Discussion on Quantum Nonlinear Optics

Michael Raymer

Oregon Center for Optics

European settlement: 1836

Statehood: 1859

Population: 3,830,000

400 Wineries

180 Micro-breweries

University of Oregon: 1873

OREGON

Sketch the q-quadrature wave function for:

1. coherent state $|\alpha\rangle$
2. n=1 state
3. squeezed vacuum state
4. Schroedinger cat state $|\alpha\rangle + |-\alpha\rangle$

Sketch the p-quadrature wave function for:

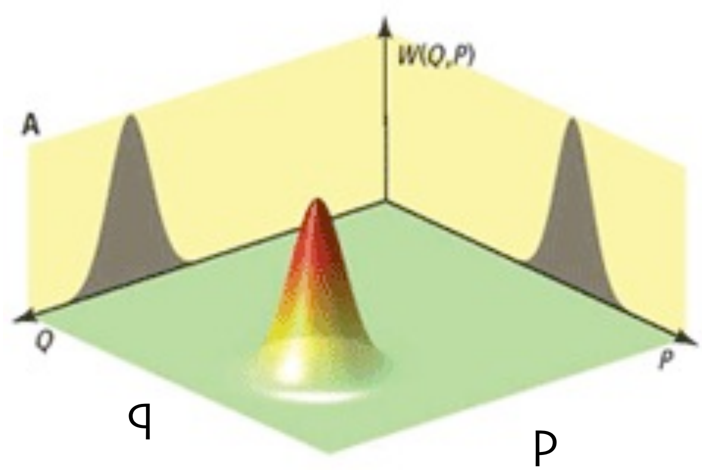
1. coherent state $|\alpha\rangle$
2. n=1 state
3. squeezed vacuum state
4. Schroedinger cat state $|\alpha\rangle + |-\alpha\rangle$

OREGON

Sketch the projected distributions Pr(q) and Pr(p) for:

1. coherent state $|\alpha\rangle$
2. n=1 state
3. squeezed vacuum state
4. Schroedinger cat state $|\alpha\rangle+|-\alpha\rangle$

for example:



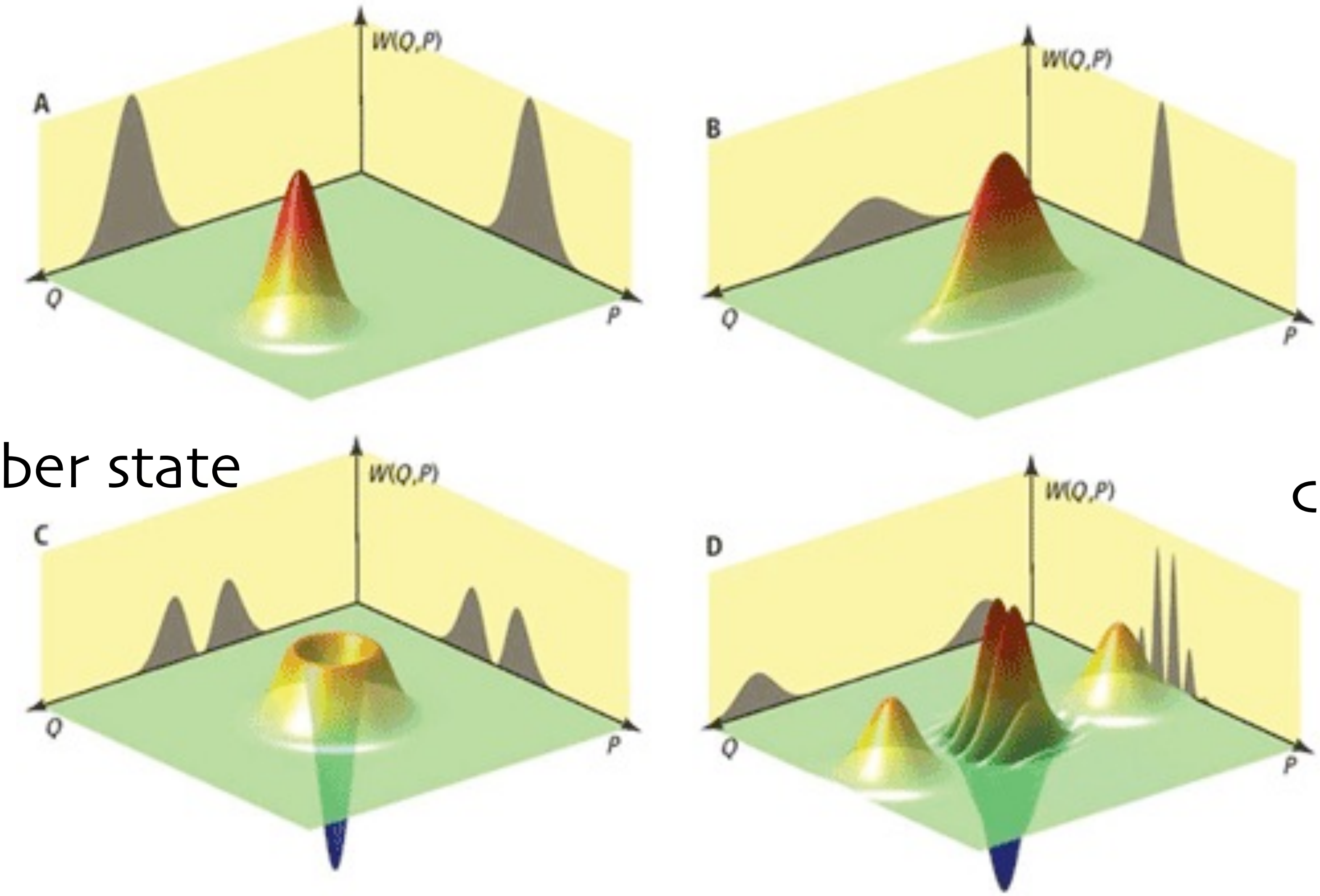
Some Wigner Distributions

coherent state

squeezed vacuum state

number state
n=1

cat state



http://www.sciencemag.org/content/332/6027/313/F1.expansion.html

CREDIT: ALEXEI OURJOUMTSEV

OREGON

What is a photon?

Think of a way, not described in the lecture, for creating a single-photon state.
How could you verify by a measurement the photon was created, without detecting it?

Discuss with your neighbor(s)

If a single photon hits a beam splitter, does it create entanglement?

Think of an argument for YES and an argument for NO.

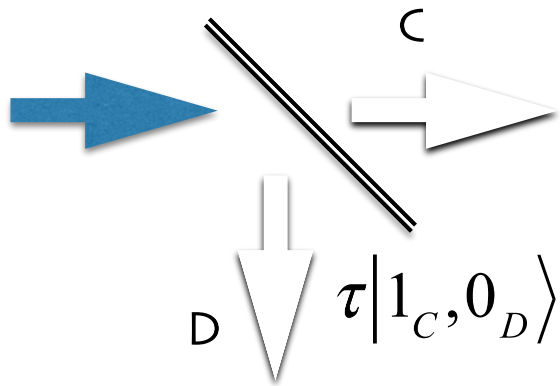
How could you verify your answer by an experiment?

If a single photon hits a beam splitter, does it create entanglement?

YES - Mode Entanglement

How could you verify your answer by an experiment?

$|1\rangle$

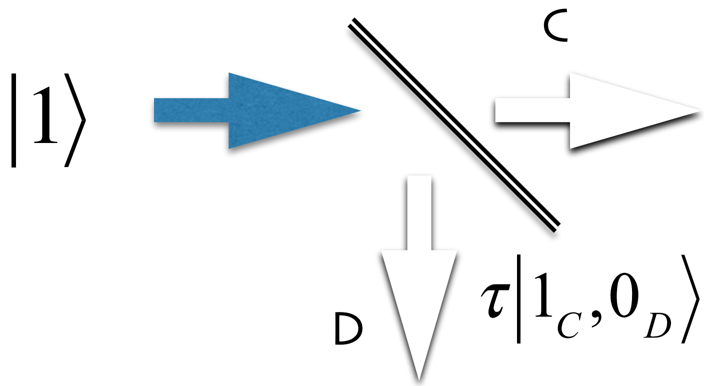


$$\tau|1_C,0_D\rangle+\rho|0_C,1_D\rangle$$

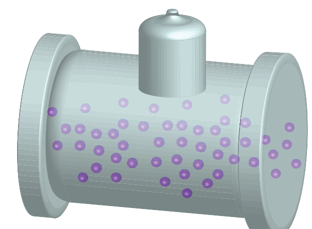
If a single photon hits a beam splitter, does it create entanglement?

