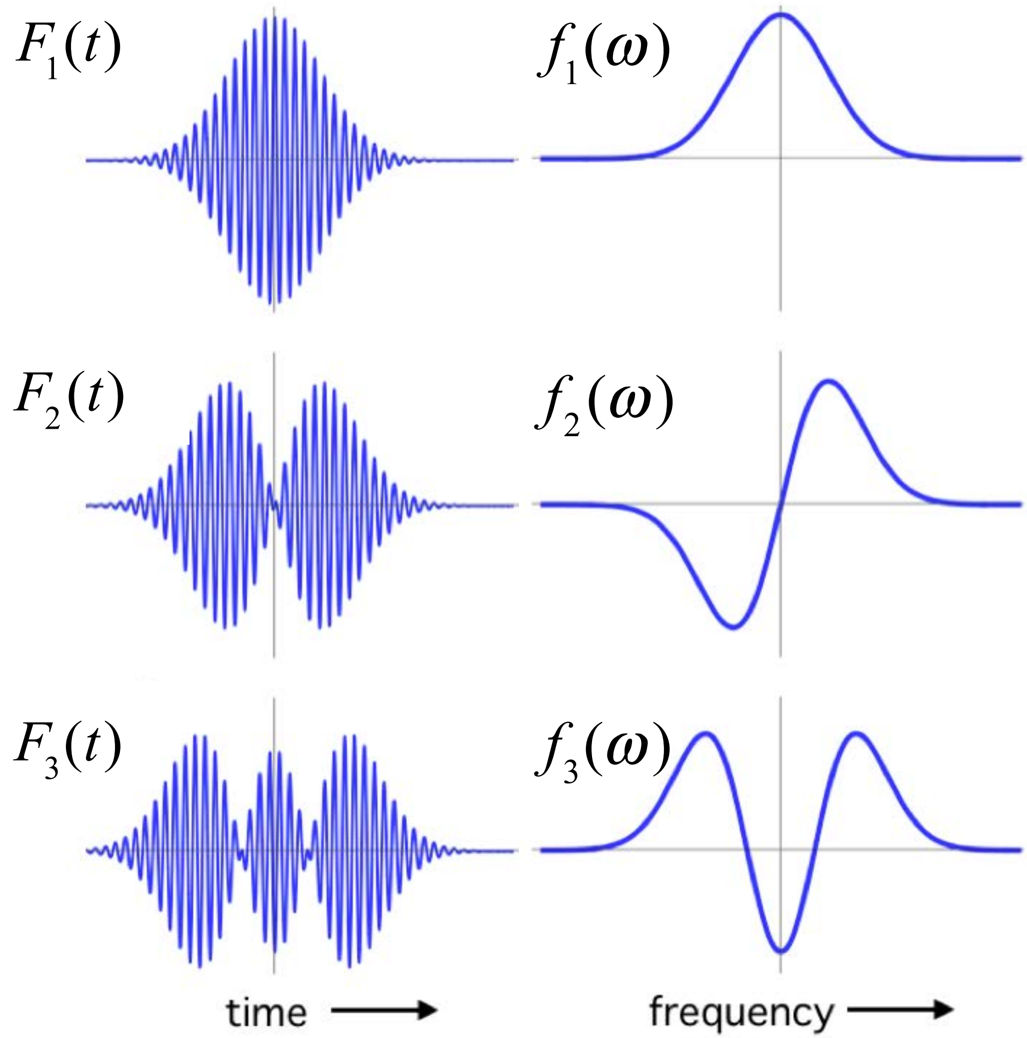
By analogy with transverse 'spatial mode' $u_j(x,y)$



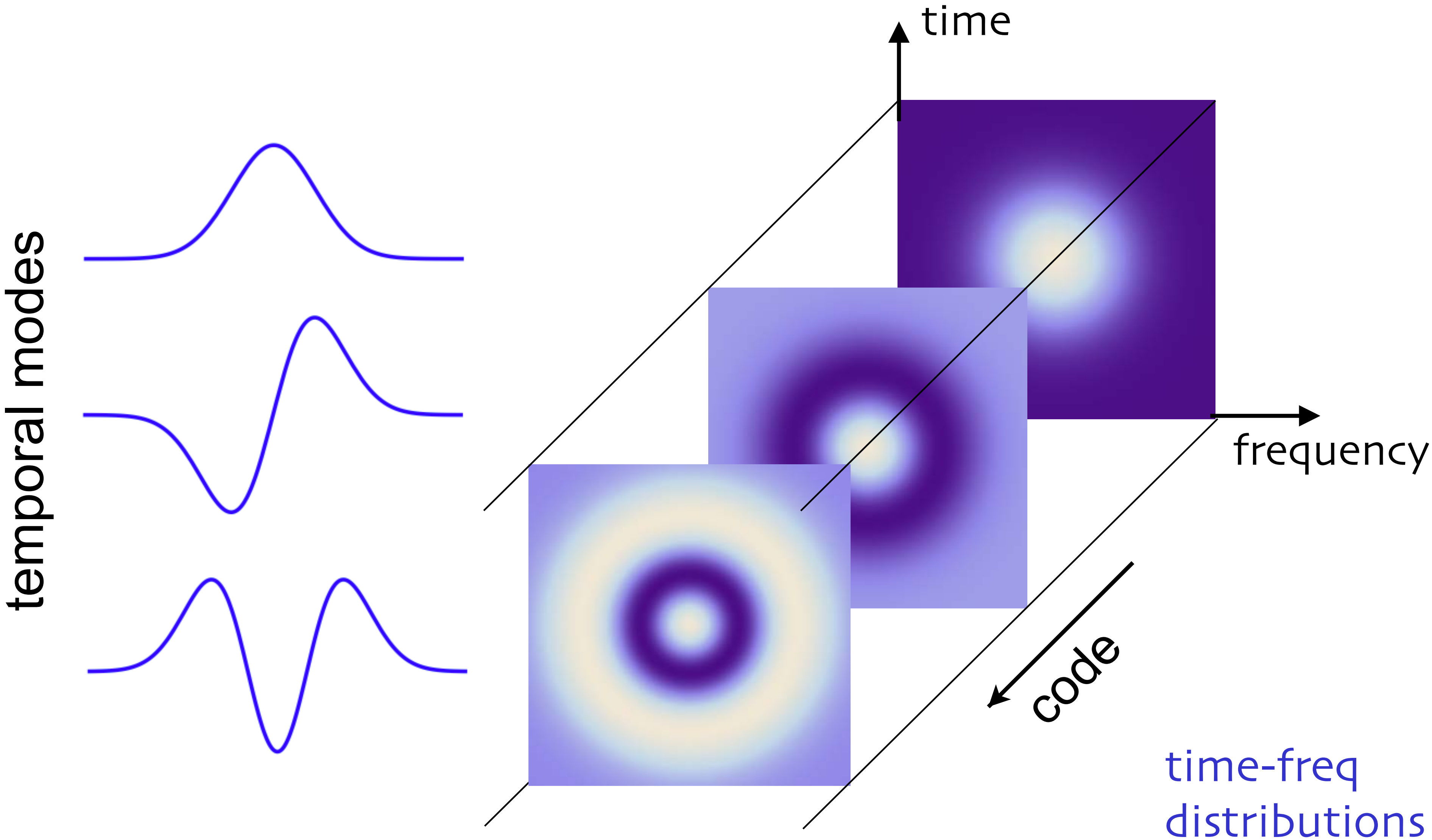
A temporal mode (TM) is one of a discrete set of orthogonal functions $F_j(t)$.



TMs overlap in time and frequency

But are still orthogonal!

$$\int F_n^*(t) F_m(t) dt = \delta_{nm}$$



EXPRESSING E FIELD IN TERMS OF TMs

narrow band scalar field:

$$\underline{\hat{E}}^{(+)}(z,t) = \frac{i}{2\pi}\sqrt{\frac{\hbar\bar{\omega}}{2\varepsilon_0 A c}}\int_0^\infty d\omega\,\hat{a}(\omega)\exp[-i\omega(t-z/c)]$$

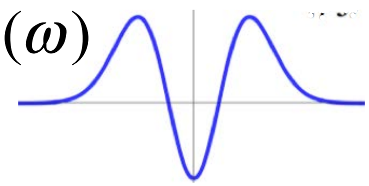
$\underbrace{\frac{i}{2\pi}\sqrt{\frac{\hbar\bar{\omega}}{2\varepsilon_0 A c}}}_{E_0}$

$\underbrace{\exp[-i\omega(t-z/c)]}$ $u_\omega(t-z/c) = \textit{monochromatic mode}$ $= u_\omega(\tau)$ where: $\tau = t - z/c$

$$\hat{E}^{(+)}(z,t) = E_0\int_0^\infty d\omega\,\hat{a}(\omega)\exp[-i\omega\tau]$$

introduce $f_j(\omega)$, *which form a complete, orthonormal set*

$f_3(\omega)$



$F_3(\tau)$

$$\hat{E}^{(+)}(z,t) = \tilde{E}_0\sum_j \hat{A}_j F_j(\tau)$$

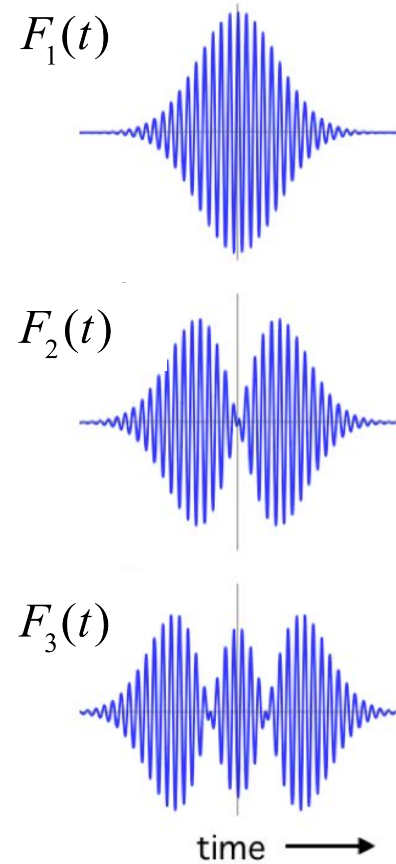
where $F_j(\tau) = FT\{f_j(\omega)\}$

and $\hat{A}_j = \frac{1}{2\pi}\int_0^\infty d\omega\, f_j^*(\omega)\hat{a}(\omega)$ = TM annihilation operators

now we have found a discrete basis in the continuum!

PROPERTIES OF TM operators

annihilation	creation
$\hat{A}_j = \frac{1}{2\pi}\int_0^\infty d\omega\, f_j^*(\omega)\hat{a}(\omega)$	$\hat{A}_j^\dagger = \frac{1}{2\pi}\int_0^\infty d\omega\, f_j(\omega)\hat{a}^\dagger(\omega)$



$$[\hat{a}(\omega),\hat{a}^\dagger(\omega')] = 2\pi\delta(\omega-\omega')$$

TM operators are bosonic:

$$[\hat{A}_j,\, \hat{A}_k^\dagger] = \delta_{jk}$$

recap:

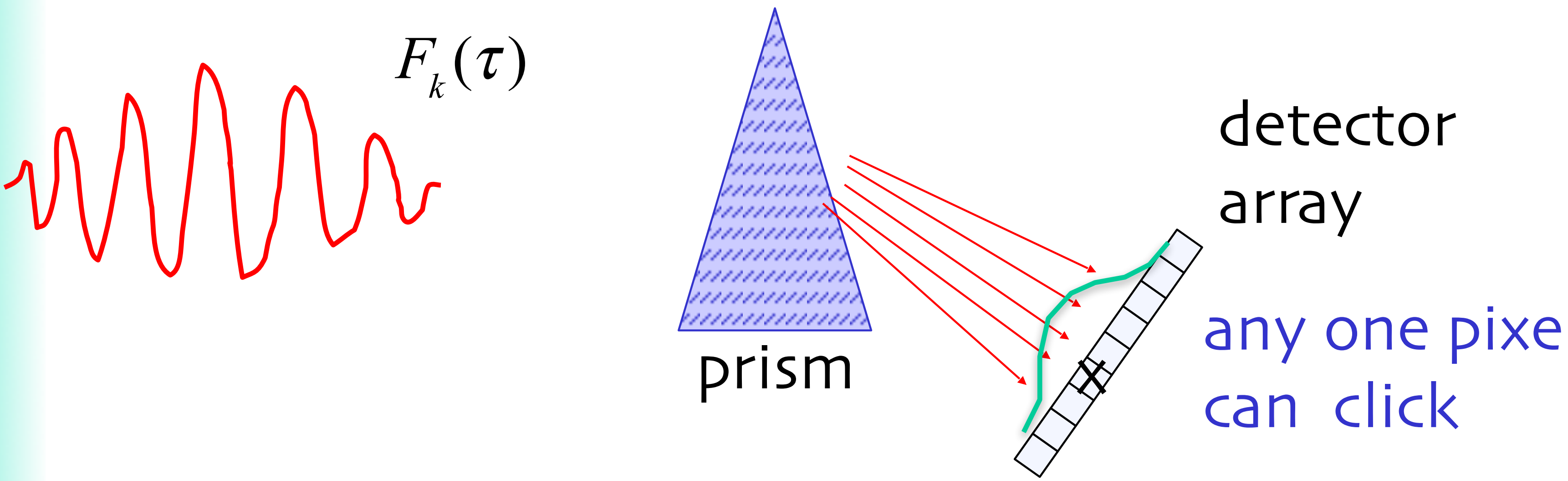
$$\hat{E}^{(+)}(z,t) = \tilde{E}_0 \sum_j \hat{A}_j F_j(\tau)$$

