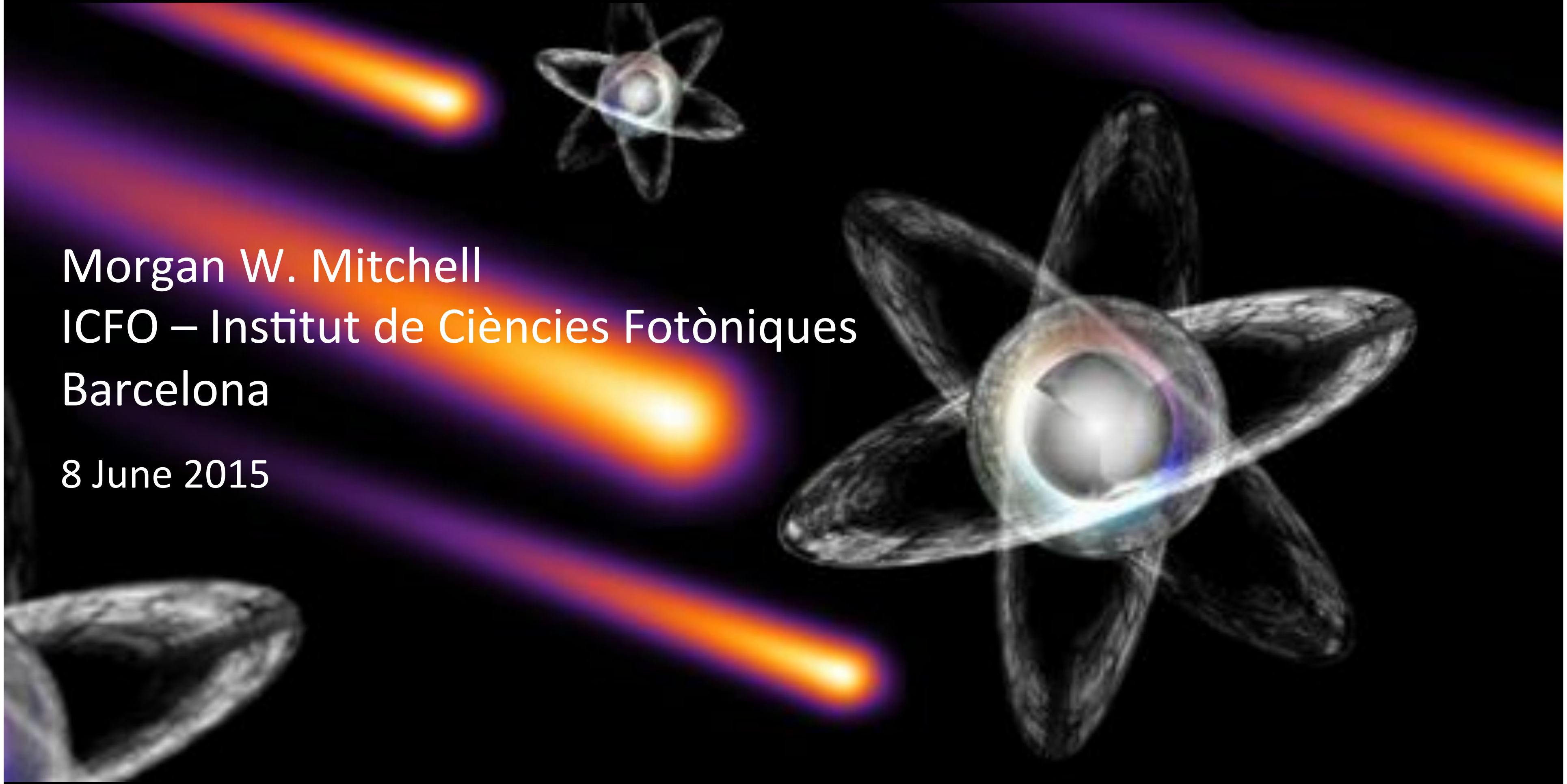
Quantum enhanced sensing with light and atoms



Morgan W. Mitchell
ICFO – Institut de Ciències Fotòniques
Barcelona

8 June 2015

Quantum Optics for the Impatient

Morgan W. Mitchell
ICFO - Institut de Ciències Fotòniques
Castelldefels

http://mitchellgroup.icfo.es

Kip Thorne

Kip Thorne

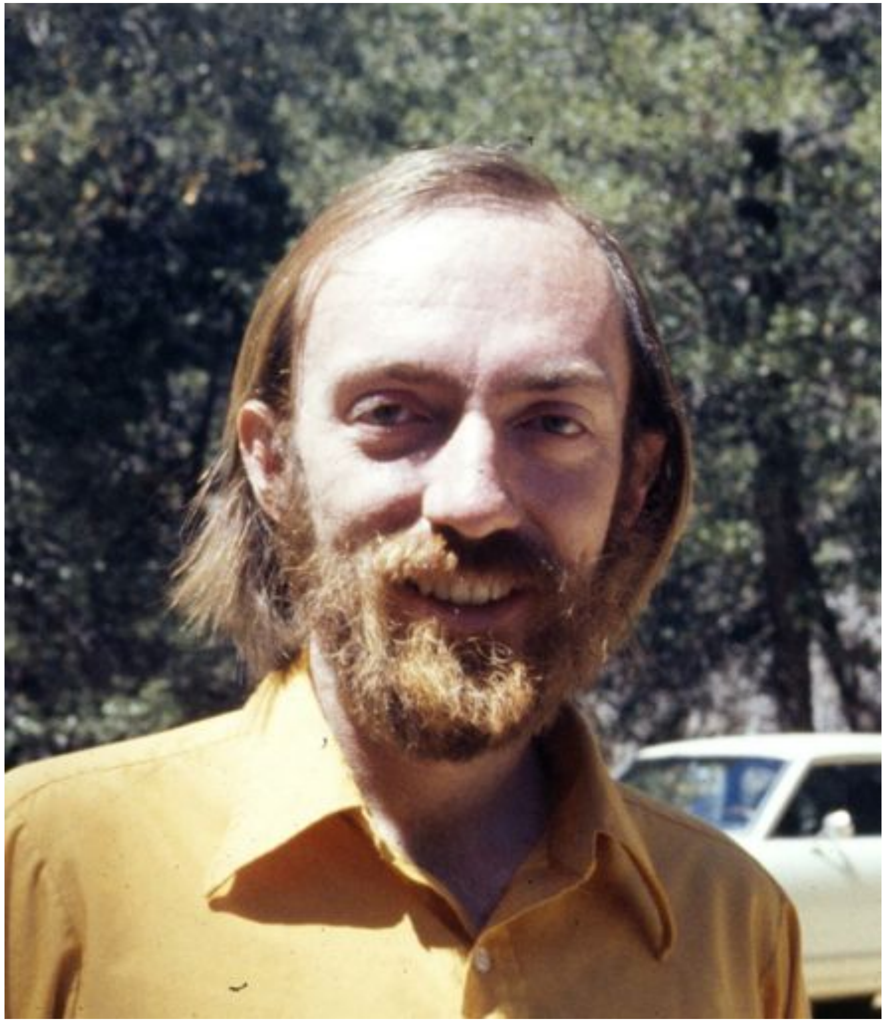
Black hole from "Interstellar"

Academy Award for Visual Effects

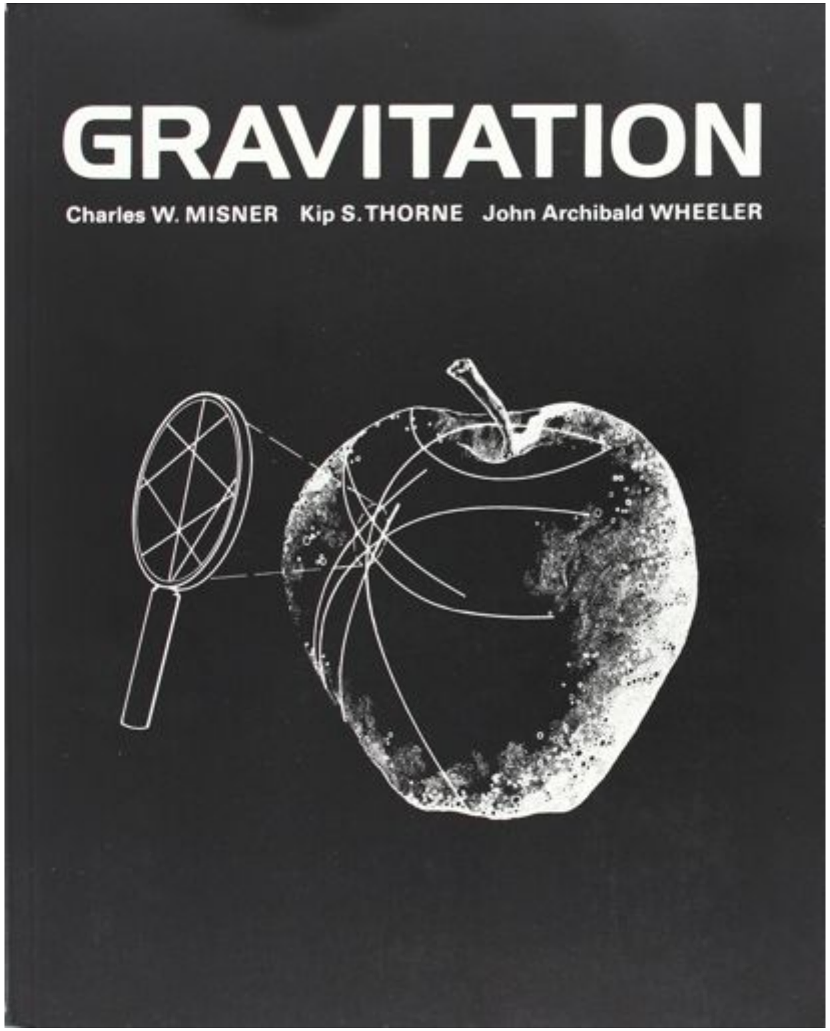
INTERSTELLAR

Paul Franklin, Andrew Lockley, Ian Hunter and Scott Fisher

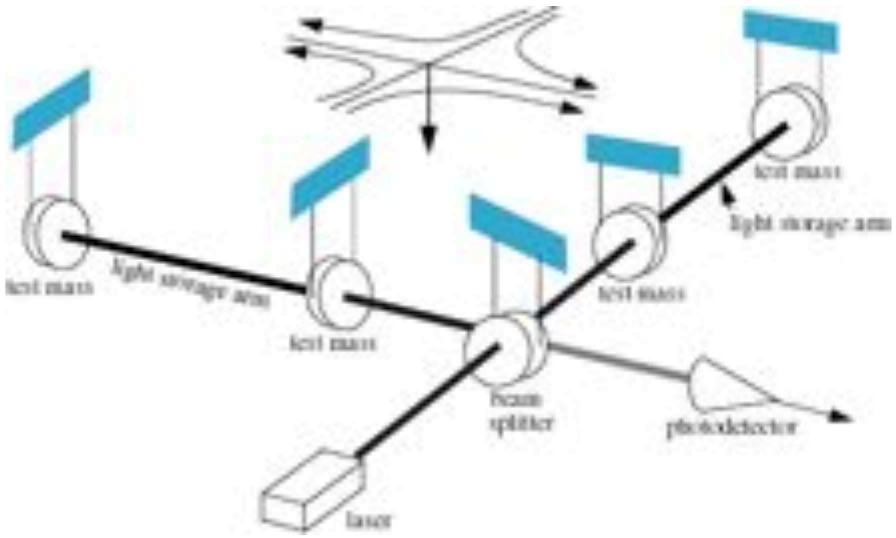
Kip Thorne

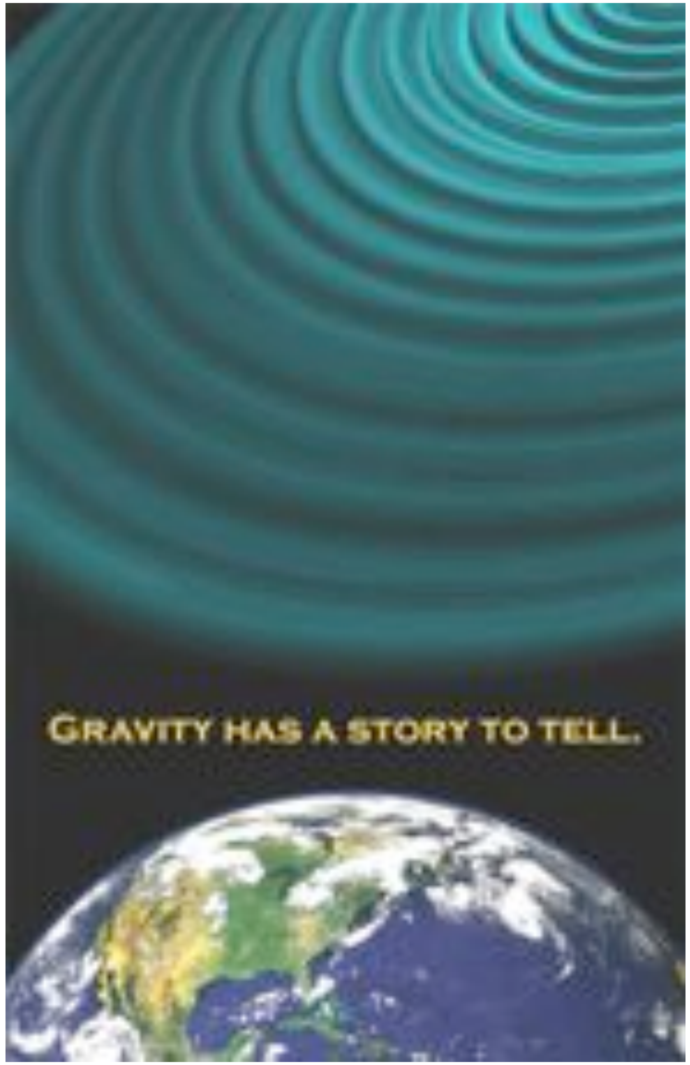


Kip Thorne 1973



Gravitational wave detection



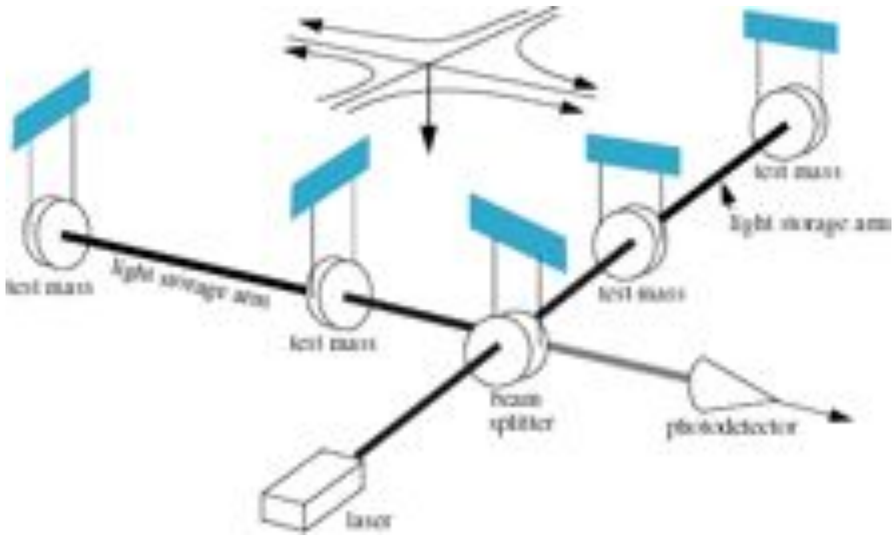


Very large Michelson interferometer,
e.g. LIGO, VIRGO, GEO, TAMA, LISA
sensitivity $\delta L = 10^{-18}$ m or $\delta L/L = 10^{-23}$

Quantum enhanced sensing (outline)

- Motivation and example
  - Gravitational wave interferometers
- Quantum sensitivity limits
  - Pre-1980
  - Caves proposal
    - Quantum optics of fields
  - Squeezing generation
- Sensing atoms with squeezed light
- Atomic ensembles as a quantum system
  - Collective variables approach
  - Quantum non-demolition measurements
  - Spin squeezing