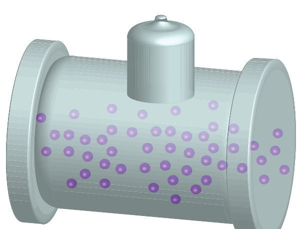
YES - Mode Entanglement

How could you verify your answer by an experiment?



$$\tau|1_C,0_D\rangle+\rho|0_C,1_D\rangle$$

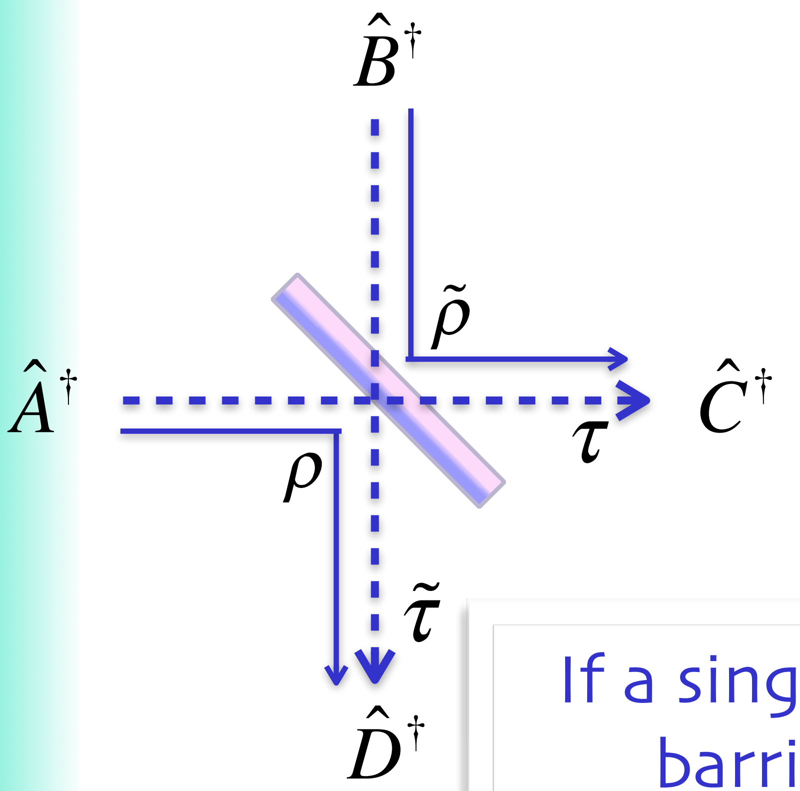




PROOF: If a single-photon state hits a beam splitter, it creates Mode Entanglement

creation operators obey inverse relation:

$$\begin{pmatrix} \hat{A}^\dagger \\ \hat{B}^\dagger \end{pmatrix} = \begin{pmatrix} \tau & \rho \\ \tilde{\rho} & \tilde{\tau} \end{pmatrix} \begin{pmatrix} \hat{C}^\dagger \\ \hat{D}^\dagger \end{pmatrix}$$



$$input\ state: \hat{A}^\dagger |vac\rangle = |1_A, 0_B\rangle$$

$$transforms\ to:$$

$$\hat{A}^\dagger |vac\rangle = \left(\tau \hat{C}^\dagger + \rho \hat{D}^\dagger\right)|vac\rangle$$

$$= \tau |1_C, 0_D\rangle + \rho |0_C, 1_D\rangle$$

There is only one photon; it is shared between modes.
The C, D modes are in an entangled state.

The BS is a 'global' operation involving both modes. Any subsequent local operations on the separated modes C, D cannot increase the entanglement.

If a single electron hits a partially reflecting barrier, does it create entanglement?

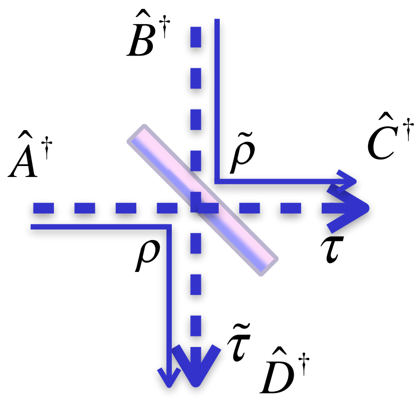
OREGON

Proof: TWO-PHOTON INTERFERENCE

Two single-photon states in identical TMs hit a 50/50 beam splitter.
Prove the output state is $\frac{1}{2}\left(\left|2_C,0_D\right\rangle+\left|0_C,2_D\right\rangle\right)$

$$\tau=\tilde{\tau}=\rho=1/\sqrt{2},\ \tilde{\rho}=-1/\sqrt{2}$$

$$\begin{pmatrix}\hat{C}\\ \hat{D}\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{A}\\ \hat{B}\end{pmatrix}$$



$$input\ state: \hat{A}^\dagger B^\dagger\left|vac\right\rangle=\left|1_A,1_B\right\rangle$$

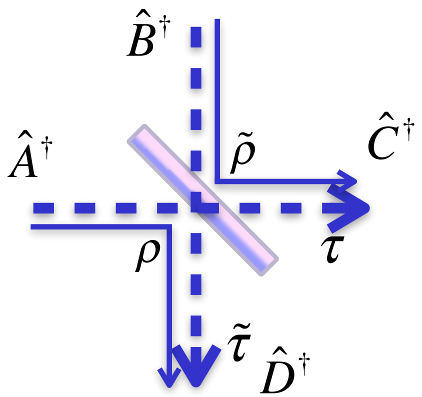
$$transforms\ to:$$

Proof: TWO-PHOTON INTERFERENCE

Two single-photon states in identical TMs hit a 50/50 beam splitter.
Prove the output state is $\frac{1}{2}\left(\left|2_C,0_D\right\rangle+\left|0_C,2_D\right\rangle\right)$

$$\tau=\tilde{\tau}=\rho=1/\sqrt{2},\ \tilde{\rho}=-1/\sqrt{2}$$

$$\begin{pmatrix}\hat{C}\\ \hat{D}\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{A}\\ \hat{B}\end{pmatrix}$$



$$\begin{pmatrix}\hat{C}\\ \hat{D}\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{A}\\ \hat{B}\end{pmatrix}$$

$$\begin{pmatrix}\hat{A}\\ \hat{B}\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ -1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{C}\\ \hat{D}\end{pmatrix}$$

$$\begin{pmatrix}\hat{A}^\dagger\\ \hat{B}^\dagger\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ -1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{C}^\dagger\\ \hat{D}^\dagger\end{pmatrix}$$

$$input\ state:\hat{A}^\dagger B^\dagger\left|vac\right\rangle=\left|1_A,1_B\right\rangle$$

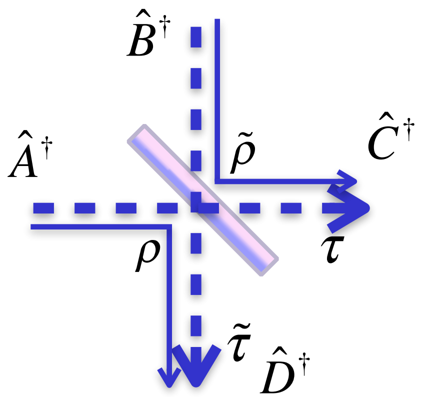
$$transforms\ to:$$

Proof: TWO-PHOTON INTERFERENCE

Two single-photon states in identical TMs hit a 50/50 beam splitter.
Prove the output state is $\frac{1}{2}\left(\left|2_C,0_D\right\rangle+\left|0_C,2_D\right\rangle\right)$

$$\tau=\tilde{\tau}=\rho=1/\sqrt{2},\ \ \tilde{\rho}=-1/\sqrt{2}$$

$$\begin{pmatrix}\hat{C}\\ \hat{D}\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{A}\\ \hat{B}\end{pmatrix}$$



$$\begin{pmatrix}\hat{C}\\ \hat{D}\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{A}\\ \hat{B}\end{pmatrix}$$

$$\begin{pmatrix}\hat{A}\\ \hat{B}\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ -1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{C}\\ \hat{D}\end{pmatrix}$$

$$\begin{pmatrix}\hat{A}^\dagger\\ \hat{B}^\dagger\end{pmatrix}=\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ -1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\hat{C}^\dagger\\ \hat{D}^\dagger\end{pmatrix}$$

$$input\ state: \hat{A}^\dagger B^\dagger\left|vac\right\rangle=\left|1_A,1_B\right\rangle$$

$$transforms\ to:$$

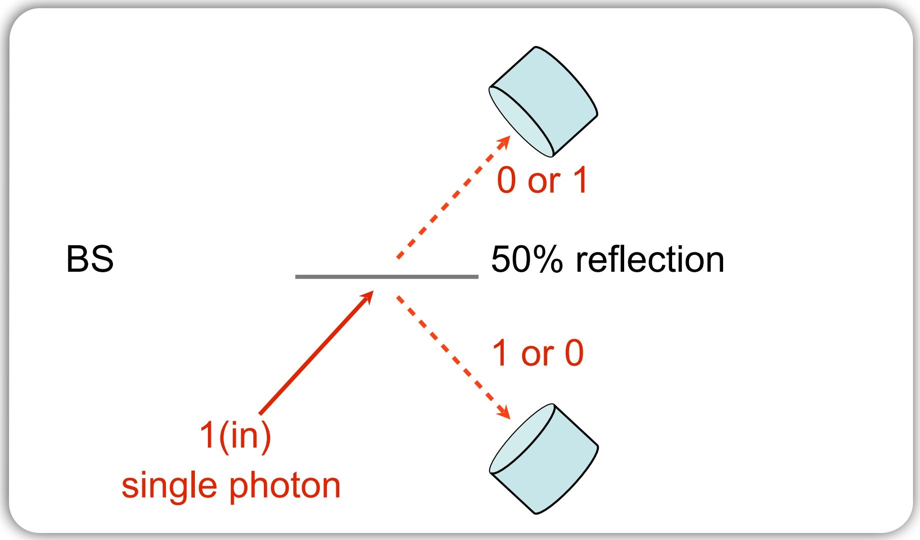
$$\hat{A}^\dagger B^\dagger\left|vac\right\rangle=\left(\frac{1}{\sqrt{2}}\hat{C}^\dagger+\frac{1}{\sqrt{2}}\hat{D}^\dagger\right)\left(\frac{-1}{\sqrt{2}}\hat{C}^\dagger+\frac{1}{\sqrt{2}}\hat{D}^\dagger\right)\left|vac\right\rangle$$

$$=\frac{1}{2}\left|2_C,0_D\right\rangle+\left(\frac{-1}{2}+\frac{1}{2}\right)\left|1_C,1_D\right\rangle+\frac{1}{2}\left|0_C,2_D\right\rangle$$

