PART 3 : Photon Temporal Modes: a Complete Framework for Quantum Information Science

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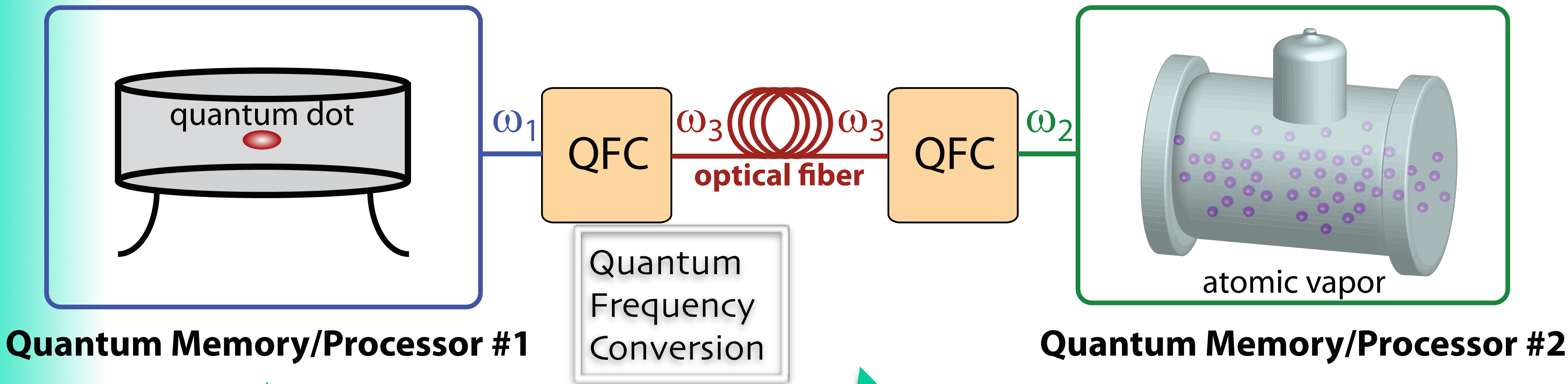
Summer School on Quantum and Nonlinear Optics
(QNLO 2015 Sørup Herregaard)

The Quantum Internet

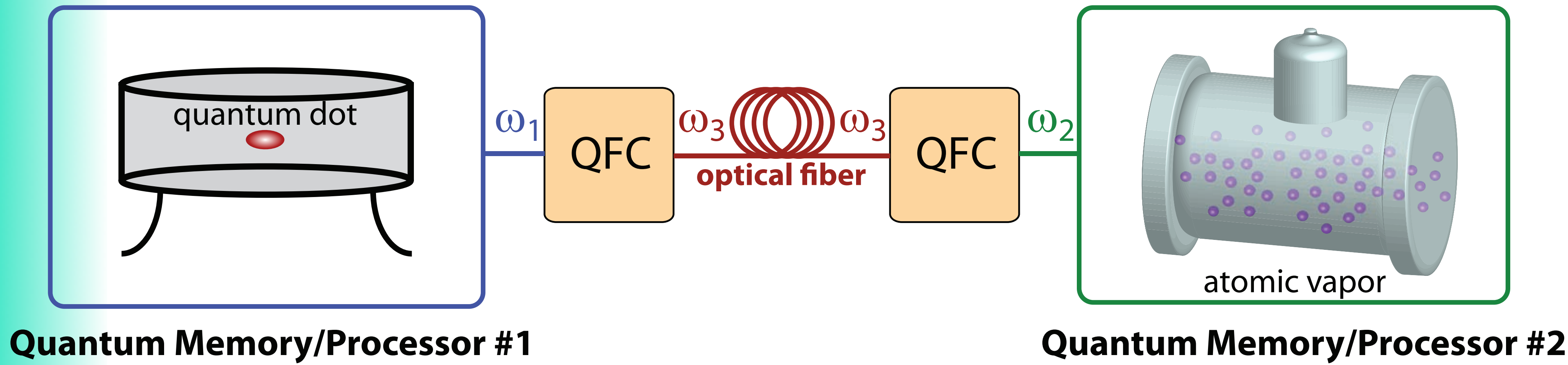


Memory and computation are implemented using material systems (atoms, electrons,…)

Communication is implemented using light (single photons, coherent states, …)

Review: M. G. Raymer and K. Srinivasan, Physics Today, **65**, 32 (2012)

The Quantum Internet



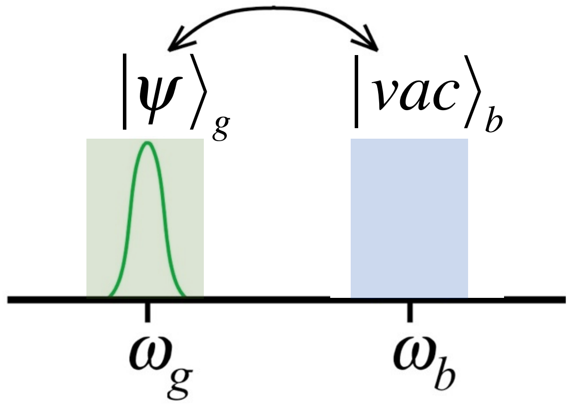
Elements needed for a complete quantum information framework :

1. well controlled generation and full characterization of qubit or qudit states
2. single- and joint logic operations on qubits or qudits
3. interconversion of stationary and flying qubits or qudits
4. transmission of flying qubits or qubits between distant locations
5. targeted manipulation of qubit or qudit states
6. efficient detection of qubit or qudit states

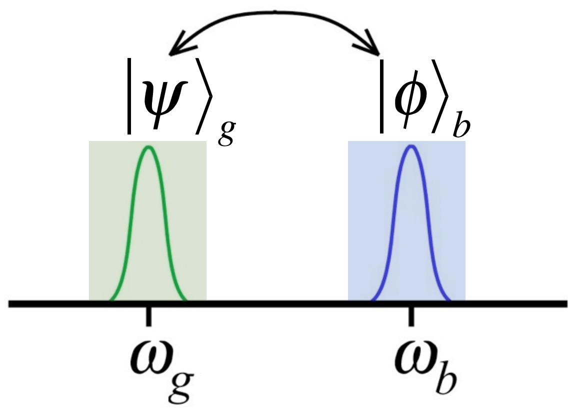
all of these may utilize frequency conversion

Brecht, Reddy, Silberhorn, Raymer, arXiv:1504.06251 Apr 2015

Quantum Frequency Conversion (QFC):
The complete or partial exchange of quantum states between two spectral bands.



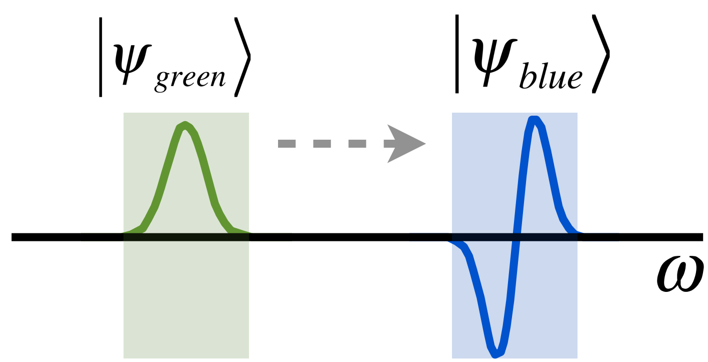
$$|\psi\rangle_g |vac\rangle_b \mapsto |vac\rangle_g |\psi\rangle_b$$



note: need phase coherence for the latter

$$|\psi\rangle_g |\phi\rangle_b \mapsto \alpha|\psi\rangle_g |\phi\rangle_b + e^{i\phi}\beta|\phi\rangle_g |\psi\rangle_b$$

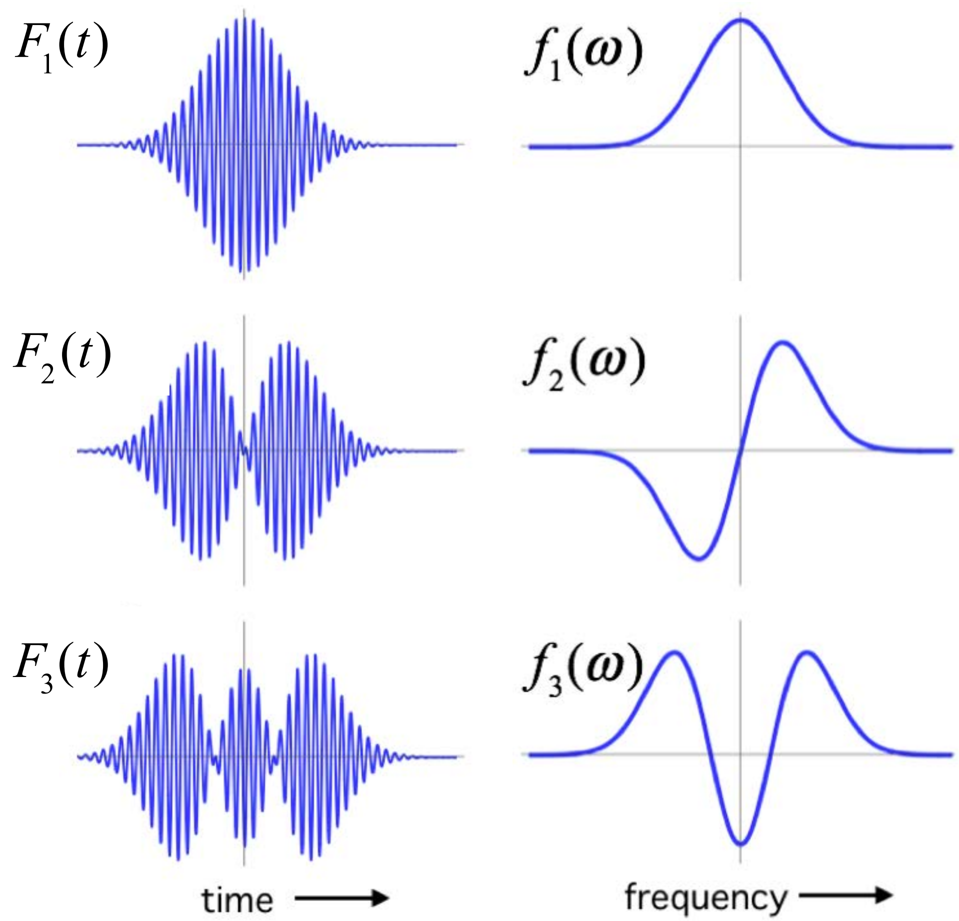
changes of photon wave-packet shape, e.g.



recall: QUANTIZATION OF EM FIELD IN TERMS OF TEMPORAL MODES

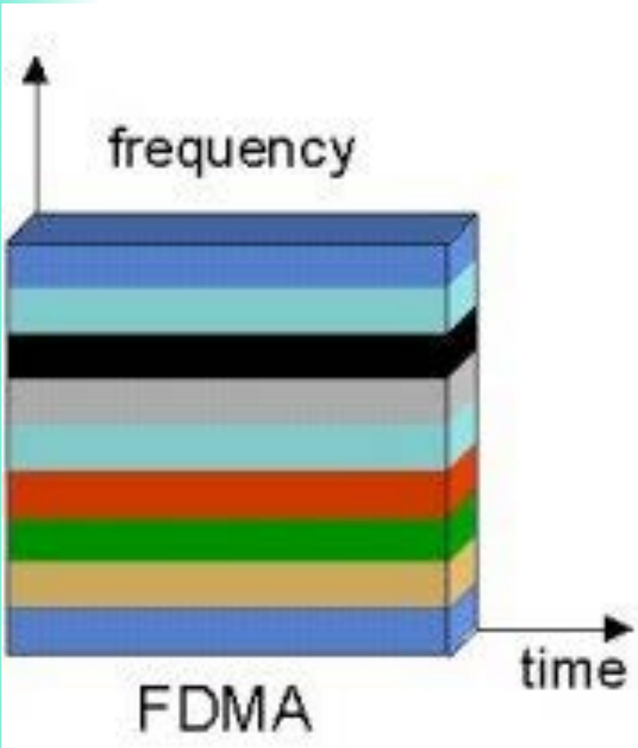
1. quantum mechanics deals with discrete degrees of freedom.
2. define a single mode from within a continuum.

A temporal mode (TM) is one of a discrete set of orthogonal functions $F_j(t)$.

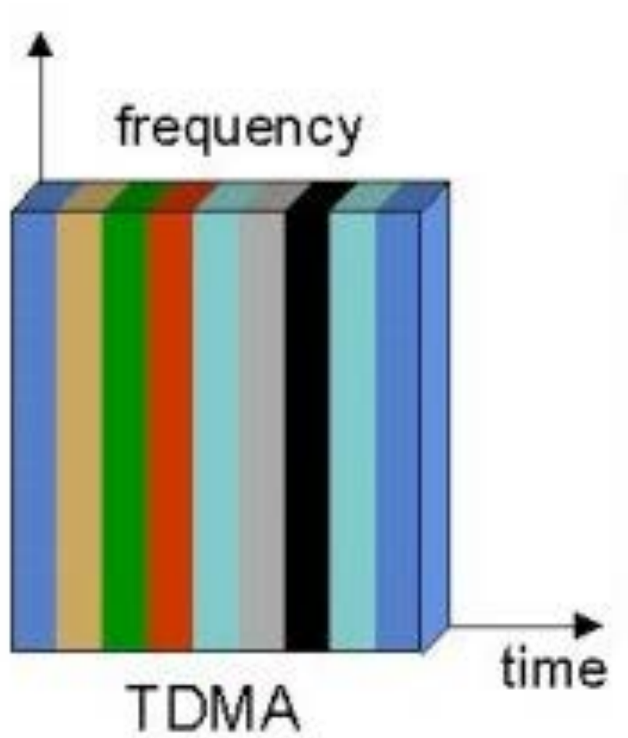


Commonly used multiplexing schemes in radio technology

Frequency-division multiple access



Time-division multiple access



FDMA and TDMA use only time-frequency space

http://www.umtsworld.com/technology/cdmabasics.htm

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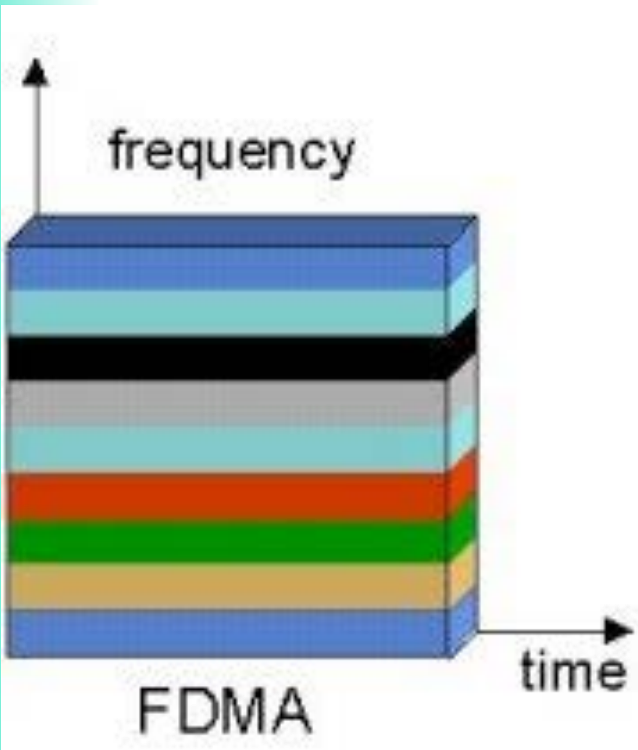
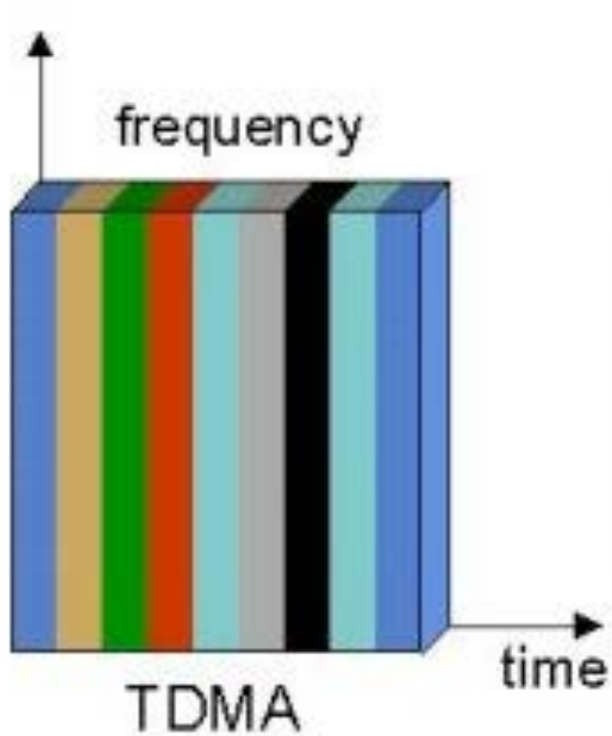
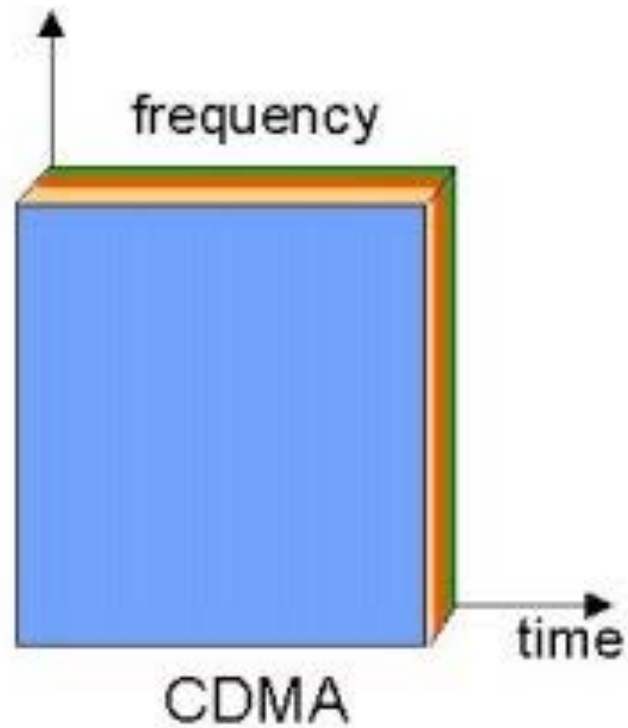