Gravitational wave detection



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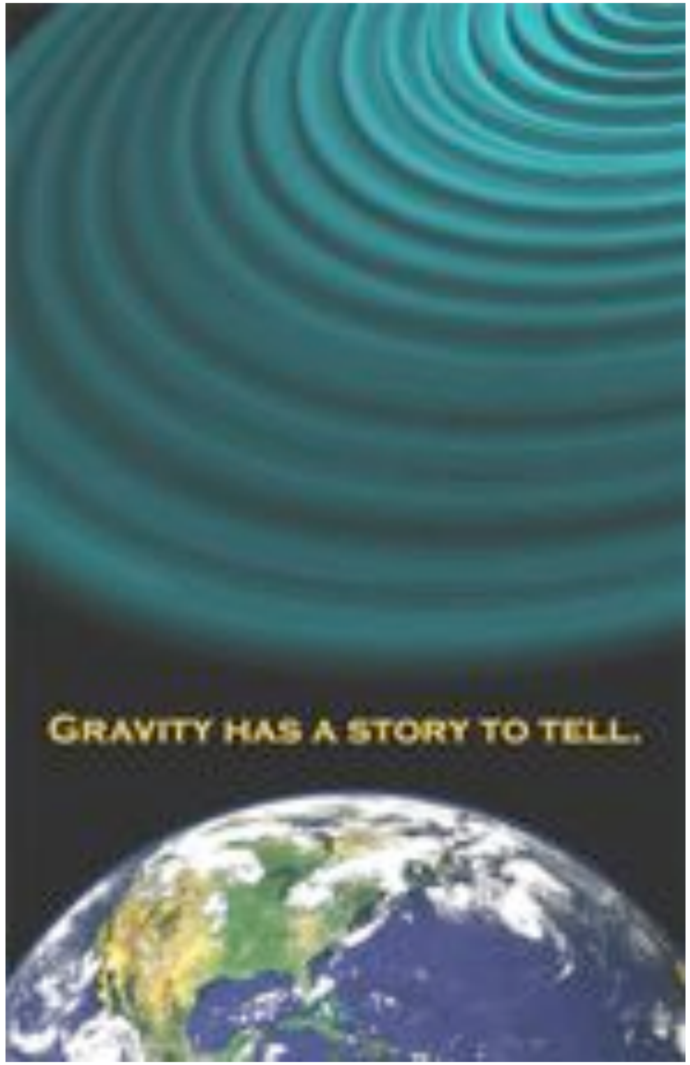
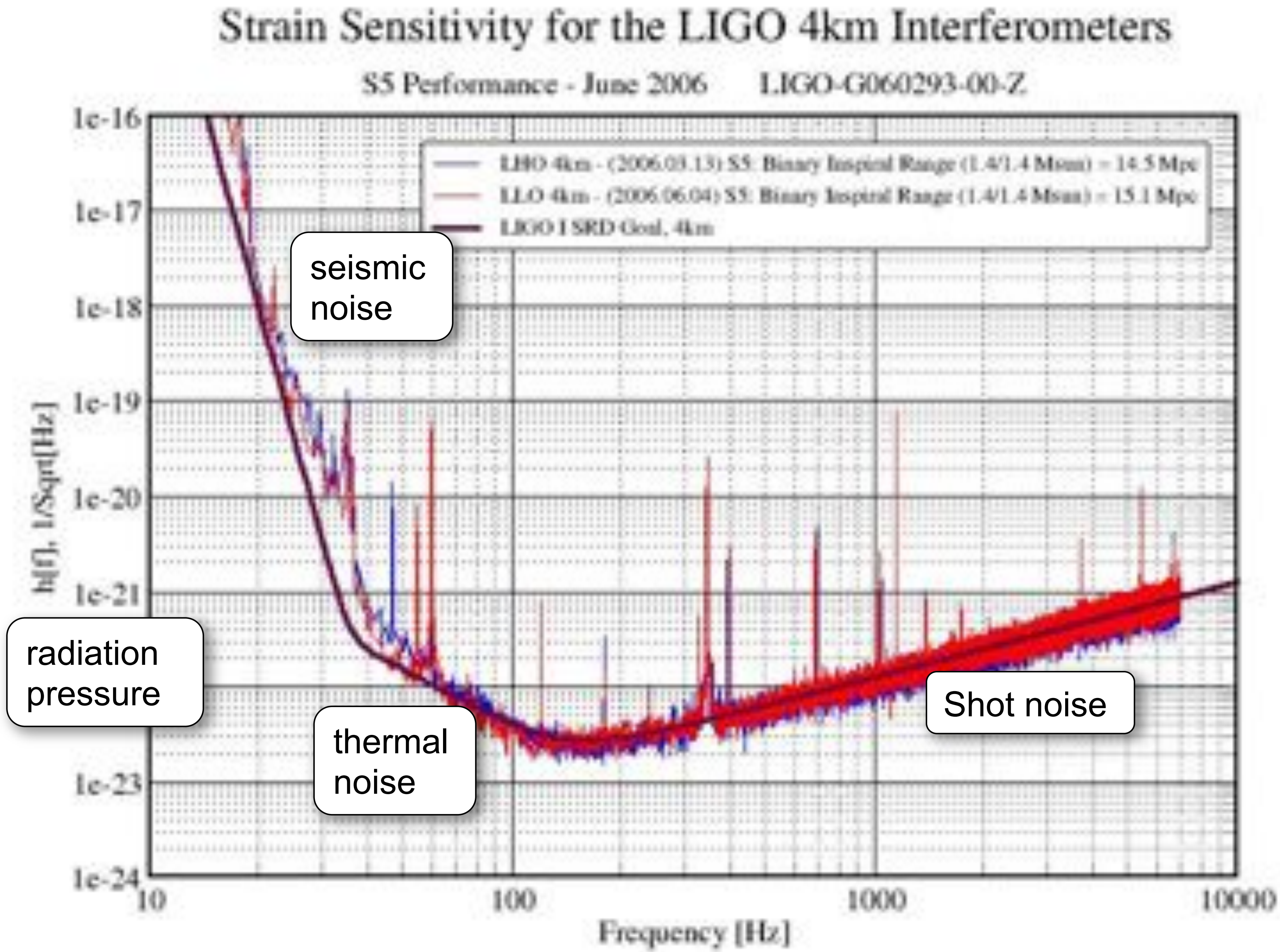
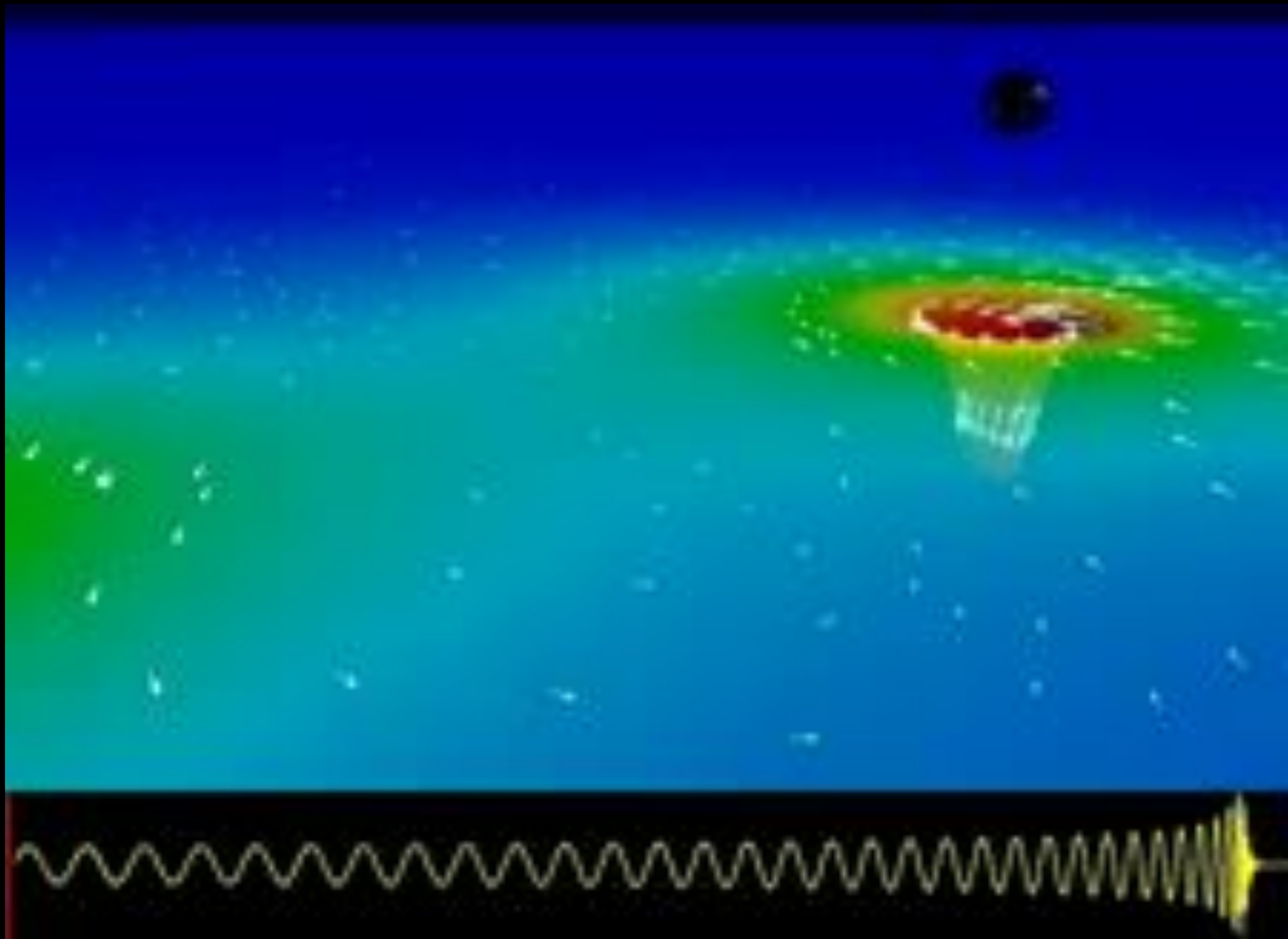


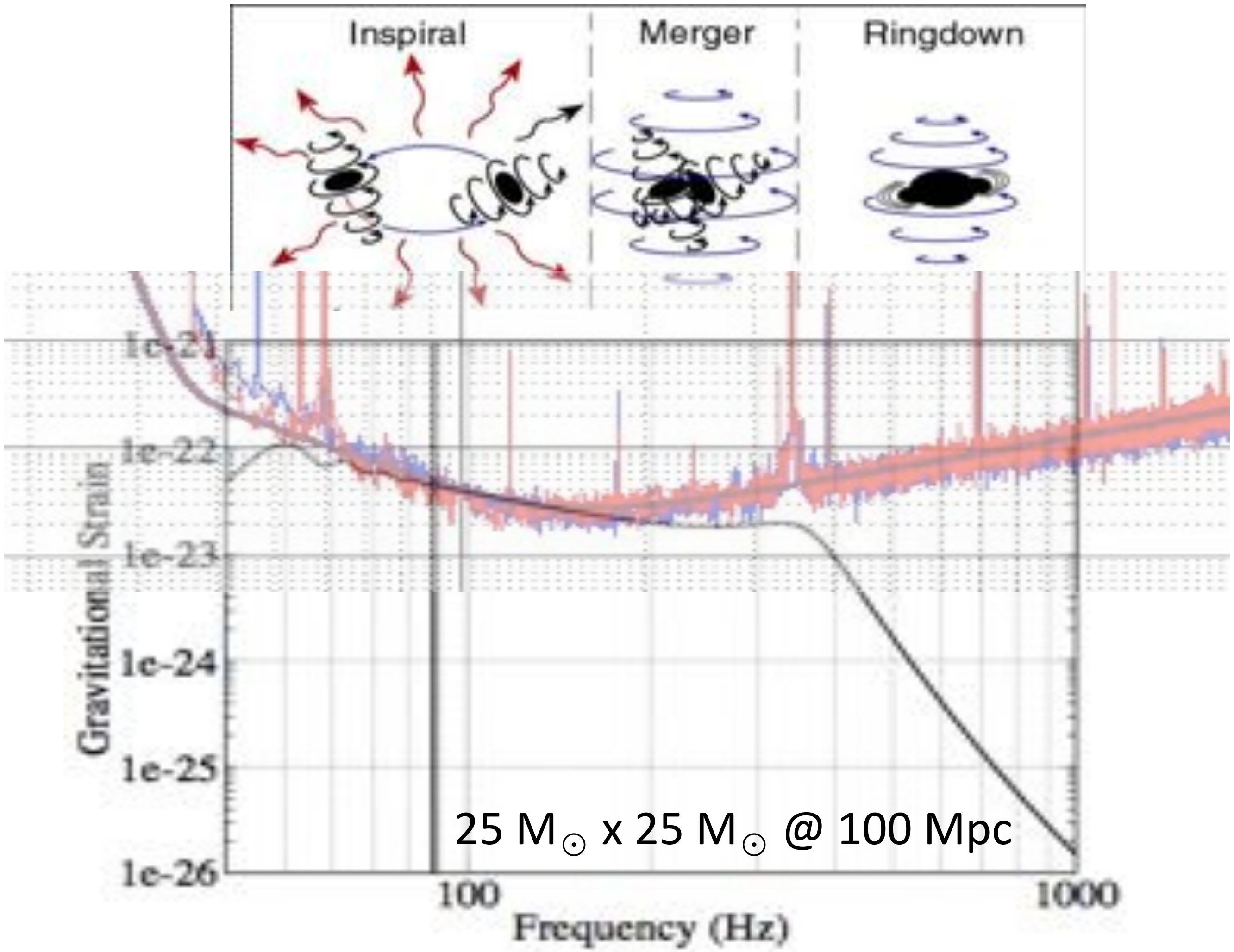

Strain Sensitivity for the LIGO 4km Interferometers
S5 Performance - June 2006 LIGO-G060293-00-Z
LHO 4km - (2006.03.13) S5: Binary Inspiral Range (1.4/1.4 Msun) = 14.5 Mpc
LLO 4km - (2006.06.04) S5: Binary Inspiral Range (1.4/1.4 Msun) = 15.1 Mpc
LIGO I SRD Goal, 4km
seismic noise
radiation pressure
thermal noise
Shot noise
h[f], 1/Sqrt[Hz]
Frequency [Hz]
1e-16
1e-17
1e-18
1e-19
1e-20
1e-21
1e-23
1e-24
10
100
1000
10000

Gravitational waves from black hole mergers



Gravitational wave detection

Search for gravitational waves from binary black hole inspiral, merger and ringdown. Phys. Rev. D 83, 122005 (2011)




Search for gravitational waves from binary black hole inspiral, merger and ringdown

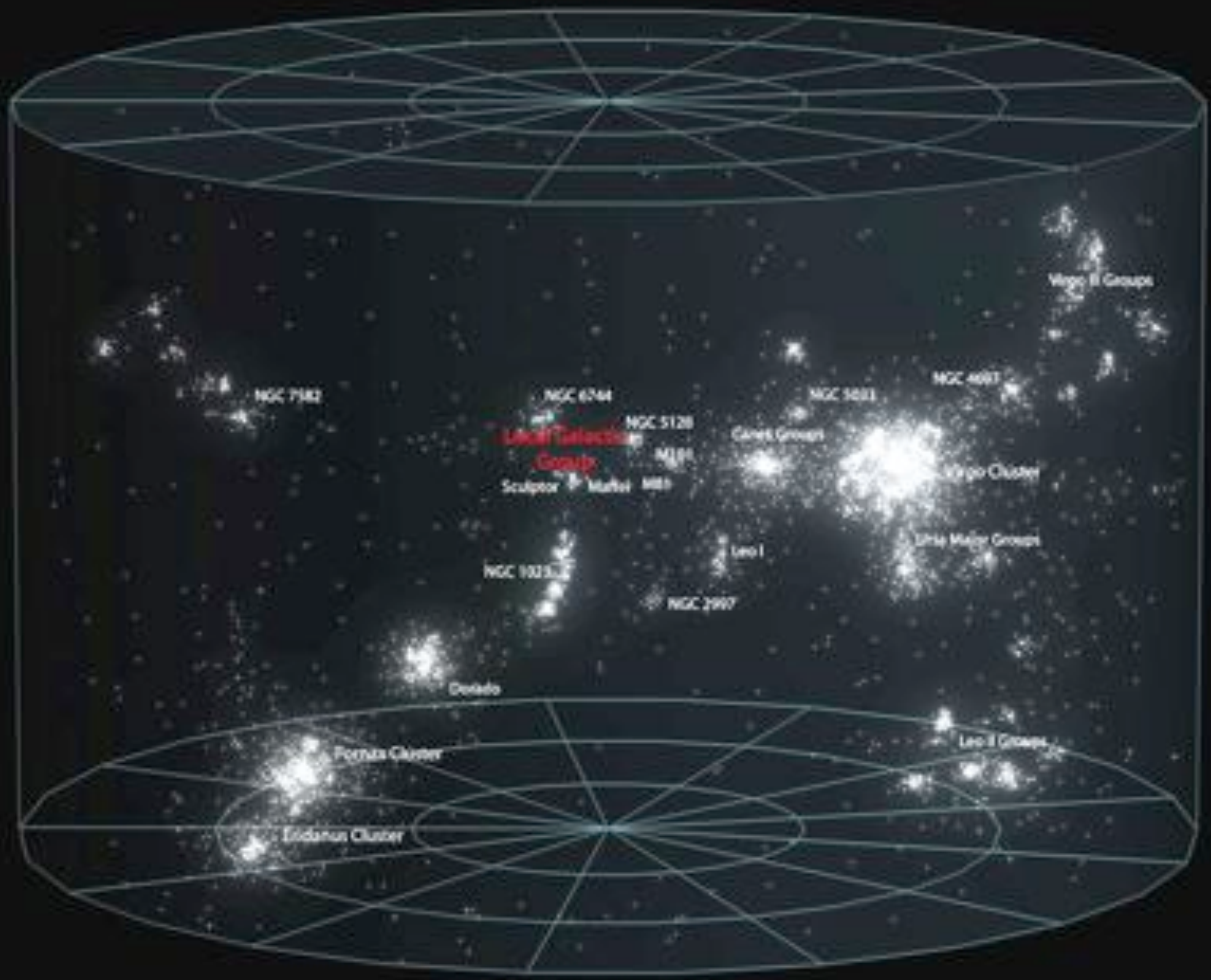
arXiv:1102.3781v1 [gr-qc] 18 Feb 2011

LIGO + VIRGO

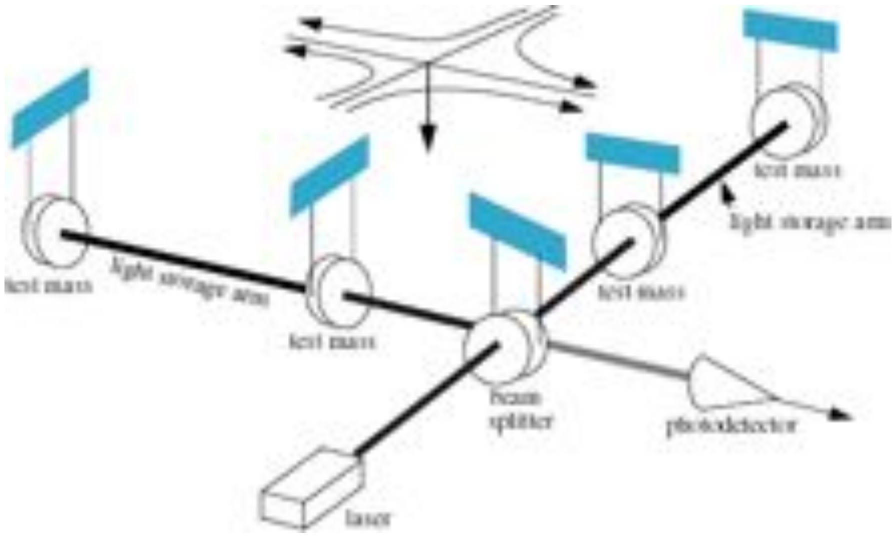
Quantum and Nonlinear Optics, Sørup Herregaard 2015