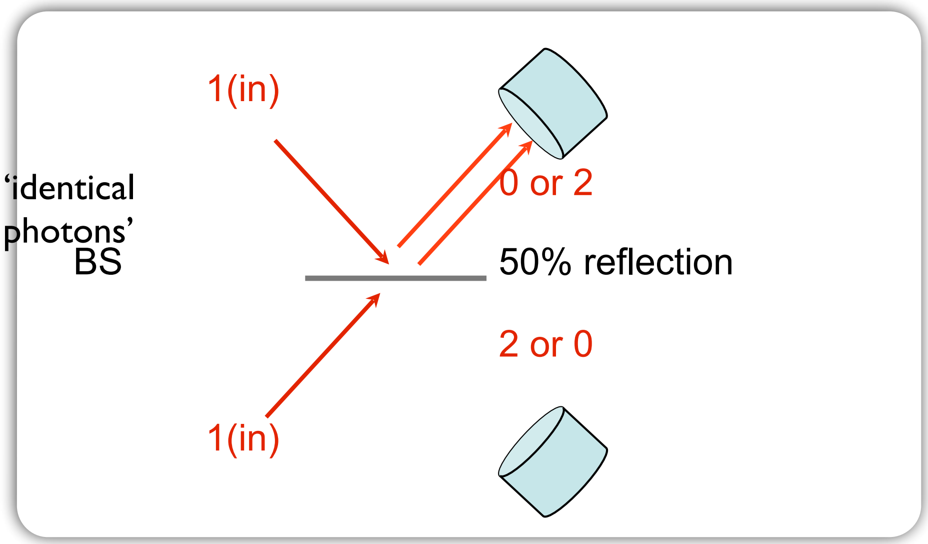
$$\hat{A}^\dagger B^\dagger\left|vac\right\rangle=\frac{\left|2_C,0_D\right\rangle+\left|0_C,2_D\right\rangle}{2}$$

Each mode has 50% chance to get both photons. (Bosons stick together)