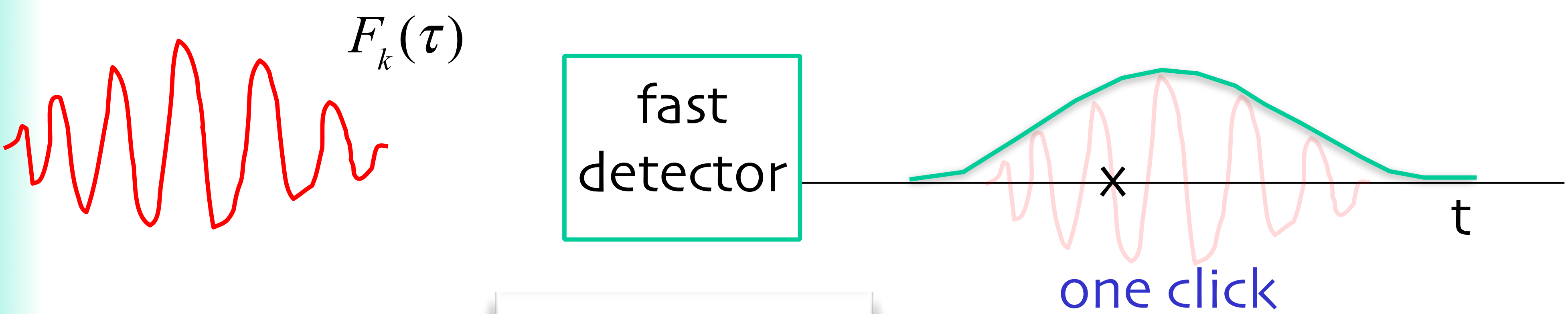
$\hat{A}_k^\dagger$ creates a non-monochromatic single-photon state in TM k

$$\hat{A}_k^\dagger|vac\rangle = \frac{1}{2\pi}\int d\omega\, f_k(\omega)\hat{a}^\dagger(\omega)|vac\rangle = \frac{1}{2\pi}\int d\omega\, f_k(\omega)|0, 0,\ldots 1_\omega, 0, 0\ldots\rangle$$

$\hat{A}_k^{\dagger}$ operator creates one photon in TM $\quad F_k(\tau) = FT\{f_k(\omega)\}$

$$\hat{A}_k^{\dagger}\left|vac\right\rangle = \frac{1}{2\pi}\int d\omega\, f_k(\omega)\,\hat{a}^{\dagger}(\omega)\left|vac\right\rangle$$

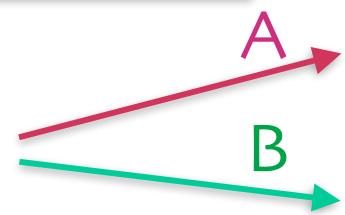




$$\Delta\omega\,\Delta t \geq 1/2$$

Joint two-photon states of spatially separated beams

one photon packet in each beam



1. Separable: $\Psi(\omega,\omega') = f_j(\omega)\cdot f_k(\omega')$

$$\left|\Psi^{(2)}\right\rangle = \hat{A}_j^{\dagger}\,\hat{B}_k^{\dagger}\left|vac\right\rangle_A \otimes \left|vac\right\rangle_B$$
$$= \int d\omega\, f_j(\omega)\,\underline{\hat{a}^{\dagger}(\omega)}\left|vac\right\rangle_A \otimes \int d\omega'\, f_k(\omega')\,\underline{\hat{b}^{\dagger}(\omega')}\left|vac\right\rangle_B$$

creation operators on distinct mode subgroups

2. Entangled: spatially separated and non-separable:

$$\left|\Psi^{(2)}\right\rangle = \int d\omega \int d\omega'\, \Psi(\omega,\omega')\,\hat{a}^{\dagger}(\omega)\left|vac\right\rangle_A \otimes \hat{b}^{\dagger}(\omega')\left|vac\right\rangle_B$$

$$\Psi(\omega,\omega') \neq f_j(\omega)\cdot f_k(\omega')$$

SCHMIDT DECOMPOSITION OF ENTANGLED STATE

Theorem: Any 2D object (function or matrix) admits a Singular-Value Decomposition (SVD):

$$M_{jk} = \sum_{n,m} U_{jn} \Lambda_{nm} V^{\dagger}_{mk} \xrightarrow{\Lambda\ diagonal} \sum_{n} U_{jn} \lambda_n V^{\dagger}_{nk}$$

U and V are unitary matrices:

$$\sum_n U_{jn} U^{\dagger}_{nk} = \delta_{jk} \qquad \sum_n V_{jn} V^{\dagger}_{nk} = \delta_{jk}$$

if M is Hermitian this reduces to an e-val decomp. with V=U

2. Function: $M(x,y) = \sum_n U_n(x)\, \lambda_n\, V^{*}_{n}(x)$

U and V are separate orthonormal function sets:

$$\int U_j^*(x)\ U_k(x)\, dx = \delta_{jk} \qquad \int V_j^*(x)\ V_k(x)\, dx = \delta_{jk}$$

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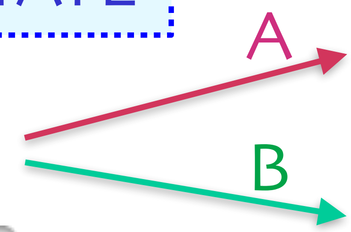
SCHMIDT DECOMPOSITION OF ENTANGLED STATE

Entangled: spatially separated and non-separable:



$$\left|\Psi^{(2)}\right\rangle = \int d\omega \int d\omega' \Psi(\omega,\omega')\, \hat{a}(\omega)^{\dagger} \left|vac\right\rangle_A \otimes \hat{b}(\omega')^{\dagger} \left|vac\right\rangle_B$$

SVD: $\Psi(\omega,\omega') = \sum_n U_n(\omega)\, \lambda_n\, V_n^{*}(\omega')$

$$\left|\Psi^{(2)}\right\rangle = \sum_n \lambda_n\, \hat{A}_n^{\dagger} \left|vac\right\rangle_A \otimes \hat{B}_n^{\dagger} \left|vac\right\rangle_B$$

$$\text{where}\quad \hat{A}_n^{\dagger} = \int d\omega\, U_n(\omega)\, \hat{a}^{\dagger}(\omega) \qquad \hat{B}_n^{\dagger} = \int d\omega'\, V_n^{*}(\omega')\, \hat{b}^{\dagger}(\omega')$$

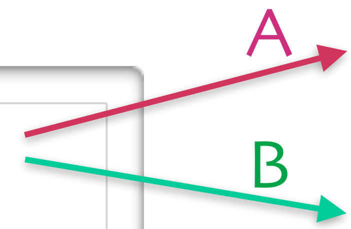
The photon states are seen to be perfectly correlated pairs of TMs $\{U_n(\omega), V_n^{*}(\omega')\}$

The double continuum integral has been replaced by a single discrete sum.

Law, Walmsley, Eberly - Phys Rev Lett, 84, 5304, 2000

OREGON

SCHMIDT DECOMPOSITION OF ENTANGLED STATE

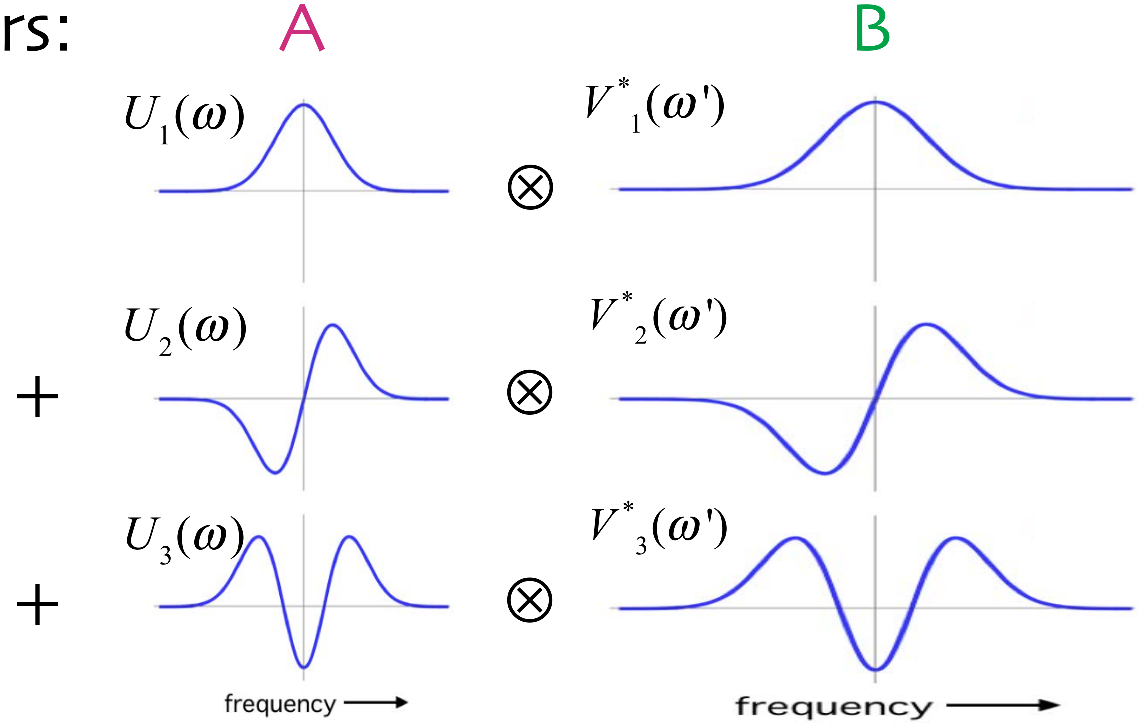


SVD:

$$\left|\Psi^{(2)}\right\rangle = \sum_n \lambda_n \hat{A}_n^\dagger \left|vac\right\rangle_A \otimes \hat{B}_n^\dagger \left|vac\right\rangle_B$$

$$where \quad \hat{A}_n^\dagger = \int d\omega U_n(\omega)\,\hat{a}^\dagger(\omega) \qquad \hat{B}_n^\dagger = \int d\omega' V_n^*(\omega')\hat{b}^\dagger(\omega')$$

Temporal Mode pairs:



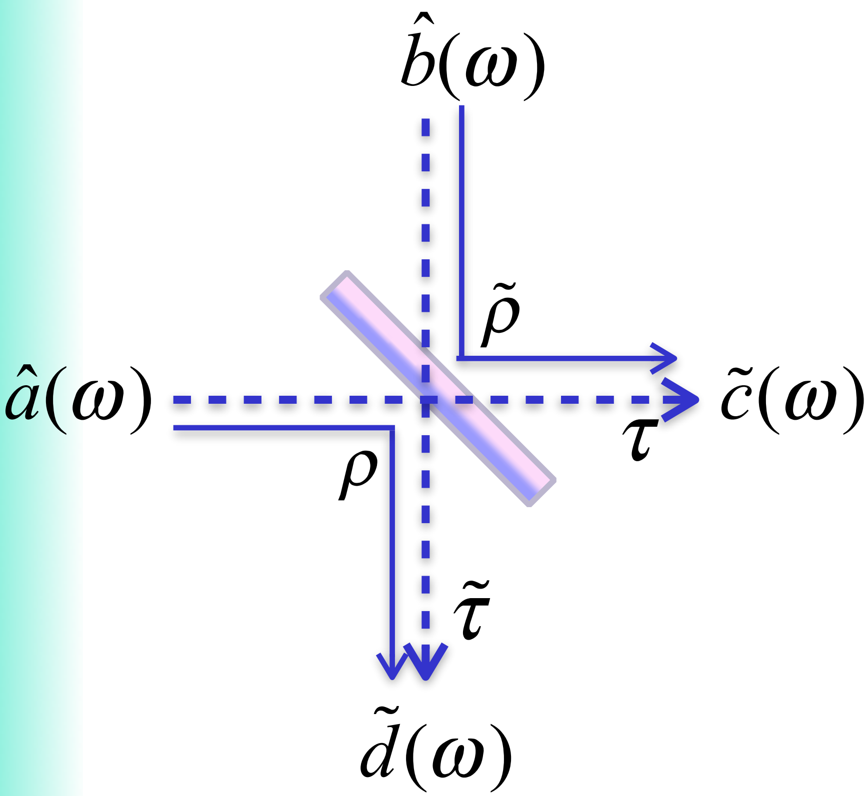
Law, Walmsley, Eberly - Phys Rev Lett, 84, 5304, 2000

6. OPTICAL BEAM SPLITTER

What happens if a quantum field hits a partially reflecting surface?