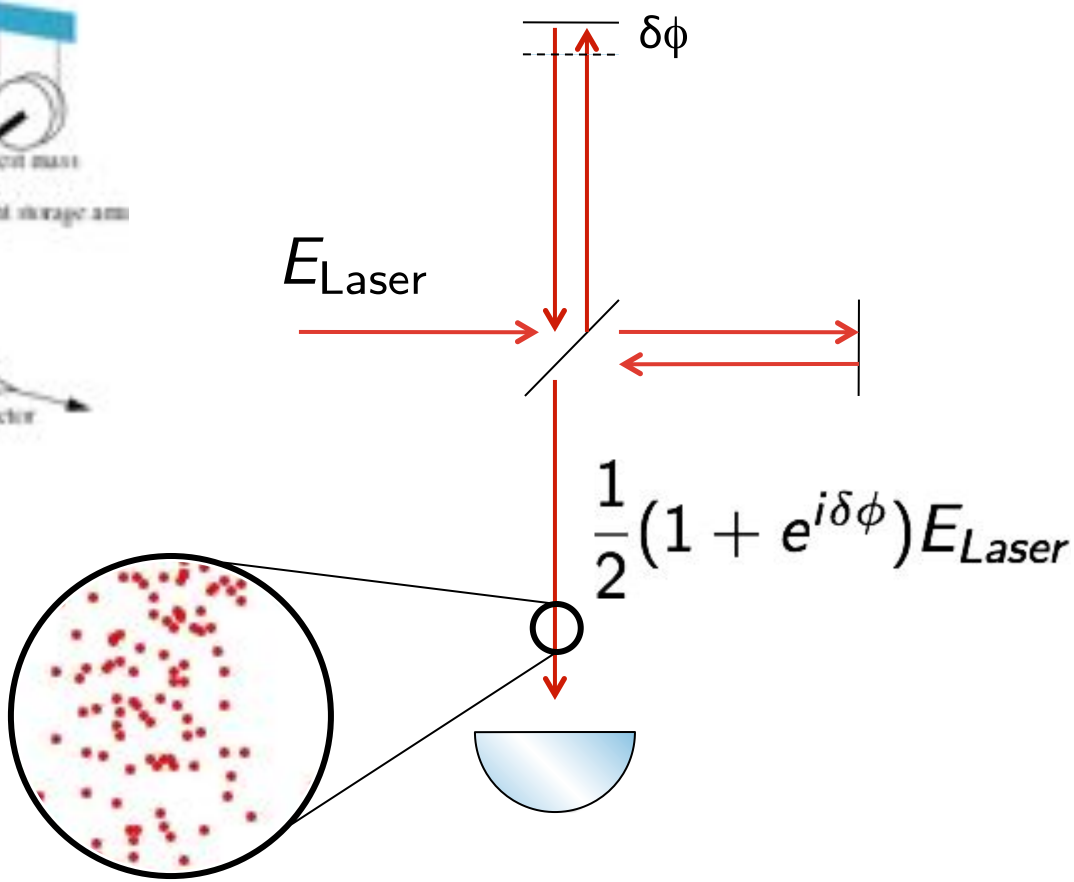
Morgan W. Mitchell

Virgo Supercluster



Shot noise in an interferometer

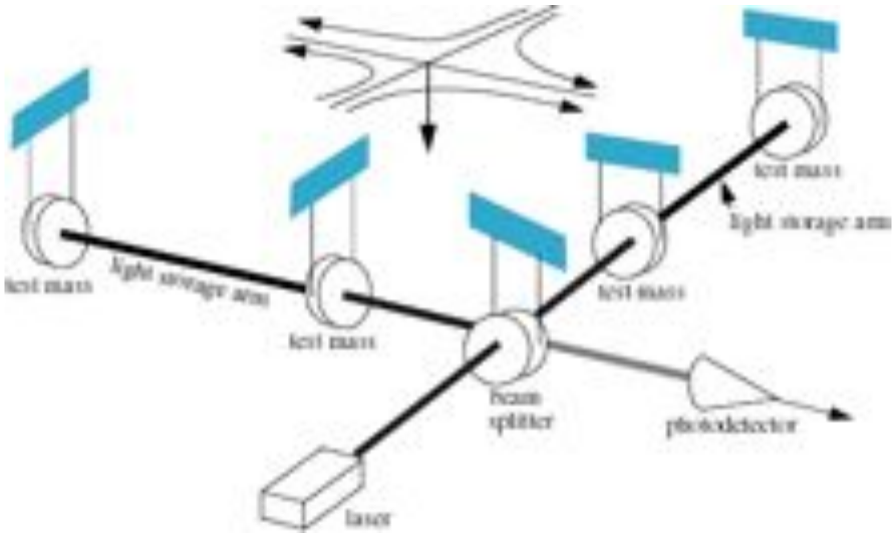


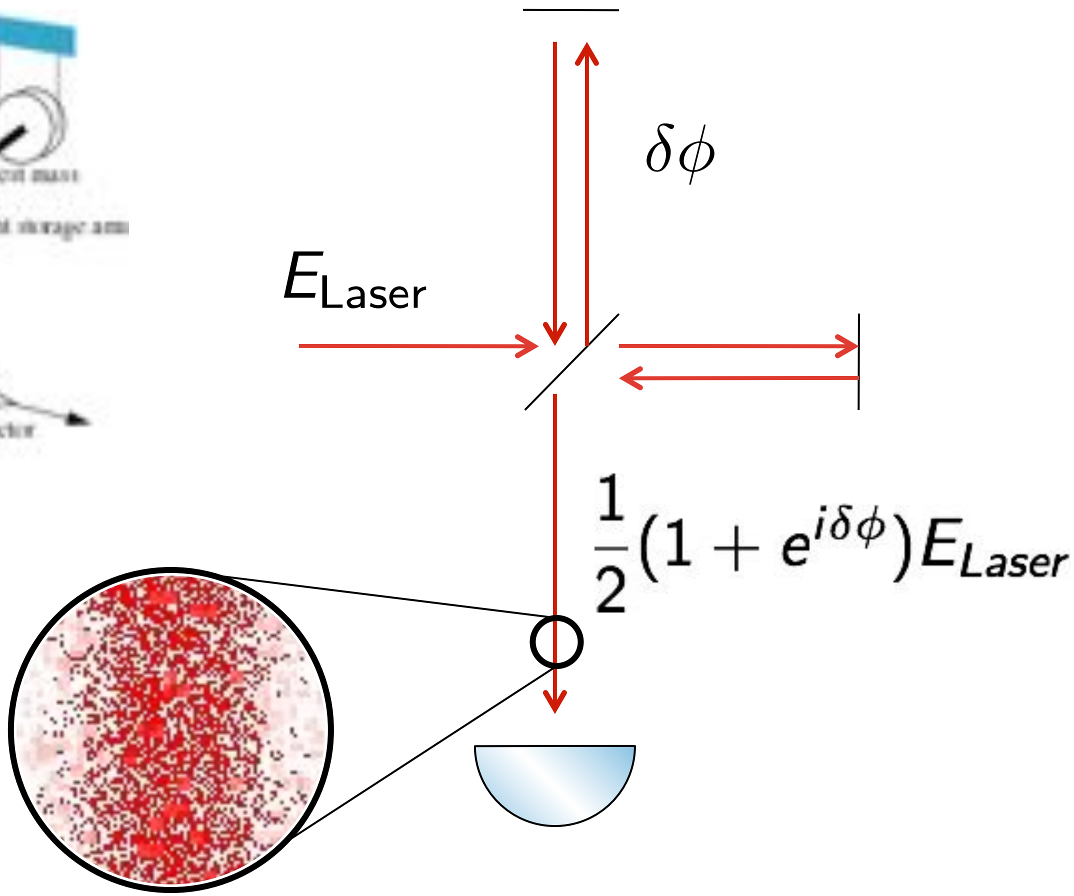


$$\langle n \rangle = \frac{P_{\text{laser}}}{\hbar\omega} t$$

$$\delta n = \sqrt{n} \qquad \delta\phi \sim n^{-1/2}$$

Shot noise in an interferomter



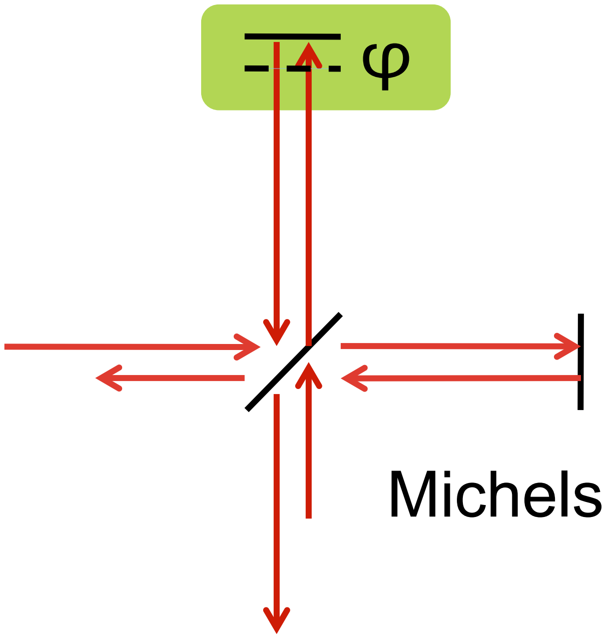


$$\langle n \rangle = \frac{P_{\text{laser}}}{\hbar\omega} t$$

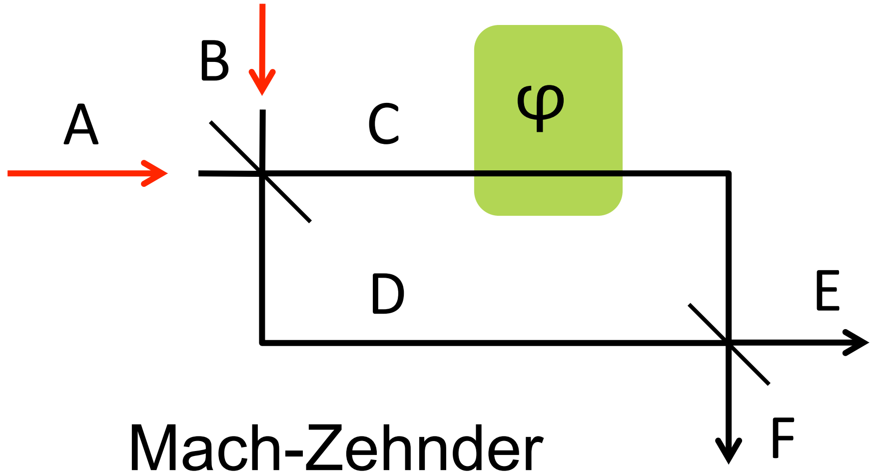
$$\delta n = \sqrt{n} \qquad \delta\phi \sim n^{-1/2}$$

$$S_P(\omega) = 2\hbar\omega\langle P \rangle$$

Michelson and other interferometers



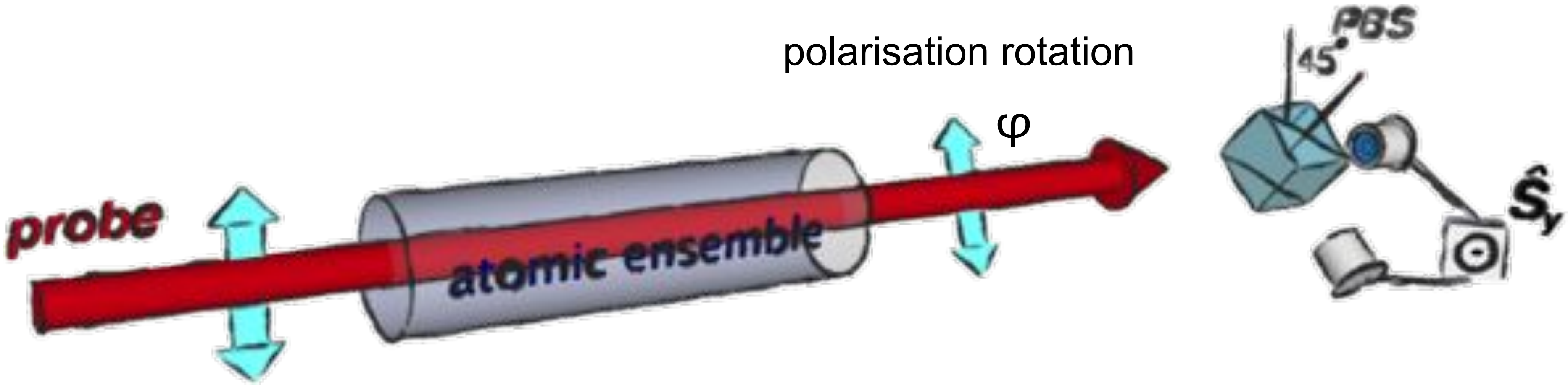
Michelson



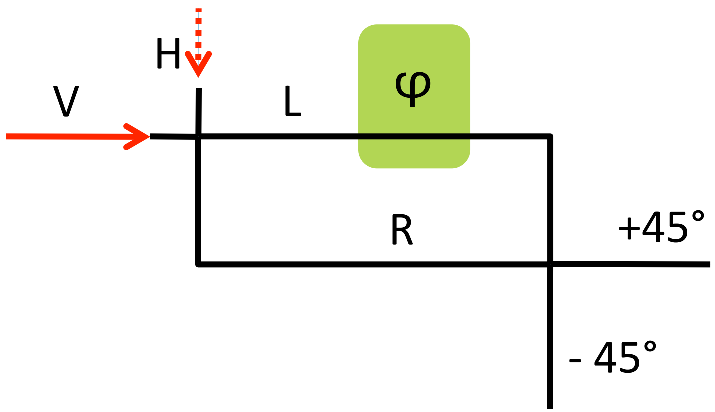
Mach-Zehnder

Michelson and other interferometers

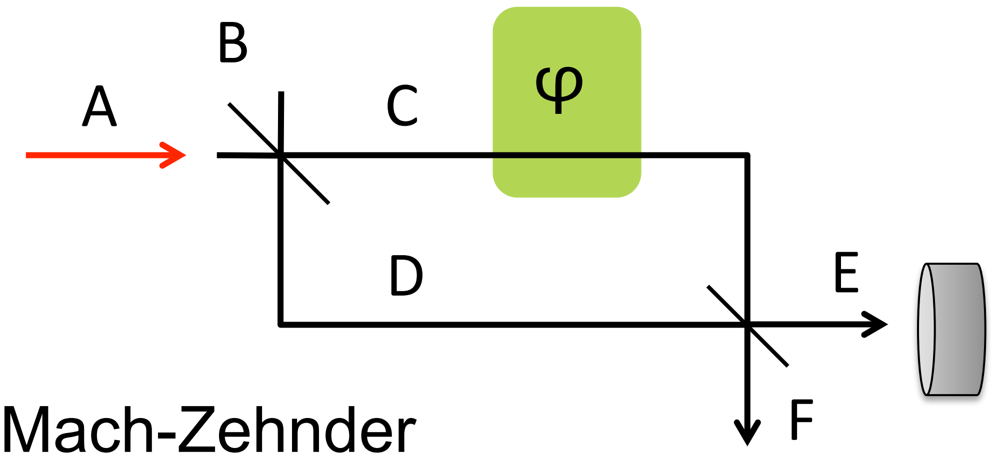
polarisation rotation



Polarisation interferometer

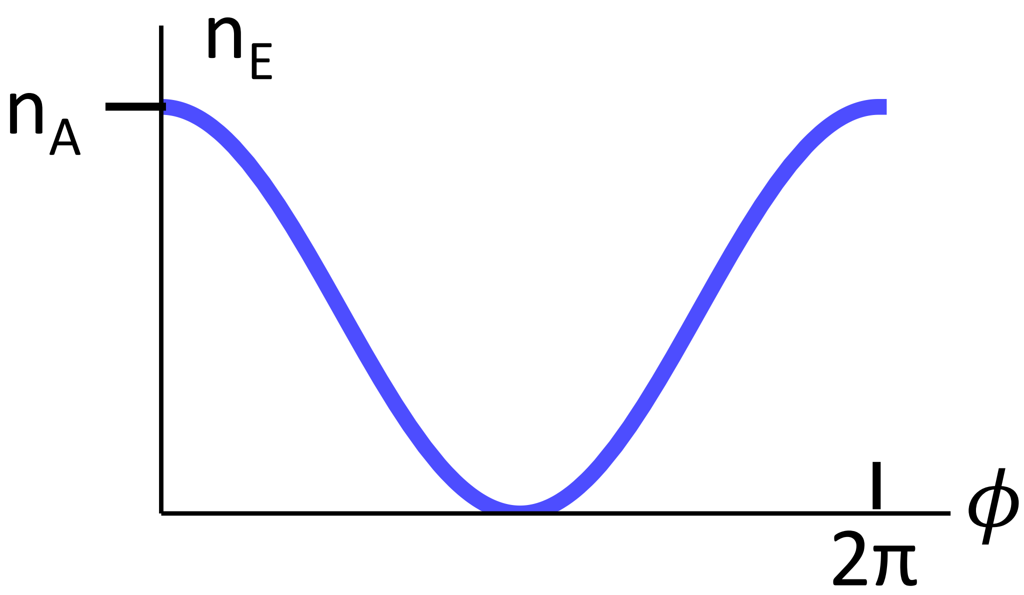


Shot noise in an interferometer (y < 1980)