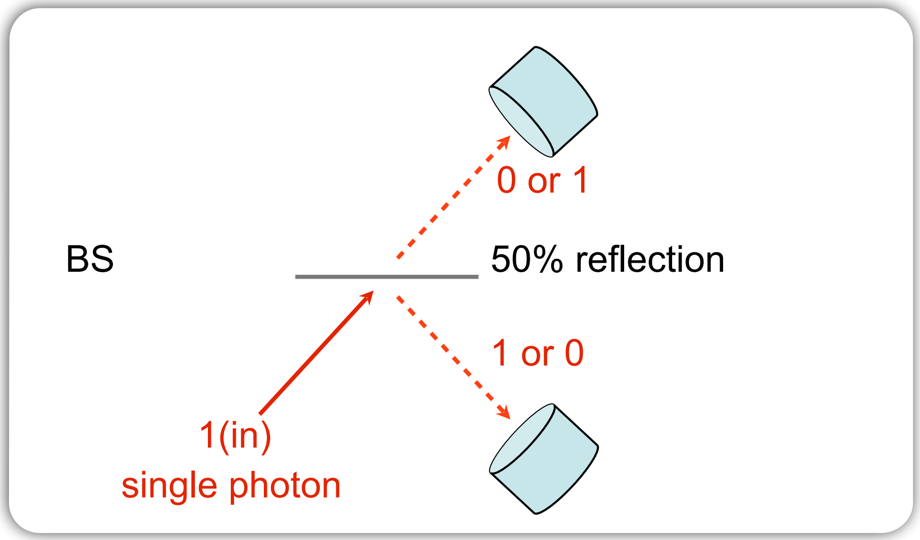
One-photon partitioning



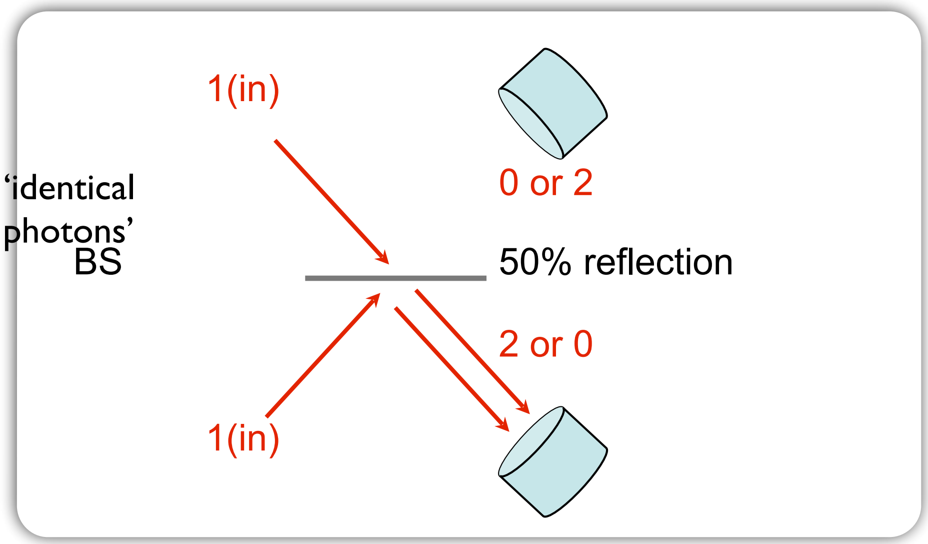
Two-photon interference



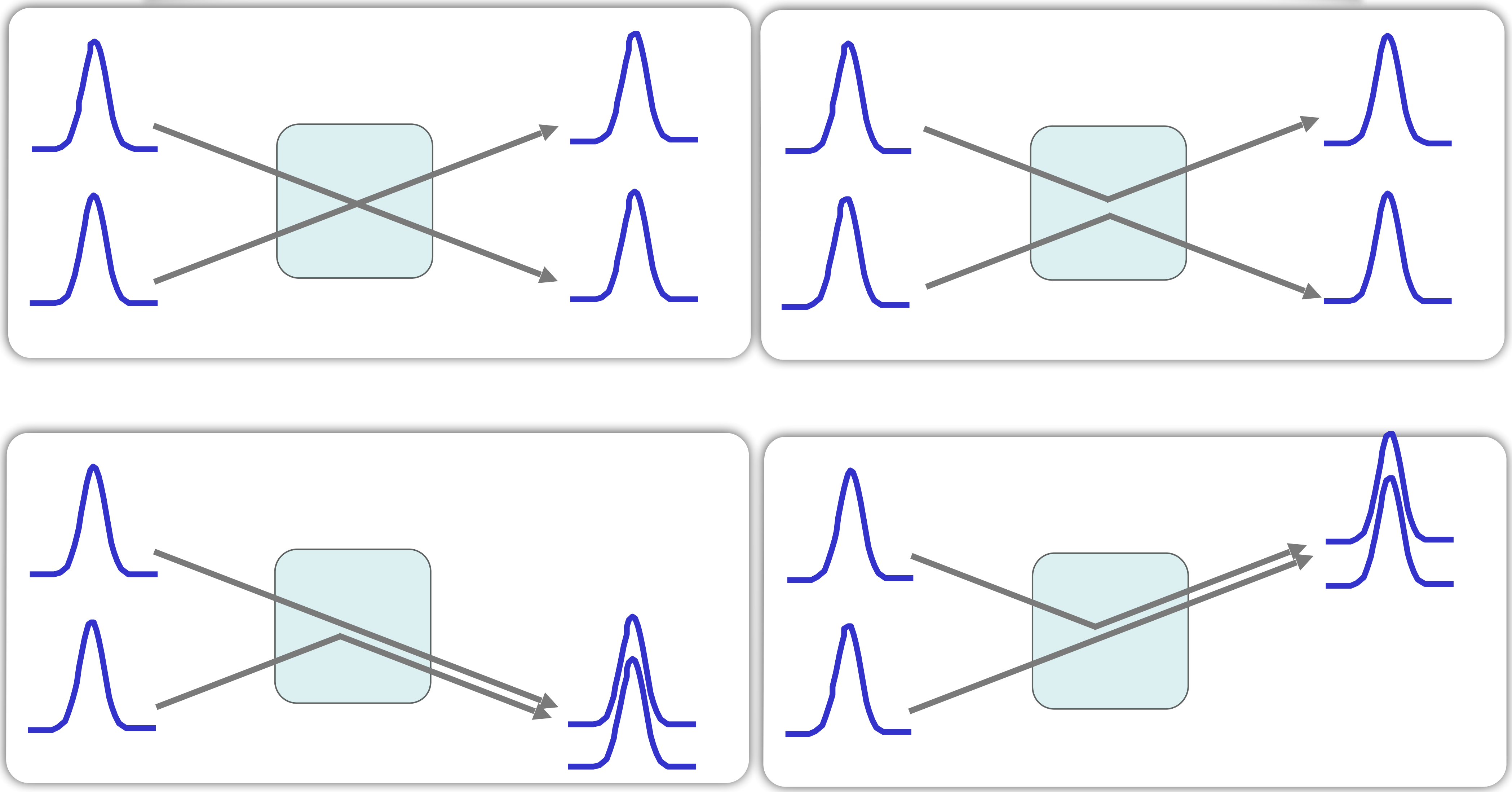
One-photon partitioning



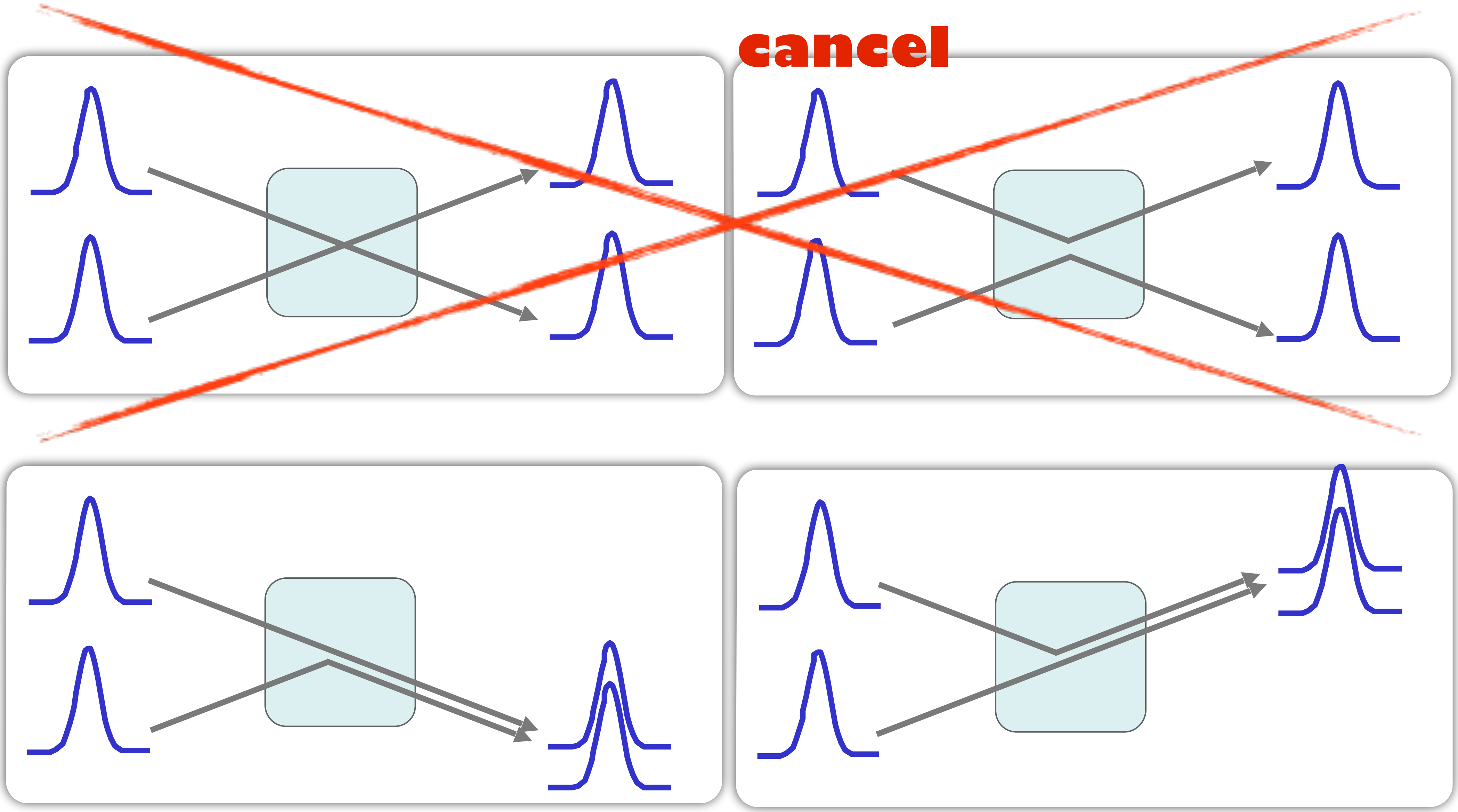
Two-photon interference



Why do two of these diagrams cancel?

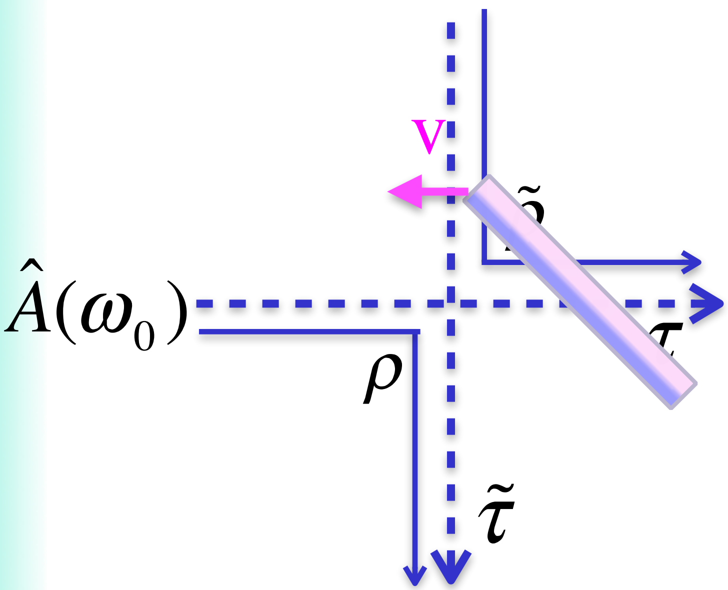


Origin of two-photon interference



MOVING BEAM SPLITTER

What happens if a quantum field hits a moving beam splitter?

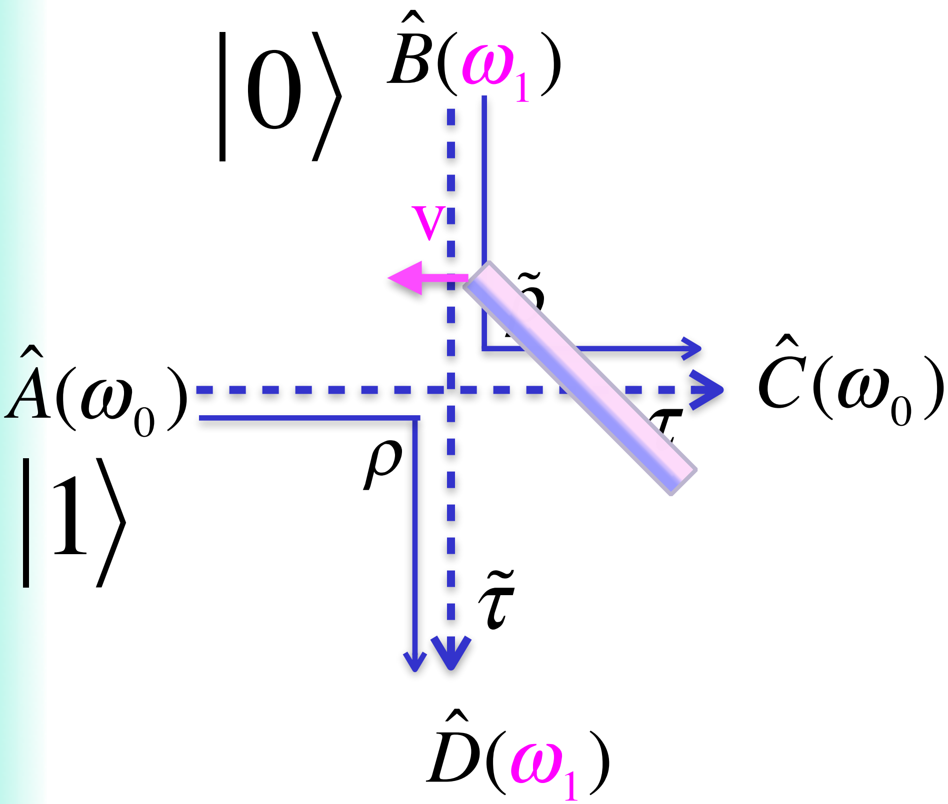


MOVING BEAM SPLITTER

What happens if a quantum field hits a moving beam splitter?

Doppler shift happens! $\omega_0 \to \omega_1 = \omega_0(1 + \mathrm{v}/c)$

if all four TMs are identical except for carrier frequency:

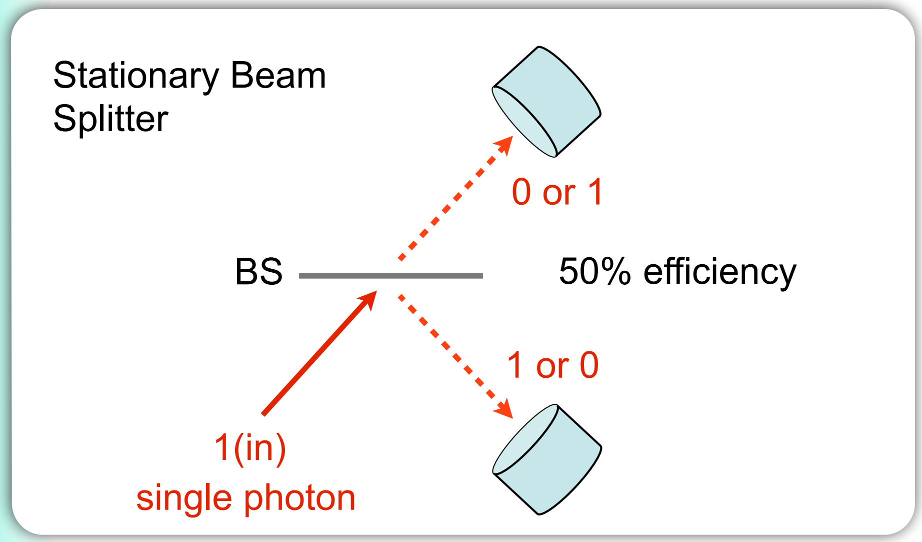


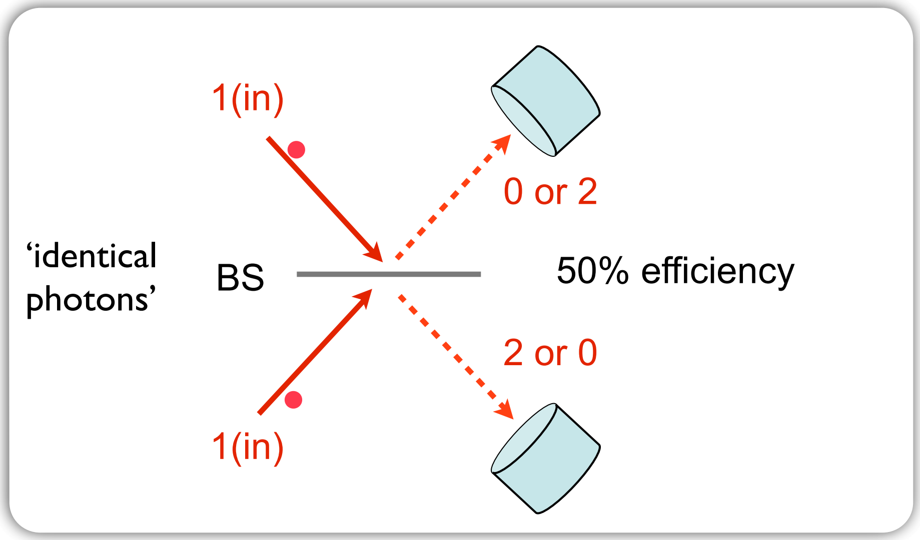
$$\begin{pmatrix} \hat{C} \\ \hat{D} \end{pmatrix} = \begin{pmatrix} \tau & \tilde{\rho} \\ \rho & \tilde{\tau} \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = \mathbf{U} \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix}$$

$$inverse: \ \mathbf{U}^{-1} = \mathbf{U}^{\dagger}$$

$$\begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = \begin{pmatrix} \tau^* & \rho^* \\ \tilde{\rho}^* & \tilde{\tau}^* \end{pmatrix} \begin{pmatrix} \hat{C} \\ \hat{D} \end{pmatrix}$$