Define the a, b, c, d beams:

$$\underline{\hat{E}}_a^{(+)}(z,t) = i\int_0^\infty d\omega \sqrt{\frac{\hbar\omega}{2\varepsilon_0 AL}}\,\hat{a}(\omega)\underline{\varepsilon}(\omega)\exp[-i\omega(t-z/c)] \qquad \text{etc.}$$



Unitarity requires for each frequency:

$$\begin{pmatrix}\tilde{c}(\omega)\\ \tilde{d}(\omega)\end{pmatrix} = \begin{pmatrix}\tau(\omega) & \tilde{\rho}(\omega)\\ \rho(\omega) & \tilde{\tau}(\omega)\end{pmatrix}\begin{pmatrix}\hat{a}(\omega)\\ \hat{b}(\omega)\end{pmatrix}$$

$$= \mathbf{U}(\omega)\begin{pmatrix}\hat{a}(\omega)\\ \hat{b}(\omega)\end{pmatrix}$$

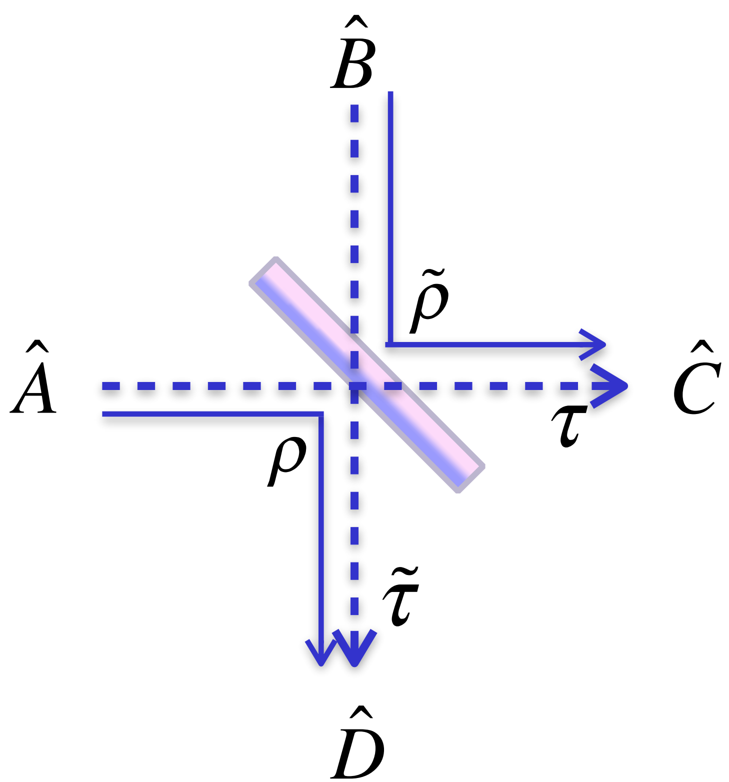
$$\mathbf{U}(\omega)^\dagger \mathbf{U}(\omega) = 1 \Rightarrow \tau^*\tilde{\rho} + \rho^*\tilde{\tau} = 0$$

$$\& \ |\tau|^2 + |\rho|^2 = |\tilde{\tau}|^2 + |\tilde{\rho}|^2 = 1$$

If a single TM hits a beam splitter for which the reflectivity is frequency-independent, the TM shape will be preserved.

$$\hat{E}_A^{(+)}(z,t) = E_0 \hat{A} F(t - z/c) \xrightarrow{\text{represent by}} \hat{A}$$

if all four TMs are identical then:



$$\begin{pmatrix} \hat{C} \\ \hat{D} \end{pmatrix} = \begin{pmatrix} \tau & \tilde{\rho} \\ \rho & \tilde{\tau} \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = \mathbf{U} \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix}$$

$$inverse: \ \mathbf{U}^{-1} = \mathbf{U}^{\dagger}$$

$$\begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = \begin{pmatrix} \tau^* & \rho^* \\ \tilde{\rho}^* & \tilde{\tau}^* \end{pmatrix} \begin{pmatrix} \hat{C} \\ \hat{D} \end{pmatrix}$$

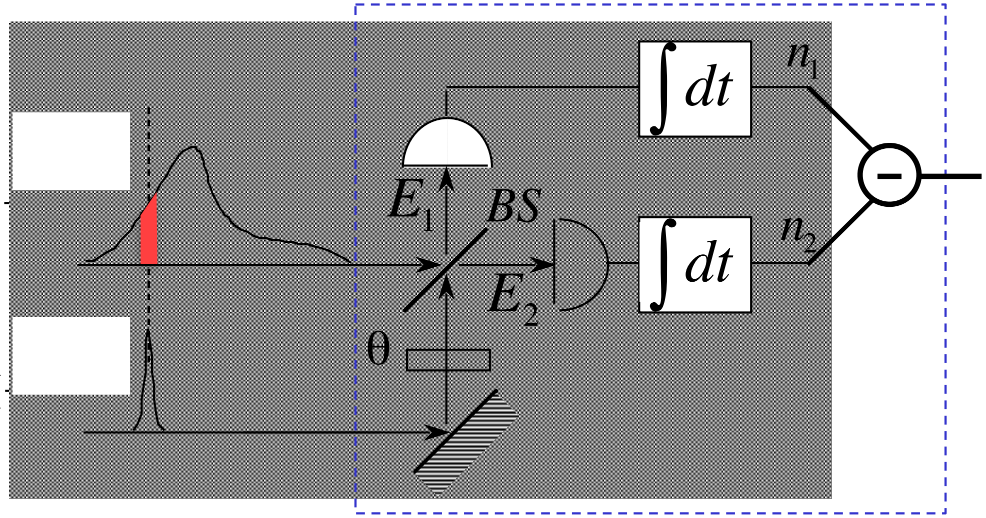
(c.c. implies time reversal)

7. BHD REVISITED

putting together beam splitter and TM concepts

signal $E_S^{(+)}$

local oscillator $E_L^{(+)}$



$$\Delta N = n_1 - n_2$$

$$\propto q_{rot,\theta}$$

quadrature operator of TM defined by the LO

$$\begin{pmatrix} E_1^{(+)} \\ E_2^{(+)} \end{pmatrix} = \begin{pmatrix} \tau E_L^{(+)} + \tilde{\rho} E_S^{(+)} \\ \rho E_L^{(+)} + \tilde{\tau} E_S^{(+)} \end{pmatrix}, \quad for\, |\rho|^2 = |\tau|^2 = 1/2:$$

$$\Delta N = \eta \int dt \left\{ \left| E_L^{(+)} e^{i\theta} + E_S^{(+)} \right|^2 - \left| E_L^{(+)} e^{i\theta} - E_S^{(+)} \right|^2 \right\}$$

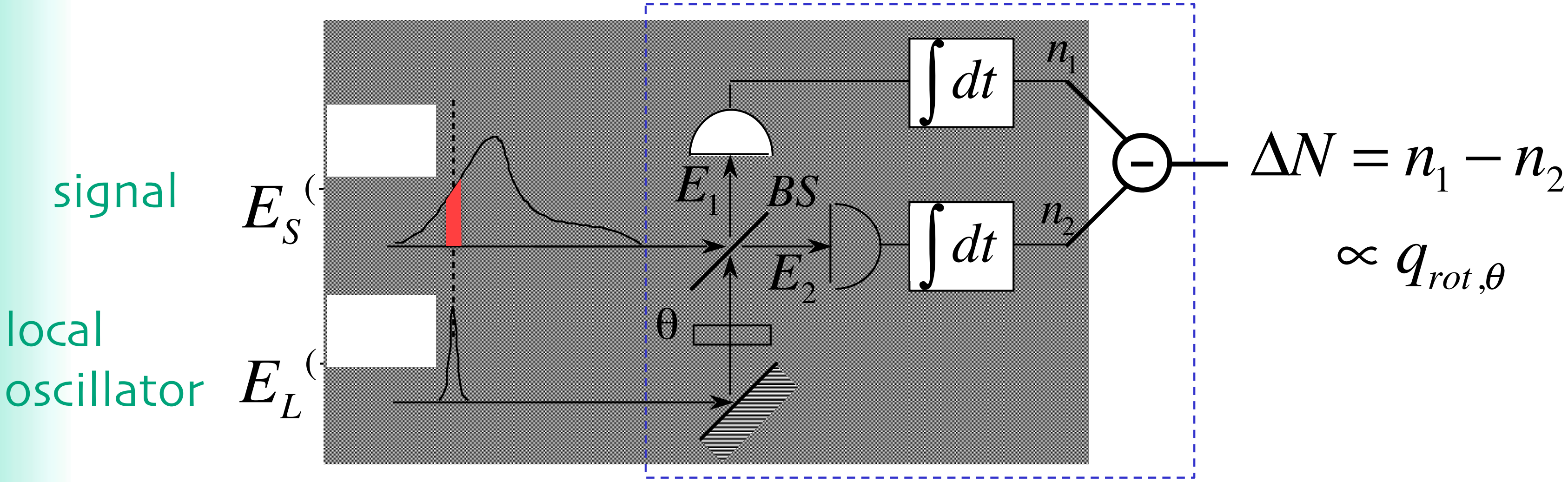
$$= \eta \int dt \left\{ e^{i\theta} E_L^{(+)}(t) E_S^{(-)}(t) + e^{-i\theta} E_L^{(-)}(t) E_S^{(+)}(t) \right\}$$

$$if\ E_L^{(+)}(t) = E_L F_{L0}(t)\ \ for\ some\ LO\ mode\ F_{L0}(t)\ then$$

$$\Delta N = \eta E_L \int dt \left\{ e^{i\theta} F_{L0}(t) E_S^{(-)}(t) + e^{-i\theta} F_{L0}^{*}(t) E_S^{(+)}(t) \right\}$$

shows the time slicing or windowing by BHD

putting together beam splitter and BHD concepts



$$\Delta N = \eta E_L \int dt \left\{ e^{i\theta} F_{L0}(t) E_S^{(-)}(t) + e^{-i\theta} F_{L0}^{\;*}(t) E_S^{(+)}(t) \right\}$$

interference with the LO projects out a single temporal mode (TM) from the multimode input signal

8. BASICS OF NONLINEAR OPTICS

$$\underline{E}(\underline{r},t) = \underline{\varepsilon}\,\mathcal{E}(\underline{r},t)\exp[ik_0 z - i\omega_0 t] + c.c.$$

electrons respond in a nonlinear manner to driving field:

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)E = \frac{1}{\varepsilon_0 c^2}\partial_t^2 P \quad (scalar)$$