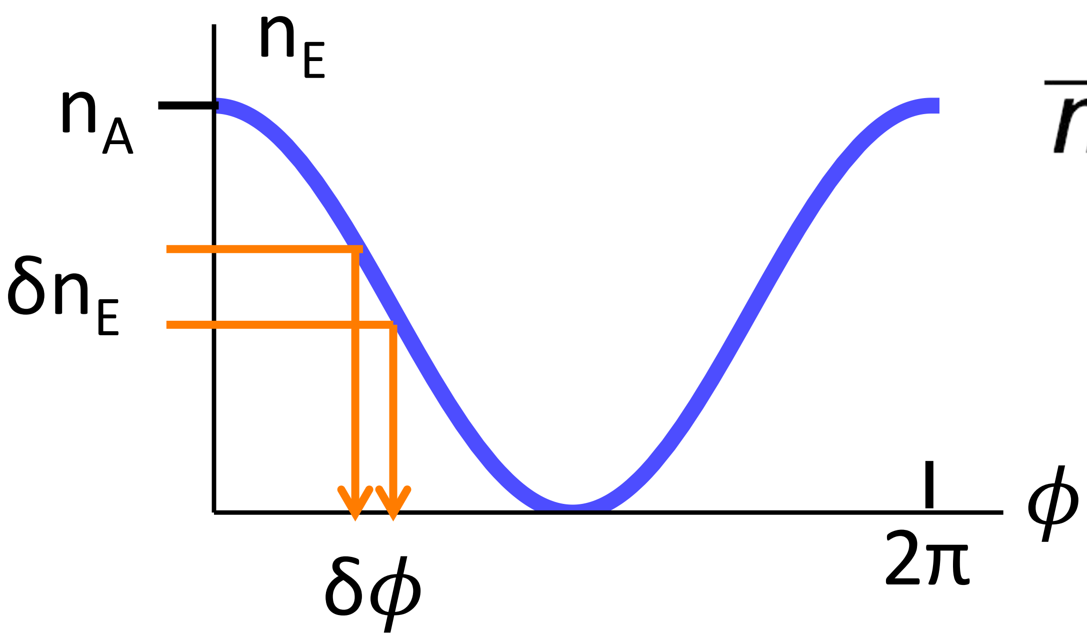


Mach-Zehnder

$$\overline{n_E} = \overline{n_A} \cos^2 \frac{\phi}{2}$$



Shot noise in an interferometer (y < 1980)



$$\overline{n_E} = \overline{n_A} \cos^2 \frac{\phi}{2}$$

$$\delta\phi = \delta n_E \left| \frac{d\overline{n_E}}{d\phi} \right|^{-1} \qquad \left| \frac{d\overline{n_E}}{d\phi} \right| = \overline{n_A} \left| \sin \frac{\phi}{2} \cos \frac{\phi}{2} \right|$$

$$\delta\phi = \frac{\delta n_E}{\overline{n_A} \left| \sin \frac{\phi}{2} \cos \frac{\phi}{2} \right|}$$

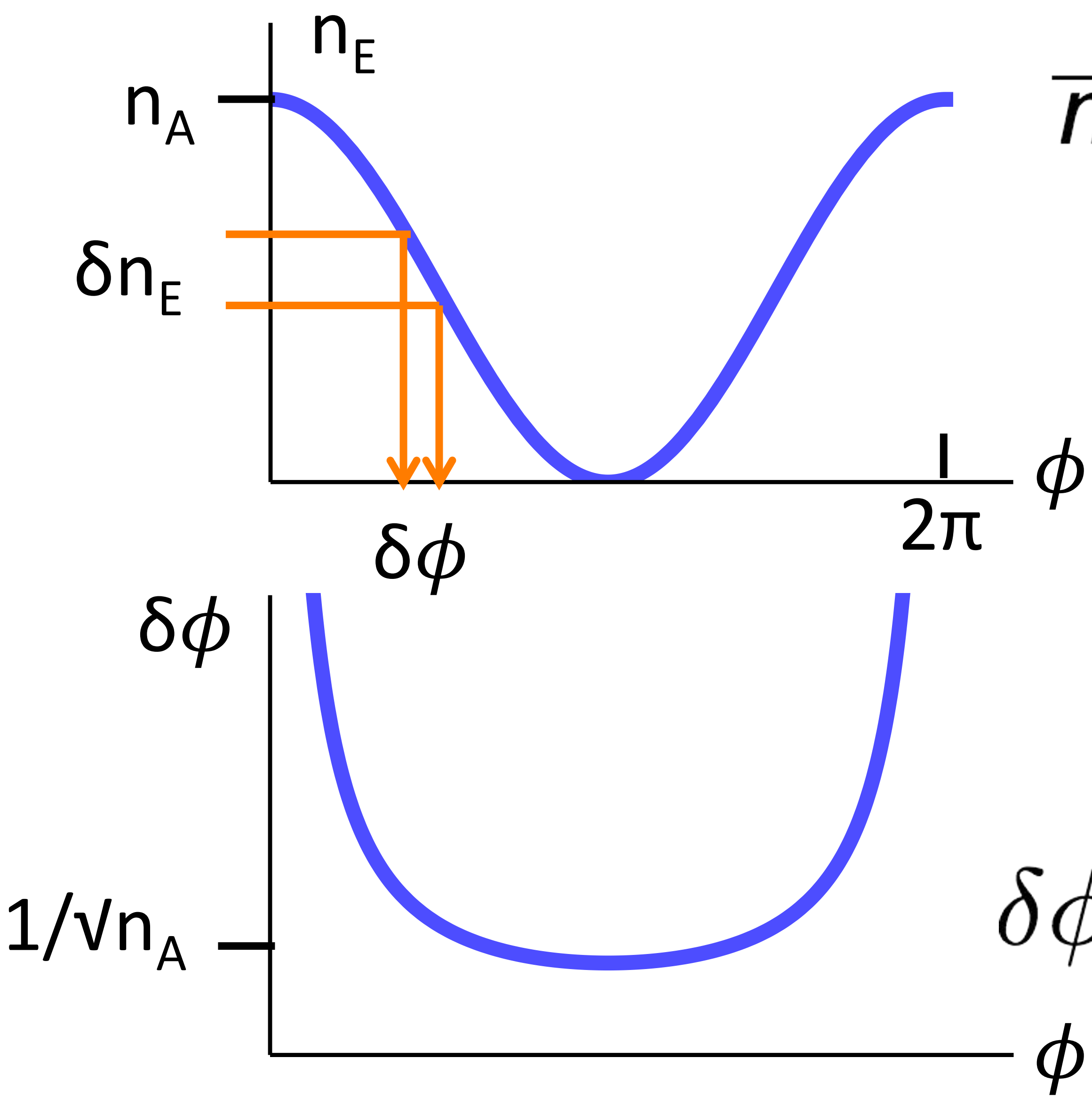
Poisson distribution

$$\overline{n_E} = \overline{n_A} \cos^2 \frac{\phi}{2}$$

$$\delta n_E = \sqrt{\overline{n_E}} = \sqrt{\overline{n_A}} \left| \cos \frac{\phi}{2} \right|$$

$$\delta\phi = \frac{\delta n_E}{\overline{n_A} \left| \sin \frac{\phi}{2} \cos \frac{\phi}{2} \right|} \qquad \delta\phi = \frac{1}{\sqrt{\overline{n_A}}} \frac{1}{\left| \sin \frac{\phi}{2} \right|}$$

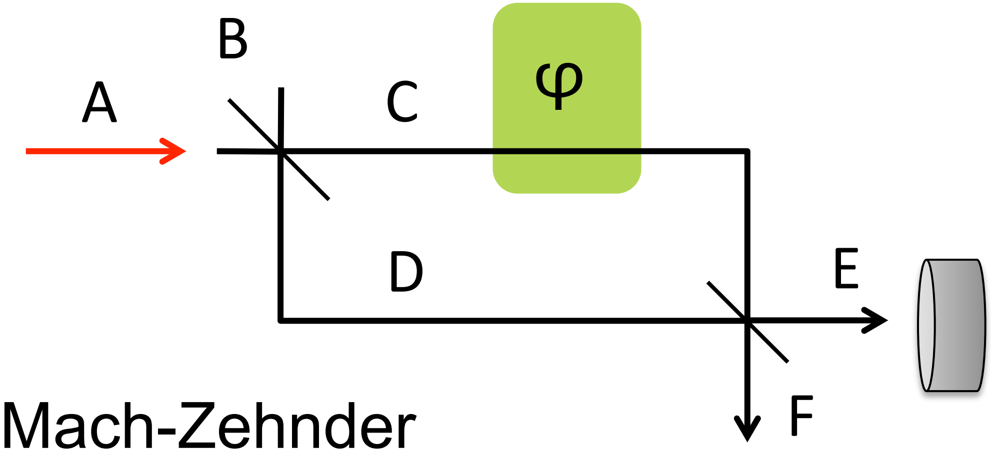
Shot noise in an interferometer (y < 1980)



$$\overline{n_E} = \overline{n_A} \cos^2 \frac{\phi}{2}$$

$$\delta\phi = \frac{1}{\sqrt{\overline{n_A}}} \frac{1}{|\sin \frac{\phi}{2}|}$$

Shot noise in an interferometer (y < 1980)



Mach-Zehnder

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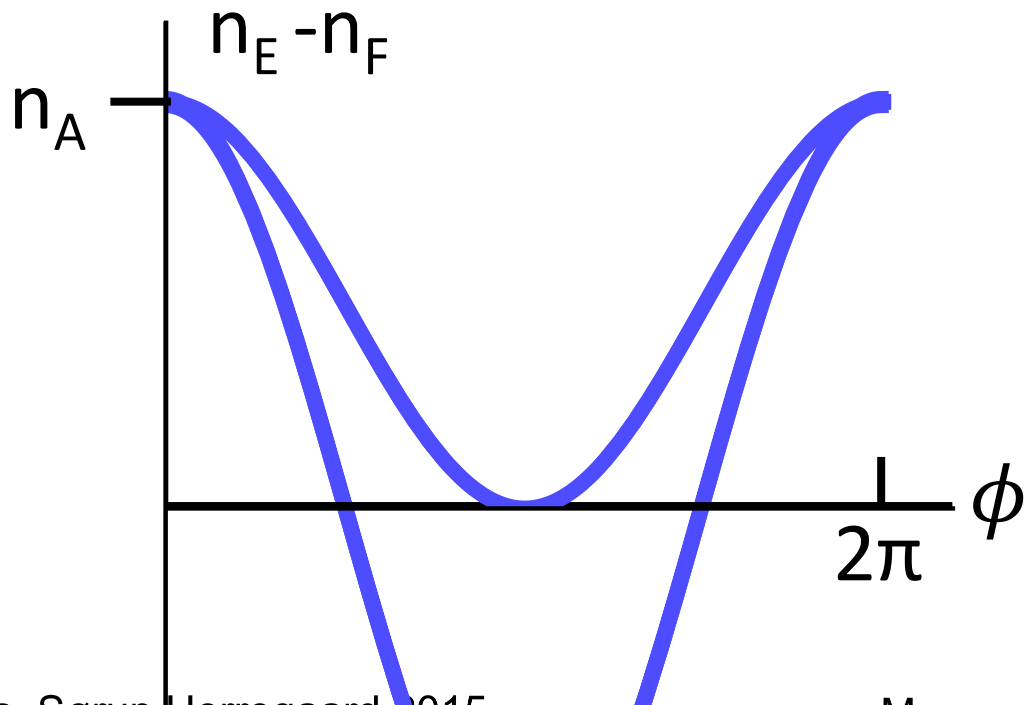
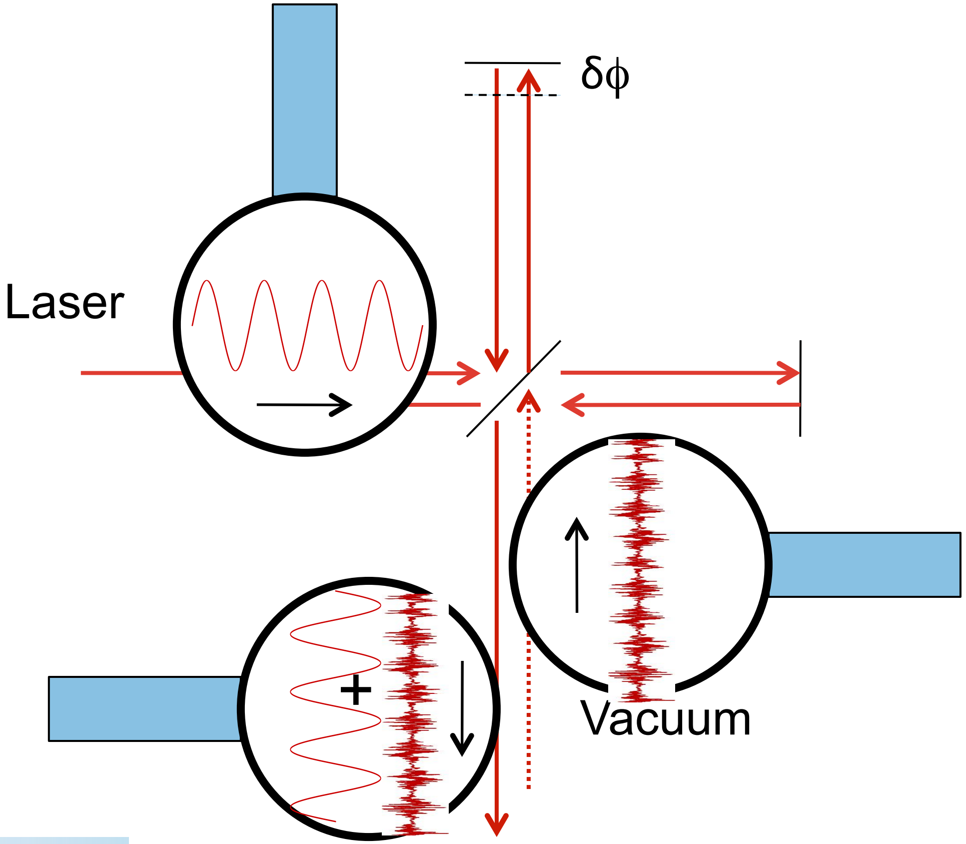
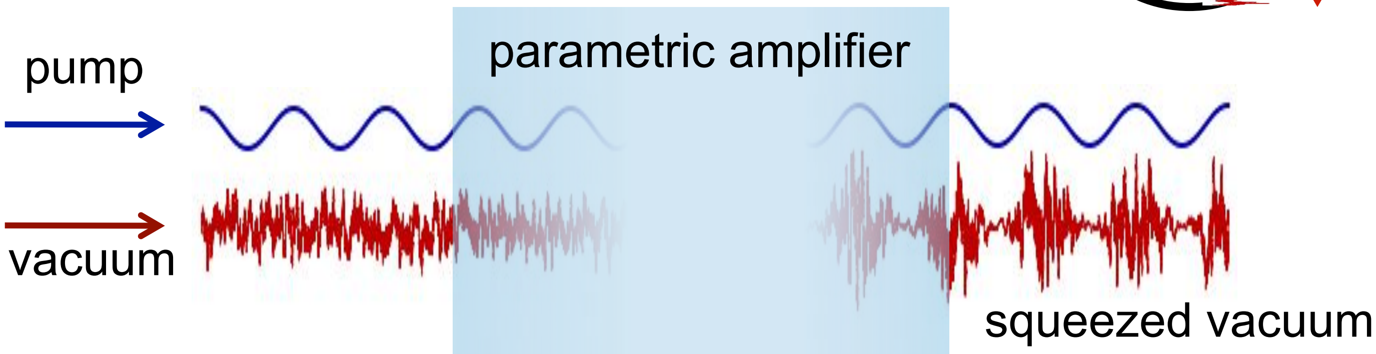


Beating shot noise in interferometry

Carlton M. Caves





Caves,
PRD 1981

Morgan W. Mitchell

Quantization of light

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial}{\partial t}\mathbf{B} \\ \nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial}{\partial t}\mathbf{E} \end{aligned}$$

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A} \\ \mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} \end{aligned}$$