Interference of Two Photons of Same Color
(Hong-Ou-Mandel)

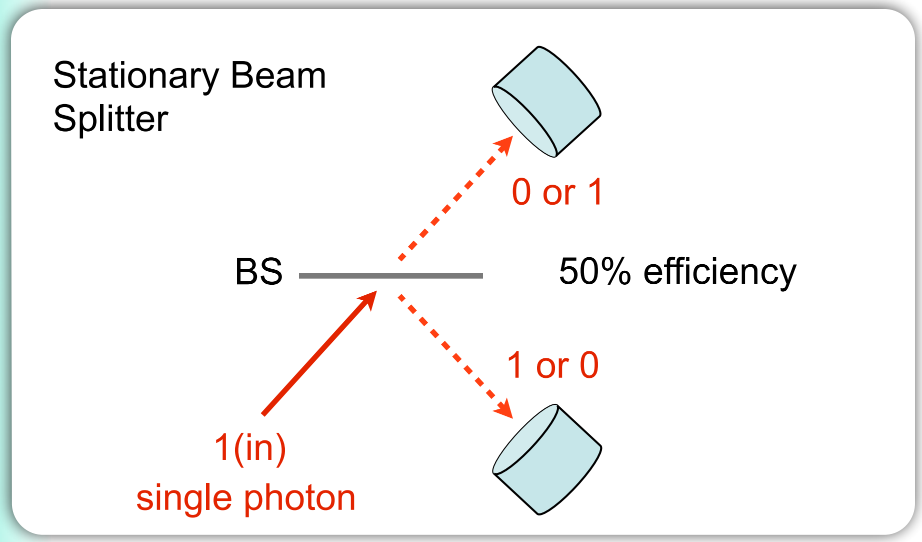


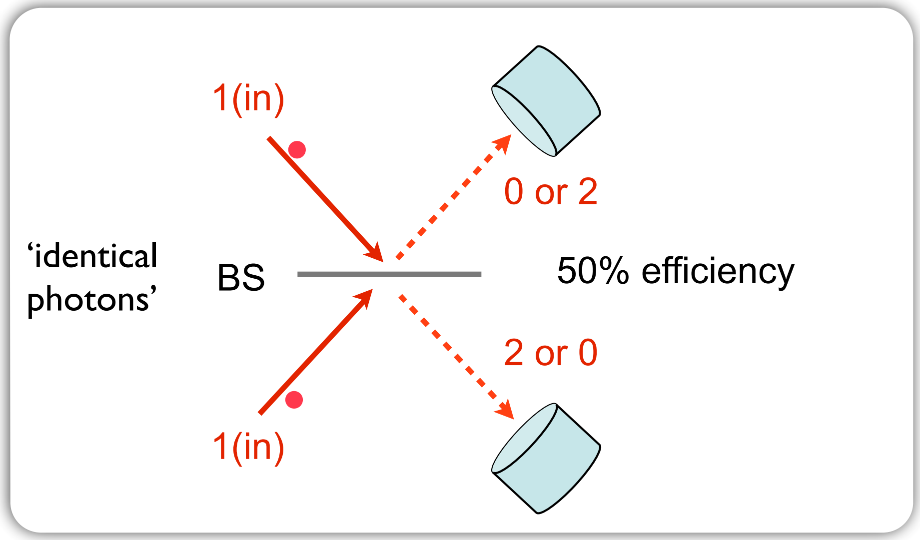


How could you create Interference of Two Photons of Different Color?

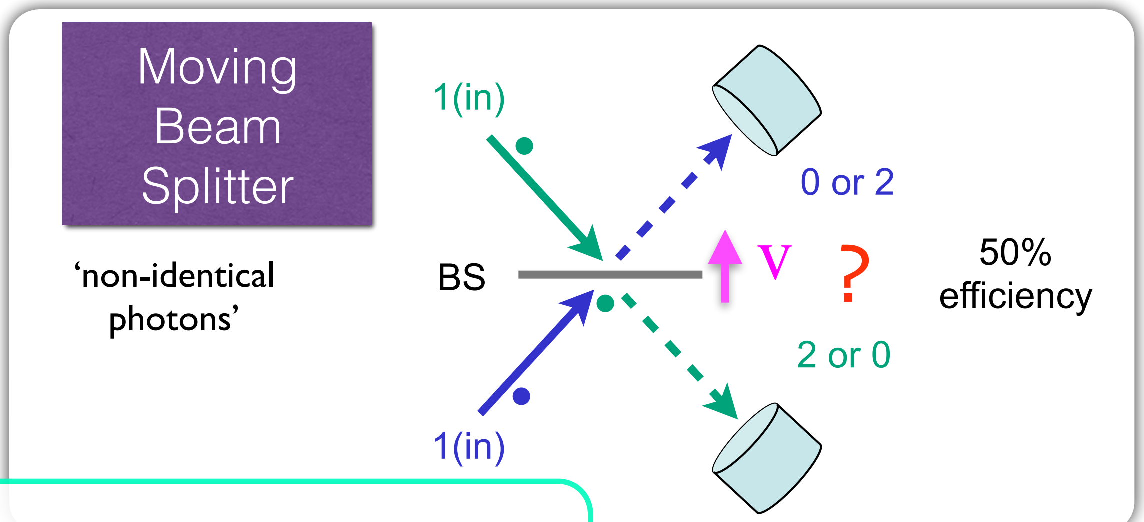
OREGON

Interference of Two Photons of Same Color
(Hong-Ou-Mandel)





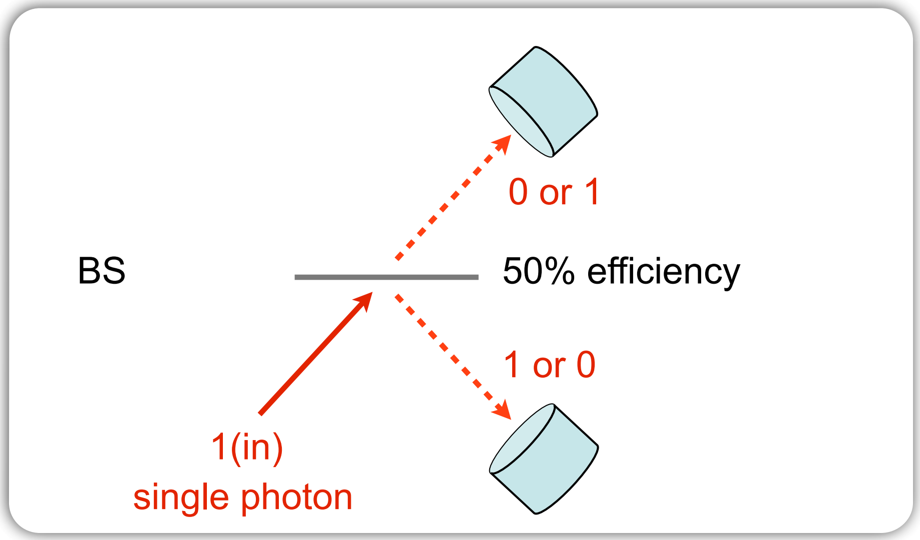
Interference of Two Photons of Different Color