Time-division multiple access



Code-division multiple access



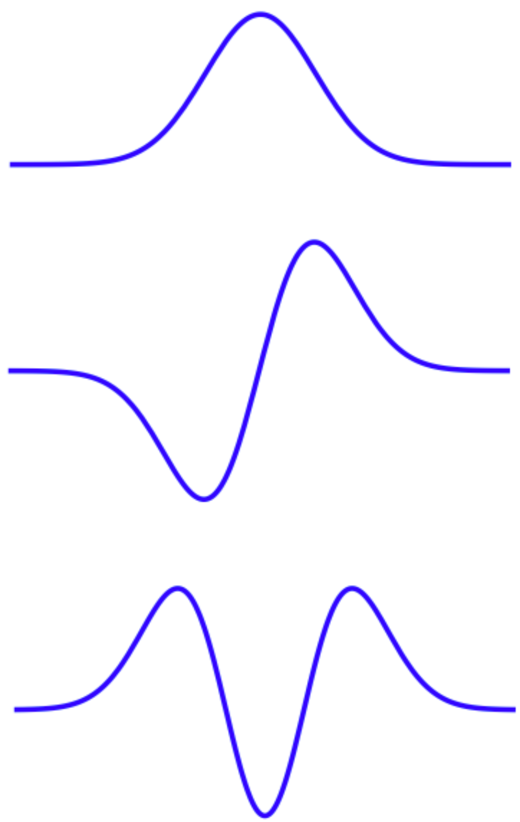
FDMA and TDMA use only time-frequency space

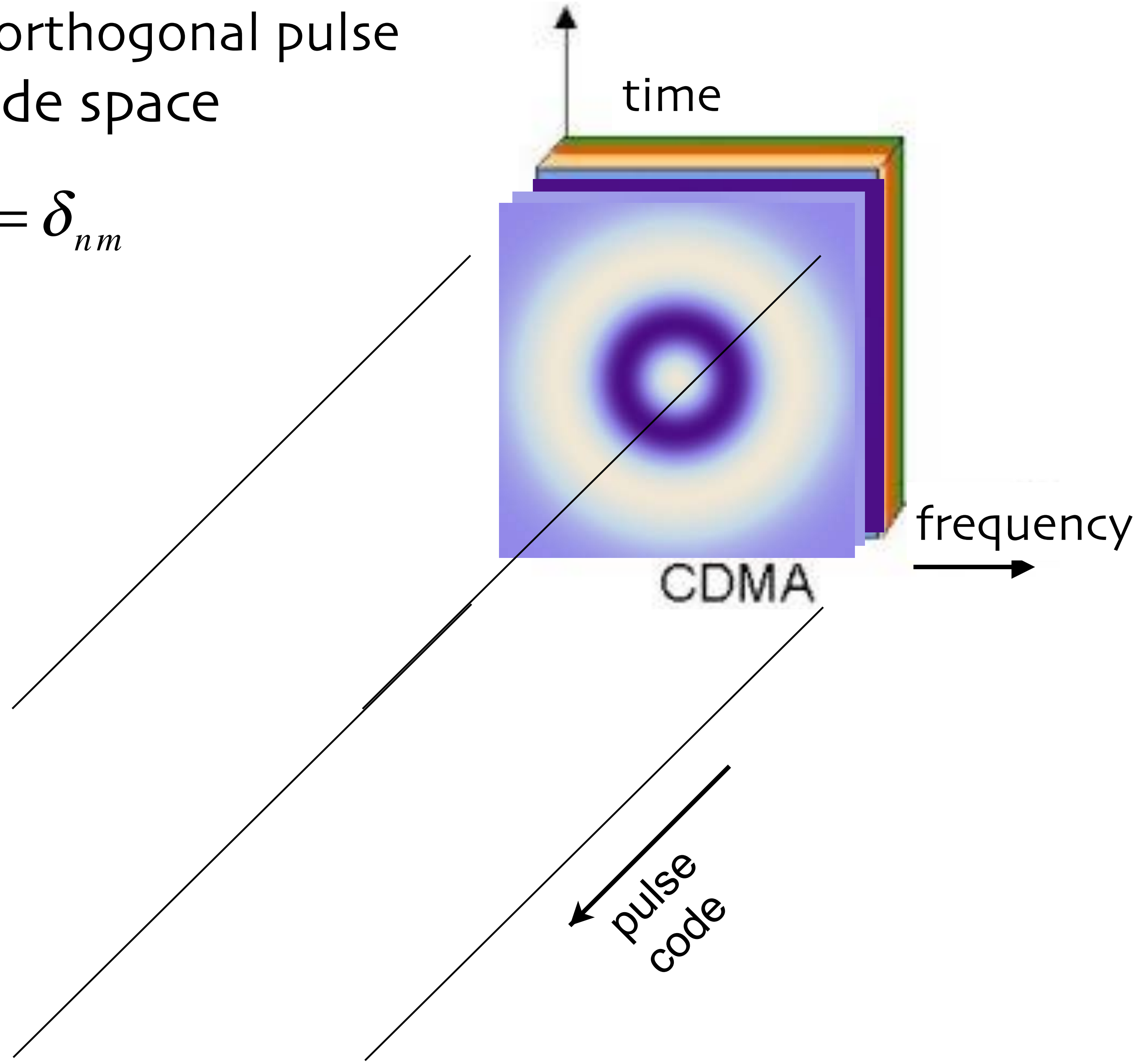
http://www.umtsworld.com/technology/cdmabasics.htm

CDMA uses field-orthogonal pulse codes in code space

$$\int F_n^*(t) F_m(t)\,dt = \delta_{nm}$$

temporal modes





In radio, field-orthogonal codes are easily demultiplexed.
In optical, there is *NO* known method to ^efficiently^ demultiplex field-orthogonal codes.

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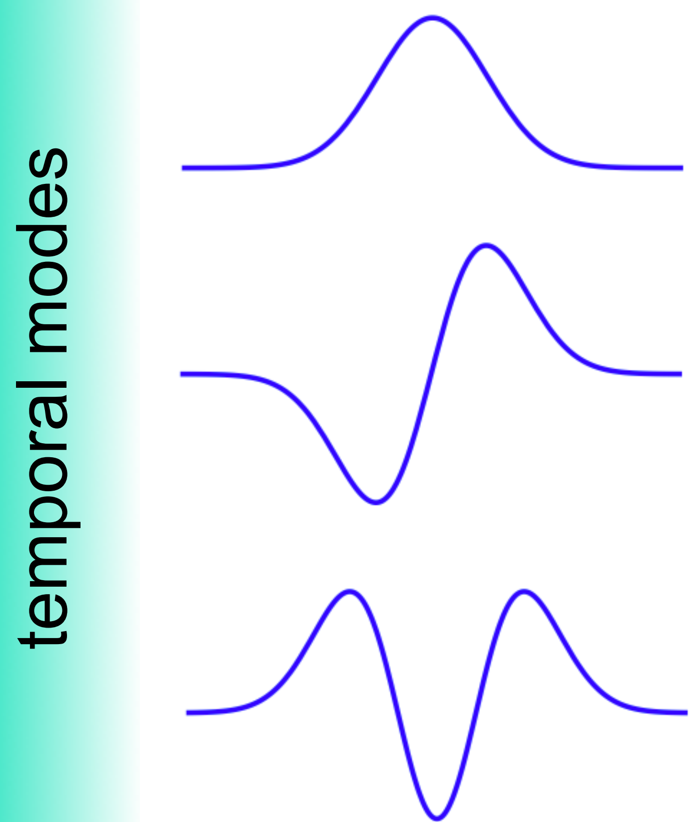

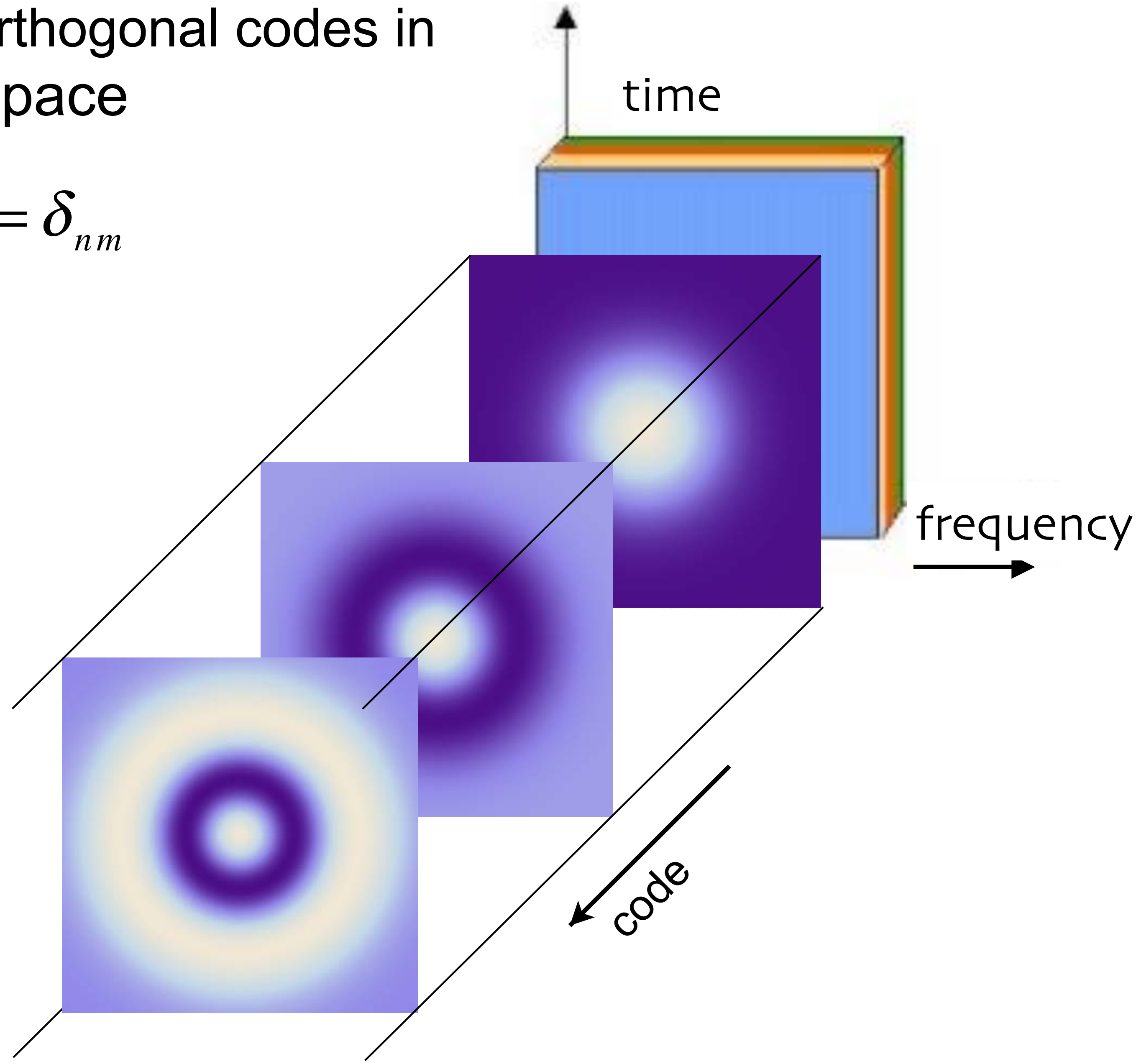

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^ efficiently

EXPRESSING FIELDS and STATES in TMs

narrow-band field:

$$\hat{E}^{(+)}(z,t) = \tilde{E}_0 \sum_j \hat{A}_j F_j(\tau)$$

$$\tau = t - z/c$$

$F_3(\tau)$

annihilation

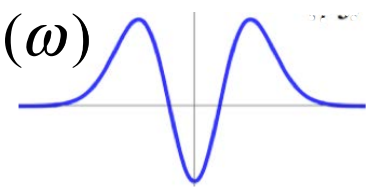
$$\hat{A}_j = \frac{1}{2\pi} \int_0^\infty d\omega \, f_j^*(\omega) \hat{a}(\omega)$$

creation

$$\hat{A}_j^\dagger = \frac{1}{2\pi} \int_0^\infty d\omega \, f_j(\omega) \hat{a}^\dagger(\omega)$$

$$F_j(\tau) = FT\{f_j(\omega)\}$$

$f_3(\omega)$



bosonic:

$$[\hat{A}_j, \hat{A}_k^\dagger] = \delta_{jk}$$

$f_j(\omega)$ are chosen to be complete and orthonormal

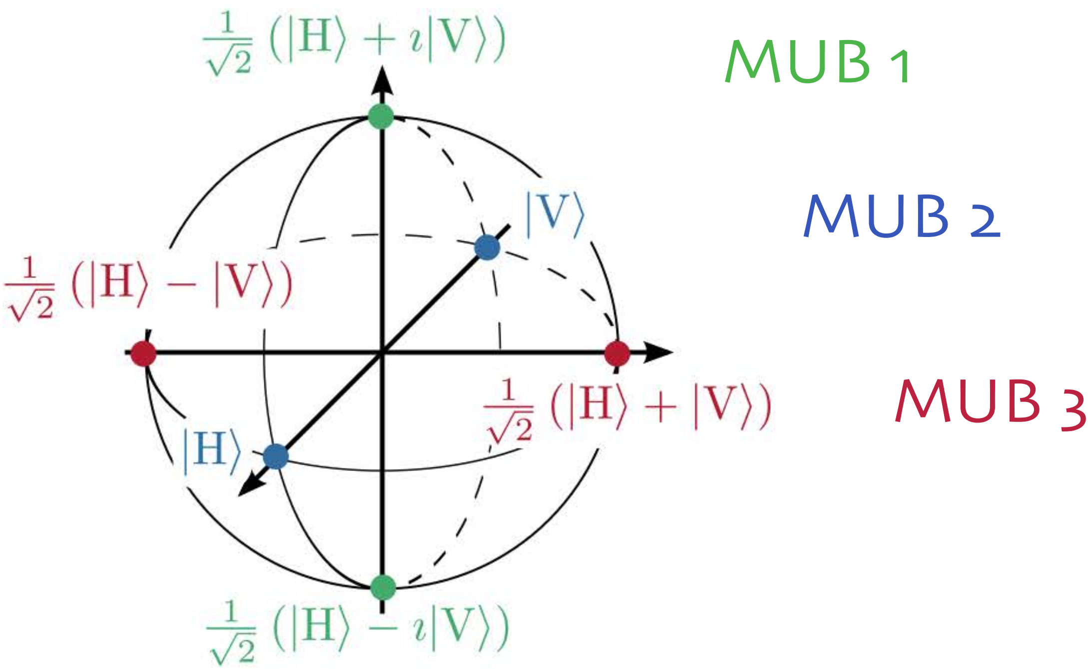
$\hat{A}_k^\dagger$ creates a single-photon state in TM k:

$$|A_j\rangle = \hat{A}_j^\dagger |vac\rangle$$

express every single-photon temporal wave-packet quantum state in a basis of TMs:

$$|\psi\rangle = \sum_{j=1}^\infty c_j |A_j\rangle$$

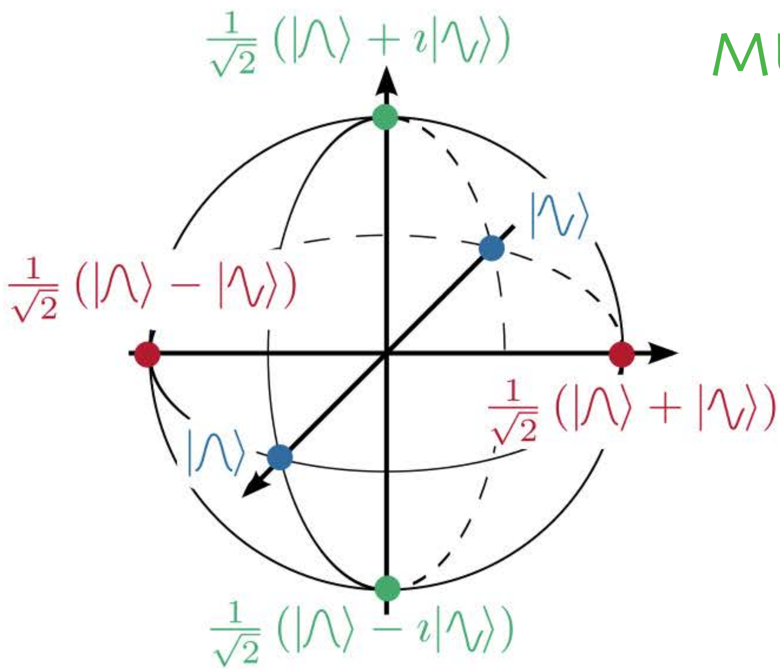
Polarization as Qubits (d=2)



Polarization (dimension-2) supports three Mutually Unbiased Bases (MUBs)

Measuring in one MUB gives no information about the other MUBs (➙ Quantum key distribution)

TMs as Qubits

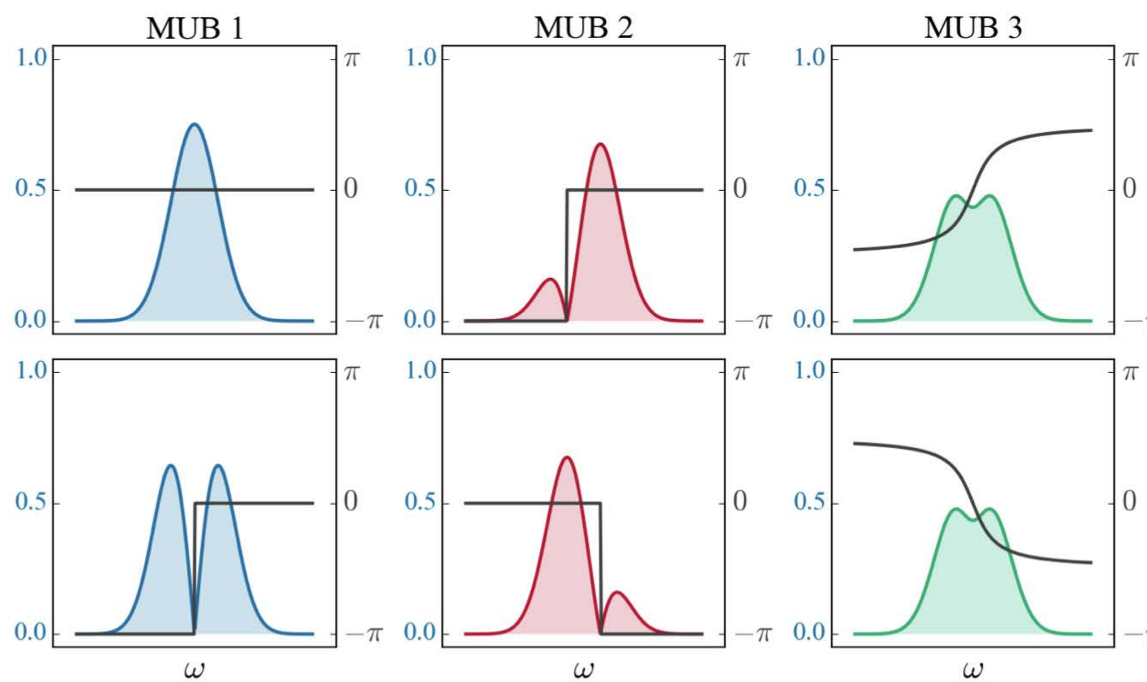


MUB 1

MUB 2

MUB 3

$$\left|\psi\right\rangle = \sum_{j=1}^{2} c_j \left|A_j\right\rangle$$



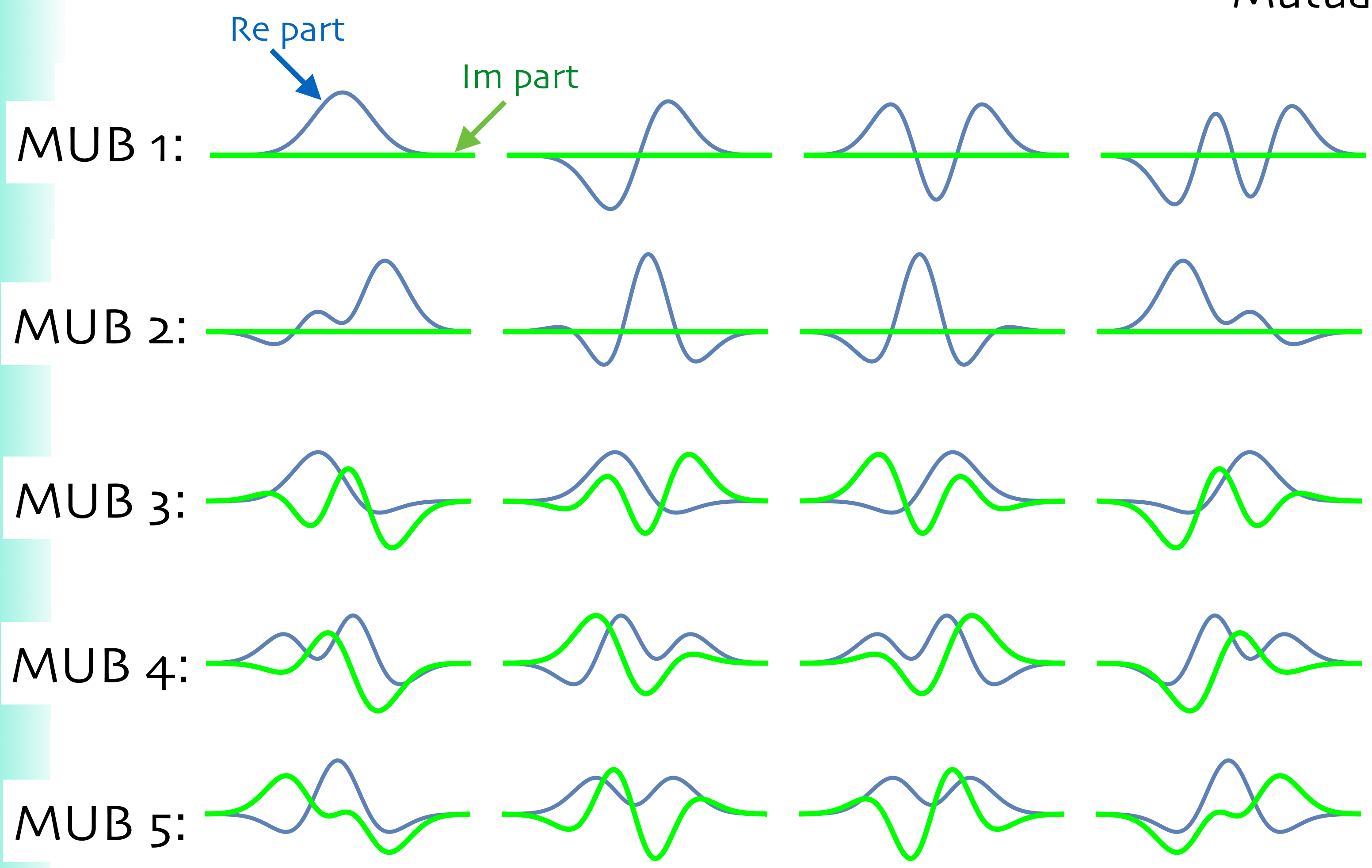
Brecht et al, arXiv:quant-ph 1504.06251

TMs as ~~Qubits~~ Qudits

Choose TMs 1 through d to create a d-dimensional state space

$$\left|\psi^{nm}\right\rangle = \sum_{j=1}^{d} \alpha_j^{nm} \left|A_j\right\rangle$$

Example: d=4
Five MUBs exist
Mutually Unbiased Bases



http://en.wikipedia.org/wiki/Mutually_unbiased_bases

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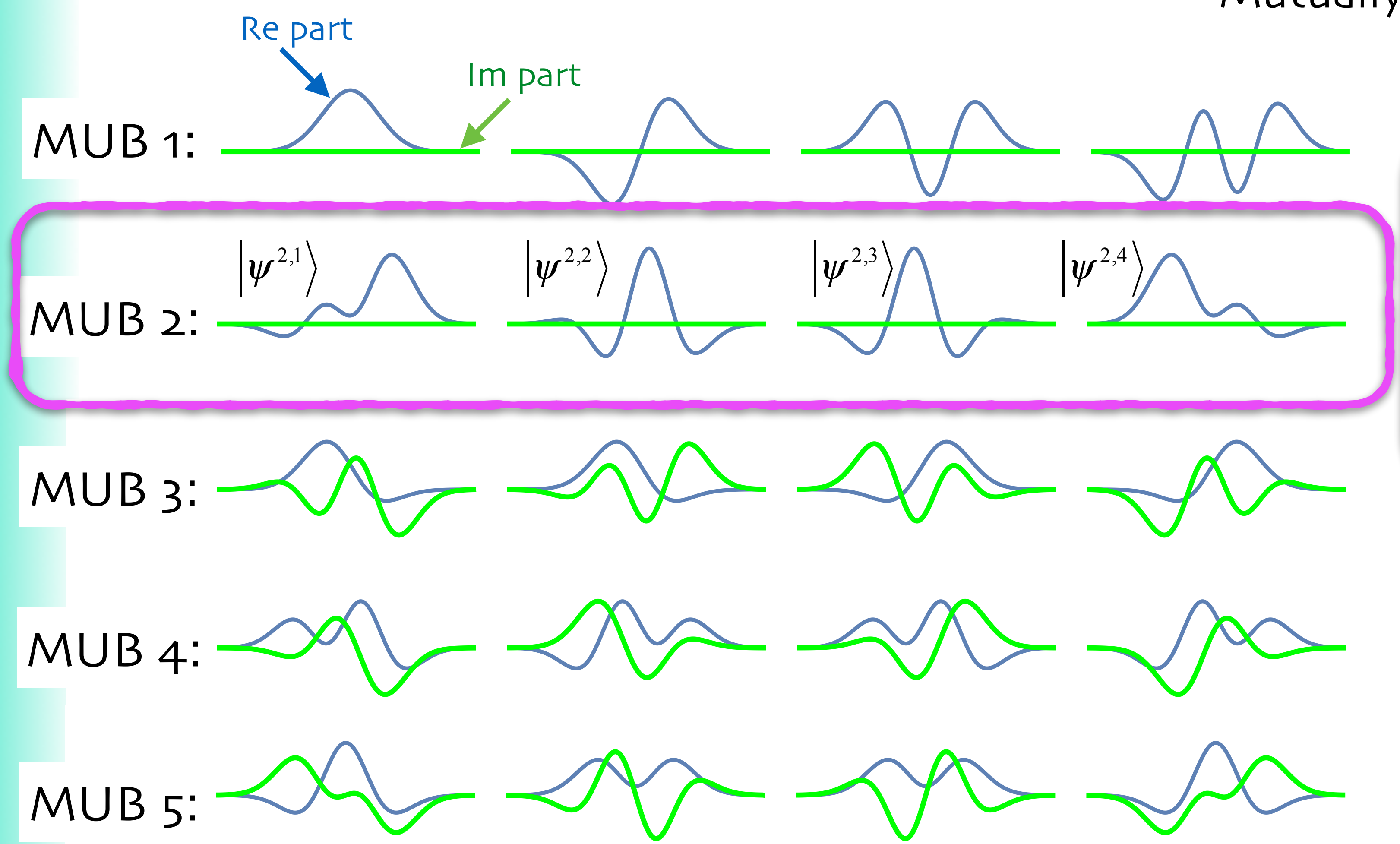

all modes within a MUB are ortho.