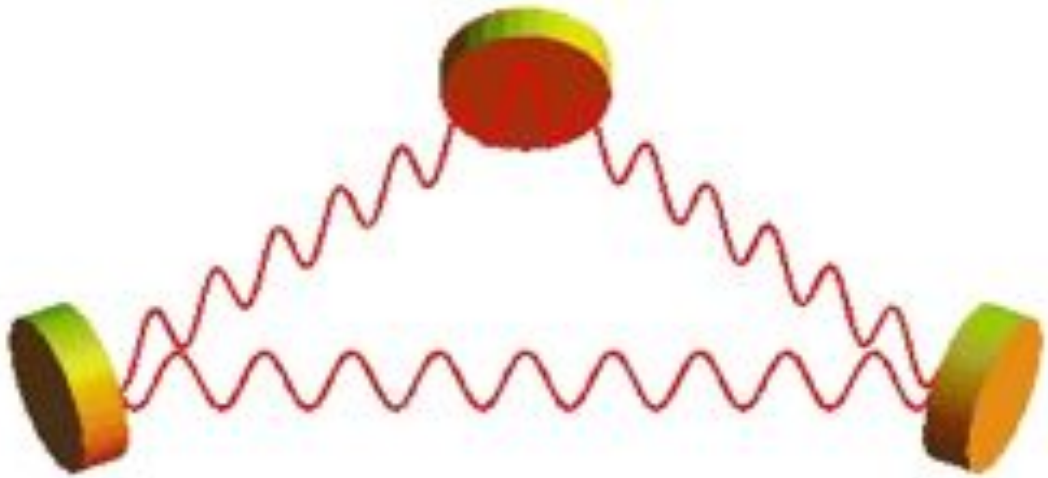
$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) \mathbf{A} = \mathbf{0}$$

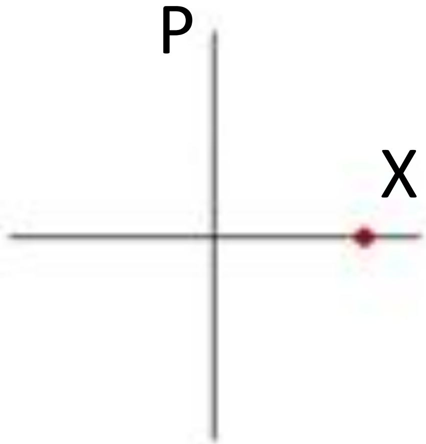
$$\mathbf{A}(\mathbf{r}, t) = \sum_{k,\alpha} q_{k,\alpha}(t) \mathbf{u}_{k,\alpha}(\mathbf{r})$$

$$\ddot{q}_{k,\alpha} = -c^2 k^2 q_{k,\alpha}$$

collection of harmonic oscillators

Light as harmonic oscillators

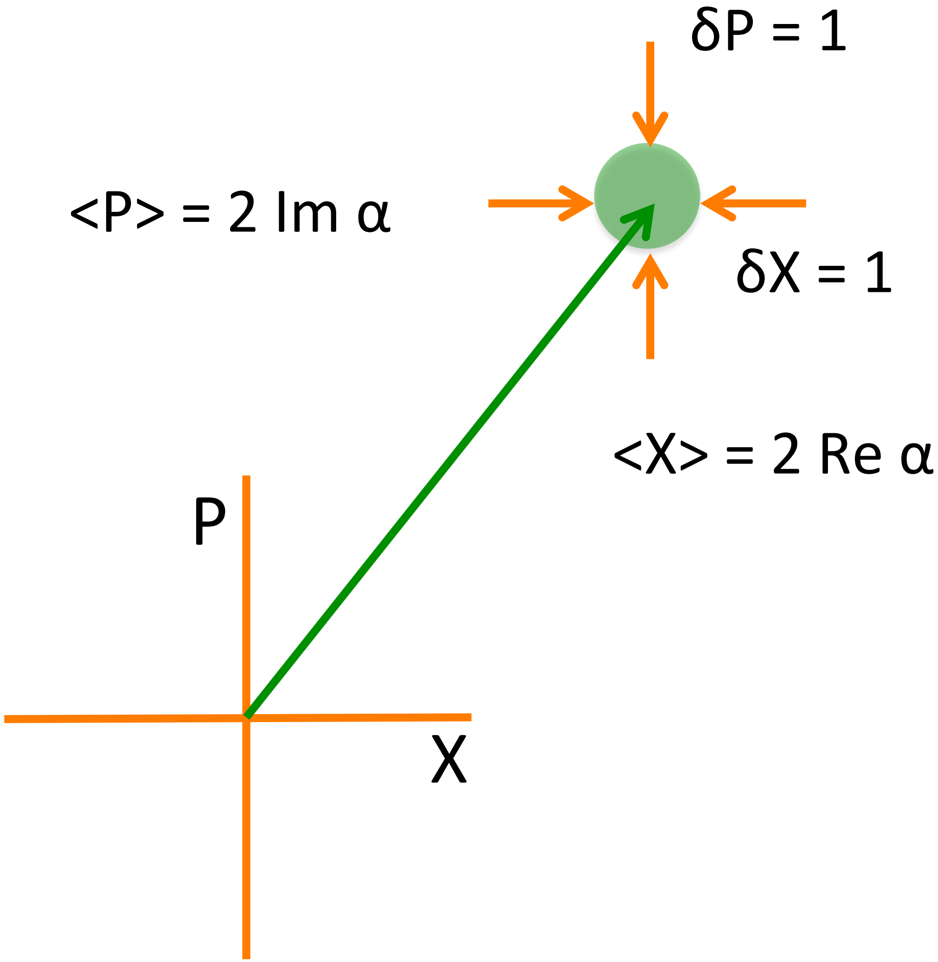




$$E(z,t) = A\cos(\omega t - kz)$$

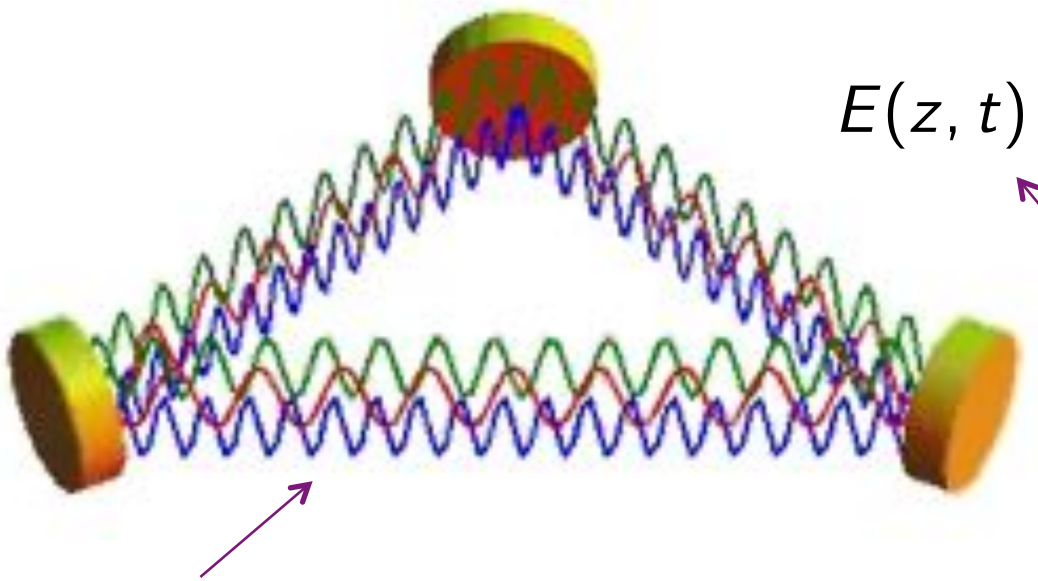
$$= X(t)\cos kz - P(t)\sin kz$$

Light as harmonic oscillators



$$|\alpha\rangle$$

Multi-mode light



$$E(z,t) = X(t)\cos kz - P(t)\sin kz$$

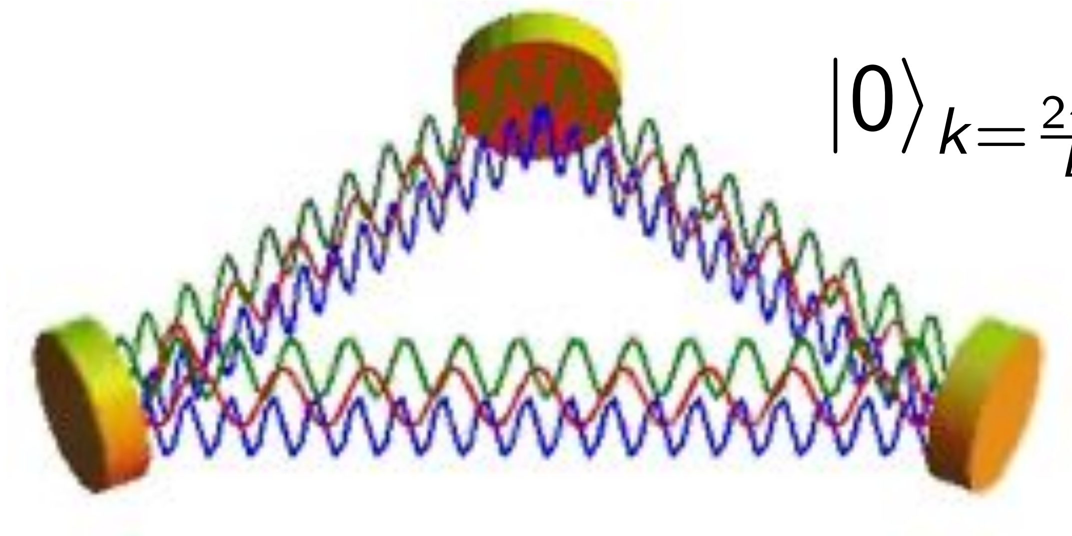
single mode

$$E(z,t) = \sum_k X_k(t)\cos kz + P_k(t)\sin kz$$

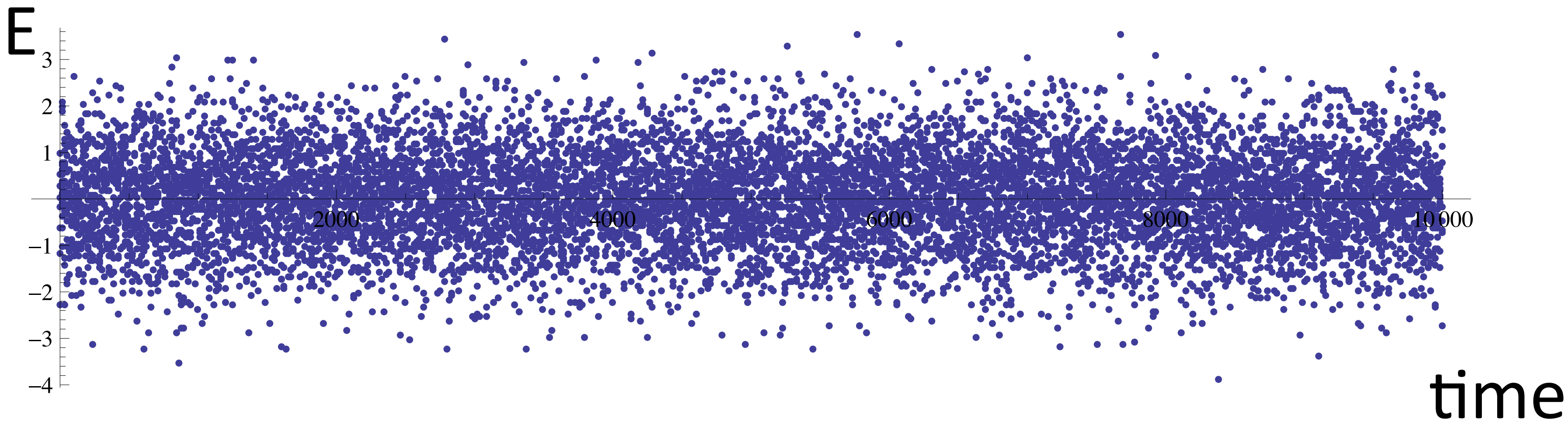
$$E(0,t) = \sum_k X_k(t)$$

$$= \sum_k \left[X_k(0)\cos ckt + P_k(0)\sin ckt\right]$$

Multi-mode light

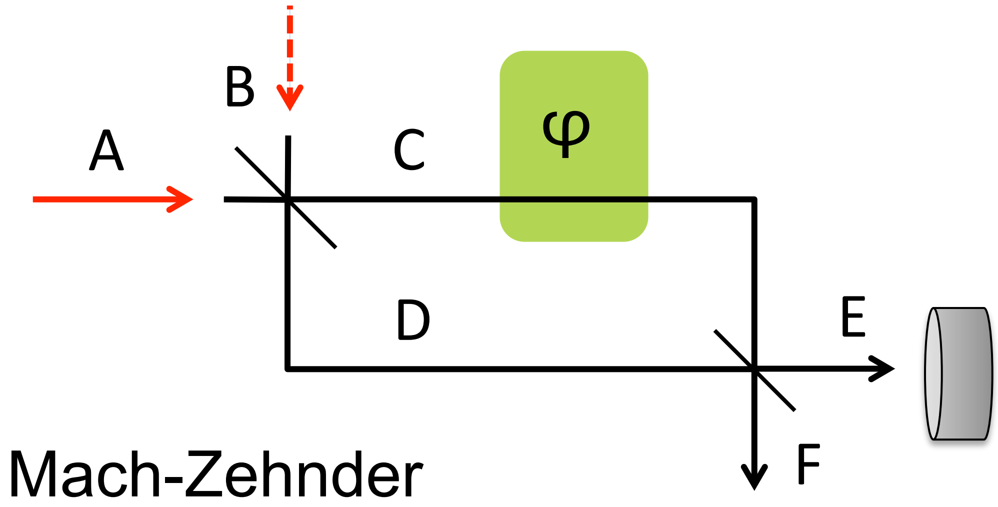


$$|0\rangle_{k=\frac{2\pi}{L}} |0\rangle_{k=\frac{4\pi}{L}} |0\rangle_{k=\frac{6\pi}{L}} \cdots$$

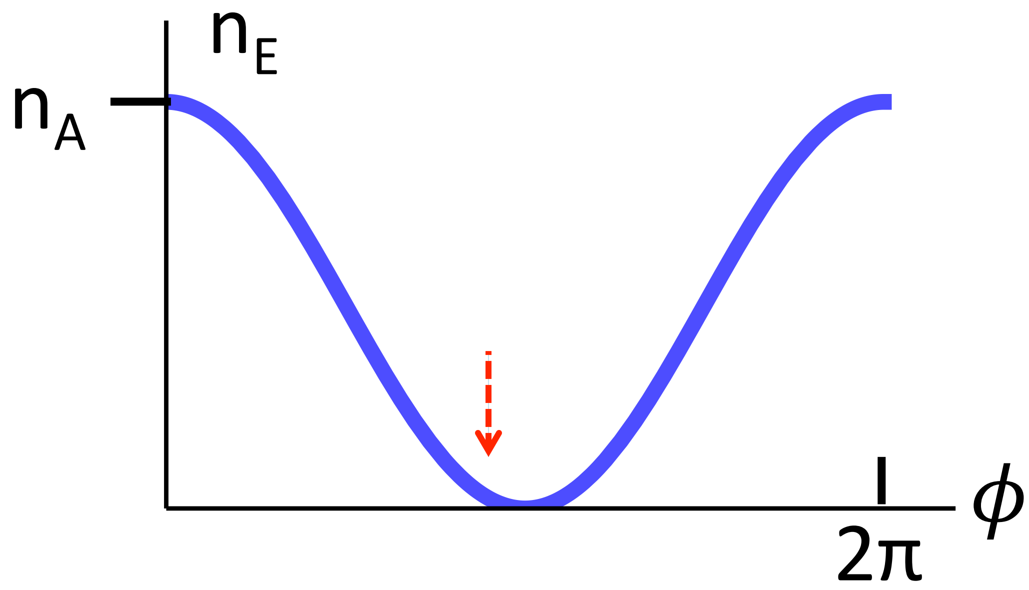


Morgan W. Mitchell

Shot noise in an interferometer (y < 1980)



$$\overline{n_E} = \overline{n_A} \cos^2 \frac{\phi}{2}$$



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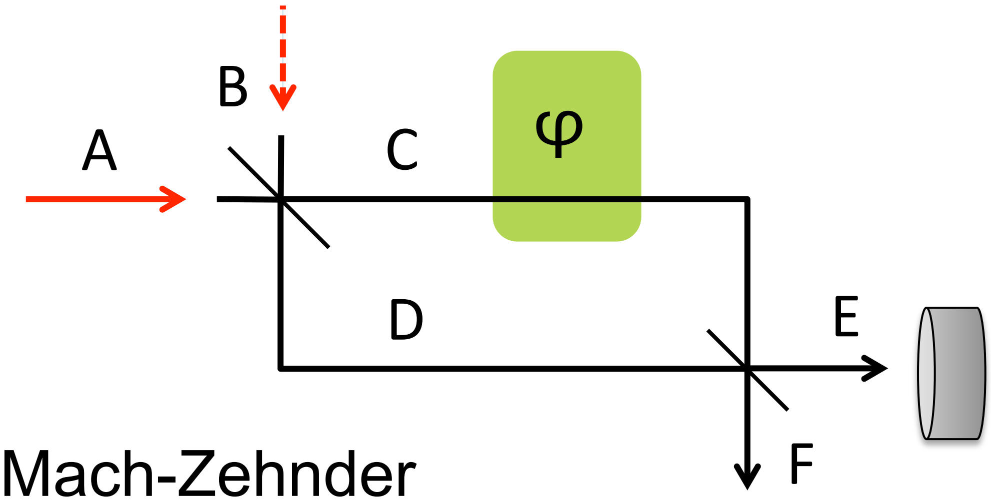