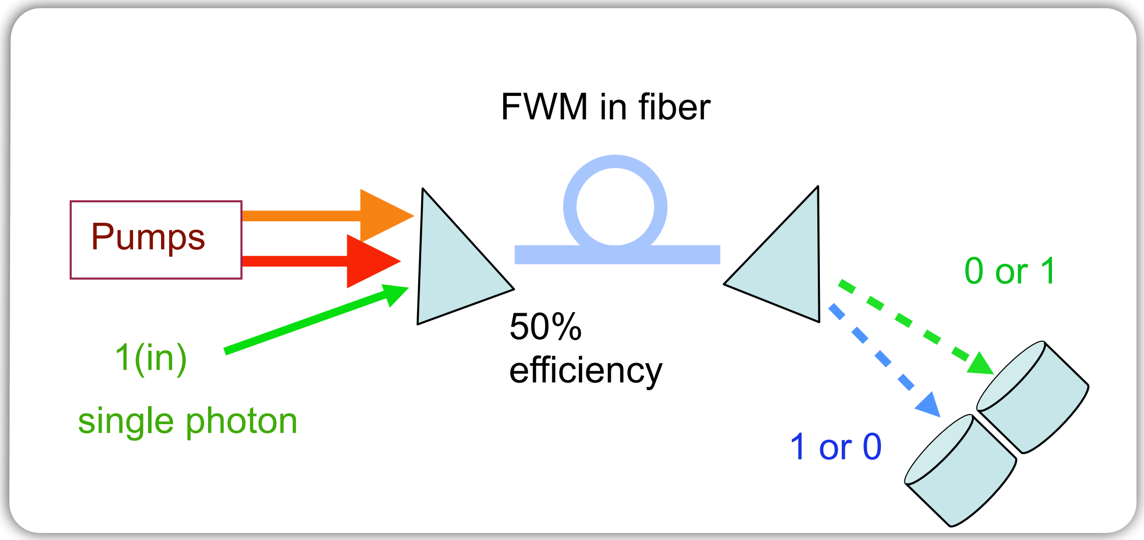


Final state is what matters

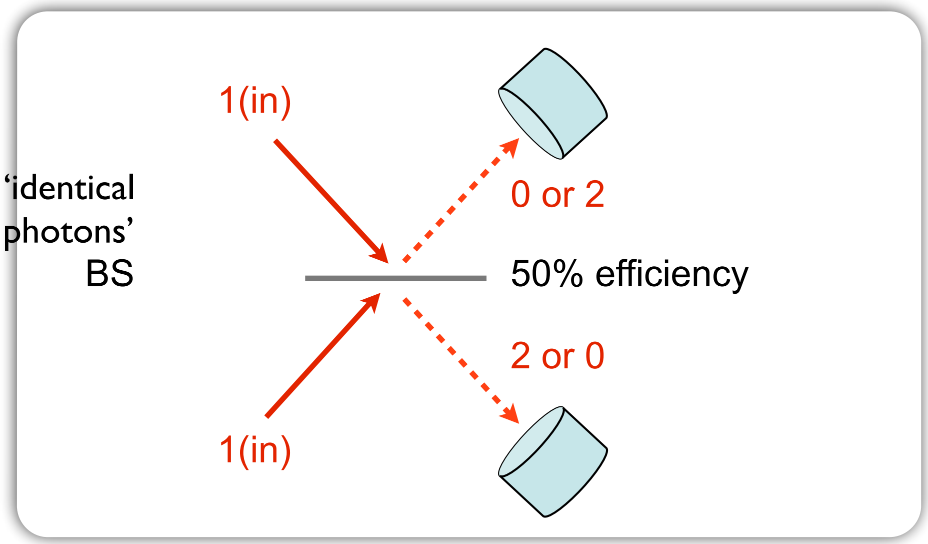
MR, HM, SVE, CM Opt. Commun. 238, 747 (2010)

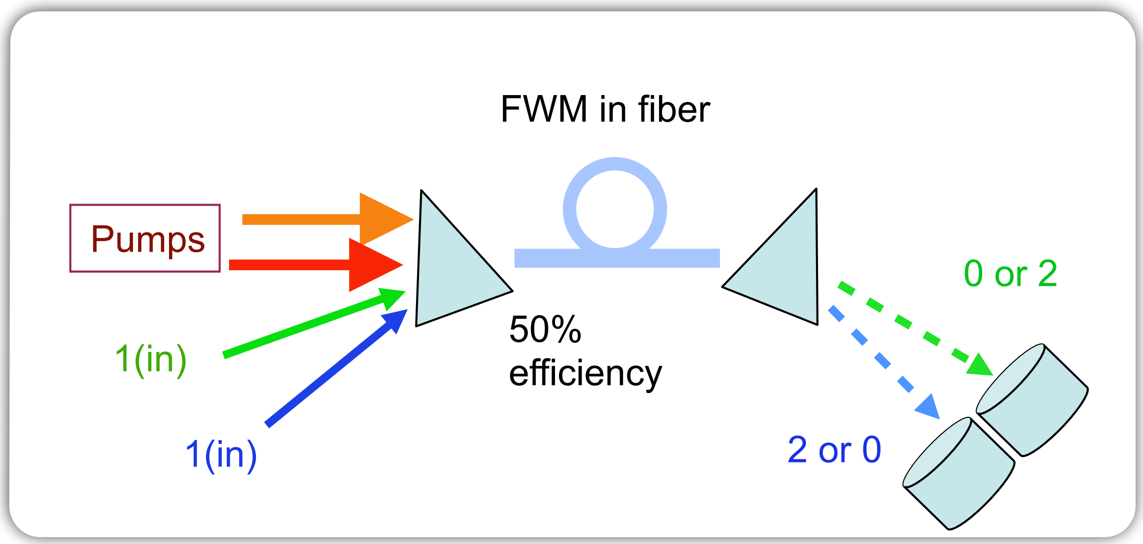
Quantum Frequency Conversion is mathematically analogous to Beam Splitting ➜ Linear Optical Operations





Suggests two-photon interference between photons of different color?

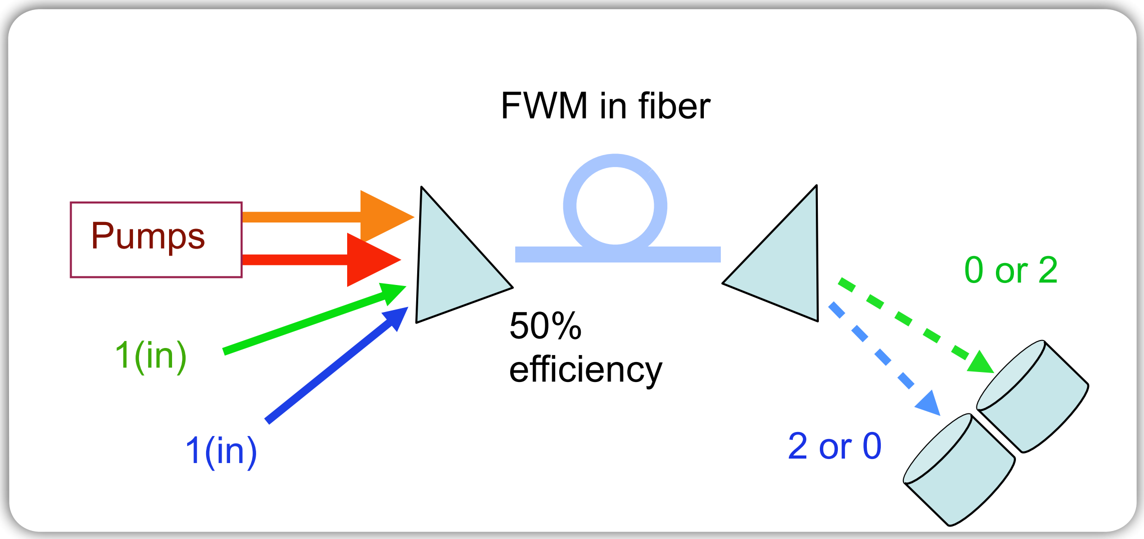




MR, HM, SVE, CM Opt. Commun. 238, 747 (2010)

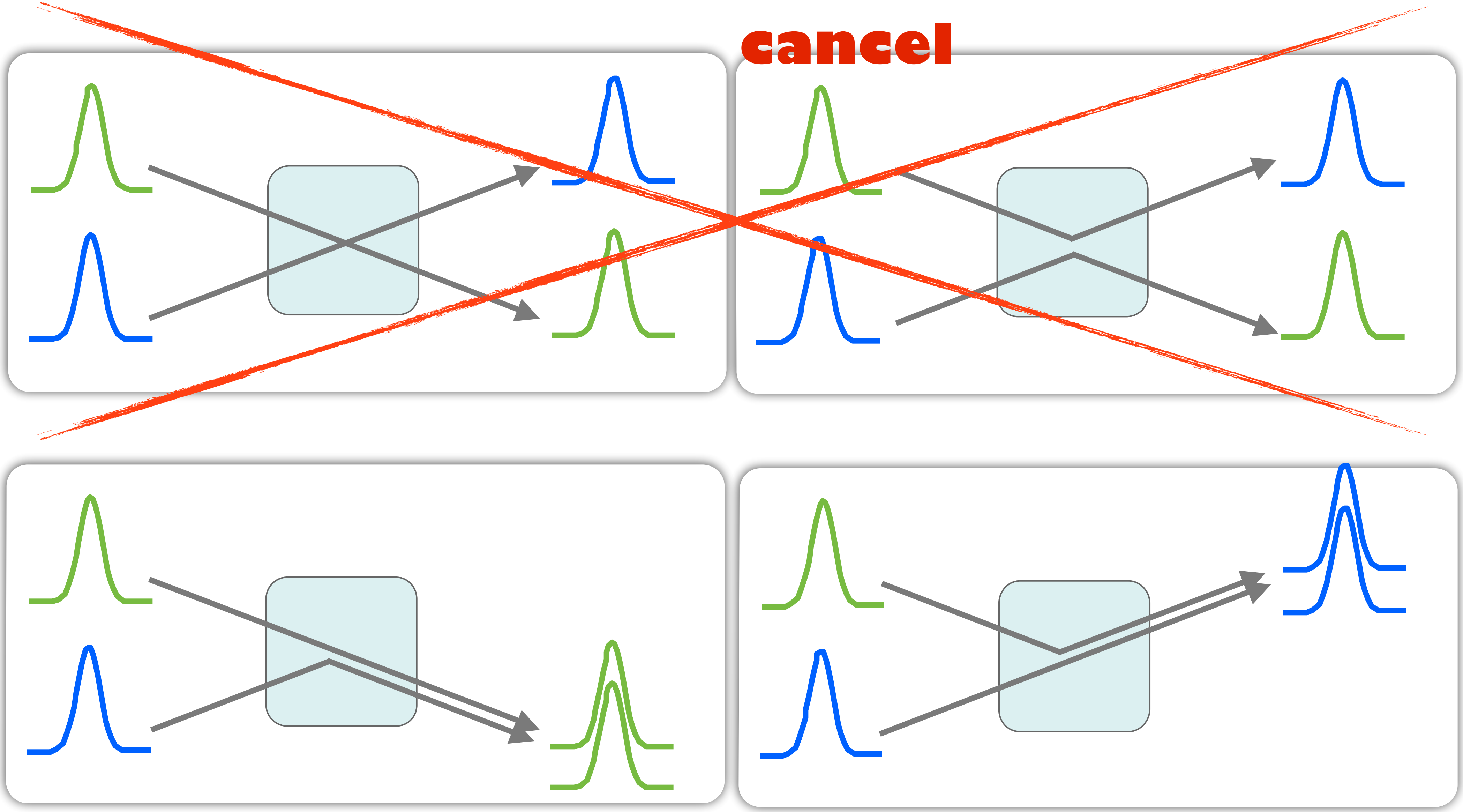
Photons are Bosons

The two photons do not need to be in identical states at the start of the process, only in the final state in order for quantum amplitudes for those processes to add and cancel.