P=nonlinear electronic polarization

$$P \approx \varepsilon_0 \breve{\chi}^{(1)}\left\{E\right\} + \varepsilon_0 \breve{\chi}^{(2)}\left\{EE\right\} + \varepsilon_0 \breve{\chi}^{(3)}\left\{EEE\right\} + \ldots$$

$$\breve{\chi}^{(n)} = nonlinear\ \ polarizability\ integral\ operator\ of\ \ order\ n$$

$$\breve{\chi}^{(1)}\left\{E\right\} = \int_{-\infty}^{t} dt'\,\chi^{(1)}(t-t')E(t')$$

linear response function

$$\breve{\chi}^{(2)}\left\{E\right\} = \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt''\,\chi^{(2)}(t,t',t'')E(t')E(t''),\ \ etc.$$

nonlinear response

incorporating the linear dispersion explicitly, and the nonlinear response as instantaneous:

$$E(z,t) = E^{(+)}(z,t) + E^{(-)}(z,t) \quad where\ E^{(+)}(z,t) = \frac{1}{2\pi}\int_0^\infty d\omega\, \tilde{E}(z,\omega)\exp[-i\omega t]$$

$$E^{(+)}(z,t) \doteq \mathcal{E}(z,t)\exp[ik_0 z - i\omega_0 t]$$

keep only terms like $\exp[-i\omega_0 t]$

$$\left(\frac{\partial}{\partial z}\mathcal{E} + k_0'\frac{\partial}{\partial t}\mathcal{E} + \sum_{n=2}^{\infty} i^{n+1}\frac{k_0^{(n)}}{n!}\left(\frac{\partial}{\partial t}\right)^n \mathcal{E}\right)\exp[ik_0 z - i\omega_0 t] \approx \frac{1}{2ik_0}\frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}P_{NL}\Big|_{\omega_0}$$

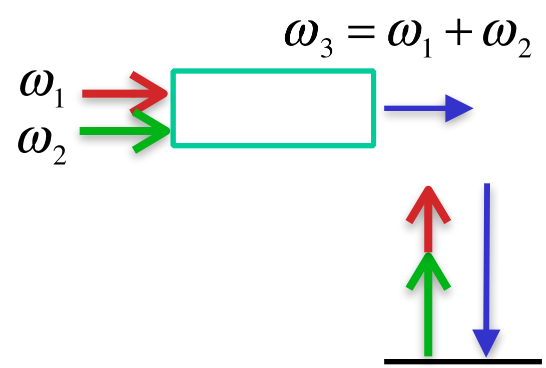
$$where\ \ k_0^{(n)} \doteq \frac{d^n k}{d\omega^n}\Big|_{\omega_0}$$

$$and\ \ P_{NL} \approx \varepsilon_0\chi^{(2)}EE + \varepsilon_0\chi^{(3)}EEE + \ldots$$

9. SECOND-ORDER NONLINEARITY

$$E^{(+)}(z,t) = \mathcal{E}_1^{(+)} \exp[ik_1 z - i\omega_1 t] + \mathcal{E}_2^{(+)} \exp[ik_2 z - i\omega_2 t] + \mathcal{E}_3^{(+)} \exp[ik_3 z - i\omega_3 t]$$

where $\omega_1 + \omega_2 = \omega_3$ *and* $k_1 \doteq k(\omega_1) = n_1\omega_1/c$ *etc.*



$$P_{NL} \approx \varepsilon_0 \chi^{(2)} E\,E = \varepsilon_0 \chi^{(2)} \left(E^{(+)} + E^{(-)}\right)\left(E^{(+)} + E^{(-)}\right)$$

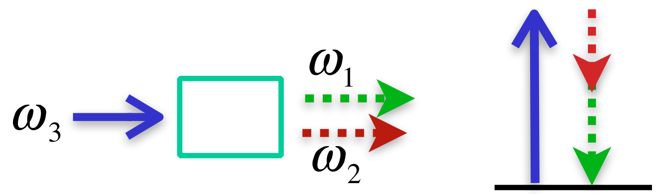
consider the field at ω_3

$$\left(\frac{\partial}{\partial z}\mathcal{E}_3^{(+)} + k_3' \frac{\partial}{\partial t}\mathcal{E}_3^{(+)} + \sum_{n=2} i^{n+1} \frac{k_3^{(n)}}{n!}\left(\frac{\partial}{\partial t}\right)^n \mathcal{E}_3^{(+)}\right) = \frac{i\omega_3}{2n_3 c}\chi^{(2)} \mathcal{E}_1^{(+)} \cdot \mathcal{E}_2^{(+)} \exp[-i\Delta k z]$$

where phase mismatch $\Delta k = k_3 - (k_1 + k_2)$

SECOND-ORDER NONLINEARITY

consider waves 1 and 2



$$\left(\frac{\partial}{\partial z}\mathcal{E}_2^{(+)} + k_2'\frac{\partial}{\partial t}\mathcal{E}_2^{(+)} + \sum_{n=2} i^{n+1}\frac{k_2^{(n)}}{n!}\left(\frac{\partial}{\partial t}\right)^n \mathcal{E}_2^{(+)}\right) = \frac{i\omega_2}{2n_2 c}\chi^{(2)}\mathcal{E}_3^{(+)}\cdot\mathcal{E}_1^{(-)}\exp[i\Delta kz]$$

$$\left(\frac{\partial}{\partial z}\mathcal{E}_1^{(+)} + k_1'\frac{\partial}{\partial t}\mathcal{E}_1^{(+)} + \sum_{n=2} i^{n+1}\frac{k_1^{(n)}}{n!}\left(\frac{\partial}{\partial t}\right)^n \mathcal{E}_1^{(+)}\right) = \frac{i\omega_1}{2n_1 c}\chi^{(2)}\mathcal{E}_3^{(+)}\cdot\mathcal{E}_2^{(-)}\exp[i\Delta kz]$$

These are the fundamental starting equations.
Let's simplify them to make solving easier.

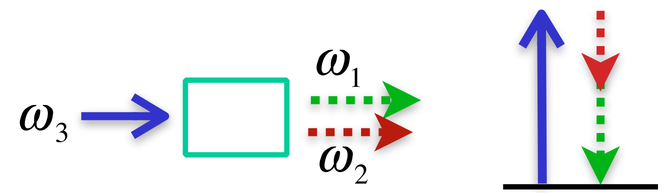
SECOND-ORDER NONLINEARITY

consider group velocities to be equal;
go into moving frame, with: $\tau = t - k_0^{'} z$

$$\frac{\partial}{\partial z}\mathcal{E}_3^{(+)}(z,\tau) = i\gamma_3 \mathcal{E}_1^{(+)}\mathcal{E}_2^{(+)} \exp[-i\Delta k z]$$

$$\frac{\partial}{\partial z}\mathcal{E}_2^{(+)}(z,\tau) = i\gamma_2 \mathcal{E}_3^{(+)}\mathcal{E}_1^{(-)} \exp[i\Delta k z]$$

$$\frac{\partial}{\partial z}\mathcal{E}_1^{(+)}(z,\tau) = i\gamma_1 \mathcal{E}_3^{(+)}\mathcal{E}_2^{(-)} \exp[i\Delta k z]$$

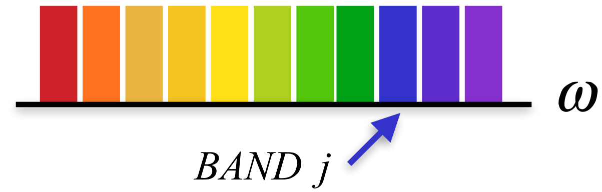


$$\Delta k = k_3 - (k_1 + k_2)$$

where: $\gamma_3 \doteq \frac{\omega_3}{2 n_3 c}\chi^{(2)}, \; etc.$

QUANTIZING with DISPERSION

divide spectrum into bands



field operator for BAND j

$$\underline{\hat{E}}_j^{(+)}(z,t) = \frac{i}{2\pi}\int_{BAND\ j} d\omega \sqrt{\frac{\hbar\omega k_j'}{2\varepsilon(\omega)A}}\,\hat{a}_j(\omega)\exp[-i\omega t + ik(\omega)z] \qquad (j = 1,2,3...)$$

$$\text{where} \quad \varepsilon(\omega) = \varepsilon_0\sqrt{1+\tilde{\chi}^{(1)}(\omega)} \quad , \quad k_j' = \left.\frac{dk}{d\omega}\right|_{\omega_j} = \frac{1}{\mathrm{v}_{gj}}$$

$$and \quad [\hat{a}_j(\omega), \hat{a}_k^{\dagger}(\omega')] = 2\pi\delta_{jk}\delta(\omega-\omega')$$

$$\mathcal{E}_j^{(+)}(z,t) = E_j^{(+)}(z,t)\exp[-ik_j z + i\omega_j t]$$ = slowly varying field

$$\mathcal{E}_j^{(+)}(z,t) \approx i\sqrt{\frac{\hbar\omega_j k_j'}{2\varepsilon(\omega_j)A}}\int_{BAND\ j}\frac{d\omega}{2\pi}\hat{a}_j(\omega)\exp[i(k(\omega)-k_j)z - i(\omega-\omega_j)t]$$

$= \hat{e}_j(z,t)$ = annihilation operator

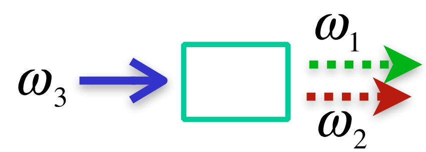
QUANTIZING SECOND-ORDER NONLINEARITY

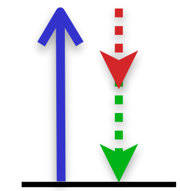
creation, annihilation operators obey:

$$\frac{\partial}{\partial z}\hat{e}_3(z,\tau) = -(\kappa/2)\,\hat{e}_1\,\hat{e}_2\exp[-i\Delta kz]$$

$$\frac{\partial}{\partial z}\hat{e}_2(z,\tau) = (\kappa/2)\,\hat{e}_3\,\hat{e}_1^{\dagger}\exp[i\Delta kz]$$

$$\frac{\partial}{\partial z}\hat{e}_1(z,\tau) = (\kappa/2)\,\hat{e}_3\,\hat{e}_2^{\dagger}\exp[i\Delta kz]$$





$$\frac{\kappa}{2} \doteq \chi^{(2)}\sqrt{\frac{\omega_1\omega_2\omega_3}{n_1 n_2 n_3}\frac{1}{\varepsilon_0{}^3 c^3 A}}$$

$$\Delta k = k_3 - (k_1 + k_2)$$

$$\tau = t - k_0' z$$

three conserved quantities:

$$energy: \quad \frac{\partial}{\partial z}\left(\hbar\omega_3\hat{e}_3^{\dagger}\hat{e}_3 + \hbar\omega_2\hat{e}_2^{\dagger}\hat{e}_2 + \hbar\omega_1\hat{e}_1^{\dagger}\hat{e}_1\right) = 0$$

$$difference\ number: \quad \frac{\partial}{\partial z}\left(\hat{e}_2^{\dagger}\hat{e}_2 - \hat{e}_1^{\dagger}\hat{e}_1\right) = 0$$

photon pair generation

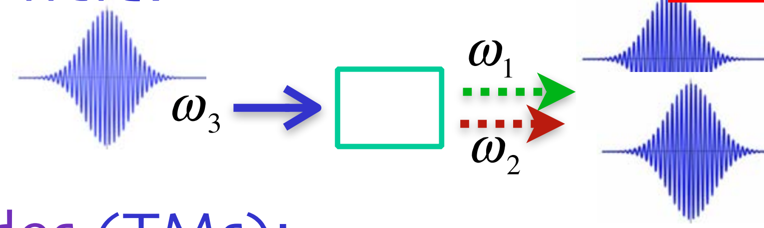
$$sum\ number: \frac{\partial}{\partial z}\left(2\hat{e}_3^{\dagger}\hat{e}_3 + [\hat{e}_2^{\dagger}\hat{e}_2 + \hat{e}_1^{\dagger}\hat{e}_1]\right) = 0$$

10. PARAMETRIC AMPLIFICATION by $\chi^{(2)}$

fix

1. treat pump as a classical, undepleted field:

$$\hat{e}_3(z,\tau) \rightarrow e_3(\tau) \; = \left|e_3(\tau)\right| \exp[-i\phi_3(\tau)]$$



2. idealize fields as single temporal modes (TMs):

$$\hat{e}_1(z,\tau) \rightarrow \hat{A}_1(z,\tau) \quad where \quad [\hat{A}_1(z,\tau), \hat{A}_1^{\dagger}(z,\tau)] = 1\,, \quad etc.$$