http://en.wikipedia.org/wiki/Mutually_unbiased_bases

TMs as ~~Qubits~~ Qudits

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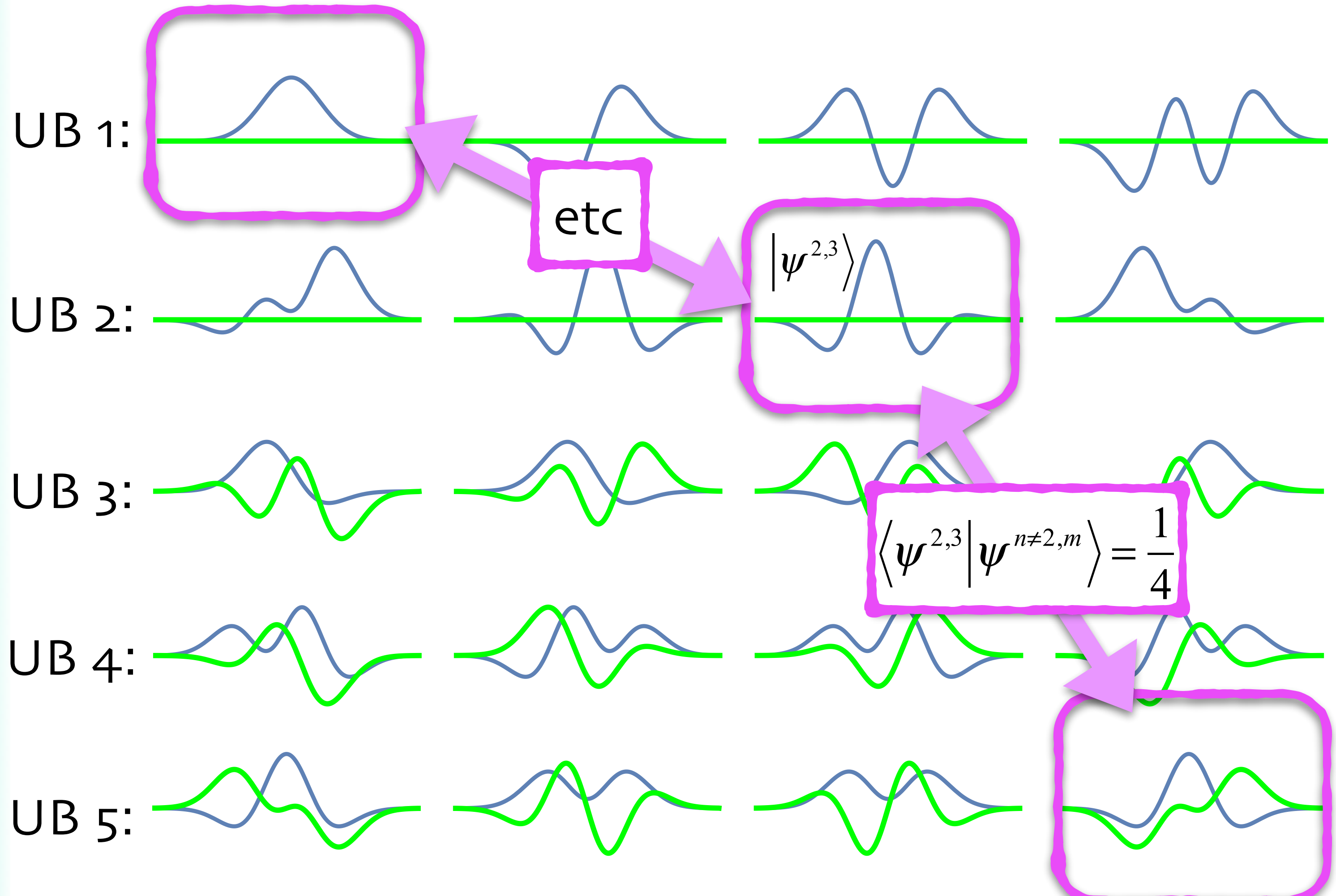
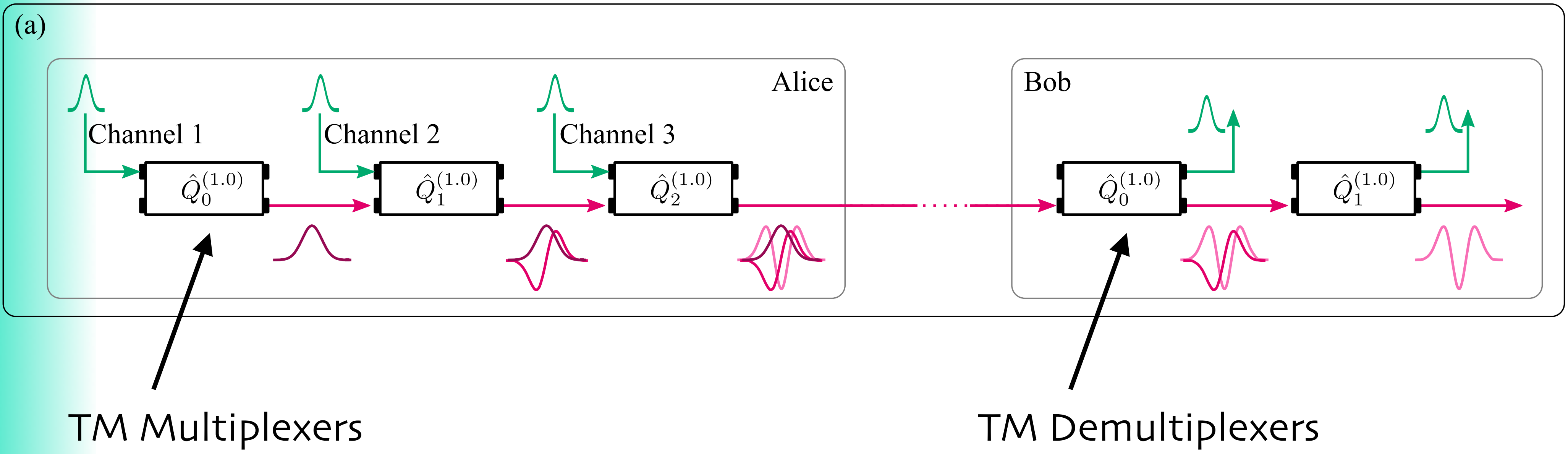


all mode pairs across different MUBs have equal inner products.

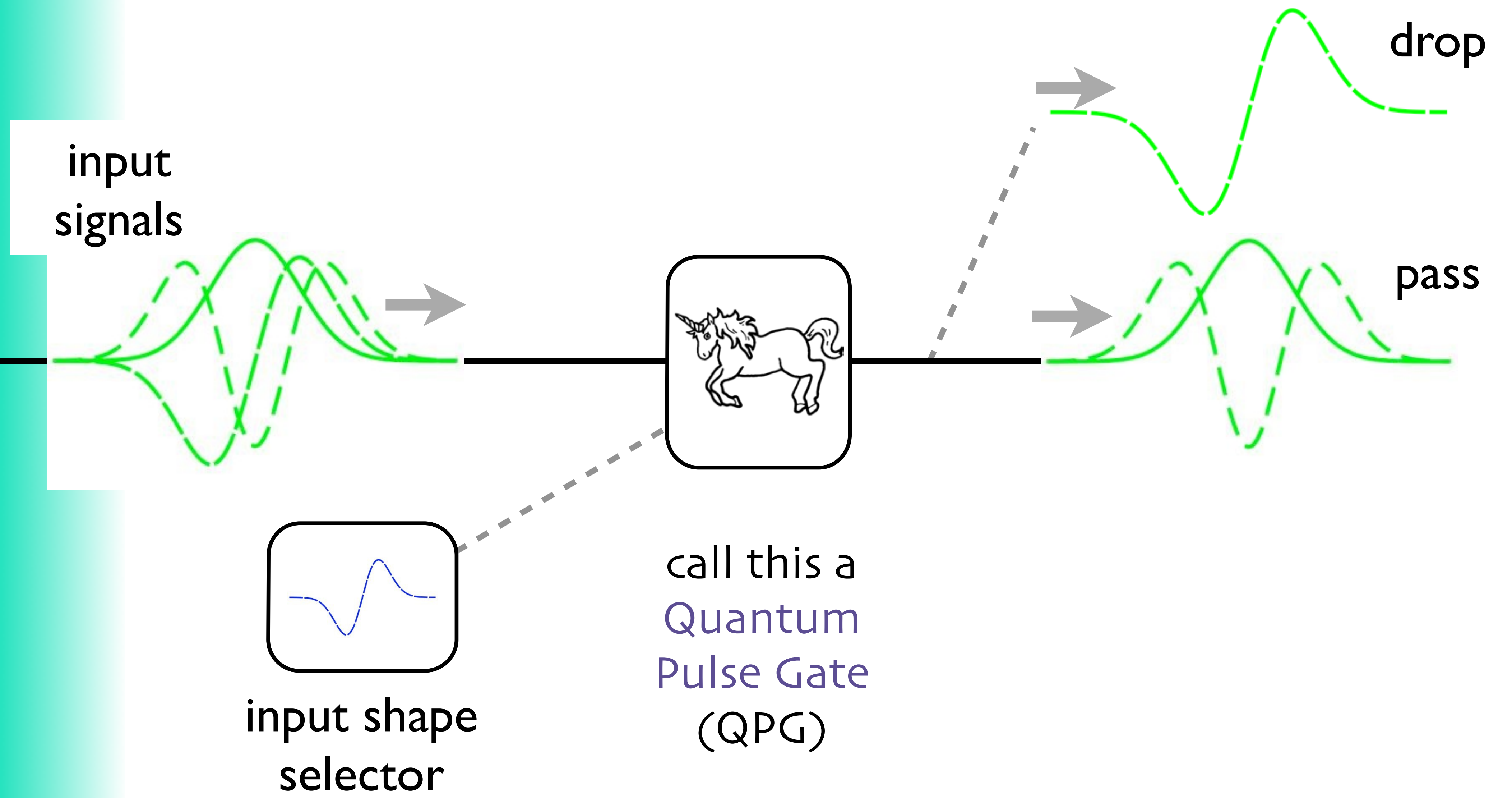
http://en.wikipedia.org/wiki/Mutually_unbiased_bases

TMs for Communication

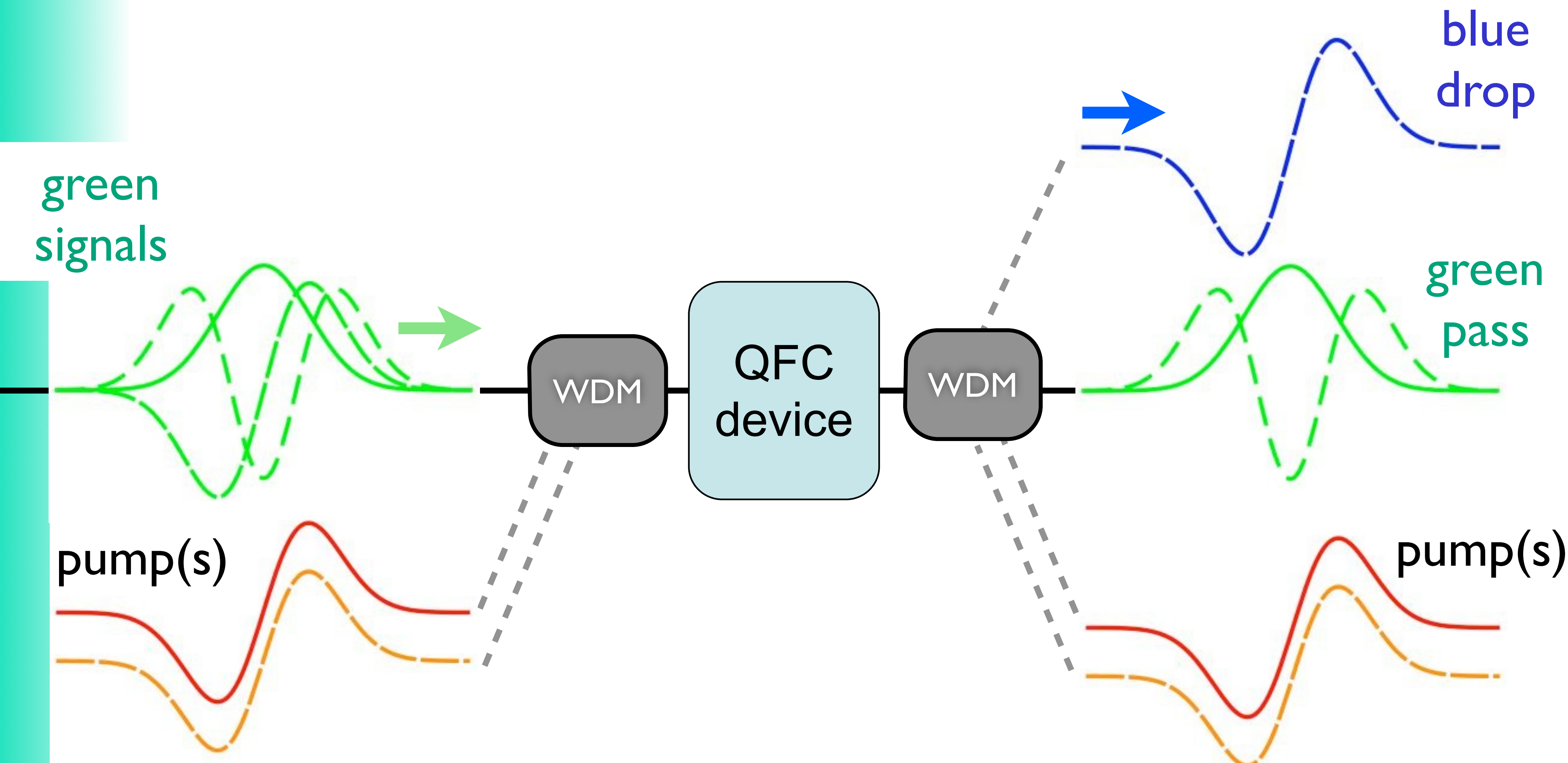


How to do TM Multiplexing?

The Mythical Device: Multiplexer/ Demultiplexer



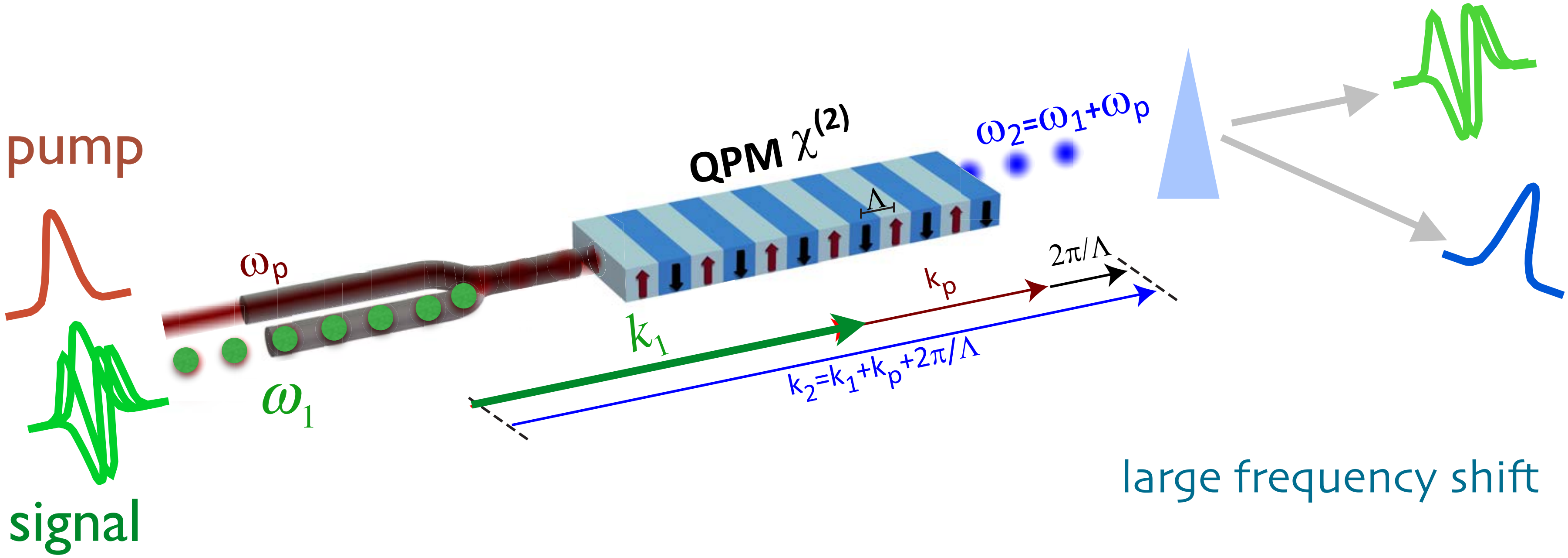
Quantum Frequency Conversion:
a method to spatially separate field-orthogonal TMs



The device is a linear-mode transformer. It treats single-photons packets the same as weak classical (coherent-state) fields.

Three-wave mixing: Eckstein, Brecht, Silberhorn, Opt. Express 19, 13770 (2010)
Four-wave mixing: McKinstrie, Mejling, Raymer, Rottwitt, Phys. Rev. A 85, 053829 (2012)

Quantum Frequency Conversion by
Three-wave mixing in NLO crystal waveguide



large frequency shift

Huang and Kumar, PRL (1992)

Rakher et al, Nat. Photonics **4,** 786 (2010)

from: MR and KS, Physics Today, **65**, 32 (2012)

Modeling QFC by Three-Wave Mixing

green signal

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_g}\frac{\partial}{\partial t}\right)A_g(z,t)=i\gamma A_p^*(z,t)A_b(z,t)$$

pump shape

blue signal

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_b}\frac{\partial}{\partial t}\right)A_b(z,t)=i\gamma A_p(z,t)A_g(z,t)$$

The equations are linear in A_g and A_b signal field operators.
Solution:

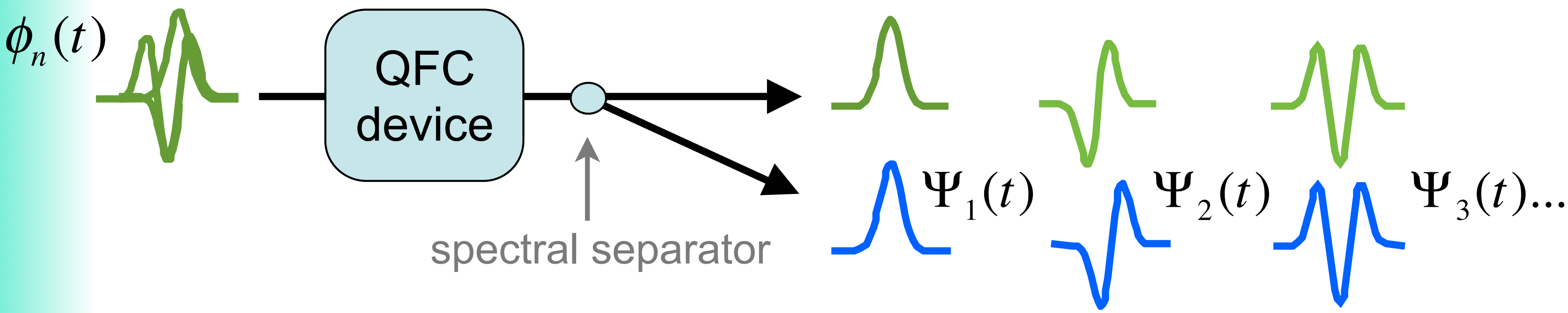
$$\begin{pmatrix} A_g(t) \\ A_b(t) \end{pmatrix}_{OUT} = \int^t dt' \begin{pmatrix} G_{gg}(t,t') & G_{gb}(t,t') \\ G_{bg}(t,t') & G_{bb}(t,t') \end{pmatrix} \begin{pmatrix} A_g(t') \\ A_b(t') \end{pmatrix}_{IN}$$

All quantum correlations can be calculated from Green functions.