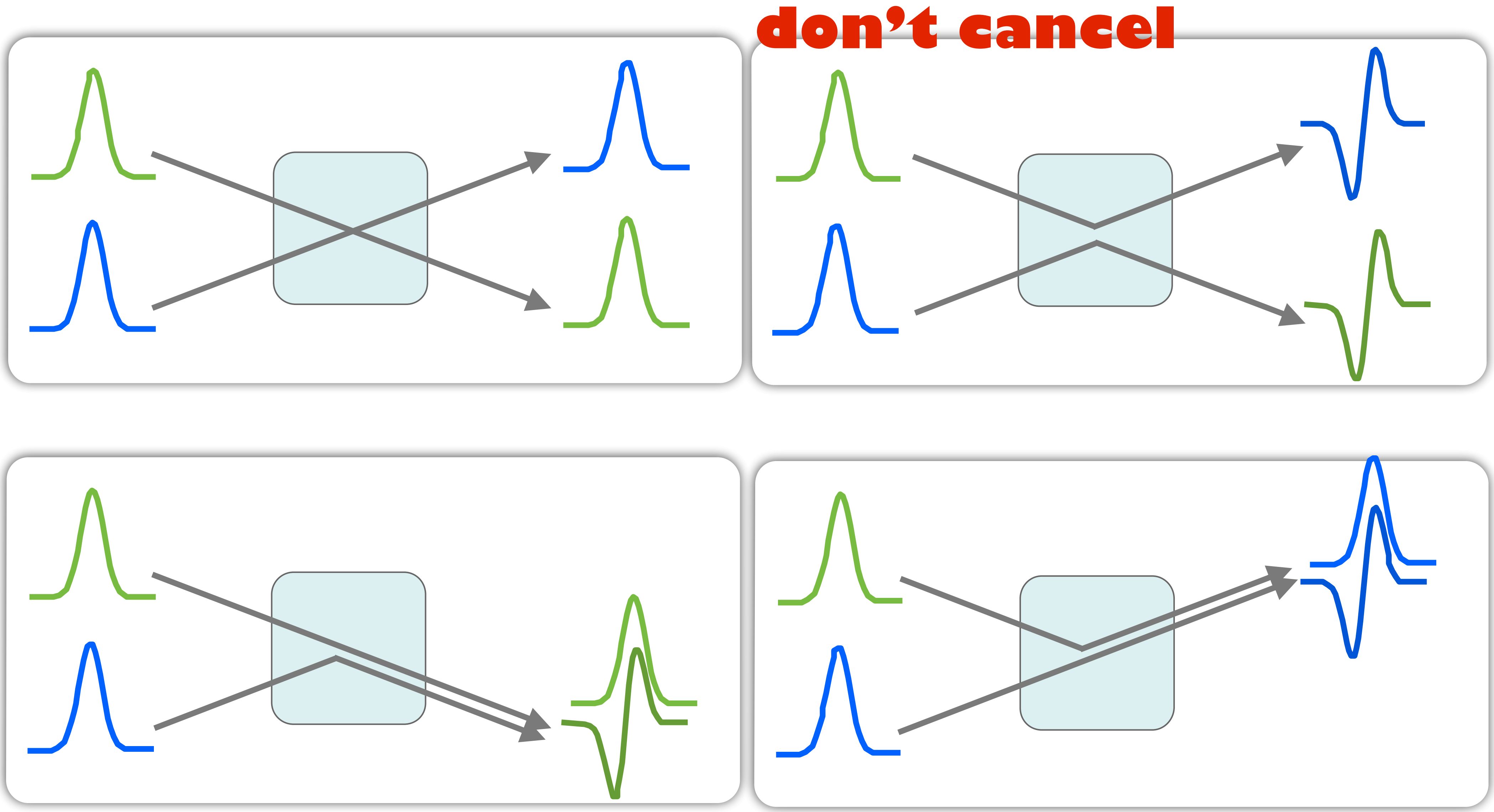
MR, HM, SVE, CM Opt. Commun. 238, 747 (2010)

Origin of two-photon interference



McGuinness, MR, CM, Opt. Express 19, 17876 (2011)

if the final states are not identical:



in this example, the modes change shape on ‘reflection’ only

McGuinness, MR, CM, Opt. Express 19, 17876 (2011)

If a Temporal Mode cannot be an eigenstate of energy, what can it be eigenstate of?

OREGON

If a Temporal Mode cannot be an eigenstate of energy, what can it be eigenstate of?

Photon Number

Can you sketch the 3D Temporal Mode that gets excited when a single atom spontaneously emits a photon? What is its mathematical form?
Is the time of the TM creation random?

add

OREGON

Can you sketch the 3D Temporal Mode
that gets excited when a single atom spontaneously emits a photon? What is its mathematical form?
Is the time of the TM creation random?

add

What shape of TM would be most efficiently absorbed by a ground-state atom?

add

OREGON

If the leading edge of a single-photon state of a Temporal Mode creates a count in a photodetector, can the voltage pulse be shorter in time than the TM, and what becomes of the trailing edge of the TM?

add

OREGON

SVD is always possible for any 2D function.

$$M(x,y) = \sum U_n(x)\,\lambda_n\,V^*_n(x)$$

Is the same true for 3D?

$$M(x,y,z) \xrightarrow{?} \sum_n U_n(x)\,\lambda_n V^*_n(x) W^*_n(z)$$

SVD is always possible for any 2D function.

$$M(x,y) = \sum U_n(x)\, \lambda_n\, V^*_n(x)$$

Is the same true for 3D?

$$M(x,y,z) \xrightarrow{?} \sum_n U_n(x)\, \lambda_n V^*_n(x) W^*_n(z)$$

No, but a double sum always exists:

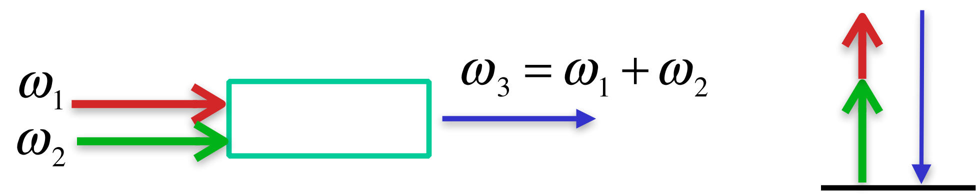
$$M(x,y,z) = \sum_n \sum_m \; U_n(x)\, \lambda_{nm} V_{nm}(x) W_{nm}(z)$$

OREGON

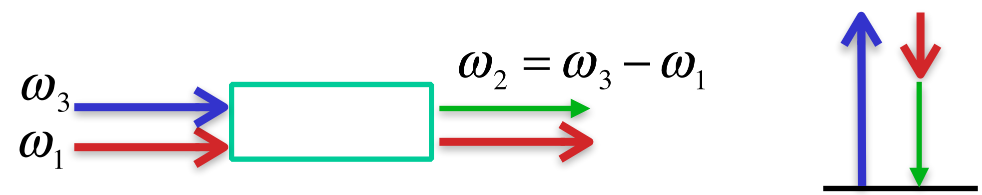
Processes of 'Classical' Nonlinear Optics

What do all of these have in common
(that is not the case for truly quantum processes)?

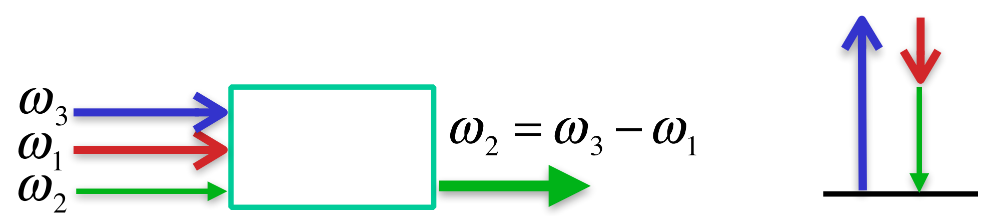
Sum frequency generation (SFG)



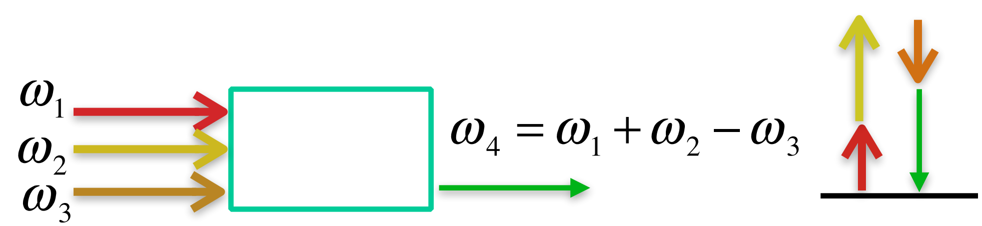
Difference frequency generation (DFG)



Optical parametric amplification (OPA)



Four-wave mixing (FWM)

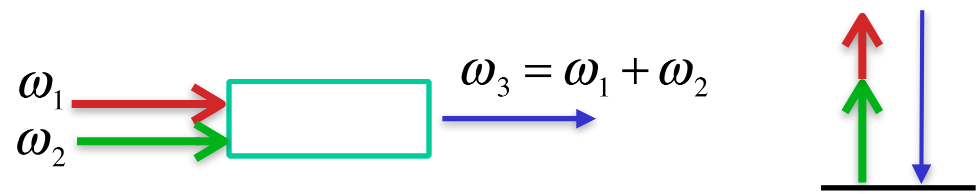


Processes of 'Classical' Nonlinear Optics

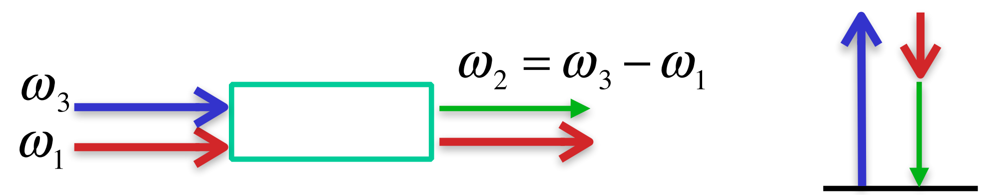
What do all of these have in common (that is not the case for truly quantum processes)?