$\hat{A}_1^{\dagger}$ creates a photon in TM $F_1(t)$

3. consider perfect phase matching: $\Delta k = 0$

$$\frac{\partial}{\partial z}\hat{A}_2 = (1/2)\, g \, \exp[-i\phi_3]\, \hat{A}_1^{\dagger}$$

$$\frac{\partial}{\partial z}\hat{A}_1 = (1/2)\, g \, \exp[i\phi_3]\, \hat{A}_2^{\dagger}$$

the mixing of A with A
non-classical effects

where gain coefficient: $g(\tau) \; = \kappa \left|e_3(\tau)\right|$

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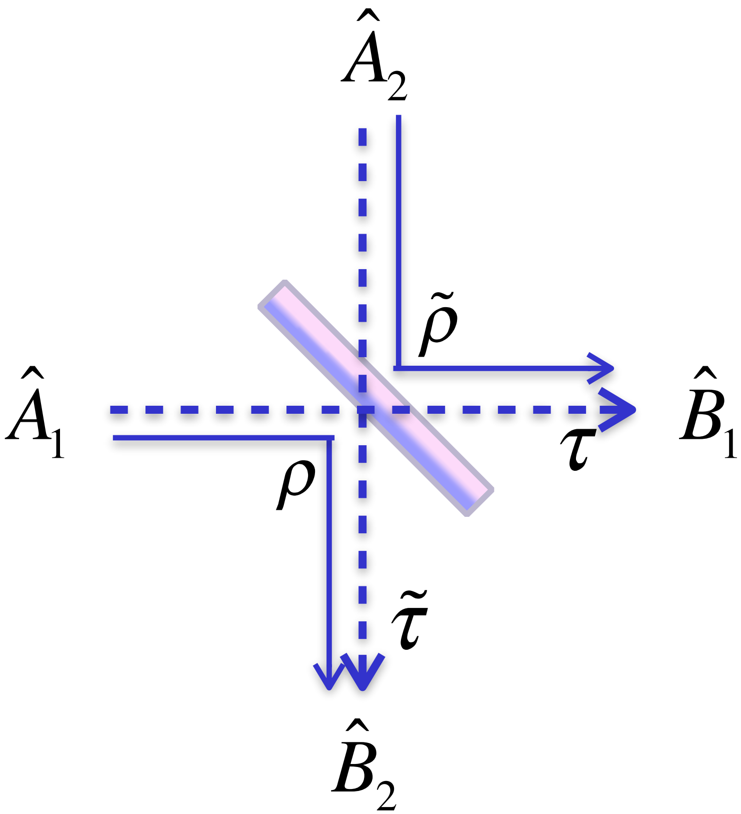
RECALL: BEAM SPLITTER

$$\hat{E}^{(+)}_{A_j}(z,t) = E_0 \hat{A}_j F(t - z/c) \xrightarrow{\text{represent by}} \hat{A}_j$$



$$\begin{pmatrix} \hat{B}_1 \\ \hat{B}_2 \end{pmatrix} = \begin{pmatrix} \tau & \tilde{\rho} \\ \rho & \tilde{\tau} \end{pmatrix} \begin{pmatrix} \hat{A}_1 \\ \hat{A}_2 \end{pmatrix}$$

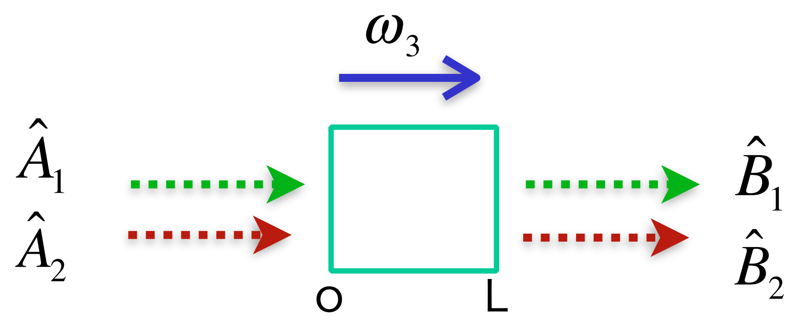
$$|\tau|^2 + |\rho|^2 = 1$$

No mixing of A with A

PARAMETRIC AMPLIFICATION

$$\frac{\partial}{\partial z}\hat{A}_2 = (1/2)\, g \exp[-i\phi_3]\, \hat{A}_1^{\dagger}$$

$$\frac{\partial}{\partial z}\hat{A}_1 = (1/2)\, g \exp[i\phi_3]\, \hat{A}_2^{\dagger}$$



solution:

$$\begin{pmatrix} \hat{B}_1 \\ \hat{B}_2^{\dagger} \end{pmatrix} = \begin{pmatrix} \mu & \nu \\ \nu^{*} & \mu \end{pmatrix} \begin{pmatrix} \hat{A}_1 \\ \hat{A}_2^{\dagger} \end{pmatrix}$$

the mixing of A with A
non-classical effects

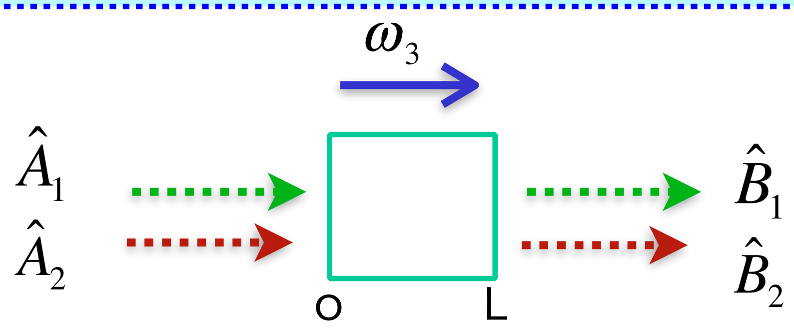
$$\mu = \cosh[gL/2] \;\; , \;\; \nu = -\exp[-i\phi_3]\sinh[gL/2]$$

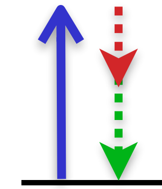
$$|\mu|^2 - |\nu|^2 = 1$$

note

PARAMETRIC AMPLIFICATION

$$\begin{pmatrix} \hat{B}_1 \\ \hat{B}_2^{\dagger} \end{pmatrix} = \begin{pmatrix} \mu & \nu \\ \nu^* & \mu \end{pmatrix} \begin{pmatrix} \hat{A}_1 \\ \hat{A}_2^{\dagger} \end{pmatrix}$$



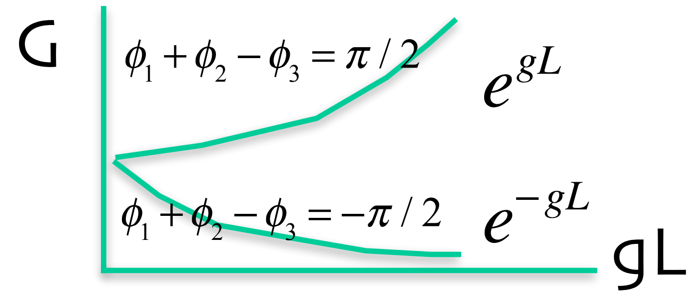


$$\mu = \cosh[gL/2] \ , \ \nu = -\exp[-i\phi_3]\sinh[gL/2]$$

phase-sensitive gain, if inputs are strong coherent states with set phases and amplitudes: $|\psi_{IN}\rangle = |\alpha \exp[i\phi_1]\rangle_1 \otimes |\alpha \exp[i\phi_2]\rangle_2$

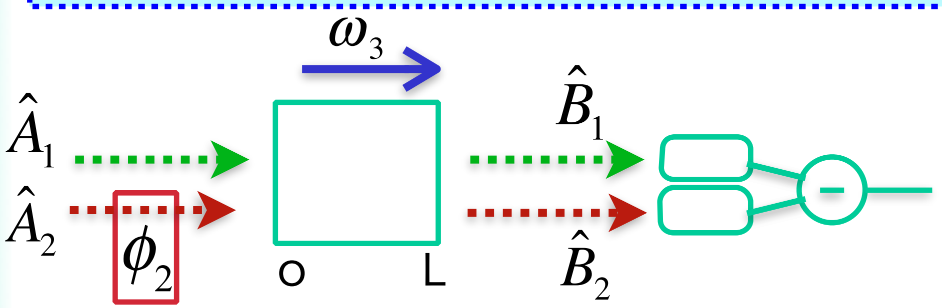
$$N_1 = \langle \hat{B}_1^{\dagger} \hat{B}_1 \rangle = G|\alpha|^2 + |\nu|^2$$

$$where\ gain = G = \mu^2 + |\nu|^2 + 2\mu|\nu| \sin(\phi_1 + \phi_2 - \phi_3)$$

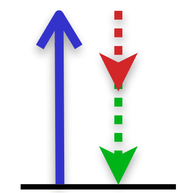


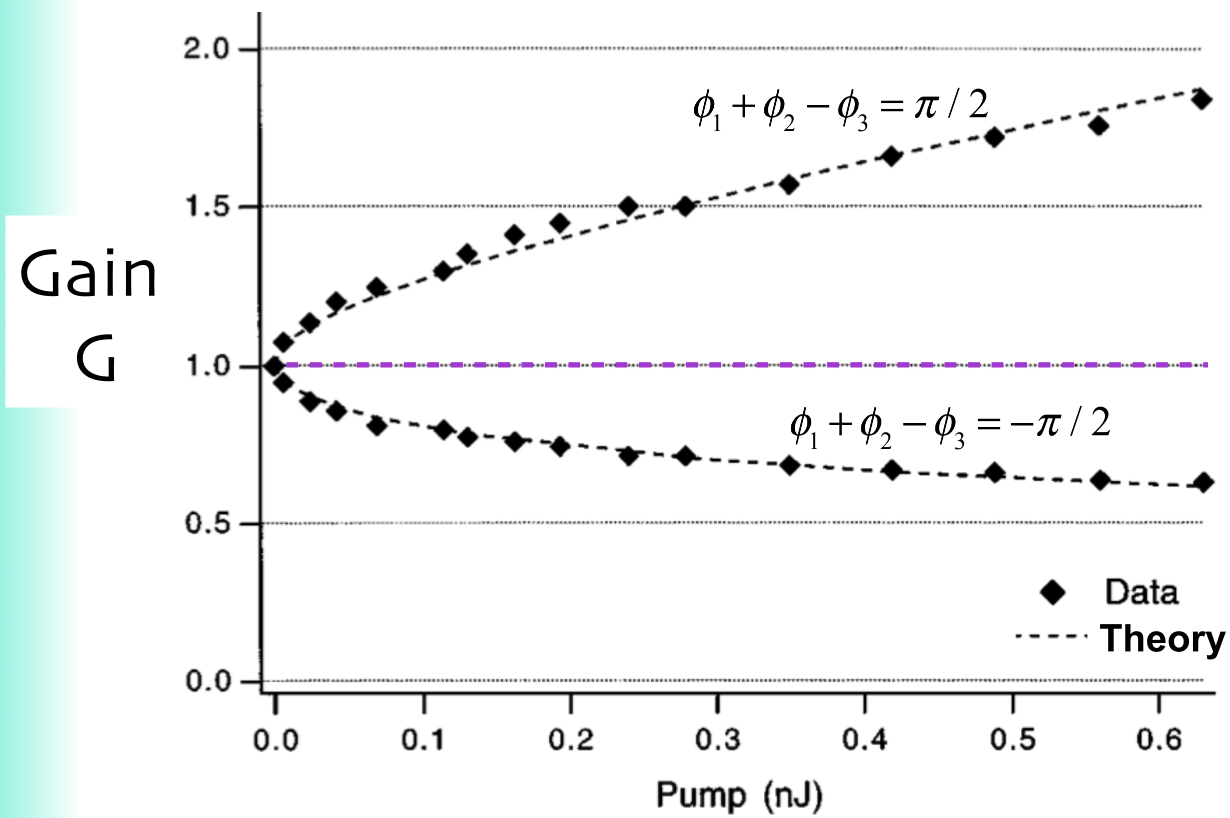
check

PARAMETRIC AMPLIFICATION



$$\Delta N = \hat{A}_2^\dagger \hat{A}_2 - \hat{A}_1^\dagger \hat{A}_1$$





M. Anderson

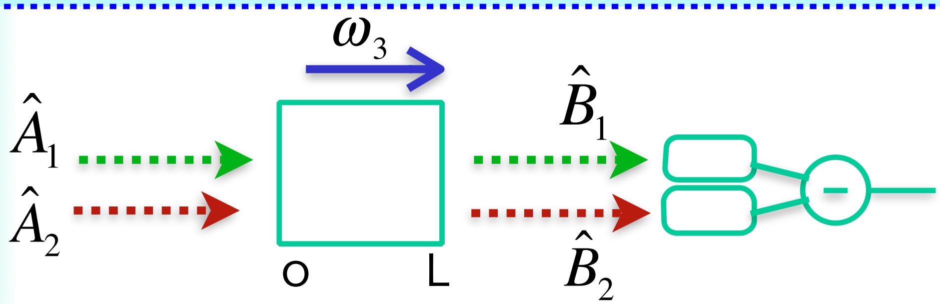
$$G = (1 - \xi) + \xi \exp[\pm\sqrt{Pump}]$$

$$\xi = 0.7\ (mode - matching\ eff.)$$

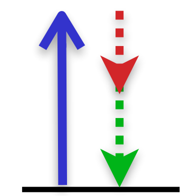
M. E. Anderson, M. Beck, M. G. Raymer, and J. D. Bierlein, Opt. Lett. **20**, 620 (1995)