Mach-Zehnder

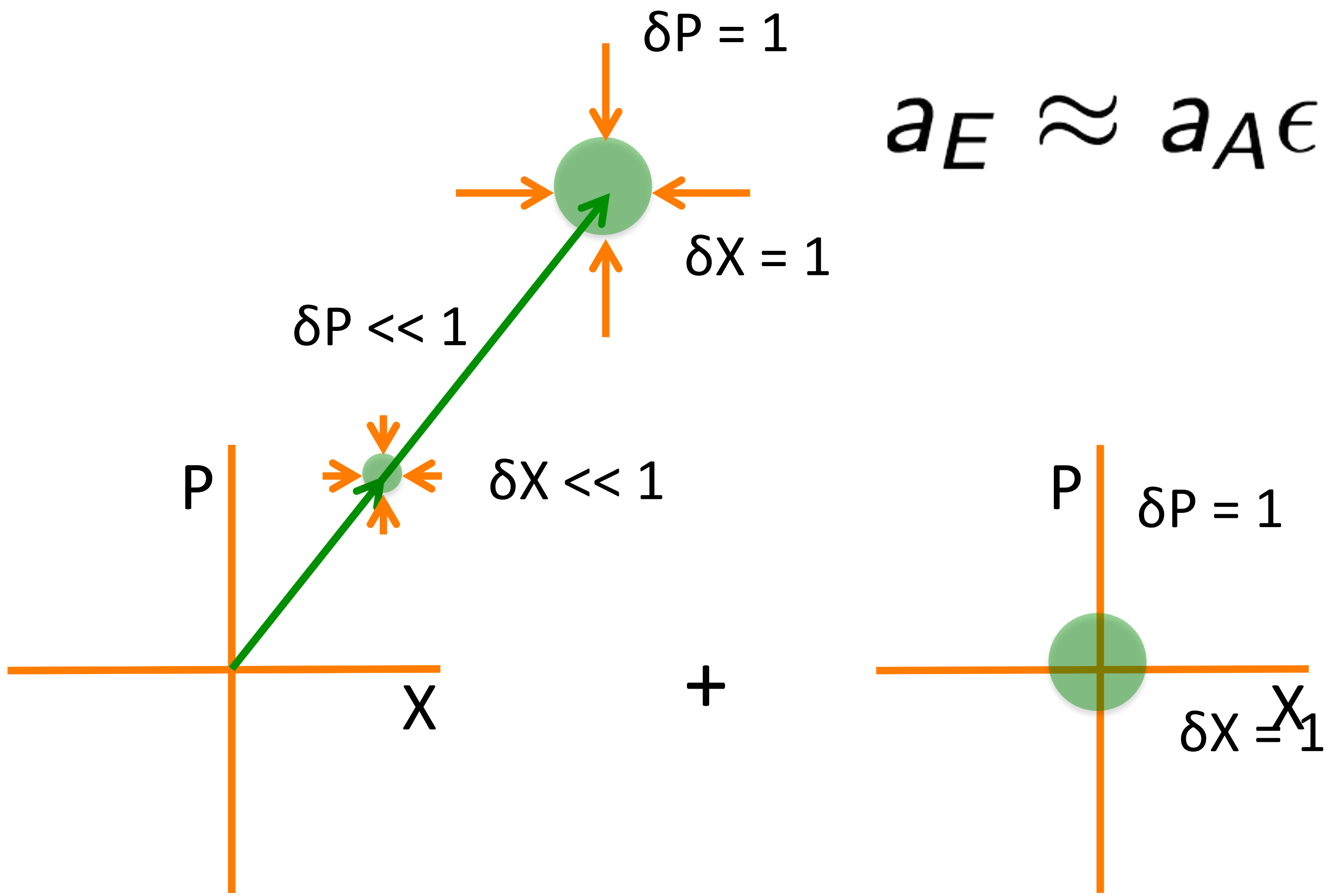
$$\overline{n_E} = \overline{n_A} \cos^2 \frac{\phi}{2}$$

$$a_E = a_A \cos \frac{\phi}{2} + a_B \sin \frac{\phi}{2}$$

$$\approx a_A \epsilon + a_B$$

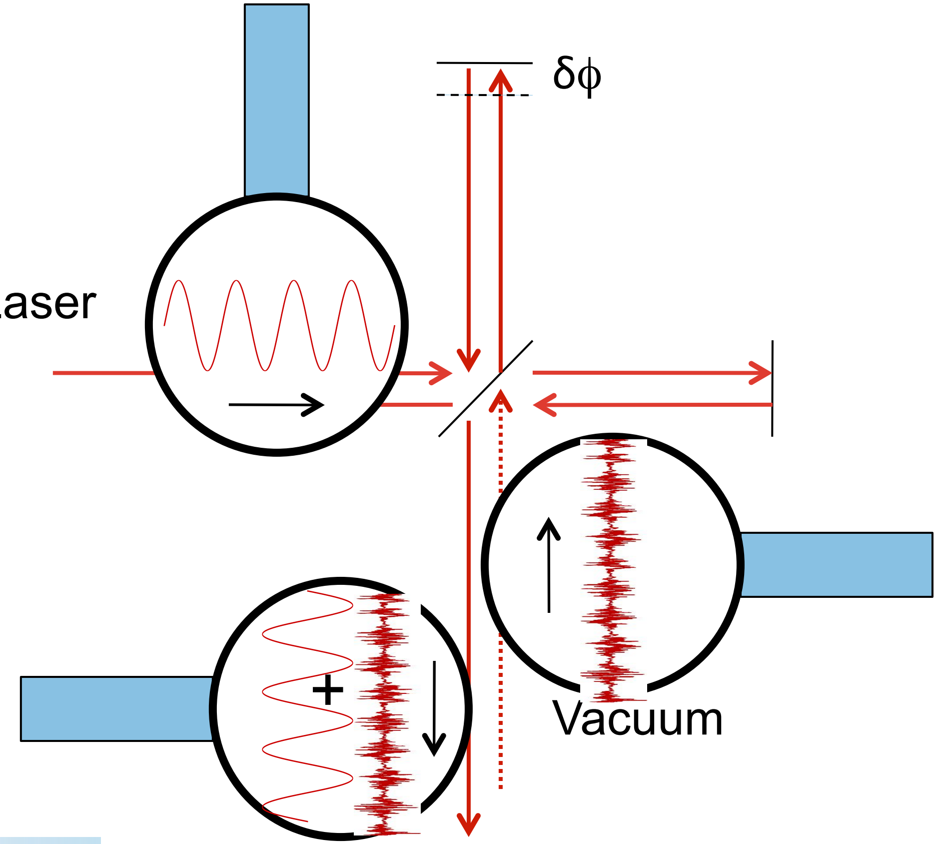
the heart of Caves' insight

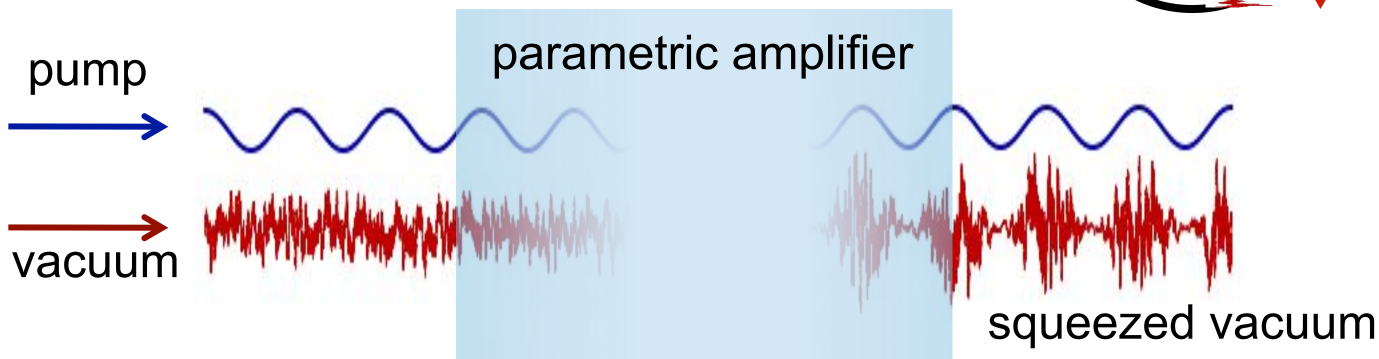
$$a_E \approx a_A \epsilon + a_B$$



Beating shot noise in interferometry

Carlton M. Caves





Caves,
PRD 1981

Proposal for squeezing (C. Caves, 1981)

Quantum-mechanical noise in an interferometer

Carlton M. Caves

Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

(Received 15 August 1980)

The interferometers now being developed to detect gravitational waves work by measuring the relative positions of widely separated masses. Two fundamental sources of quantum-mechanical noise determine the sensitivity of such an interferometer: (i) fluctuations in number of output photons (photon-counting error) and (ii) fluctuations in radiation pressure on the masses (radiation-pressure error). Because of the low power of available continuous-wave lasers, the sensitivity of currently planned interferometers will be limited by photon-counting error. This paper presents an analysis of the two types of quantum-mechanical noise, and it proposes a new technique—the "squeezed-state" technique—that allows one to decrease the photon-counting error while increasing the radiation-pressure error, or vice versa. The key requirement of the squeezed-state technique is that the state of the light entering the interferometer's normally unused input port must be not the vacuum, as in a standard interferometer, but rather a "squeezed state"—a state whose uncertainties in the two quadrature phases are unequal. Squeezed states can be generated by a variety of nonlinear optical processes, including degenerate parametric amplification.

C. Caves, "Quantum Mechanical Noise In an Interferometer" Phys. Rev. D **23** 1693 1981

Proposal for squeezing (C. Caves, 1981)

$$S^\dagger(\zeta) a S(\zeta) = a \cosh r - a^\dagger e^{i\theta} \sinh r\,,$$
$$S^\dagger(\zeta) a^\dagger S(\zeta) = a^\dagger \cosh r - a e^{-i\theta} \sinh r\,, \qquad (2.8)$$

$$S^\dagger(re^{i\theta}) a S(re^{i\theta}) = a \cosh r - a^\dagger e^{i\theta} \sinh r$$
$$S^\dagger(re^{i\theta}) a^\dagger S(re^{i\theta}) = a^\dagger \cosh r - a e^{i\theta} \sinh r$$

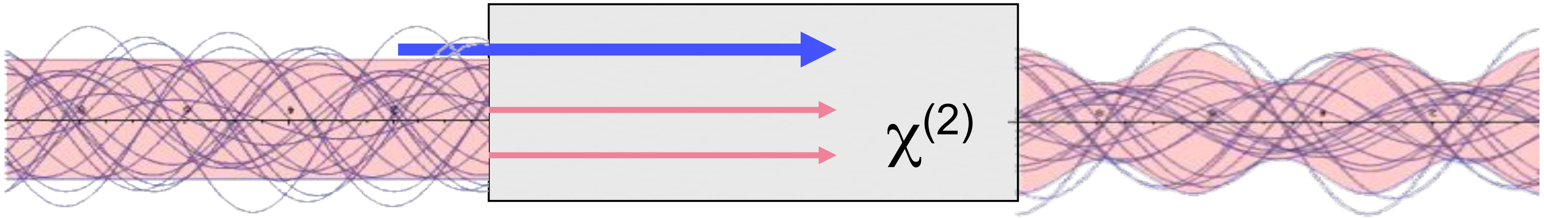
FIG. 2. Graphs of electric field versus time for three states of the electromagnetic field. In each graph the dark line is the expectation value of the electric field, and the shaded region represents the uncertainty in the electric field. To the right of each graph is the corresponding "error box" in the complex-amplitude plane. (a) Coherent state $|\alpha\rangle$ (α real). This state exhibits neither bunching nor antibunching ($g_{12}^{(2)}=1$). (b) Squeezed state $|\alpha, r\rangle$ (α real) with $r>0$. This state exhibits antibunching ($g_{12}^{(2)}<1$) as long as $0<r\le\frac{1}{4}\ln(8\alpha^2)$. (c) Squeezed state $|\alpha, r\rangle$ (α real) with $r<0$. This state exhibits bunching.

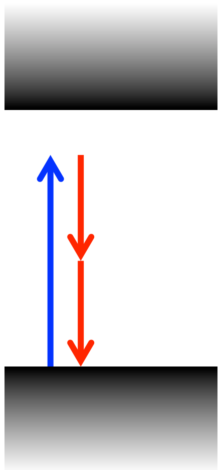
FIG. 1. (a) Error circle in complex-amplitude plane for coherent state $|\alpha\rangle$. (b) Error ellipse in complex-amplitude plane for squeezed state $|\alpha, re^{i\theta}\rangle$ ($r>0$).

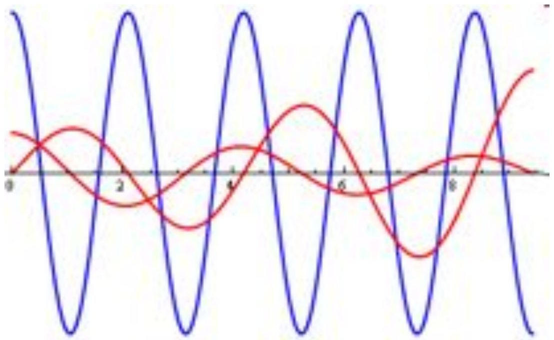
C. Caves, "Quantum Mechanical Noise In an Interferometer" Phys. Rev. D **23** 1693 1981

Parametric amplification

vacuum fluctuations   parametric amplifier   “squeezed vacuum”