Three-wave mixing: Reddy, MR, CM, AM, KR, Opt. Express 21, 13840 (2013)
Christ, Brecht, Mauerer, Silberhorn (NJP 2013)
Four-wave mixing: McGuinness, MR, CM, Opt. Express 19, 17876 (2011)

Figure of Merit for Temporal-mode Selectivity

$$G_{bg}(t,t') = -\sum_n \rho_n \Psi_n(t)\, \phi^*_n(t')$$



$$\eta_n = |\rho_n|^2 = \textit{conversion efficiency}$$

$$separability \equiv \frac{\eta_{Target}}{\sum_n \eta_n} \leq 1$$

$$S \equiv Selectivity \equiv separability \times \eta_{Target}$$

$$S = \frac{|\eta_{Target}|^2}{\sum_n \eta_n} \leq 1$$

ideally: $S = 1$

Reddy, MR, CM, AM, KR, Opt. Express 21, 13840 (2013)

Apply to three-wave mixing:

Schmidt Mode Decomposition of the Green functions

(singular-value decomposition)

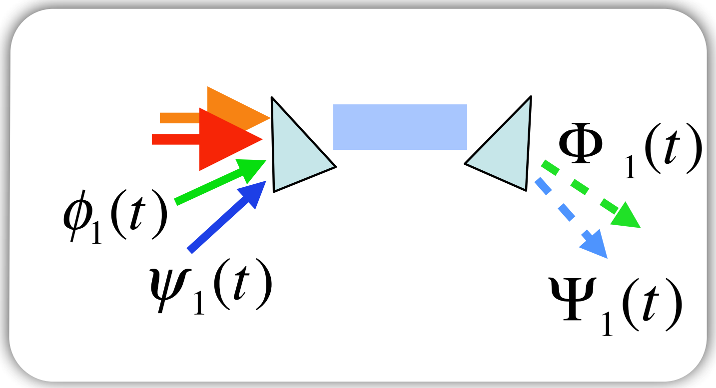
$$\begin{pmatrix} \hat{A}_g(t) \\ \hat{A}_b(t) \end{pmatrix}_{OUT} = \sum_n \int^t dt' \begin{pmatrix} \tau_n \Phi_n(t)\phi_n^*(t') & \rho_n \Phi_n(t)\psi_n^*(t') \\ -\rho_n \Psi_n(t)\phi_n^*(t') & \tau_n \Psi_n(t)\psi_n^*(t') \end{pmatrix} \begin{pmatrix} \hat{A}_g(t') \\ \hat{A}_b(t') \end{pmatrix}_{IN}$$

for each mode pair: $\rho_n^2 + \tau_n^2 = 1$ ρ_n^2 = conversion, τ_n^2 = nonconversion

Temporal Schmidt Modes reduce the problem to low-dimensional state space:

$$if \begin{pmatrix} \hat{A}_g(t') \\ \hat{A}_b(t') \end{pmatrix}_{IN} = \begin{pmatrix} \hat{a}_g \phi_1(t') \\ \hat{a}_b \psi_1(t') \end{pmatrix}$$

$$then \begin{pmatrix} \hat{A}_g(t) \\ \hat{A}_b(t) \end{pmatrix}_{OUT} = \begin{pmatrix} (\tau_1 \hat{a}_g + \rho_1 \hat{a}_b)\ \Phi_1(t) \\ (-\rho_1 \hat{a}_g + \tau_1 \hat{a}_b)\ \Psi_1(t) \end{pmatrix}$$



Operators undergo pair-wise beam-splitter-like transformation (Bloch-Messiah thm)

MR, HM, SVE, CM Opt. Commun. 238, 747 (2010)

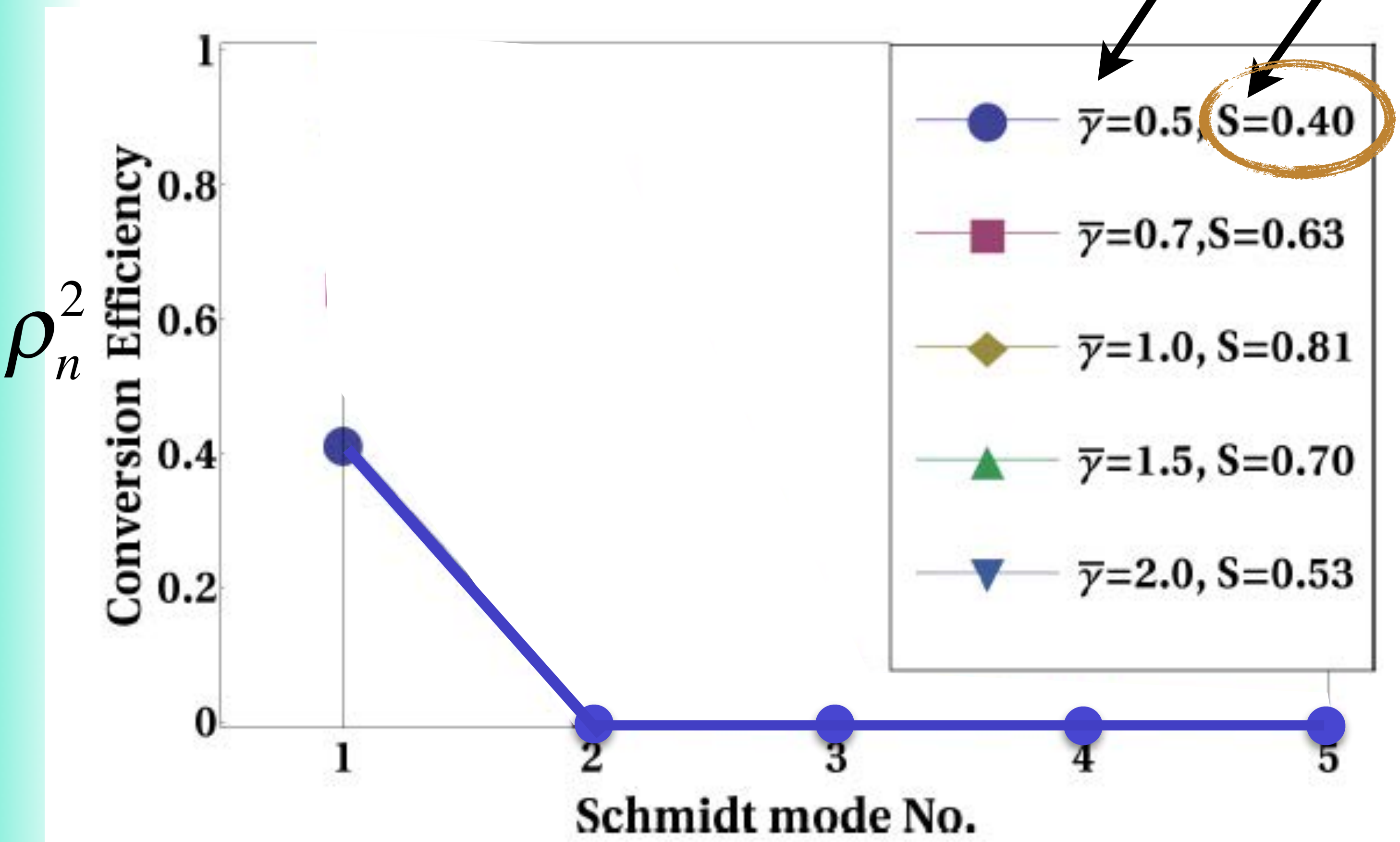
Three-Wave Mixing
conversion efficiency ρ_n^2

Optimum case: Pump velocity matches green signal velocity

ultrashort pump pulse

vary pump strength $\bar{\gamma} = \gamma \Big/ \left(\frac{1}{v_g} - \frac{1}{v_b} \right)$

S = selectivity



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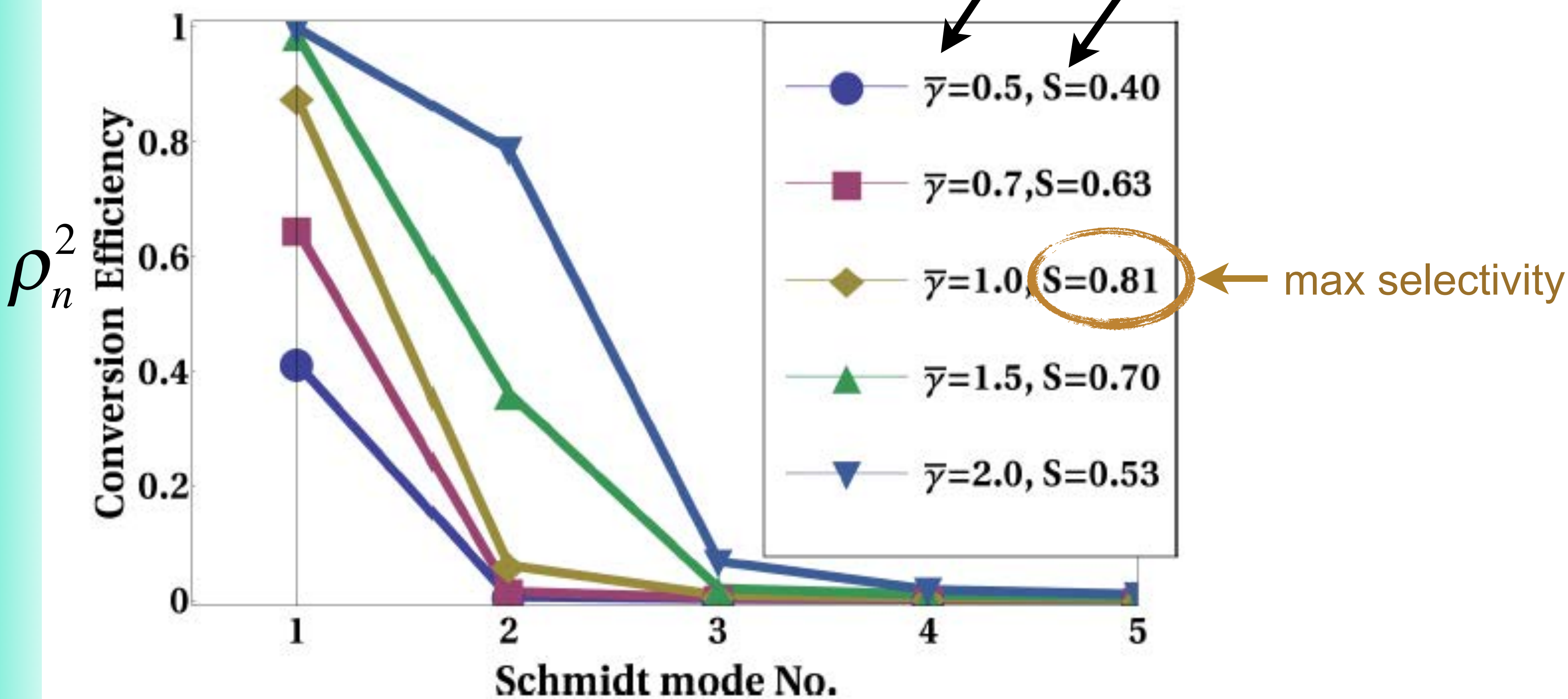
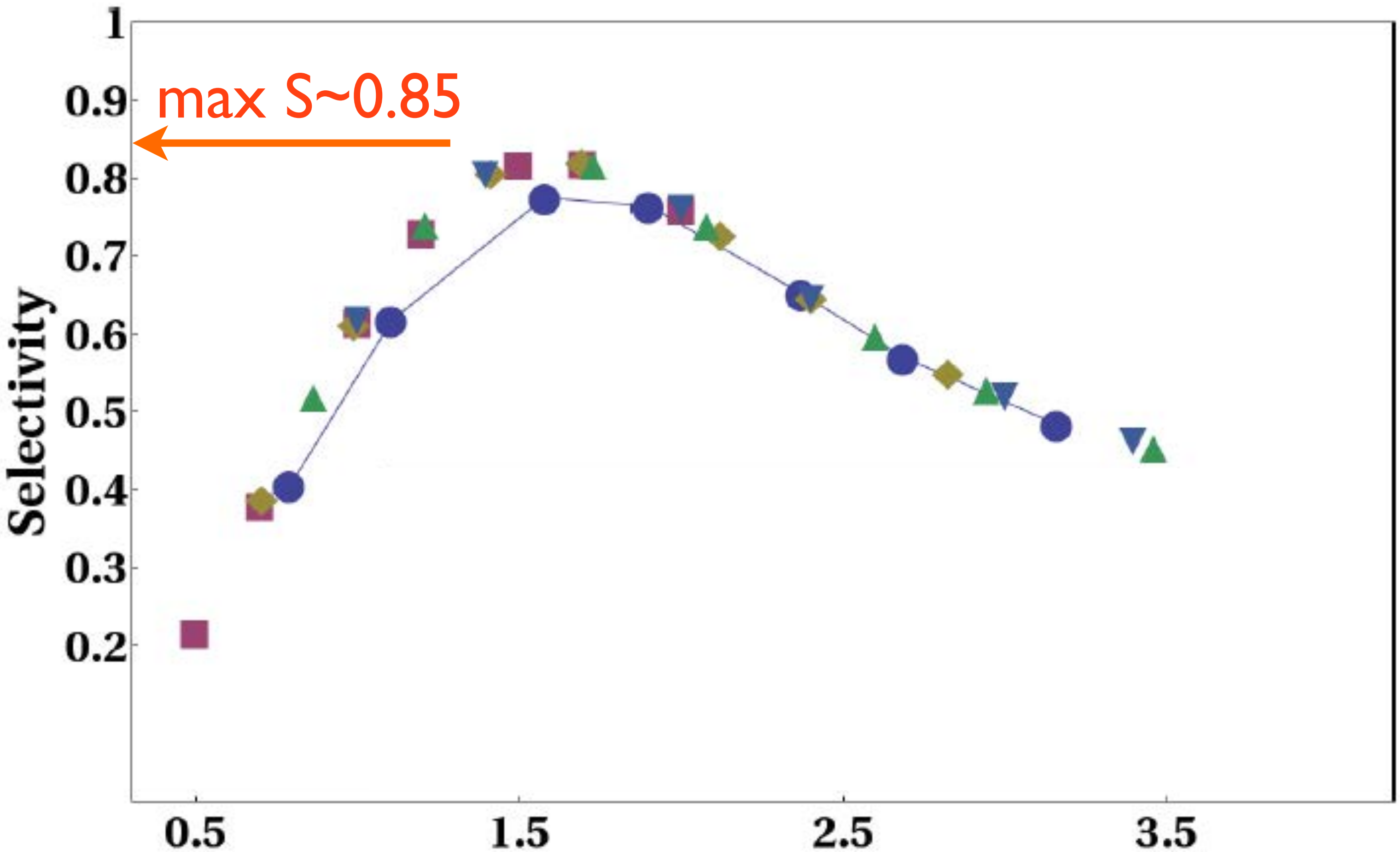