OREGON

Conservation of Difference Number by PARAMP



$$\Delta N = \hat{A}_2^\dagger \hat{A}_2 - \hat{A}_1^\dagger \hat{A}_1$$



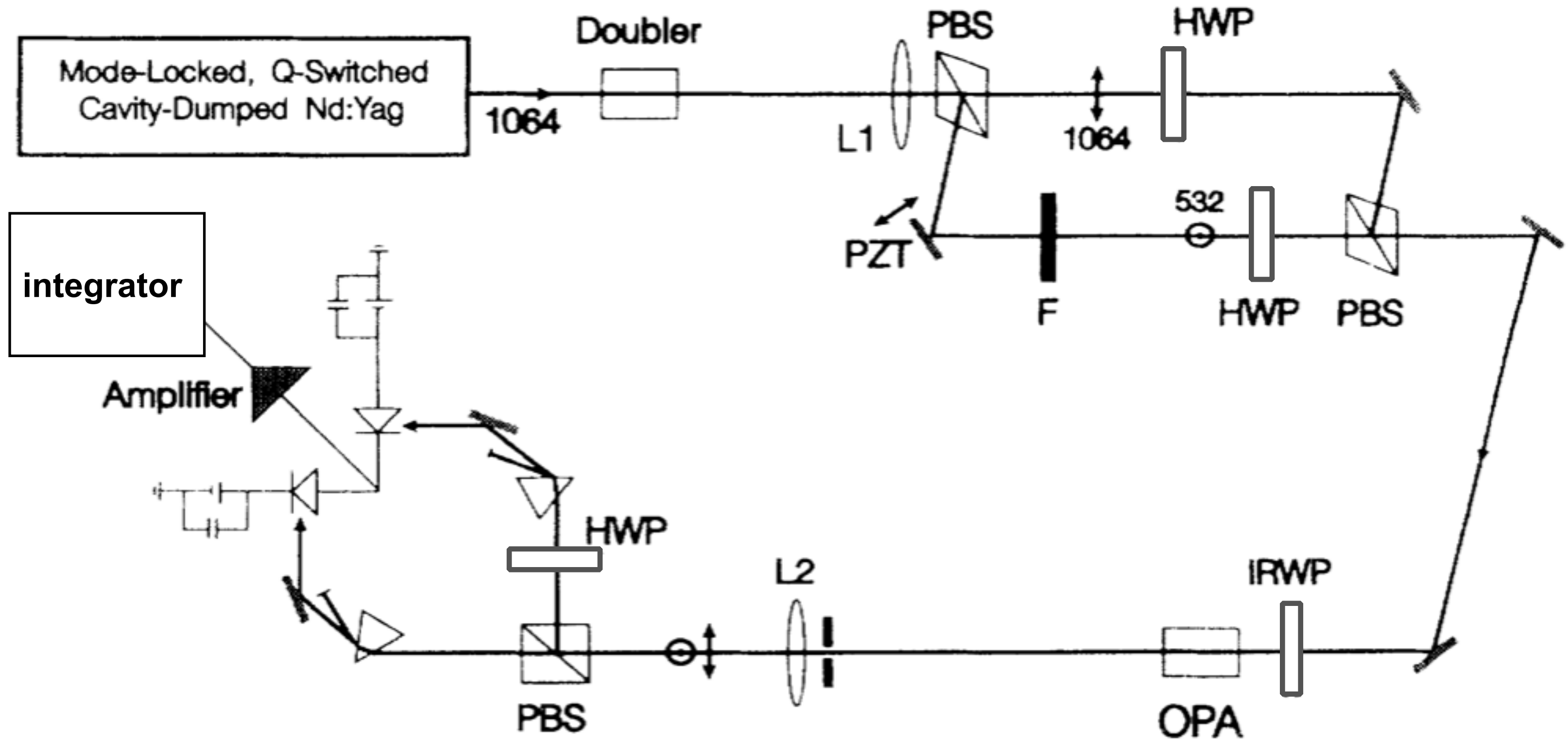
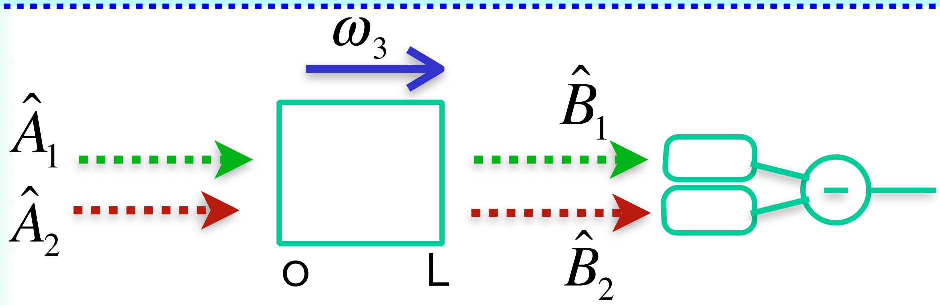


FIG. 1. The experimental setup for whole-pulse detection of sub-SNL intensity correlations.

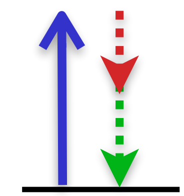
M. Beck

D. T Smithey, M. Beck, M. Belsley, and M. G. Raymer, Phys. Rev. Lett. **69**, 2650 (1992).

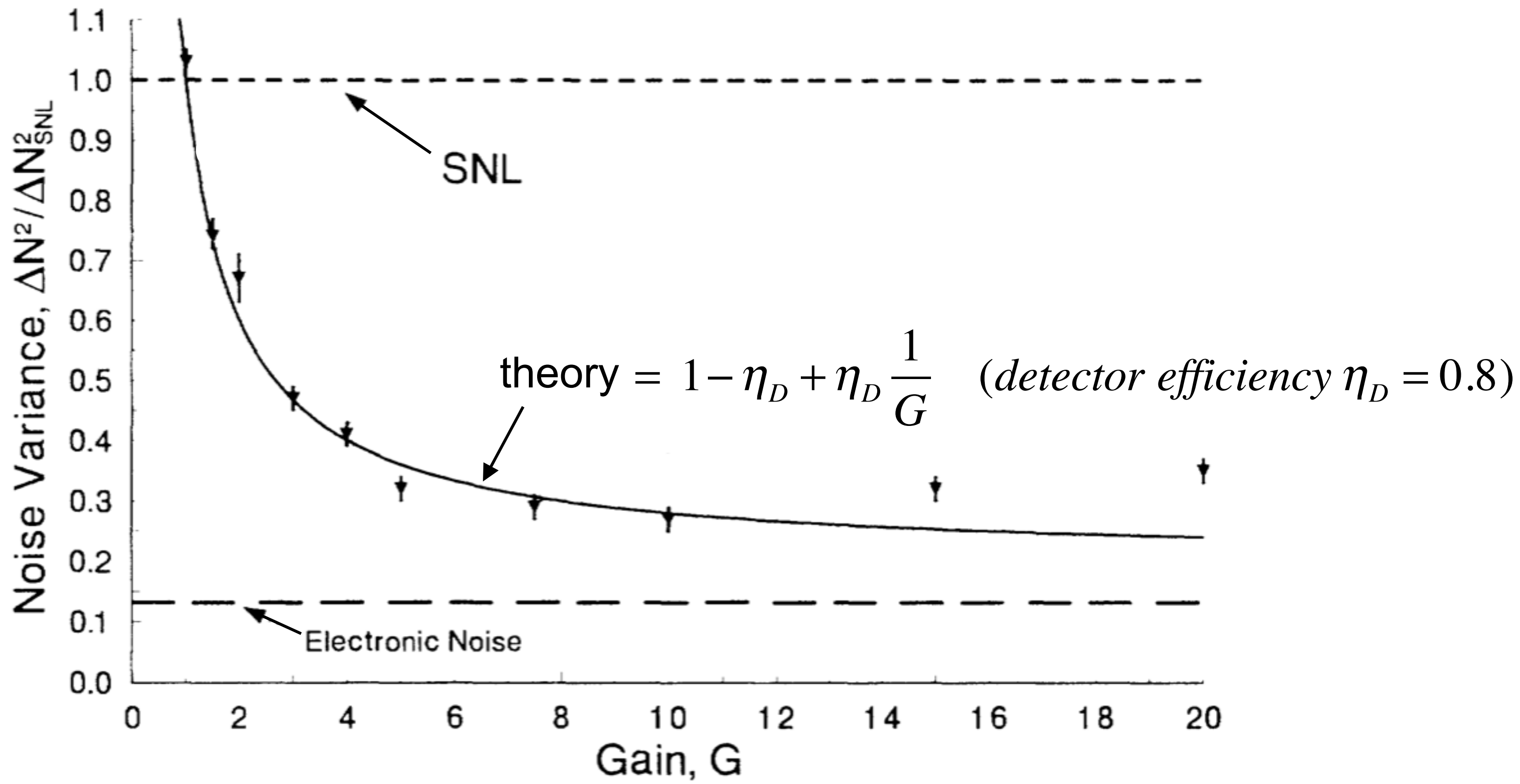
Conservation of Difference Number by PARAMP



$$\Delta N = \hat{A}_2^{\dagger}\hat{A}_2 - \hat{A}_1^{\dagger}\hat{A}_1$$



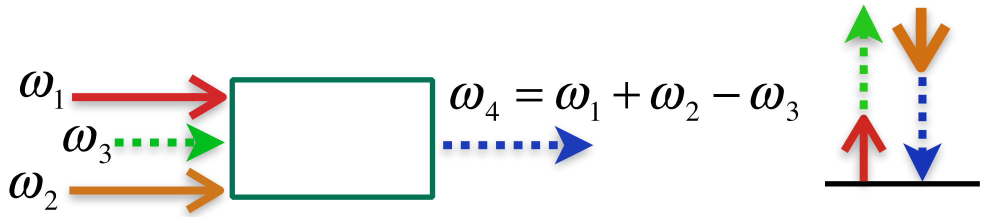
variance of difference number: $\dfrac{\mathrm{var}(\Delta N)}{\langle N_1\rangle + \langle N_2\rangle} = \dfrac{1}{G} = \dfrac{1}{e^{gL}}$