All have $\langle P_{SIGNAL\ FREQUENCY} \rangle \neq 0$, *where* $P =$ *electronic polarization*

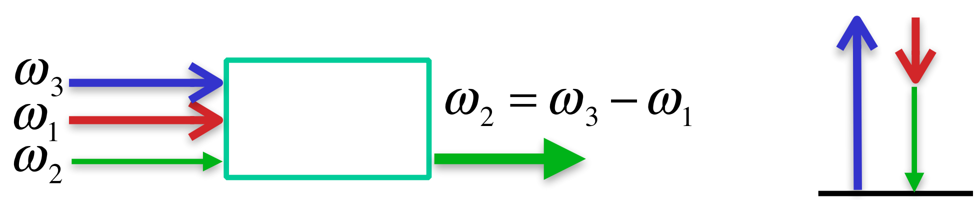
Sum frequency generation (SFG)



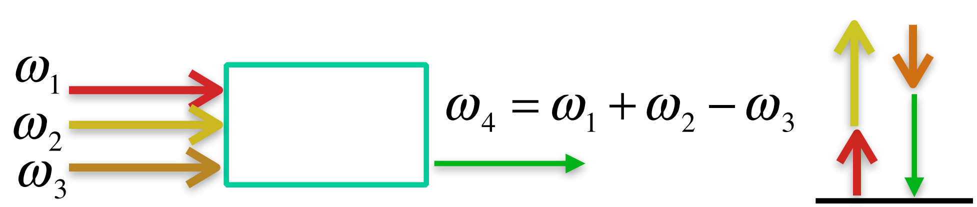
Difference frequency generation (DFG)



Optical parametric amplification (OPA)



Four-wave mixing (FWM)



Processes of 'Quantum' Nonlinear Optics

What are examples of processes that have zero mean electronic polarization at the generated signal frequency?

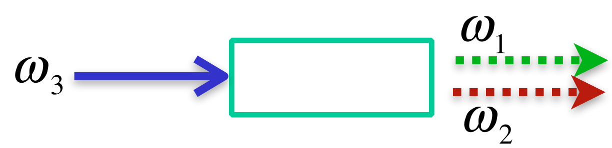
$$\langle P_{SIGNAL\ FREQUENCY} \rangle = 0\ ,\ \ where\ P = electronic\ \ polarization$$

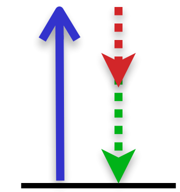
OREGON

Processes of 'Quantum' Nonlinear Optics

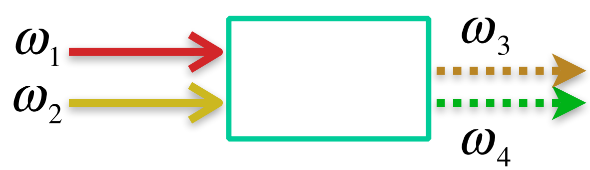
$$\langle P_{SIGNAL\ FREQUENCY} \rangle = 0\ ,\ \ where\ P = electronic\ polarization$$

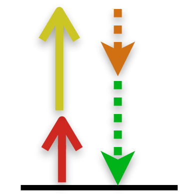
Spontaneous parametric downconversion (SPDC)





Spontaneous four-wave mixing (SFWM)

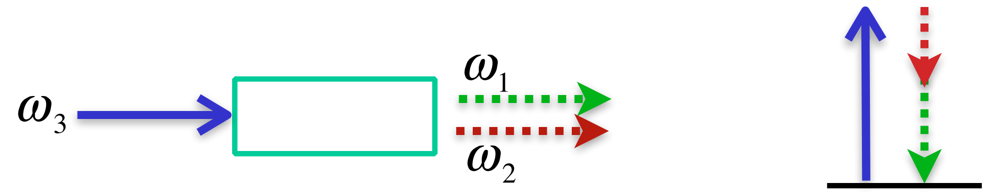




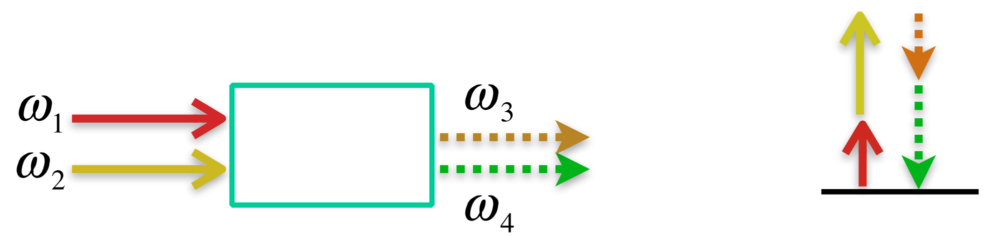
Processes of 'Quantum' Nonlinear Optics

$$\langle P_{SIGNAL\ FREQUENCY} \rangle = 0\ ,\ \ where\ P = electronic\ polarization$$

Spontaneous parametric downconversion (SPDC)

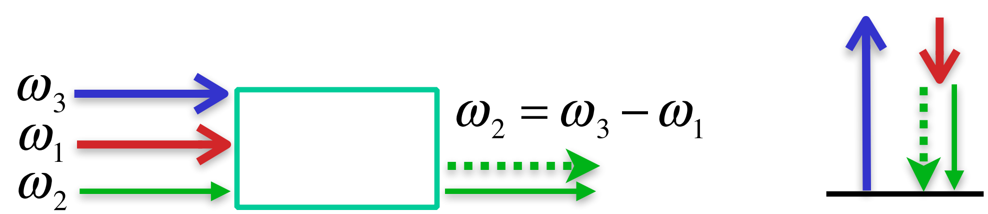


Spontaneous four-wave mixing (SFWM)

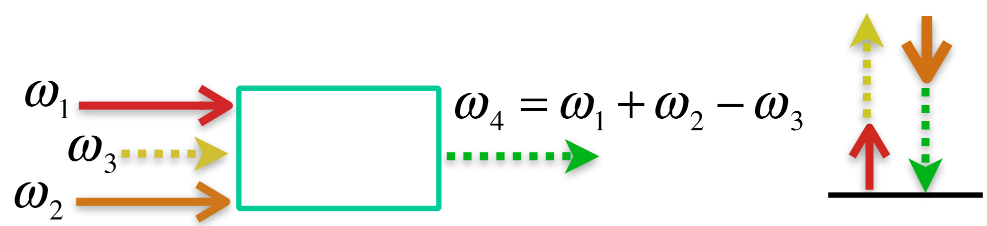


Hybrid 'Classical-Non-Classical' Nonlinear Optics

Optical parametric amplification (OPA)

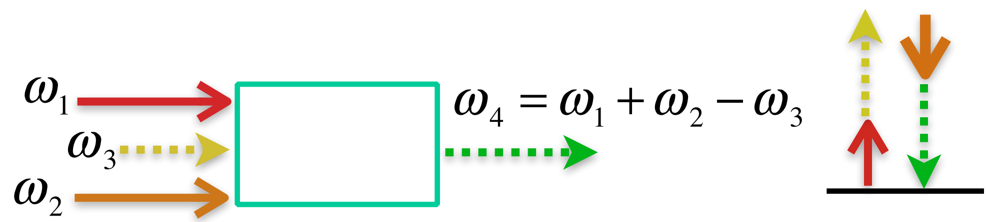


Quantum frequency conversion (QFC)

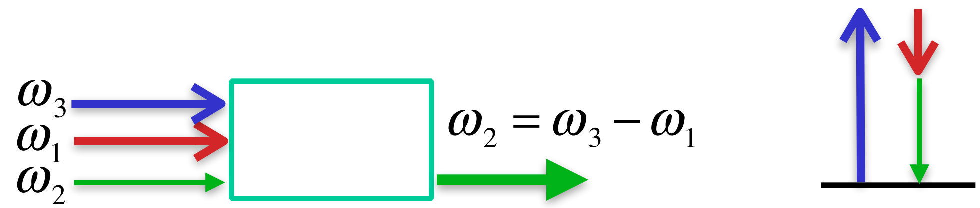


Why is frequency conversion background free while parametric amplification is not?

Quantum frequency conversion (QFC)



Optical parametric amplification (OPA)



What is the origin of second- and third-order optical nonlinear medium response?

For instantaneous medium response
(or monochromatic fields):

$$P(z,t) \approx \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E E + \varepsilon_0 \chi^{(3)} E E E + \ldots$$

$$\chi^{(n)} = nonlinear\ \ polarizability\ coefficient\ of\ \ order\ n$$

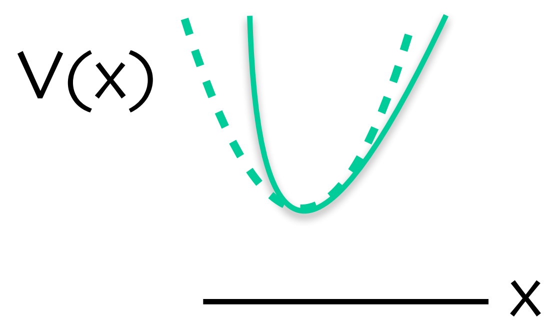
For instantaneous medium response (or monochromatic fields):

$$P(z,t) \approx \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E E + \varepsilon_0 \chi^{(3)} E E E + \ldots$$

$$\chi^{(n)} = nonlinear\ \ polarizability\ coefficient\ of\ \ order\ n$$

Origin of nonlinear response

$\chi^{(2)}$ is non-zero only for non-centro-symmetric media



$\chi^{(3)}$ is non-zero for any medium