Three-Wave Mixing - Selectivity

Optimum case: Pump velocity matches green signal velocity

$$S = \frac{\left|\eta_{Target}\right|^2}{\sum_n \eta_n}$$



pump strength, $\gamma\sqrt{\frac{L}{v_g^{-1} - v_b^{-1}}}$

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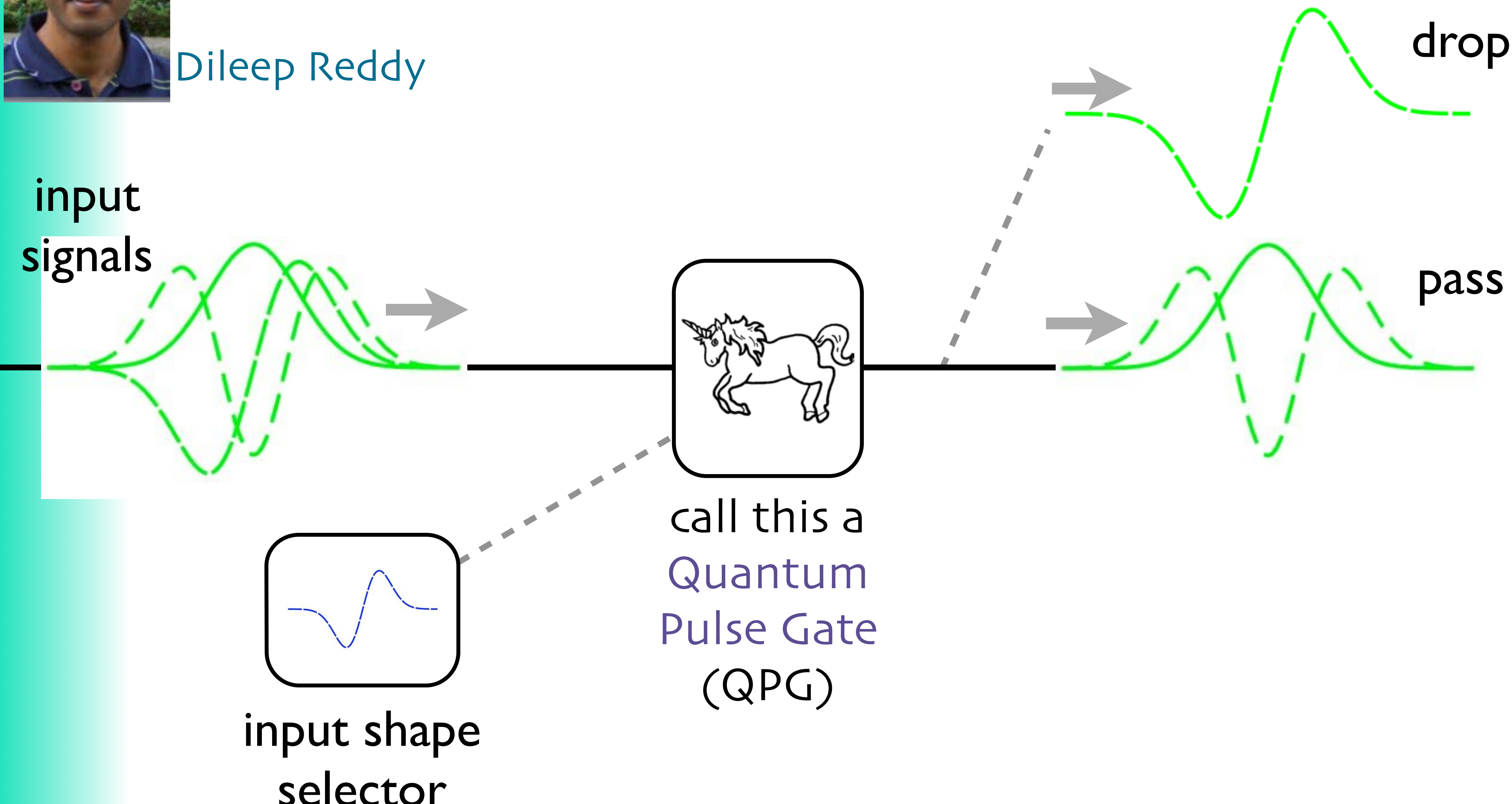
Efficient sorting of quantum-optical wave packets by temporal-mode interferometry

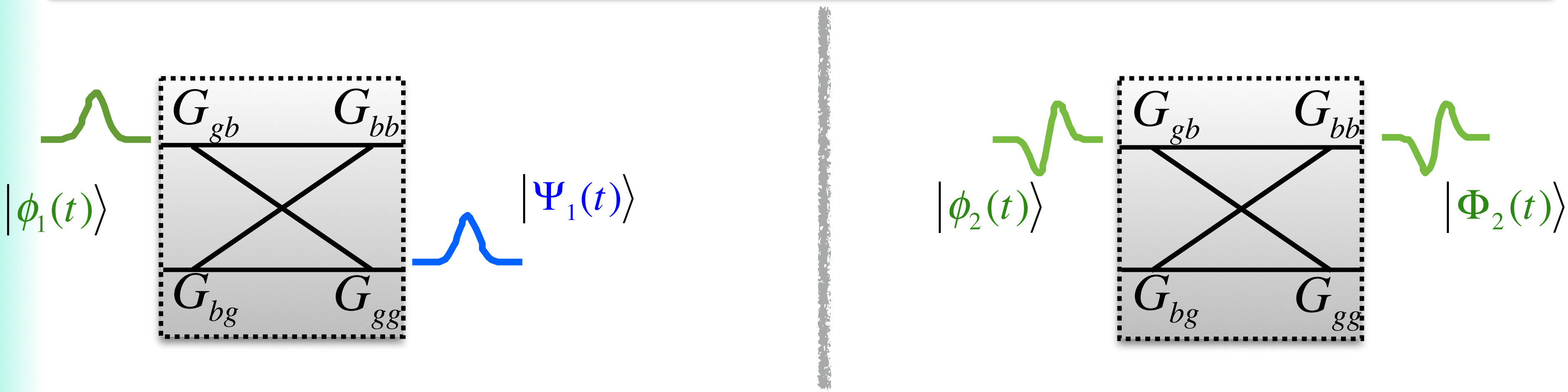
D. V. Reddy,[1] M. G. Raymer,[1,*] and C. J. McKinstrie[2]

Dileep Reddy



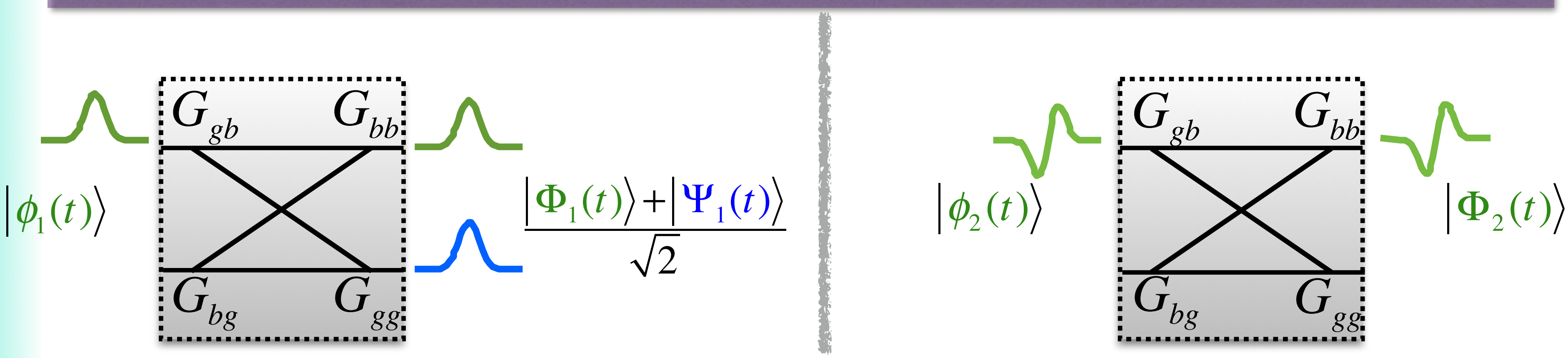
Achieving a drop/add device with 100% Selectivity

what we want:



100% conversion of target mode; zero conversion of all others

what we have:

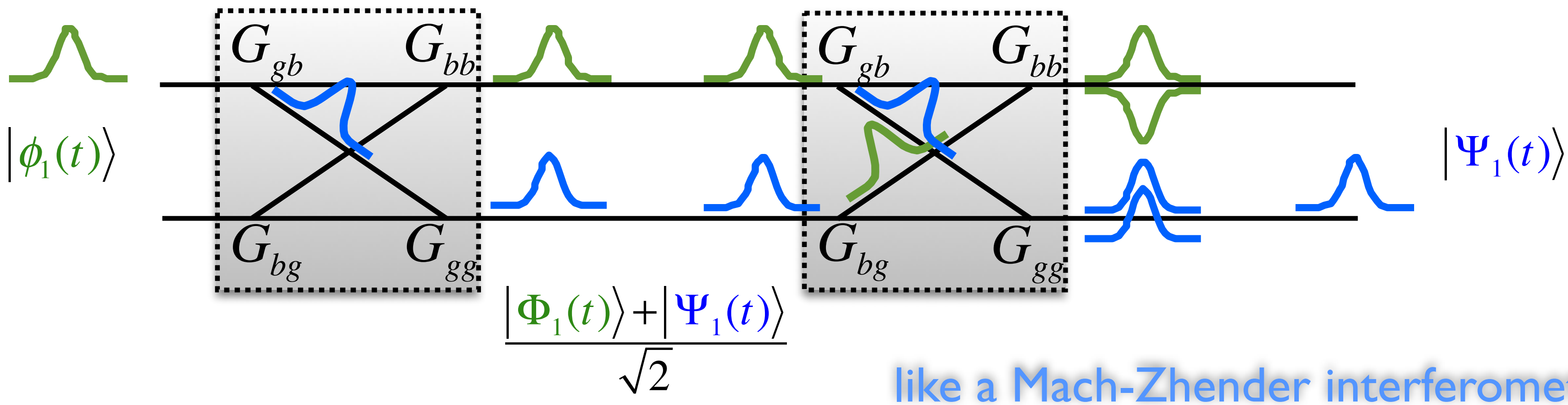


50% conversion of target mode; zero conversion of all others

Achieving a drop/add device with 100% Selectivity

combine two stages:

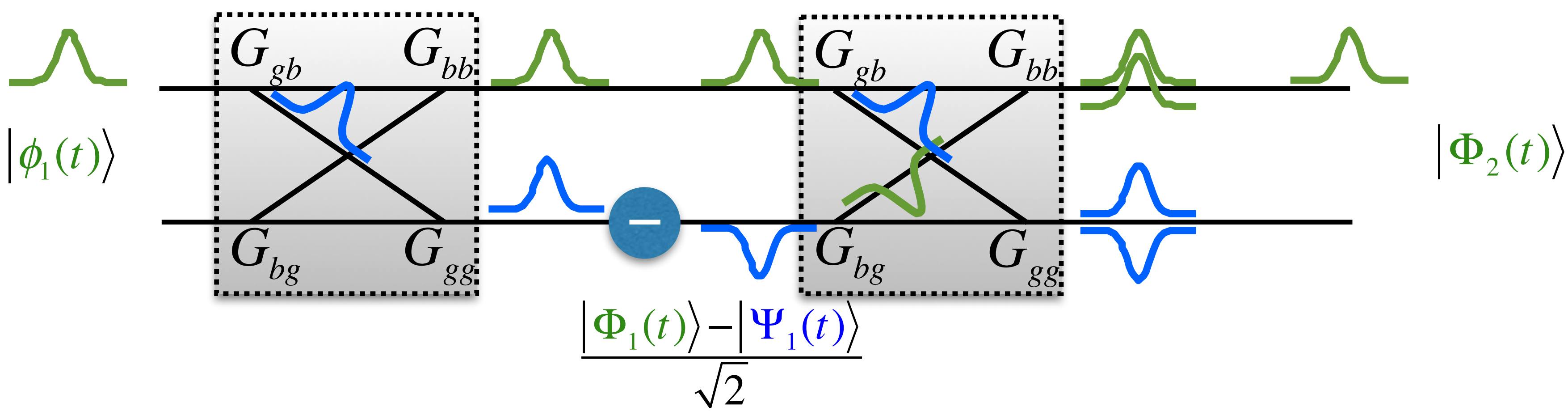
50% conversion of target mode in each stage



like a Mach-Zhender interferometer

flip the blue phase:

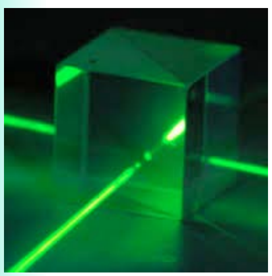
100% non-conversion of target mode



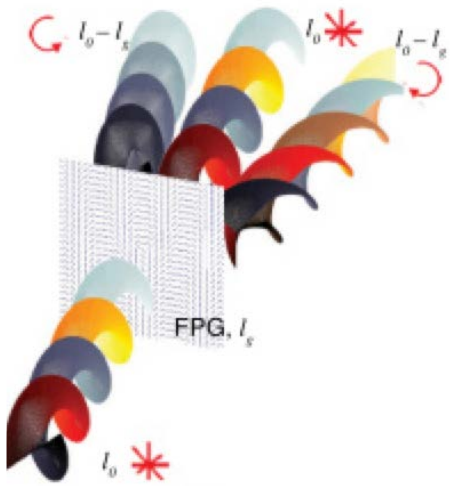
Completing the Tool Kit for Photons as a Quantum Information Resource

Photons have four degrees of freedom:

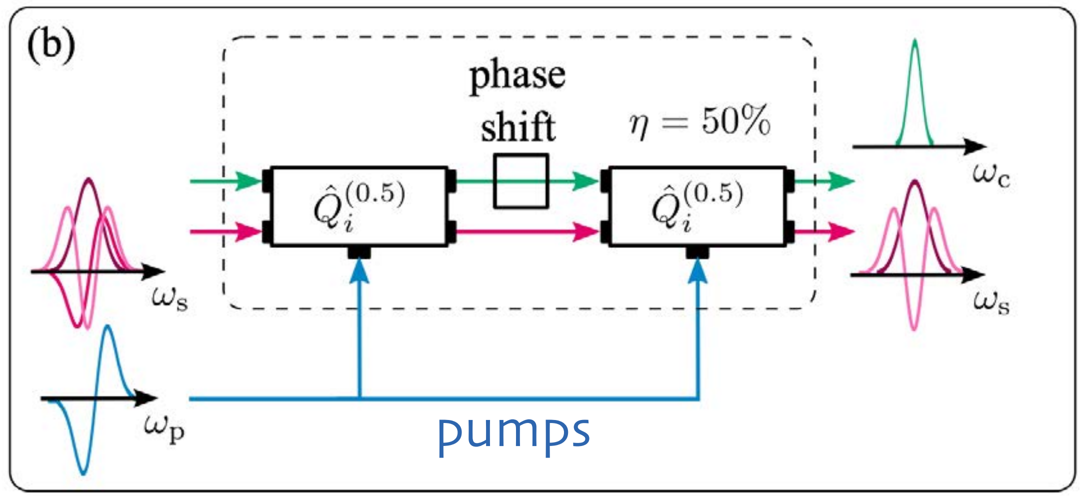
1. polarization
2&3. x,y transverse mode
4. energy or frequency



polarizing beam splitter (PBS)



forked hologram



quantum pulse gate (QPG)

Y. Li, Applied Optics 51, 34, 8236 (2012)

color labeling is changed from here on

What else is in the tool kit for Photon Temporal Modes?