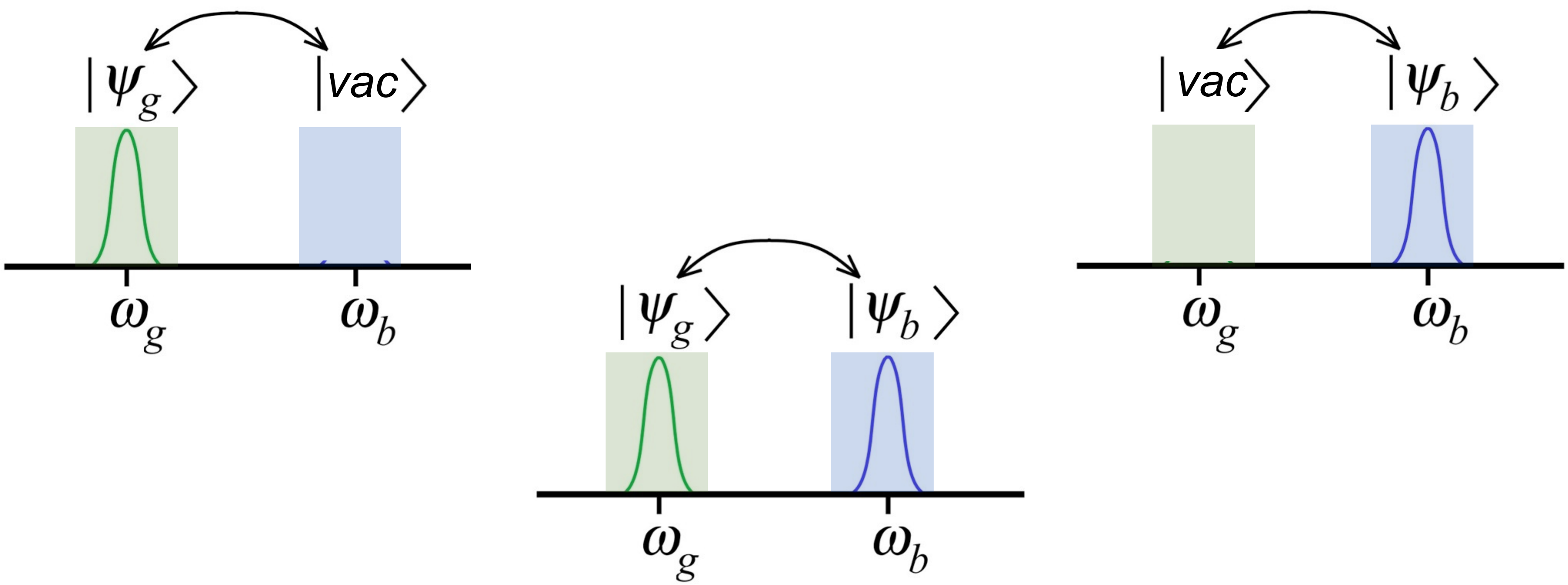


M. Beck

D. T Smithey, M. Beck, M. Belsley, and M. G. Raymer, Phys. Rev. Lett. **69**, 2650 (1992).

11. Quantum Frequency Conversion (QFC): complete or partial exchange of quantum states between two spectral bands.

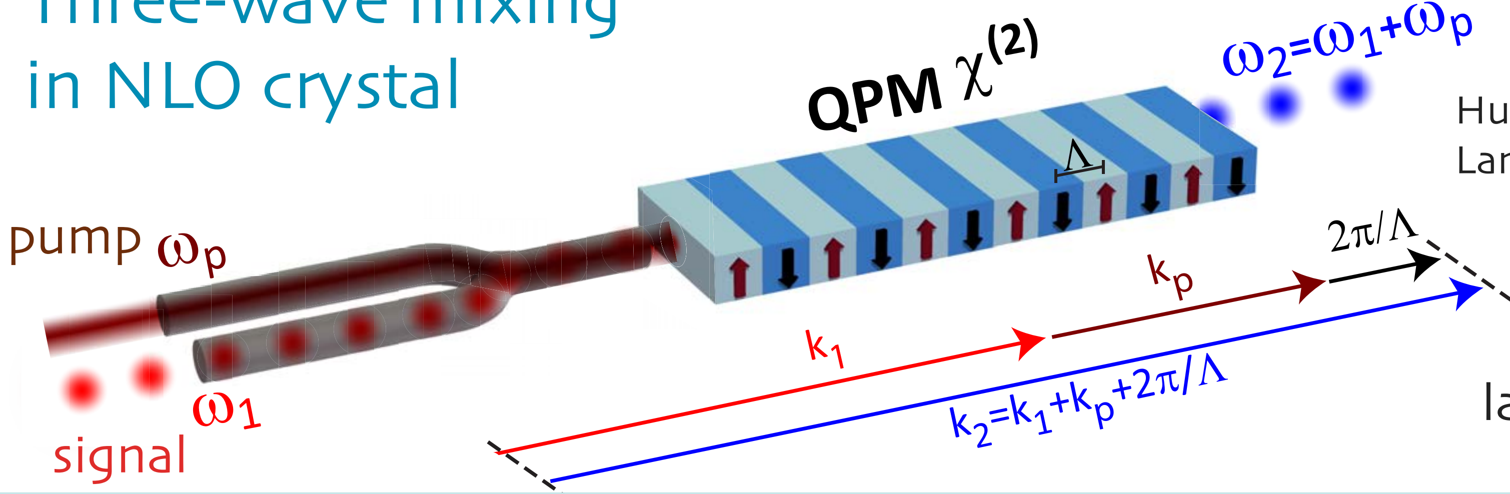




$$|\psi_g\rangle|\psi_b\rangle \mapsto \alpha|\psi_g\rangle|\psi_b\rangle + \beta|\psi_b\rangle|\psi_g\rangle$$

Methods for Quantum Frequency Conversion

Three-wave mixing in NLO crystal



Huang and Kumar, PRL (1992)
Langrock, Optics Letters (2005)

large frequency shift

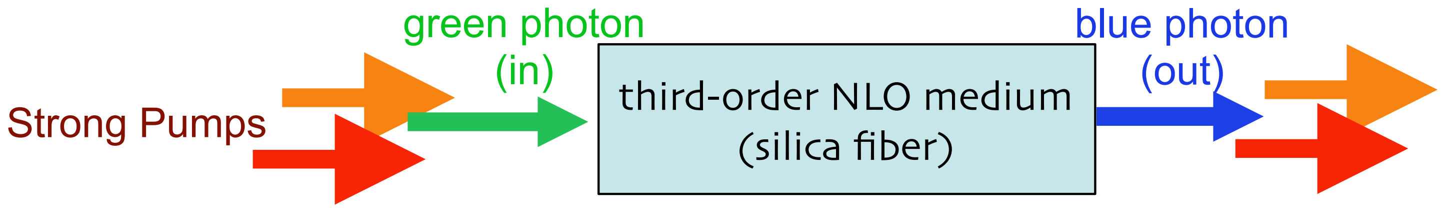
Four-wave mixing in optical fiber

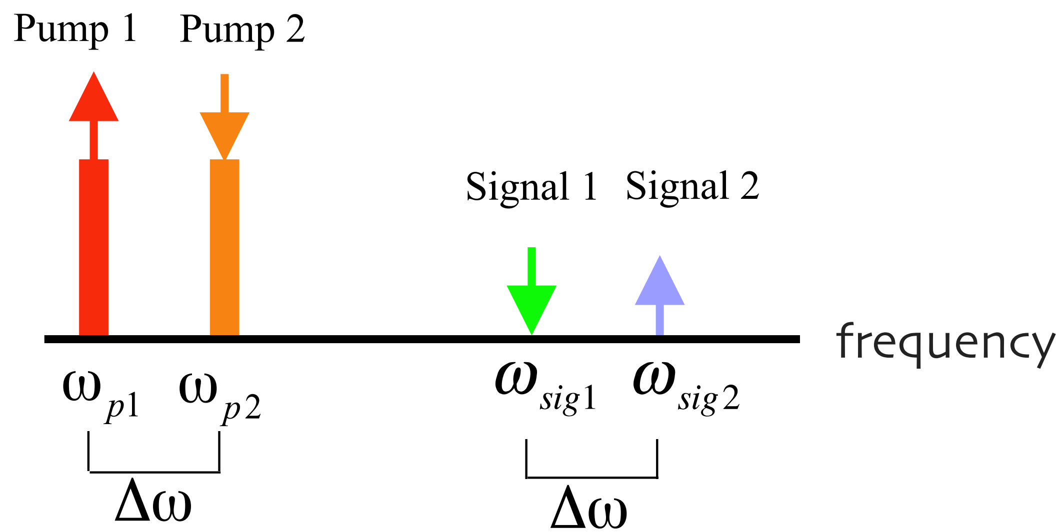
McKinstrie, Opt Ex (2005)
McGuinness PRL (2010)

allows small frequency shift

from: Raymer and Srinivasan, Physics Today, 65, 32 (2012)

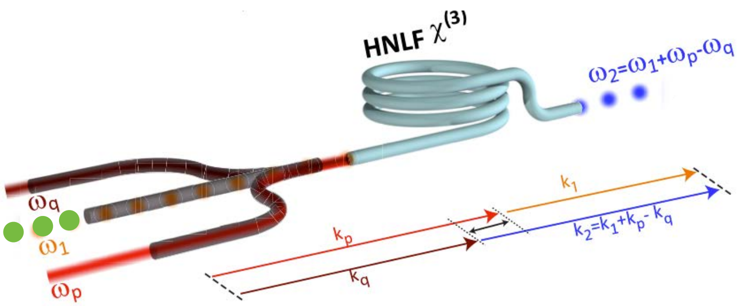
Frequency Conversion of <u>single-photon states</u> by four-wave mixing in optical fiber





Hayden McGuinness
and Wallace

Modeling QFC by Four-Wave Mixing in Optical Fiber



dispersion   pumps

$$\frac{\partial}{\partial z}\hat{A}_g = i\sum_m \beta_m \frac{\partial^m}{\partial t^m}\hat{A}_g + i\gamma A_p^{*} A_q \hat{A}_b$$

$$\frac{\partial}{\partial z}\hat{A}_b = i\sum_m \beta_m \frac{\partial^m}{\partial t^m}\hat{A}_b + i\gamma A_p A_q^{*} \hat{A}_g$$

The equations are linear in A_g and A_b signal field operators

$$\begin{pmatrix} \hat{A}_g(t) \\ \hat{A}_b(t) \end{pmatrix}_{OUT} = \int^t dt' \begin{pmatrix} G_{gg}(t,t') & G_{gb}(t,t') \\ G_{bg}(t,t') & G_{bb}(t,t') \end{pmatrix} \begin{pmatrix} \hat{A}_g(t') \\ \hat{A}_b(t') \end{pmatrix}_{IN}$$

No mixing of A and A^+ : Like a Beam-Splitter transformation (background-free)

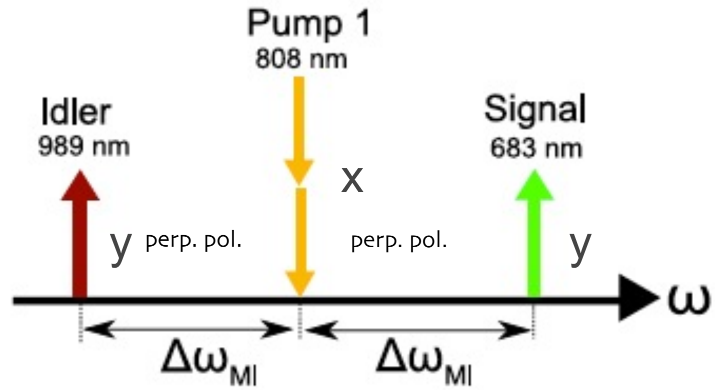
All quantum correlations can be calculated from Green functions.

McGuinness, MR, CM, Opt. Express 19, 17876 (2011)