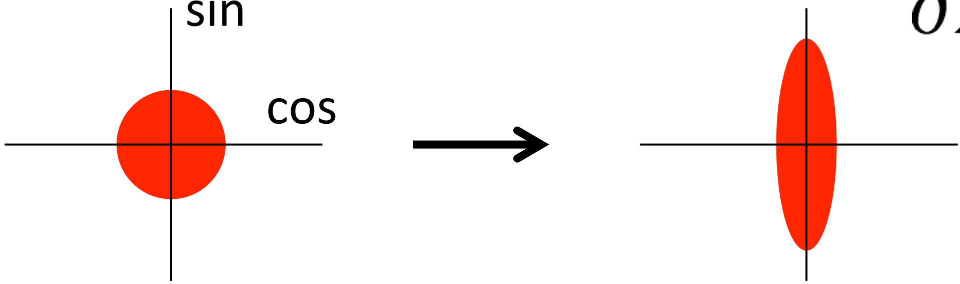




sine – amplified

cosine - deamplified

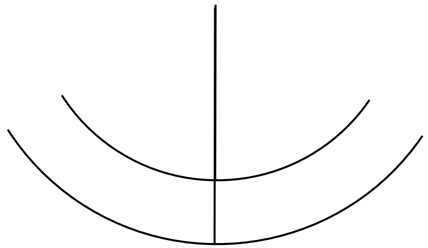
Phase-sensitive amplification



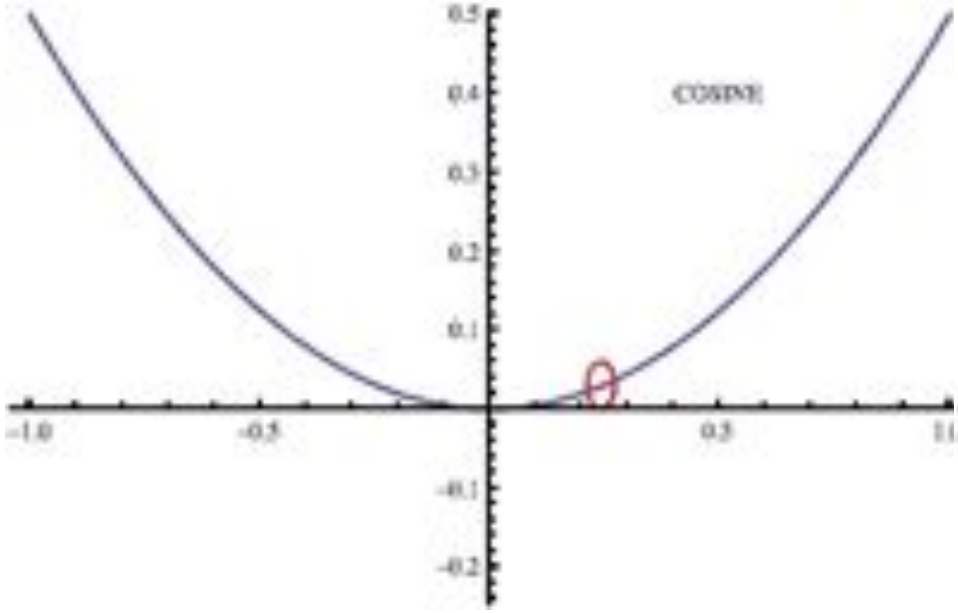
$$\delta x \delta p \geq \frac{1}{2}$$

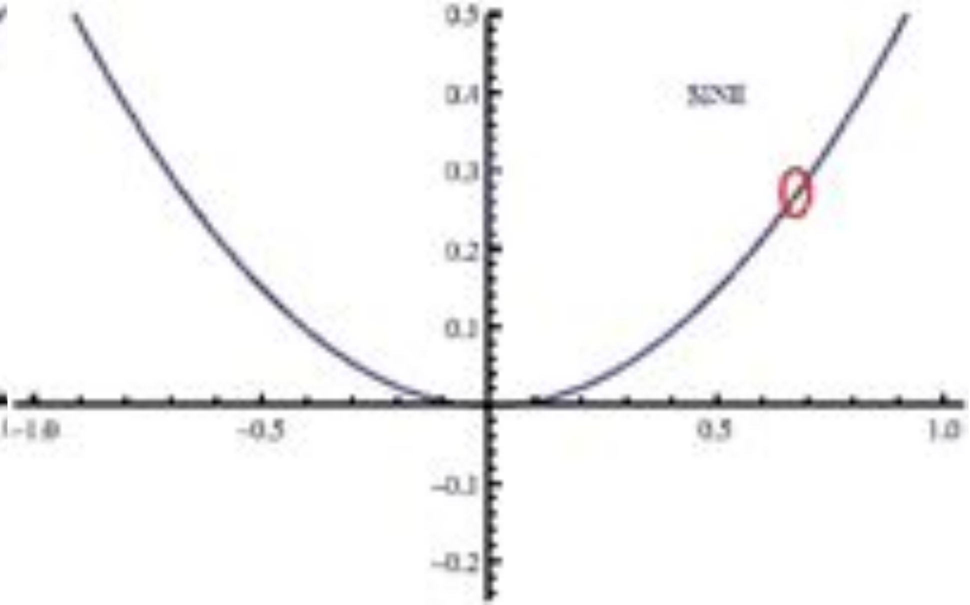
parametric amplifier

$$\frac{d^2}{dt}\theta = -\frac{g}{L}\theta$$

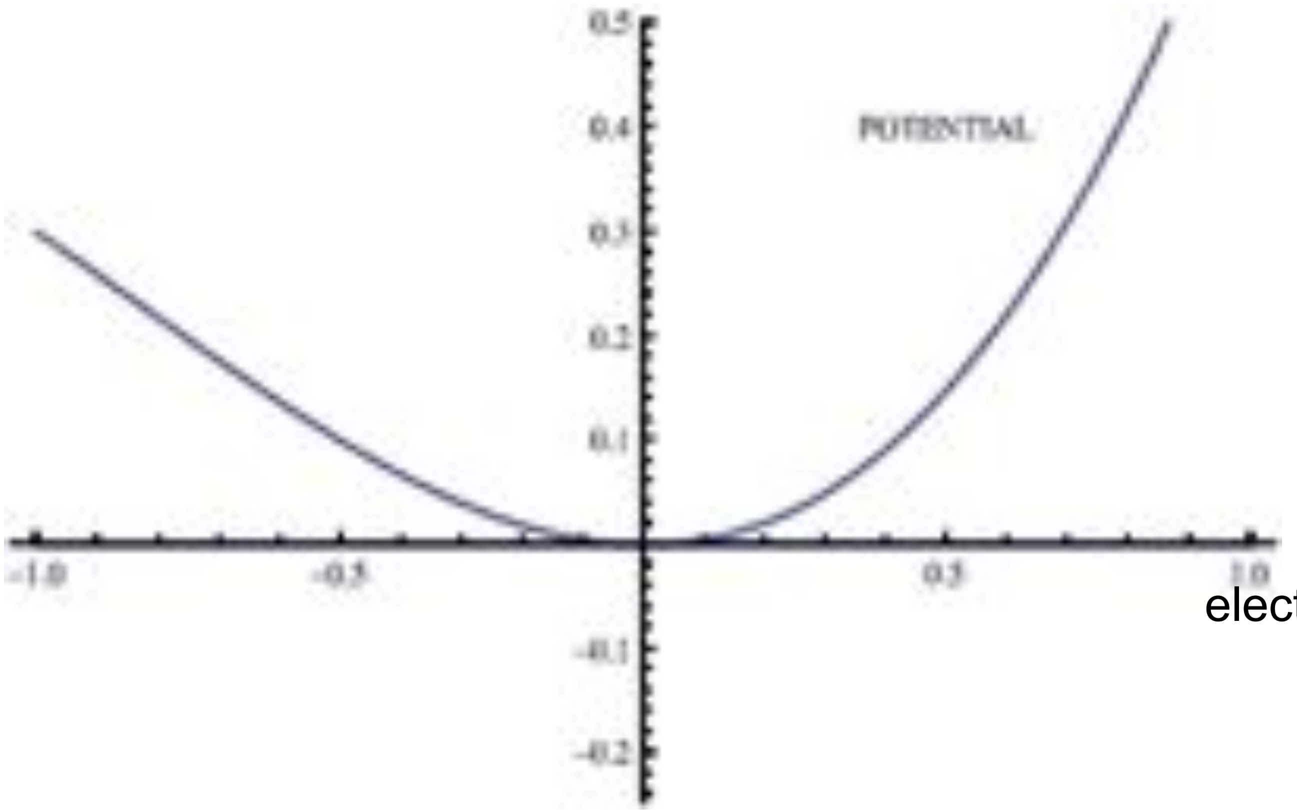


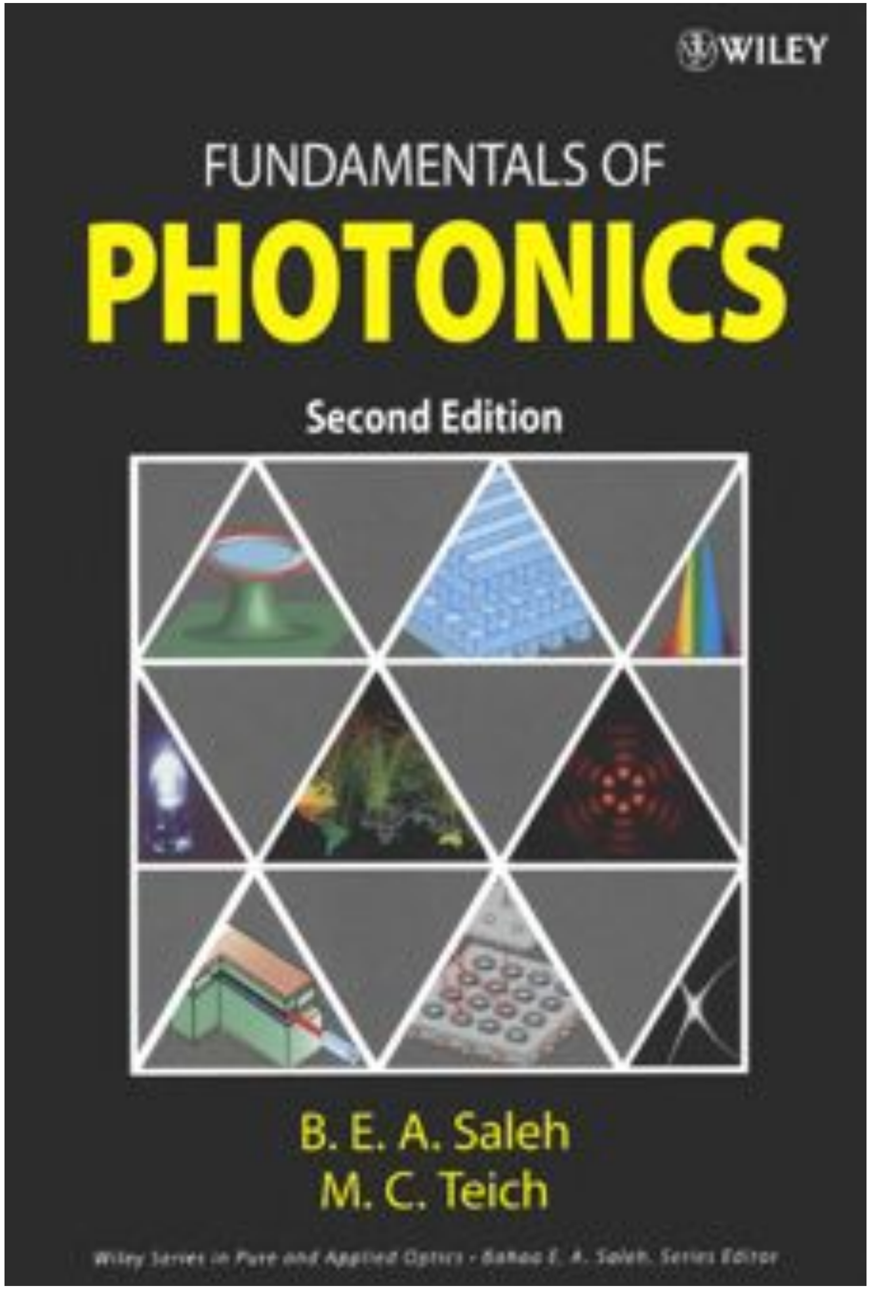
parametric amplification





optical parametric amplifier

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Second Edition
B. E. A. Saleh
M. C. Teich

Saleh + Teich “Fundamentals of Photonics” p. 914

Optical Parametric Amplifier (OPA)

The OPA uses three-wave mixing in a nonlinear crystal to provide optical gain [Fig. 21.4-3(*a*)]. The process is governed by the same three coupled equations (21.4-20) with the waves identified as follows. Wave 1 is the **signal** to be amplified; it is incident on the crystal with a small intensity $I_1(0)$. Wave 3, the **pump**, is an intense wave that provides power to the amplifier. Wave 2, called the **idler**, is an auxiliary wave created by the interaction process.

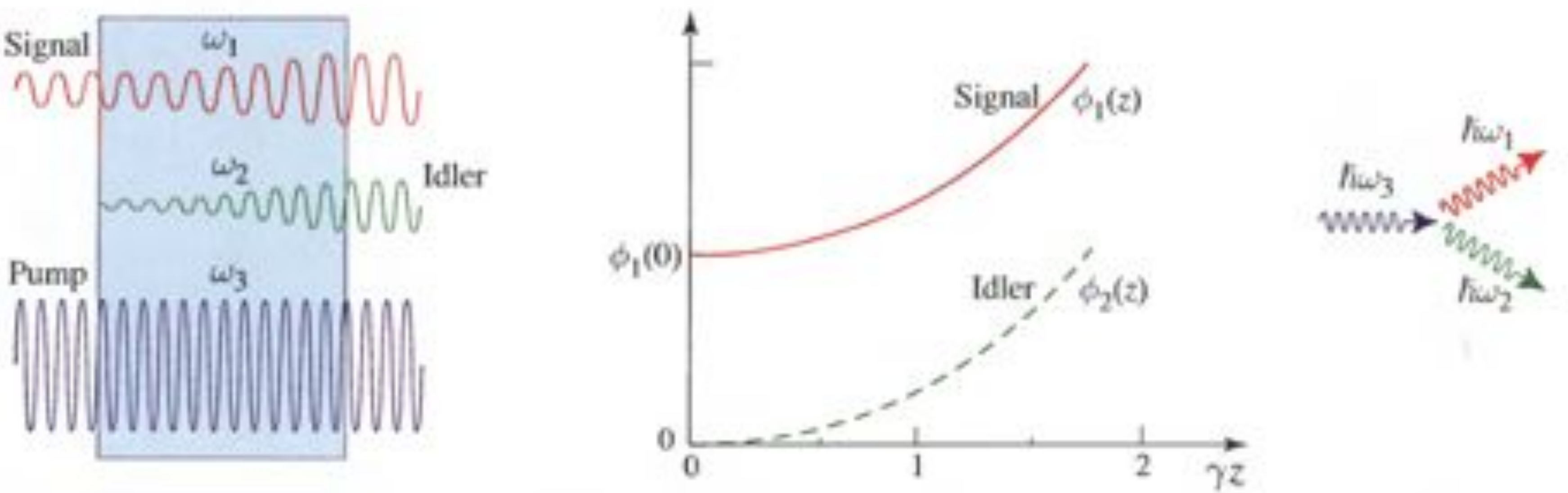


Figure 21.4-3 The optical parametric amplifier: (*a*) wave mixing; (*b*) photon flux densities of the signal and the idler (the pump photon-flux density is assumed constant); (*c*) photon mixing.

$$(\nabla^2 + k_1^2)E_1 = -2\mu_o\omega_1^2 \mathrm{d}\, E_3 E_1^*, \qquad (21.4\text{-}16a)$$

$$(\nabla^2 + k_3^2)E_3 = -\mu_o\omega_3^2 \mathrm{d}\, E_1 E_1. \qquad (21.4\text{-}16b)$$

SHG Coupled Equations

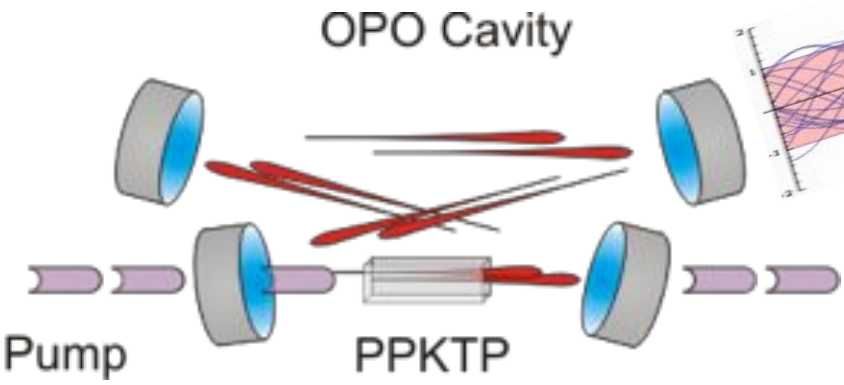
$$E_q = \sqrt{2\eta\hbar\omega_q}\,\mathrm{a}_q \exp(-jk_q z), \qquad q = 1, 2, 3,$$

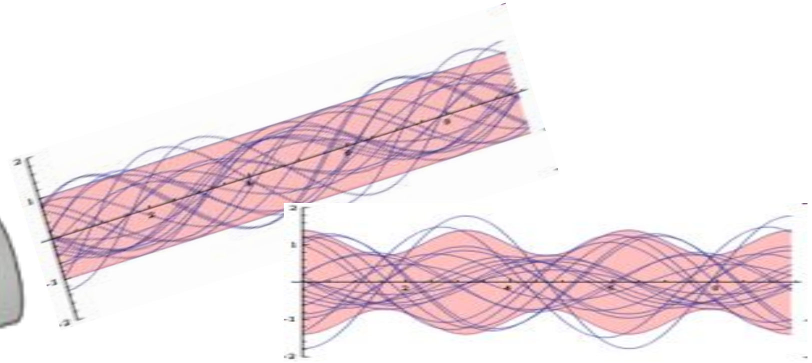
$$S^\dagger(re^{i\theta})aS(re^{i\theta}) = a\cosh r - a^\dagger e^{i\theta}\sinh r$$
$$S^\dagger(re^{i\theta})a^\dagger S(re^{i\theta}) = a^\dagger\cosh r - ae^{i\theta}\sinh r$$

where $\gamma = 2g\mathfrak{a}_3(0)$. If $\mathfrak{a}_3(0)$ is real, γ is also real, and the differential equations have the solution

$$\mathfrak{a}_1(z) = \mathfrak{a}_1(0)\cosh\frac{\gamma z}{2} - j\mathfrak{a}_2^*(0)\sinh\frac{\gamma z}{2} \quad (21.4\text{-}45a)$$
$$\mathfrak{a}_2(z) = -j\mathfrak{a}_1^*(0)\sinh\frac{\gamma z}{2} + \mathfrak{a}_2(0)\cosh\frac{\gamma z}{2}. \quad (21.4\text{-}45b)$$