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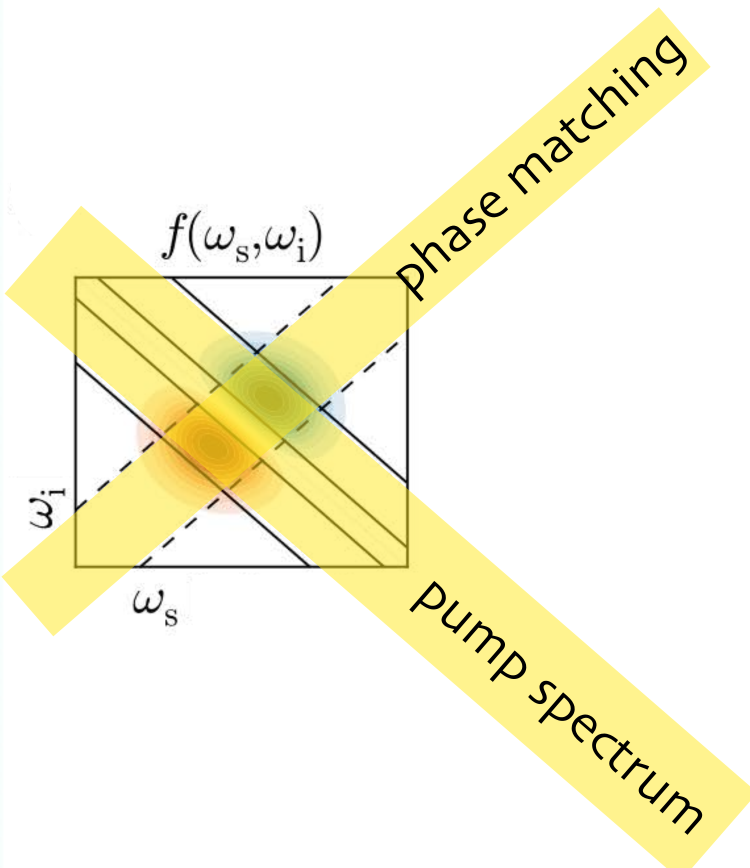
Generation of TM 'Entangled' Bi-Photon State

$$|\Psi\rangle = \sqrt{1-\varepsilon^2}\,|vac\rangle + \varepsilon\,\frac{|A_0\rangle_{signal}\,|A_0\rangle_{idler} + |A_1\rangle_{signal}\,|A_1\rangle_{idler}}{\sqrt{2}}$$



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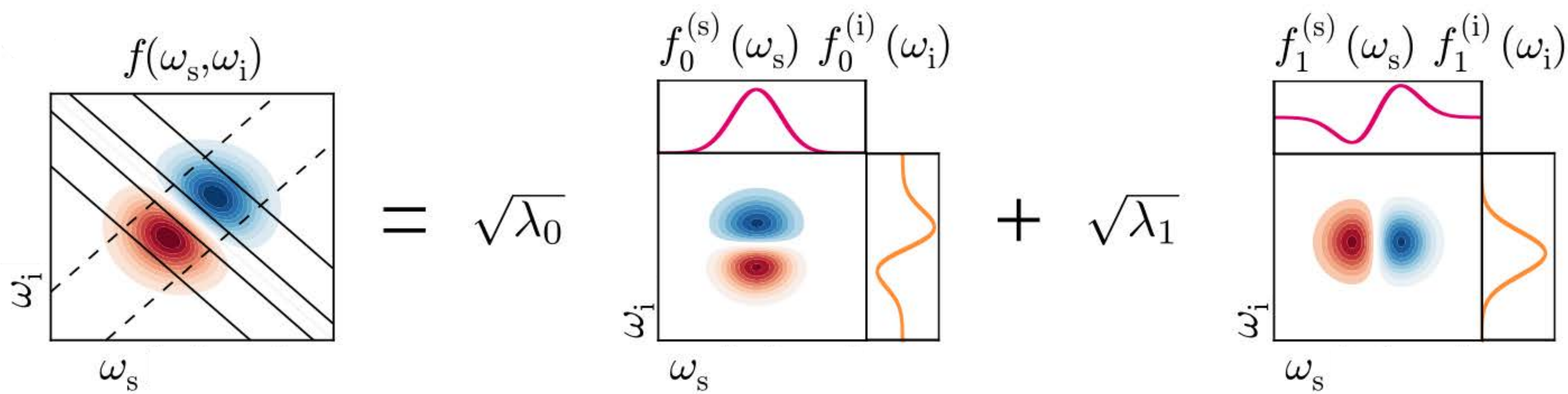
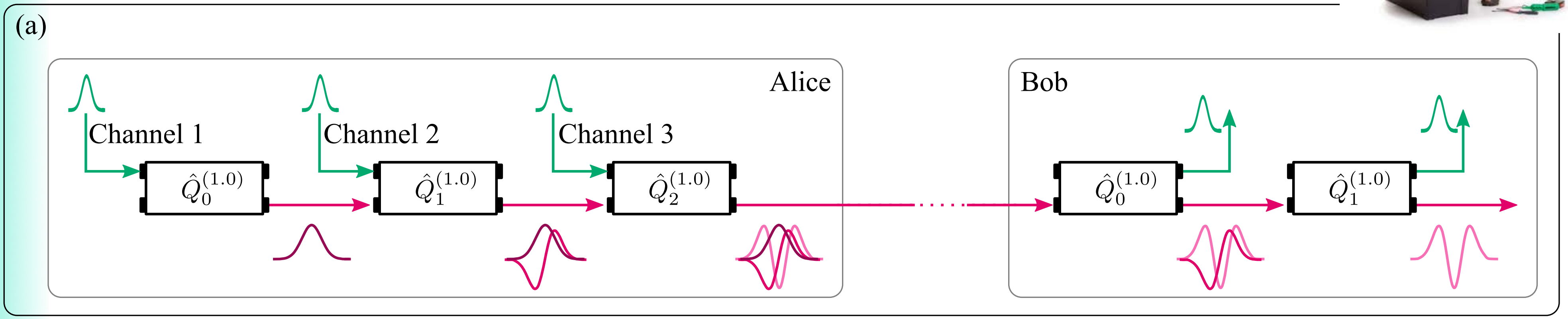
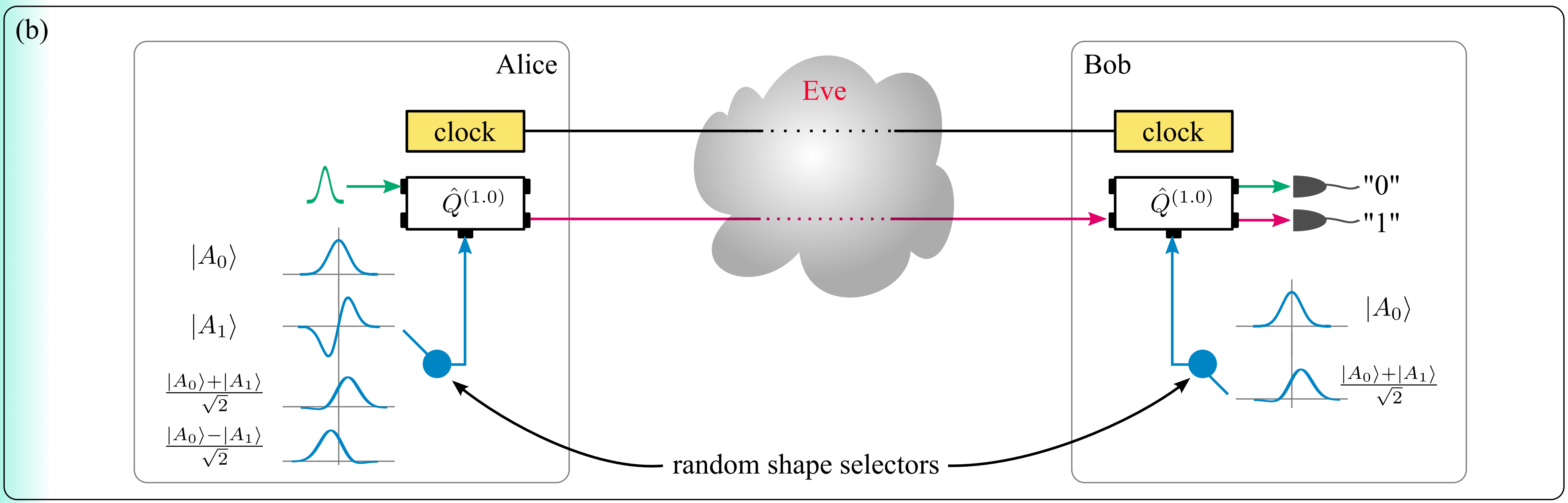


TMs for Communication

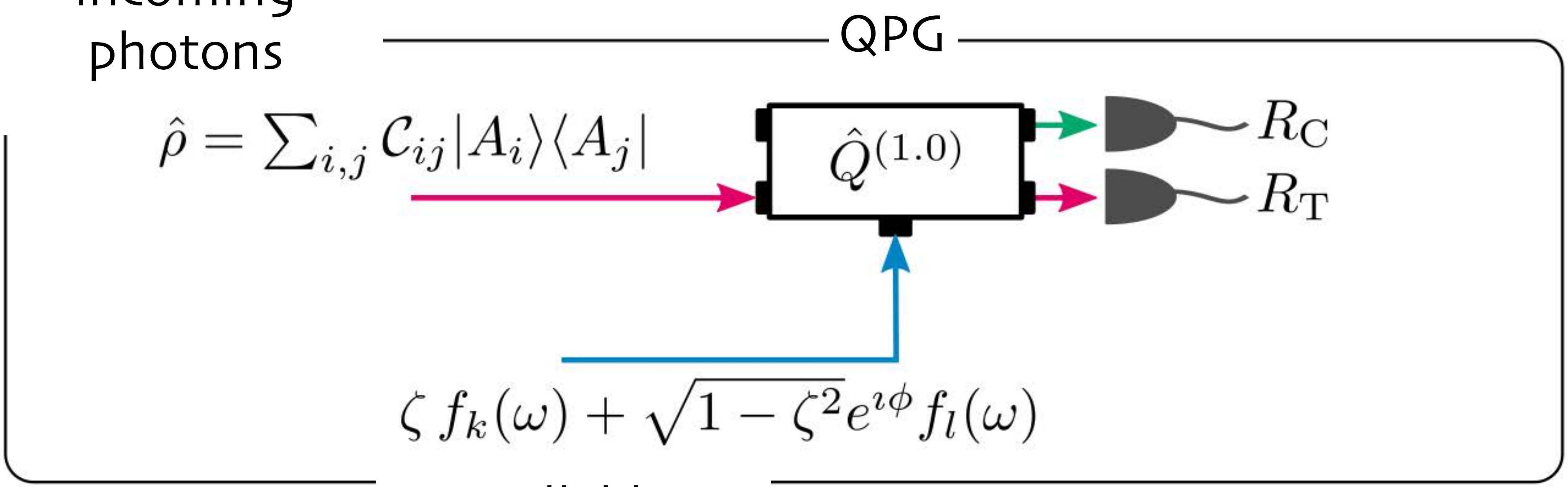


TMs for QKD

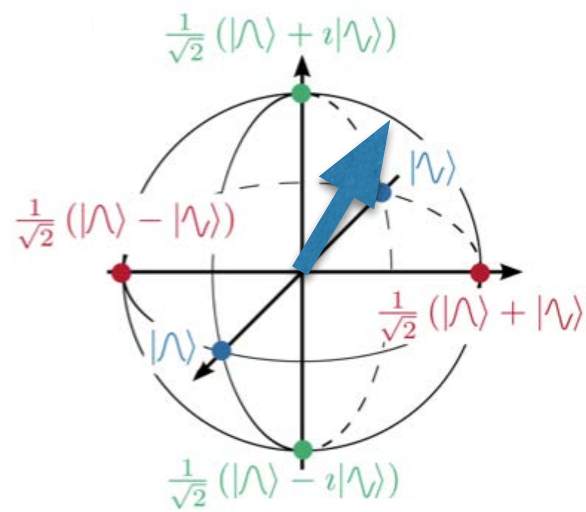


TM Quantum State Tomography

ensemble of incoming photons



controllable pump shapes

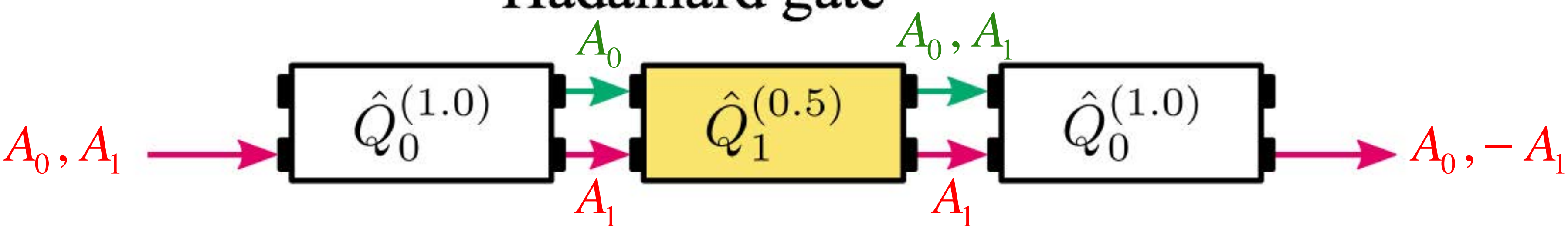


All Single-Photon Gate Operations

fix

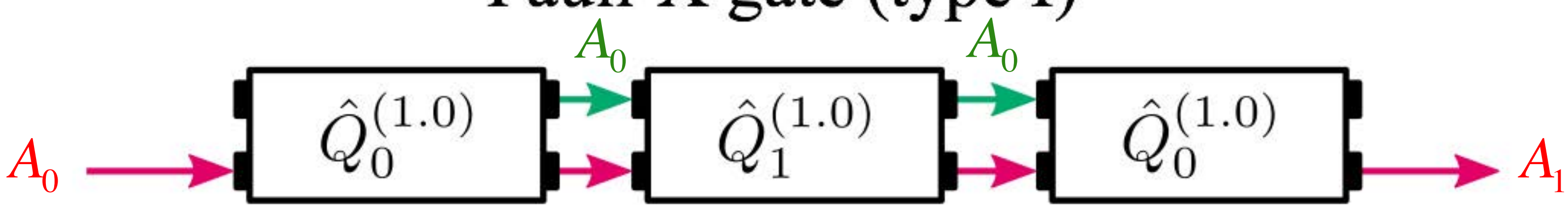
for example:

Hadamard gate



$$\hat{H} = \frac{|A_0\rangle + |A_1\rangle}{\sqrt{2}} \langle A_0| + \frac{|A_0\rangle - |A_1\rangle}{\sqrt{2}} \langle A_1|$$

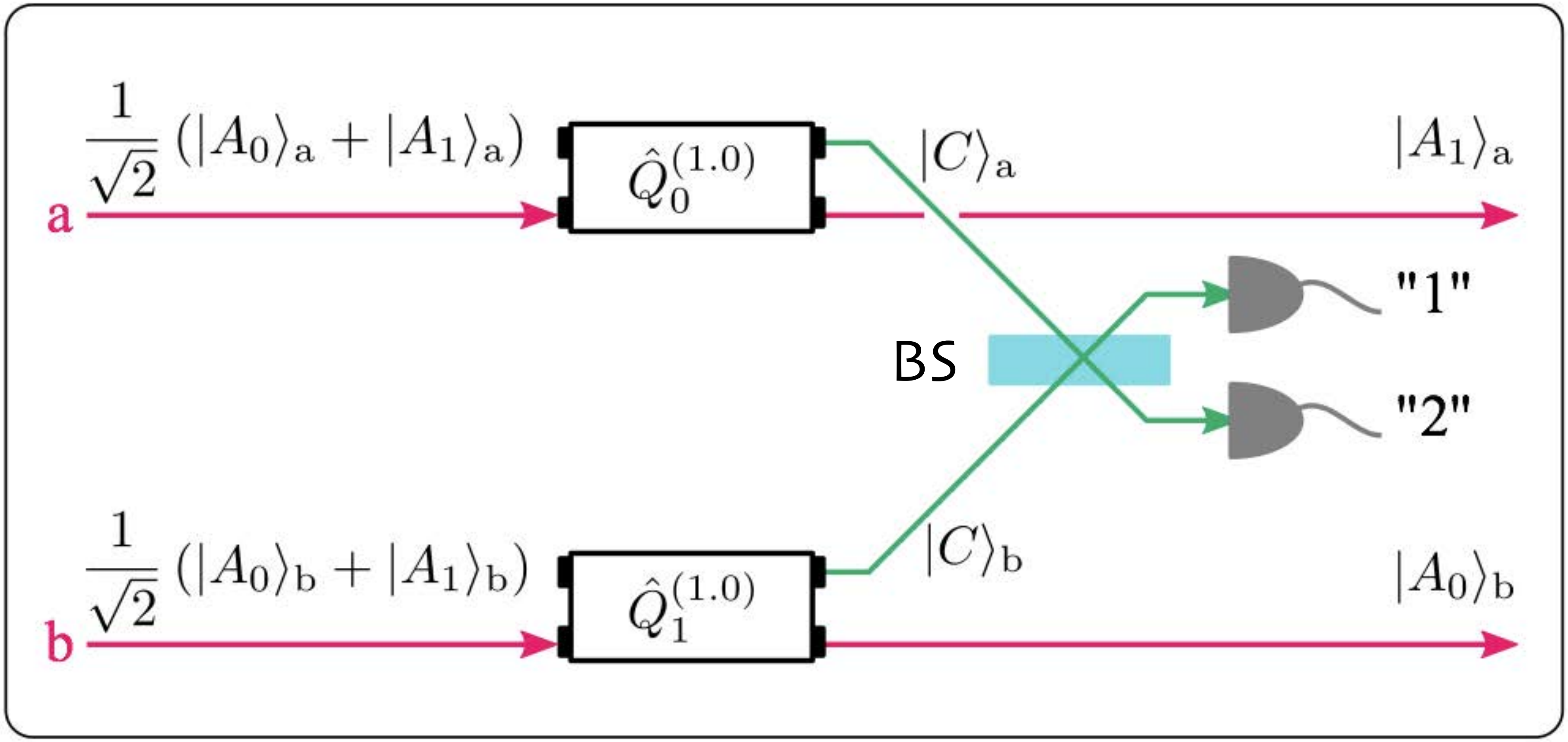
Pauli-X gate (type I)



$$\hat{X} = |A_1\rangle \langle A_0| + |A_0\rangle \langle A_1|$$

Multi-Photon Gate Operations

Cluster-state operations:

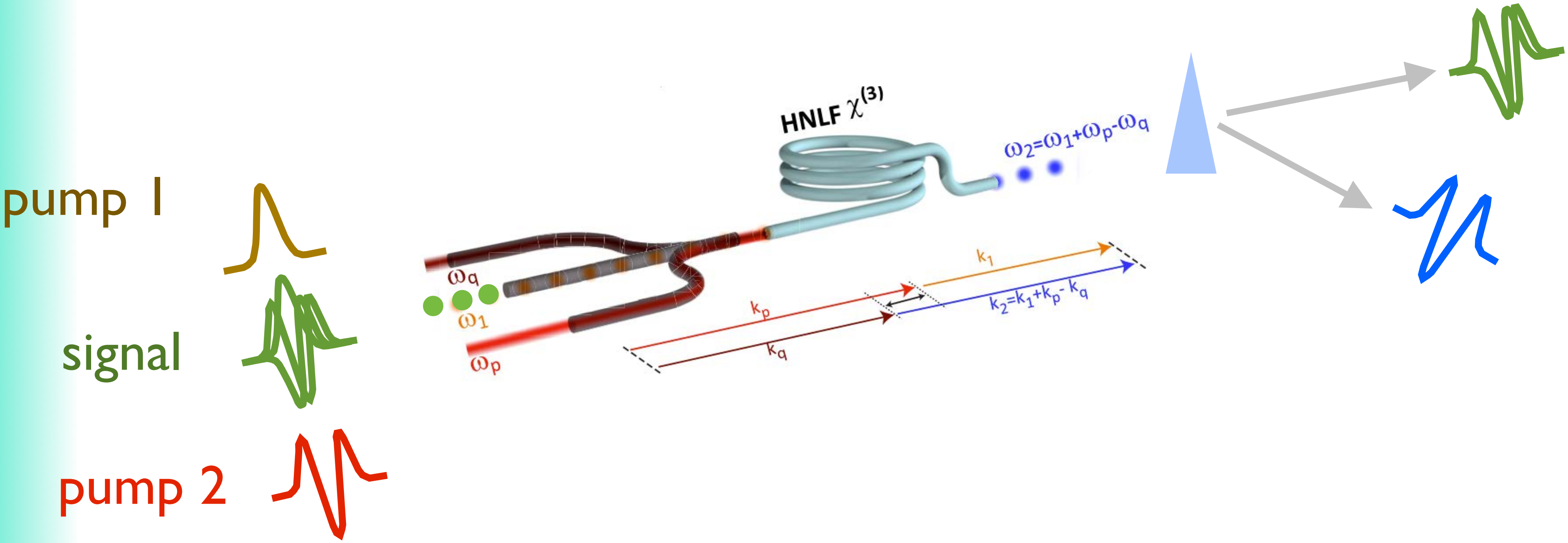


Two TM qubits in spatial beams a and b are fused using two QPGs, which select different "red" TM components and selectively frequency convert them. Then, the "green" outputs of the QPGs are interfered at a 50/50 beamsplitter and detected.

One More Tool?

QPG using Four-Wave Mixing

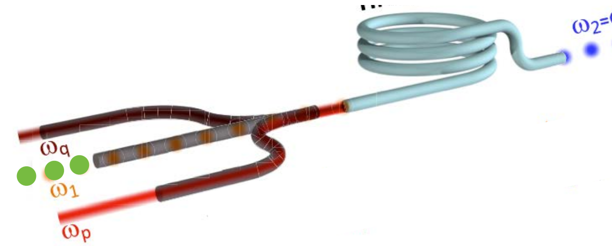
Lasse Mejling, Jesper Christensen,
Karsten Rottwitt, Colin McKinstrie



One pump selects the input mode shape;
Other pump determines the output mode shape.

CM, LM, MR, KR, PRA 053829 (2012)

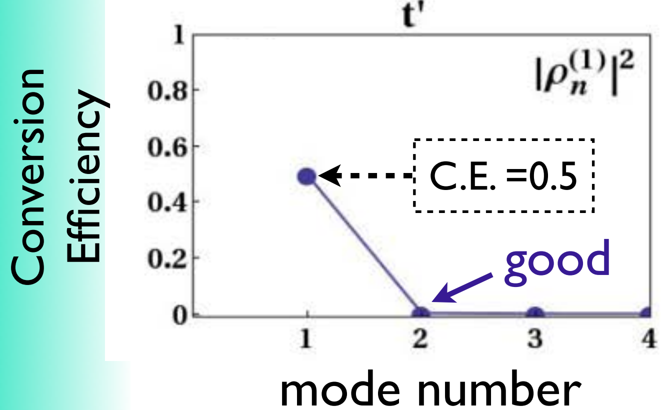
Four-Wave Mixing

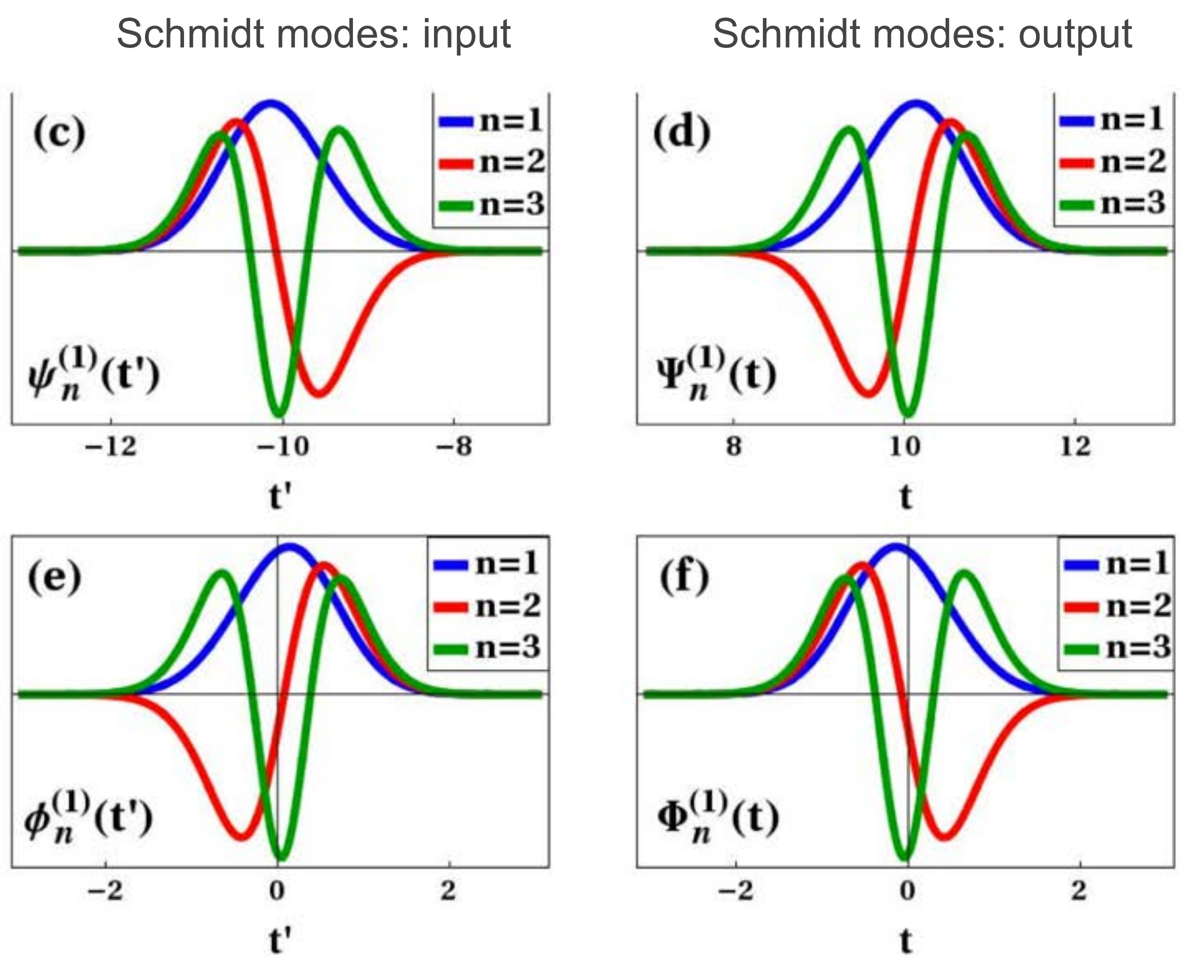


Much the same as TWM, with the shape of the medium replaced by the shape of the second pump.

Optimum case: pump 1 velocity matches green signal velocity and pump 2 matches blue signal velocity. Complete collision occurs.

$\bar{\gamma} = 0.83$





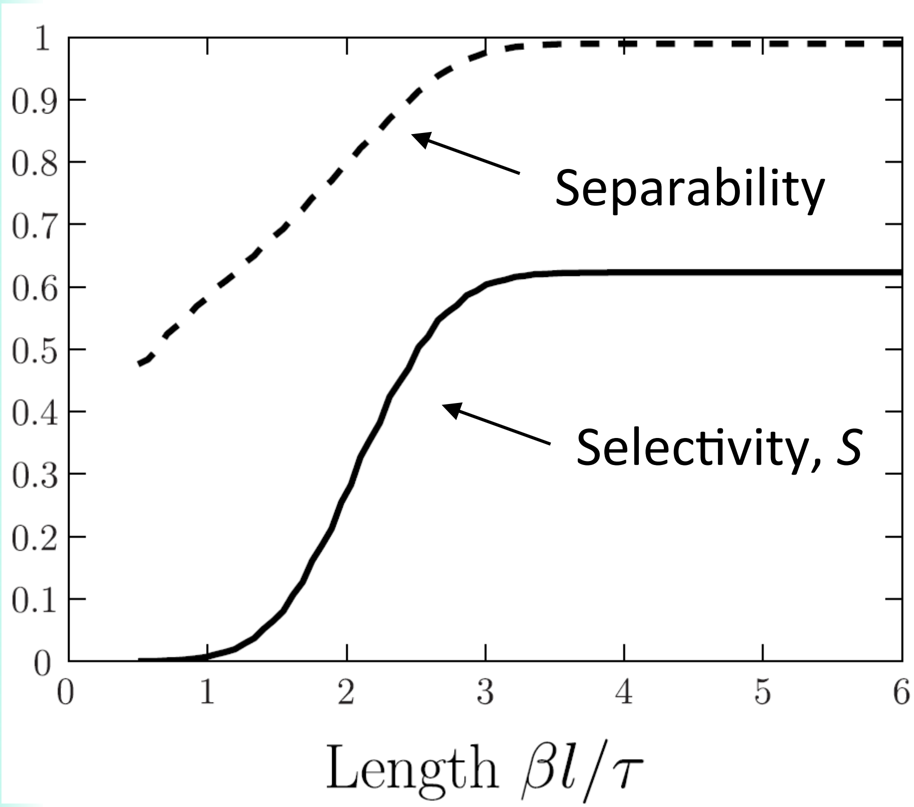
Selectivity ~ 0.5

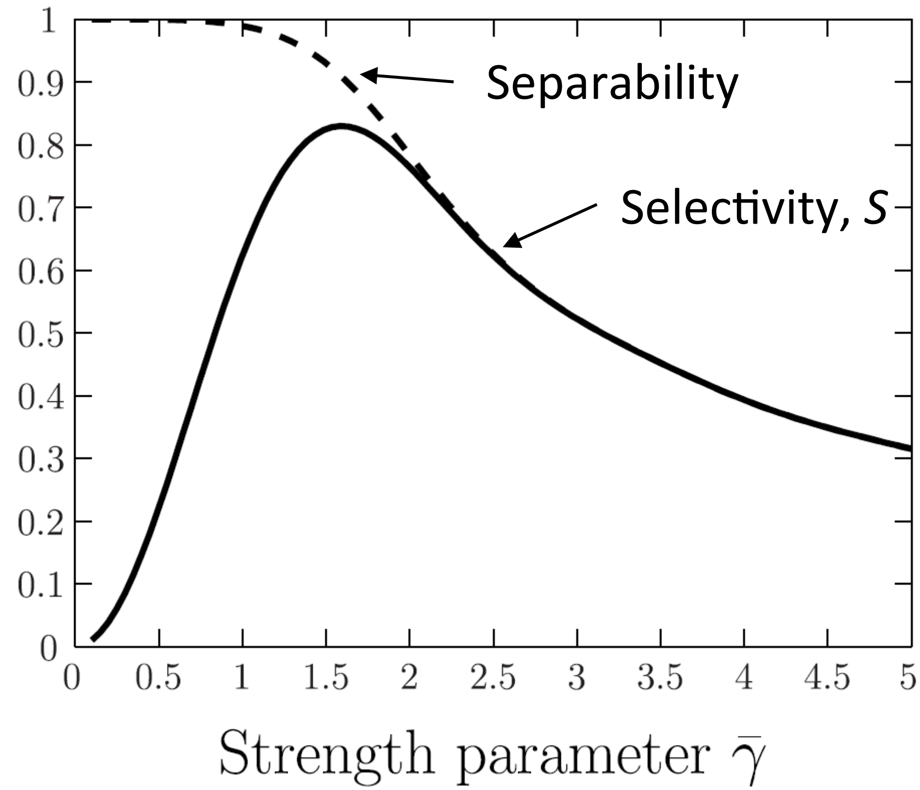
increase $\bar{\gamma}$

Four-Wave Mixing

$$separability \equiv \frac{\eta_{Target}}{\sum_n \eta_n} \leq 1$$

$$Selectivity \quad S = \frac{\left|\eta_{Target}\right|^2}{\sum_n \eta_n} \leq 1$$



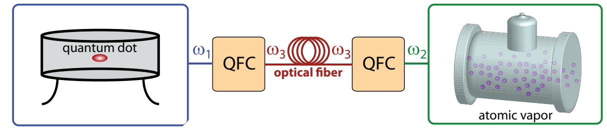


With single stage cannot exceed S = 0.85

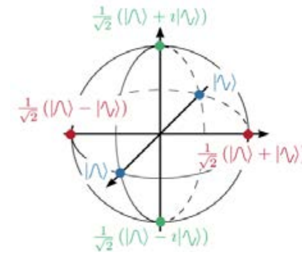
With two stages can exceed S = 0.95

Jesper Christensen, L Mejling, K Rottwitt, CM, DR, MR

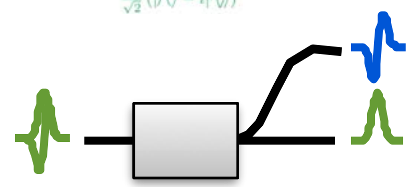
Quantum conversion between frequency channels, for Quantum Internet



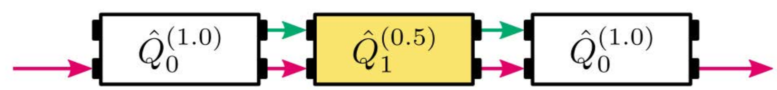
Temporal-mode qubits and qudits.



Quantum pulse gate: Temporal-mode sorting and analysis.



→ A new framework for quantum information



Dileep Reddy, C. J. McKinstrie, Benjamin Brecht, Christine Silberhorn, Lasse Mejling, Jesper Christensen, Karsten Rottwitt