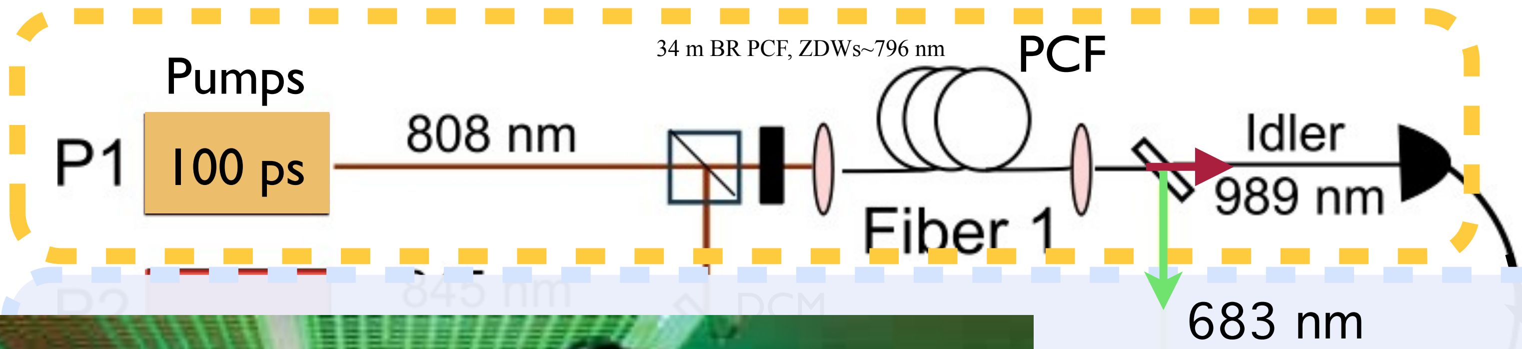
Experiment

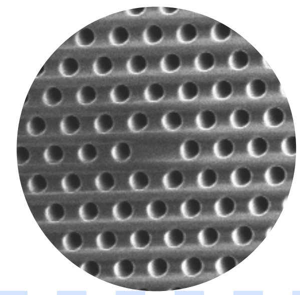
PRL, *105, 093604 (2010)*

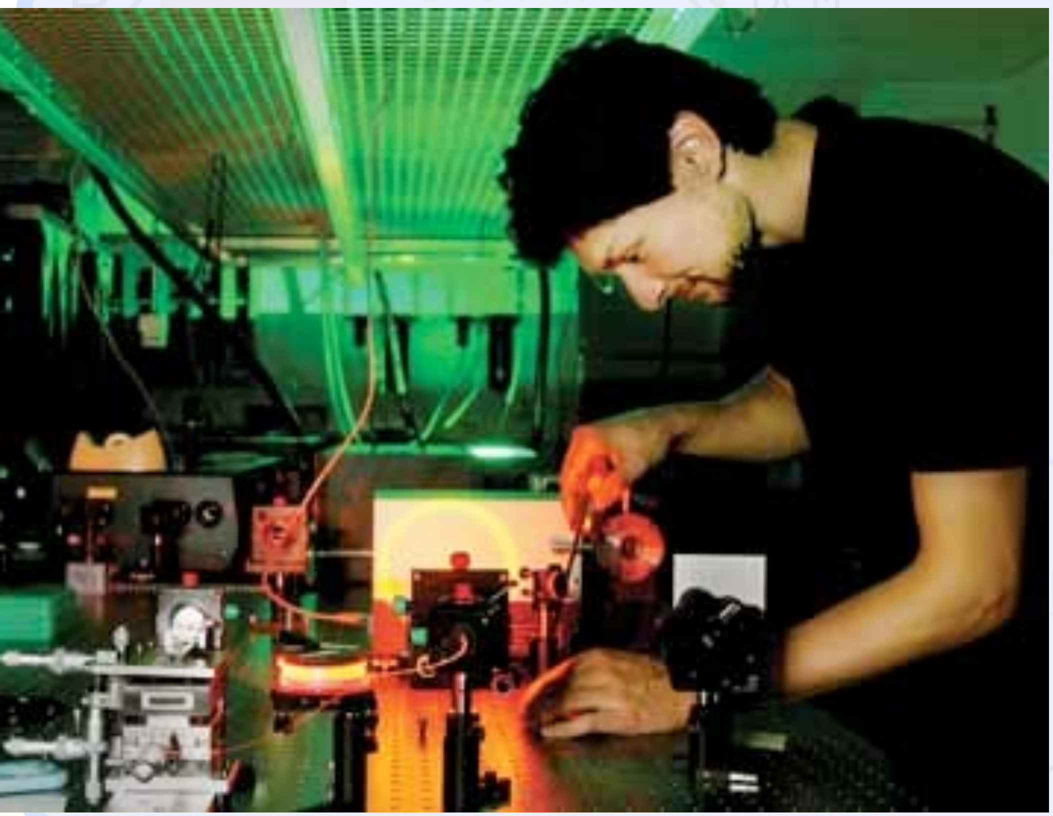
1. Pair Creation

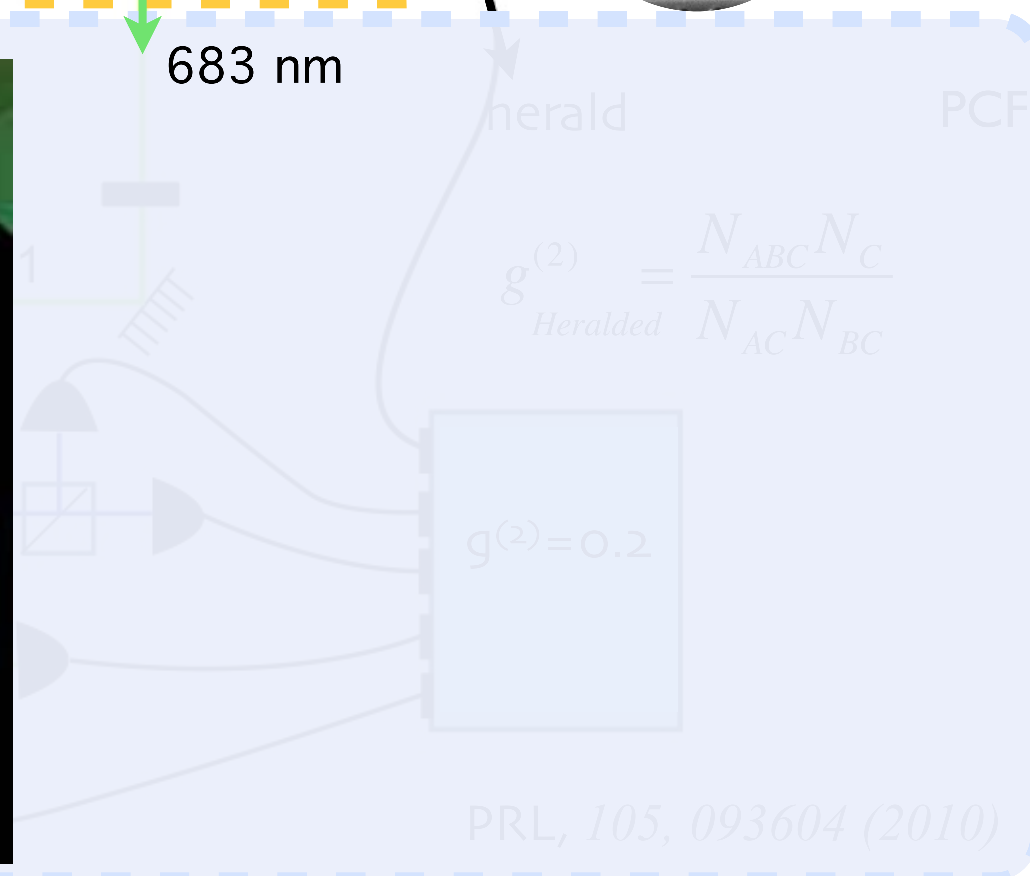


Generate heralded single photon

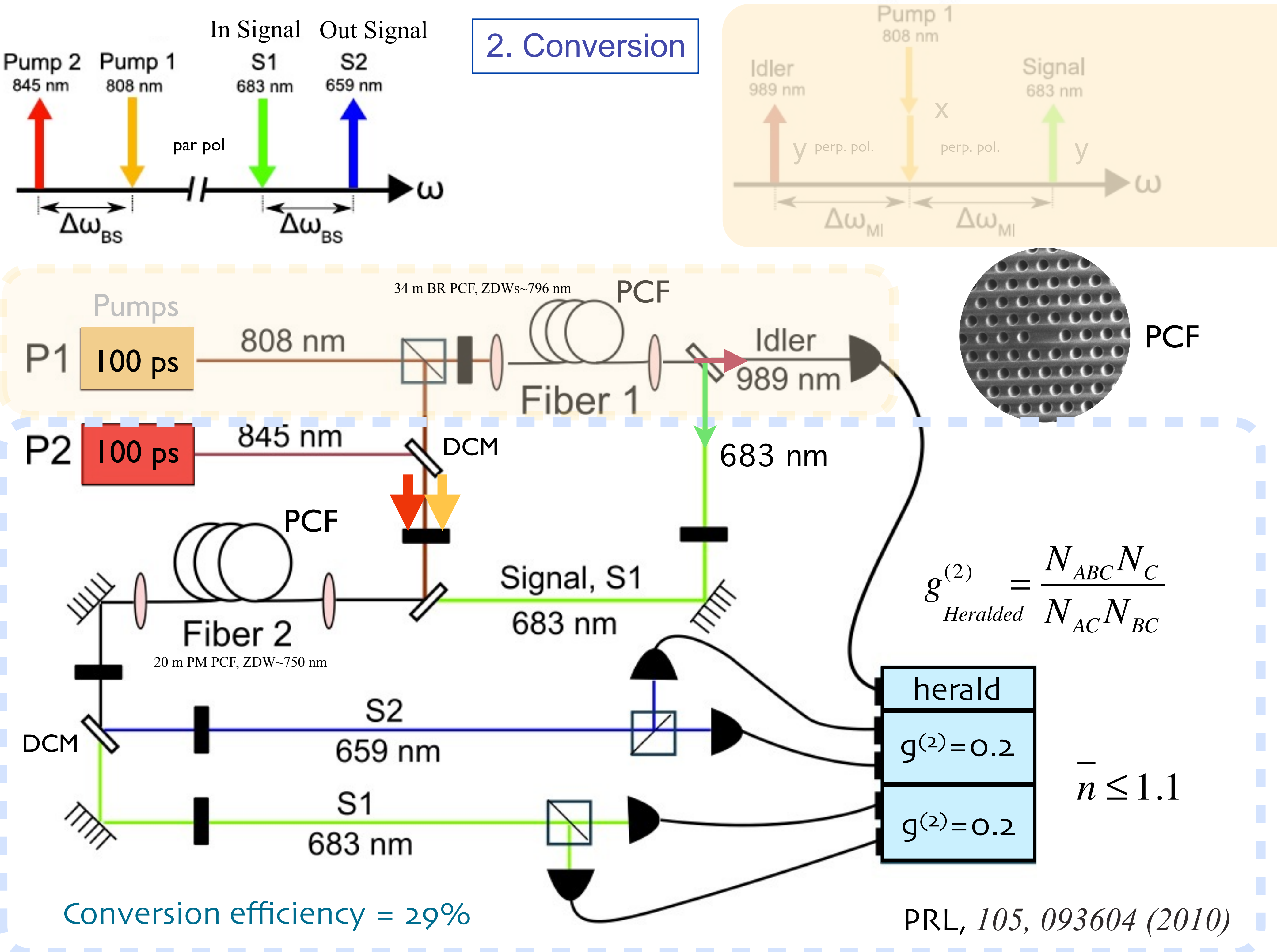








2. Conversion



$$g^{(2)}_{Heralded} = \frac{N_{ABC} N_C}{N_{AC} N_{BC}}$$

$$\bar{n} \leq 1.1$$

Conversion efficiency = 29%

PRL, *105, 093604 (2010)*

Singular-Value Decomposition of the Green functions

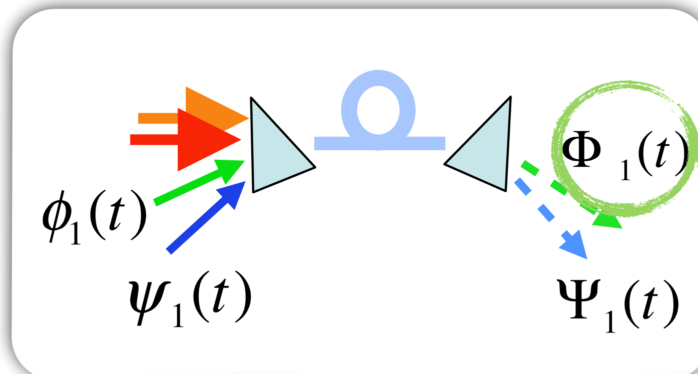
$$\begin{pmatrix} \hat{A}_g(t) \\ \hat{A}_b(t) \end{pmatrix}_{OUT} = \sum_n \int^t dt' \begin{pmatrix} \tau_n \Phi_n(t)\phi^*_n(t') & \rho_n \Phi_n(t)\psi^*_n(t') \\ -\rho_n \Psi_n(t)\phi^*_n(t') & \tau_n \Psi_n(t)\psi^*_n(t') \end{pmatrix} \begin{pmatrix} \hat{A}_g(t') \\ \hat{A}_b(t') \end{pmatrix}_{IN}$$

for each mode pair: $\rho_n^2 + \tau_n^2 = 1$ ρ_n^2 = conversion, τ_n^2 =nonconversion

Temporal Schmidt Modes reduce the problem to low-dimensional state space:

$$if \quad \begin{pmatrix} \hat{A}_g(t') \\ \hat{A}_b(t') \end{pmatrix}_{IN} = \begin{pmatrix} \hat{a}_g \phi_1(t') \\ \hat{a}_b \psi_1(t') \end{pmatrix}$$

$$then \quad \begin{pmatrix} \hat{A}_g(t) \\ \hat{A}_b(t) \end{pmatrix}_{OUT} = \begin{pmatrix} (\tau_1 \hat{a}_g + \rho_1 \hat{a}_b)\, \Phi_1(t) \\ (-\rho_1 \hat{a}_g + \tau_1 \hat{a}_b)\, \Psi_1(t) \end{pmatrix}$$



Operators undergo pair-wise beam-splitter-like transformat

green OUT has same shape regardless of its origin ➜ 2-photon interf.

MR, HM, SVE, CM Opt. Commun. 238, 747 (2010)

OREGON

Summary of PART 1:

1. Field can be quantized in monochromatic modes (Dirac), or non-monochromatic temporal modes (Glauber)

2. A single TM can be excited into photon-number states, coherent states, or squeezed states.

3. Quantum state tomography can determine the properties of these TM states.

4. Nonlinear optics can be used to create and manipulate quantum states of TMs.