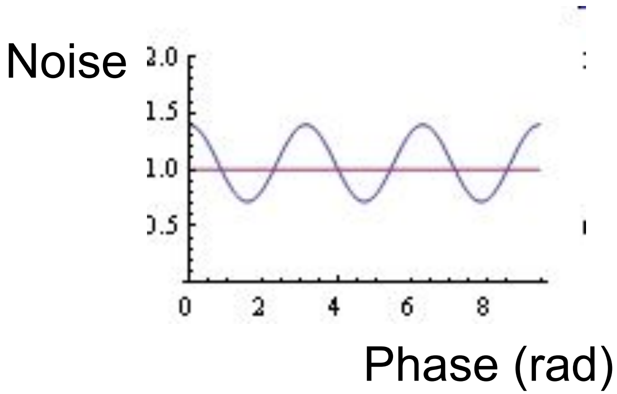
Reducing the vacuum fluctions





cosine amplified

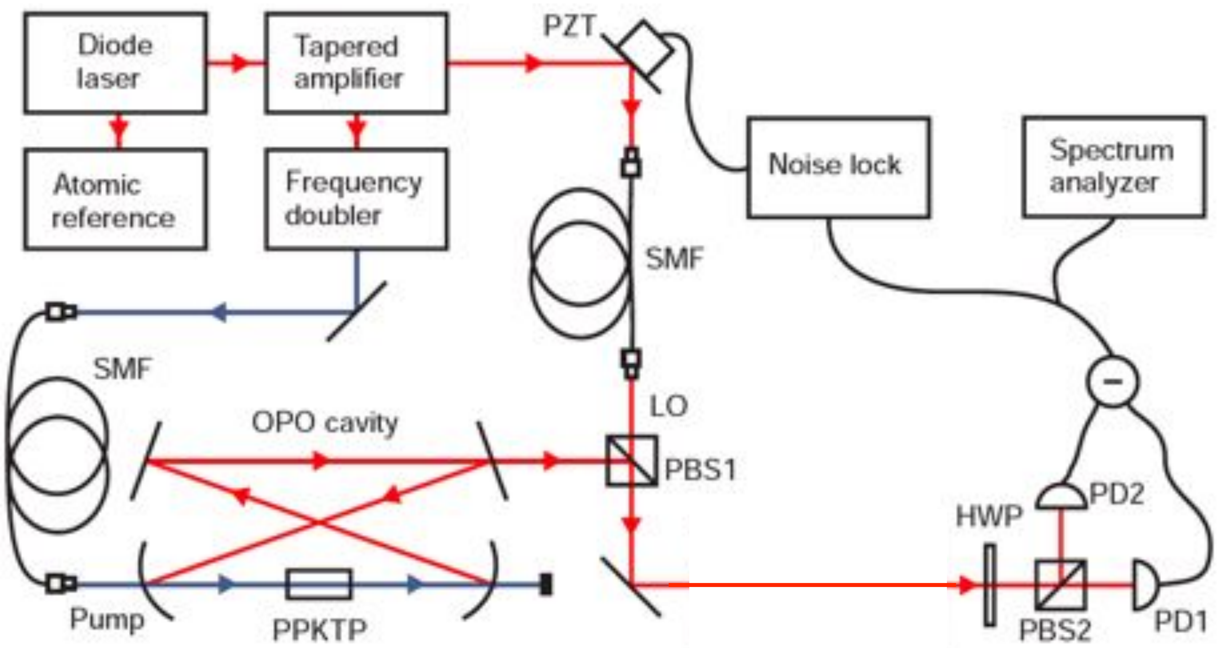
sine de-amplified



Squeezed-light in Barcelona

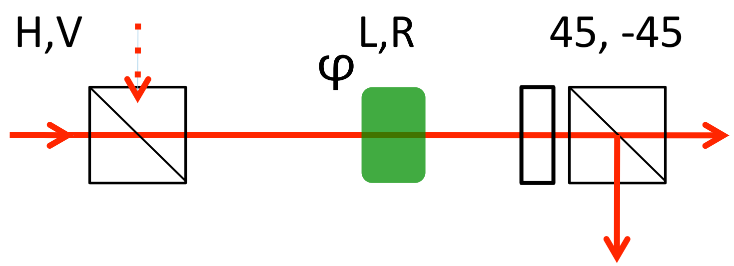
External cavity diode laser
795 nm (Rb D_1 line)

Quantum
noise lock

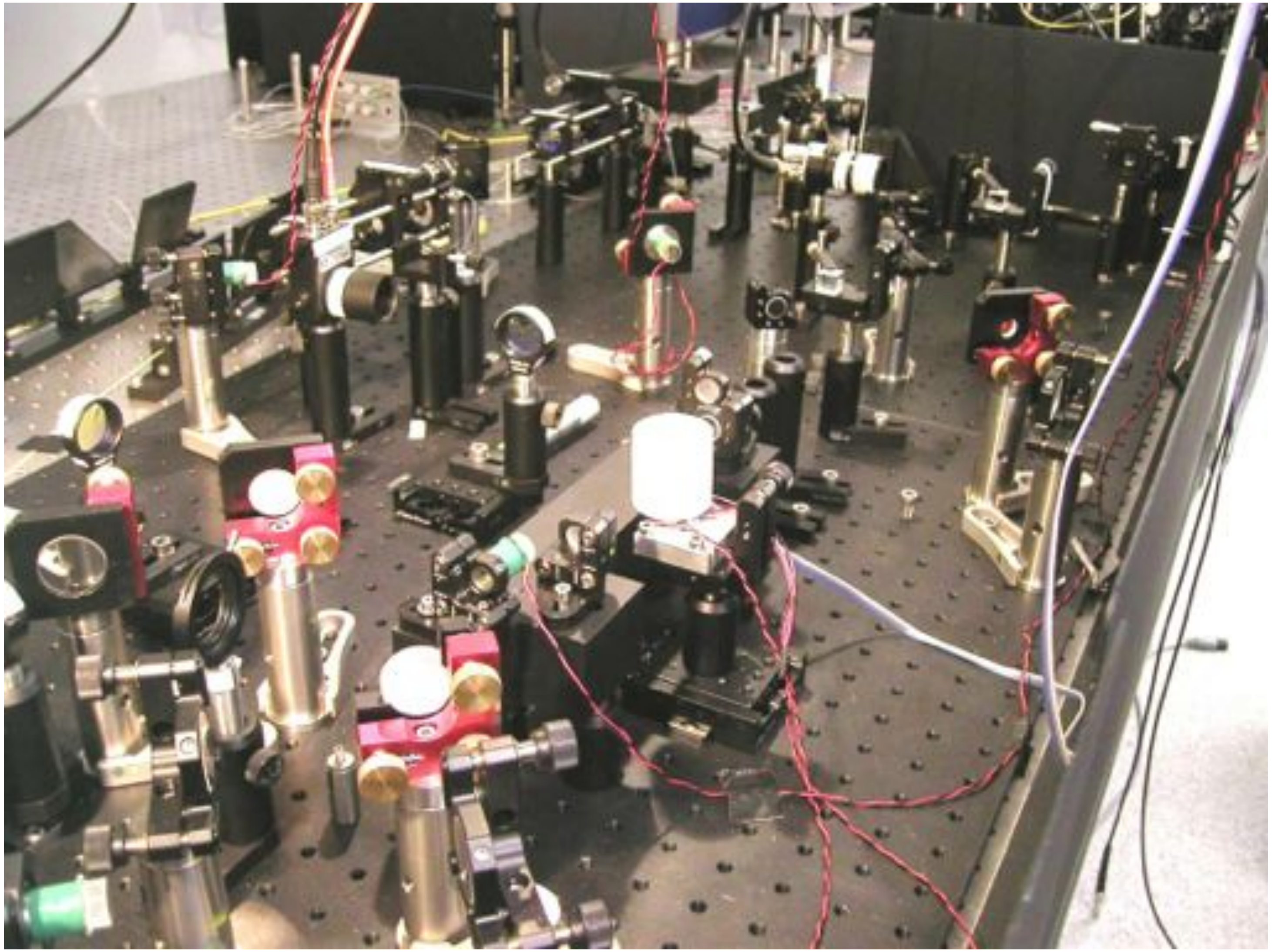


PPKTP OPO
cavity bandwidth 8 MHz
Parametric gain 4.6

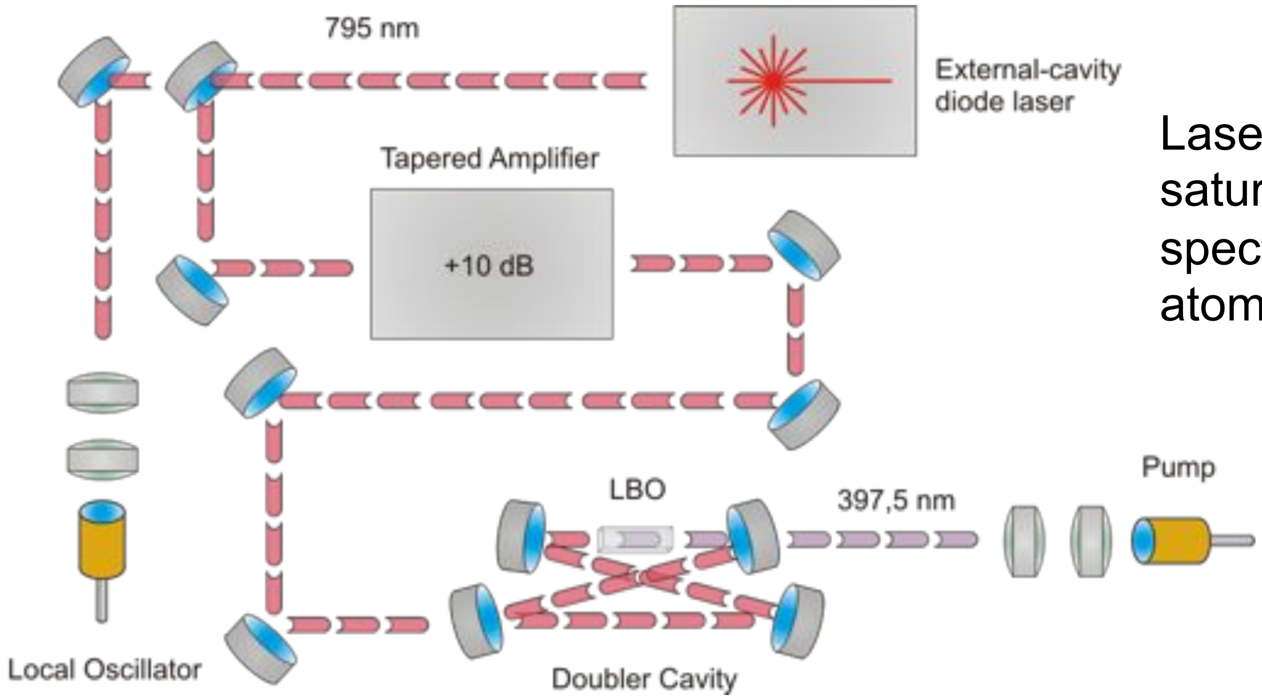
Shot-noise limited
balanced polarimeter



Squeezing experiments at ICFO



Construction of OPO: Pump system

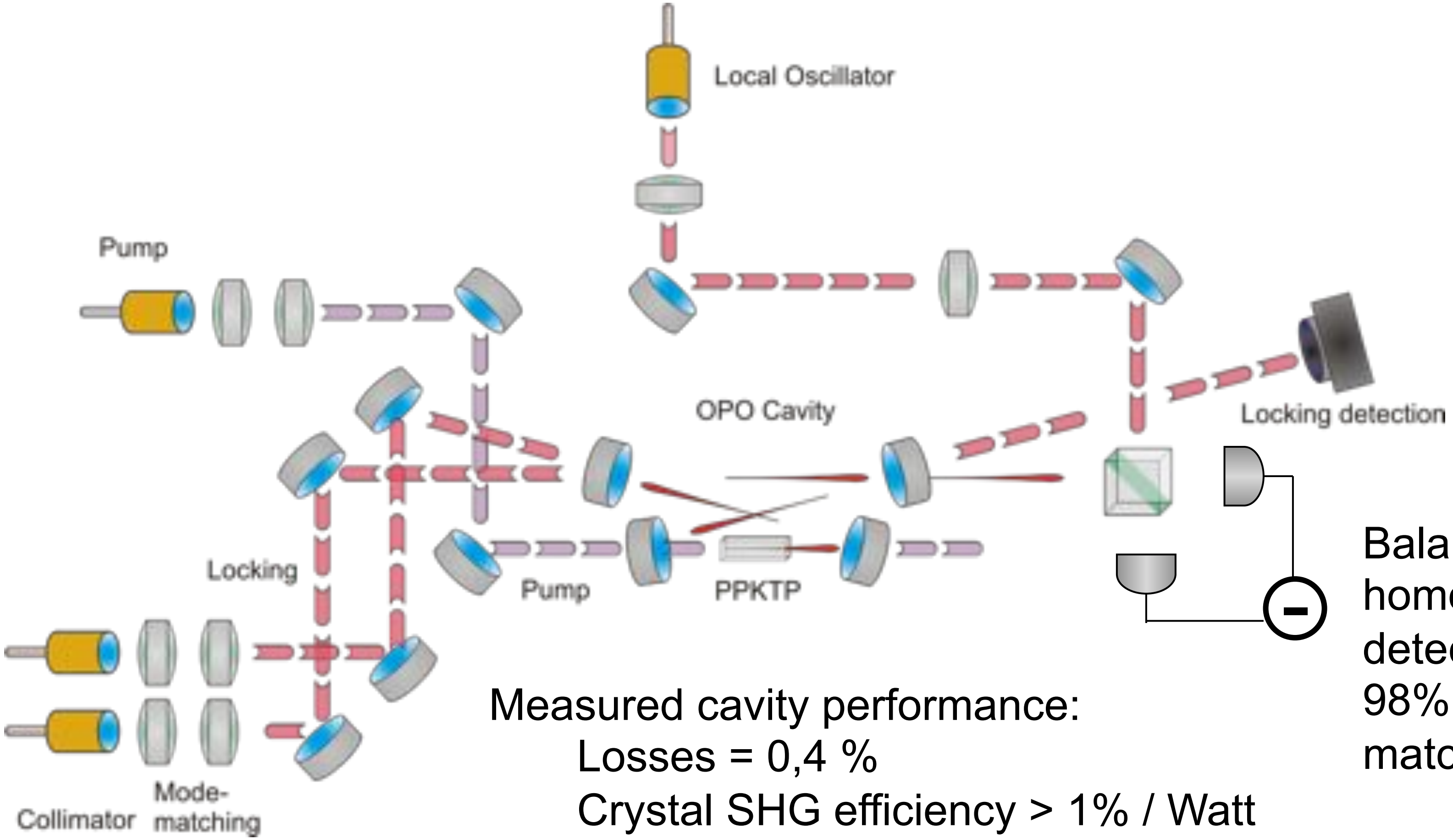


Laser is stabilized by saturated absorption spectroscopy to D_1 line of atomic rubidium.

LO Output: 10 mW at 795 nm
Measured linewidth: 400 kHz

Blue output: 100 mW at 397,5 nm

Construction of OPO: OPO and locking



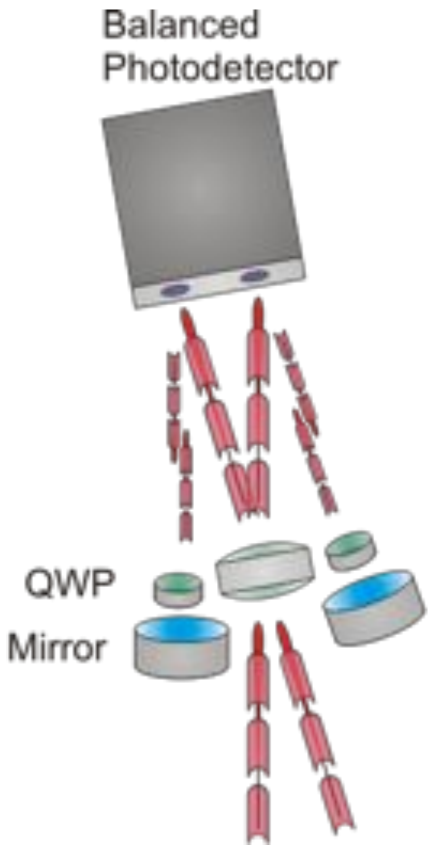
Balanced homodyne detection: 98% mode-match.

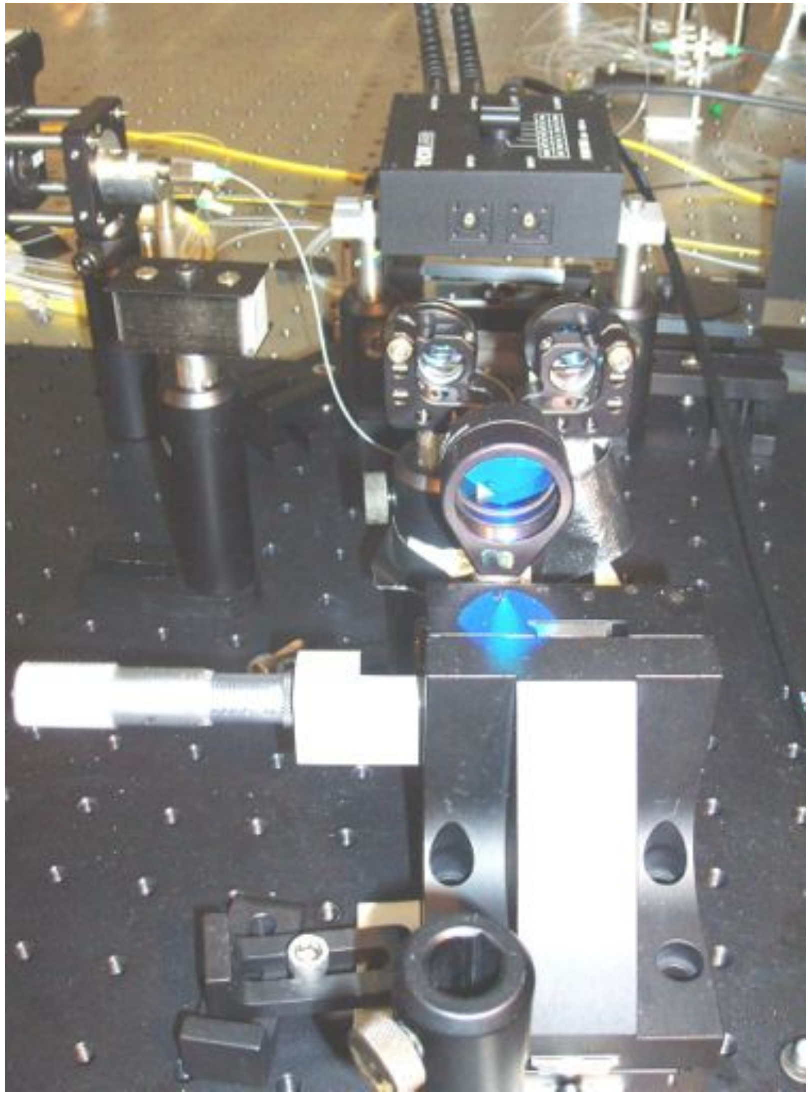
Measured cavity performance:

Losses = 0,4 %
Crystal SHG efficiency > 1% / Watt
OPO gain 3 @ 45 mW
Linewidth 8 MHz
Free-spectral range 504 MHz.

Cavity is stabilized to laser wavelength.

Construction of OPO: detection

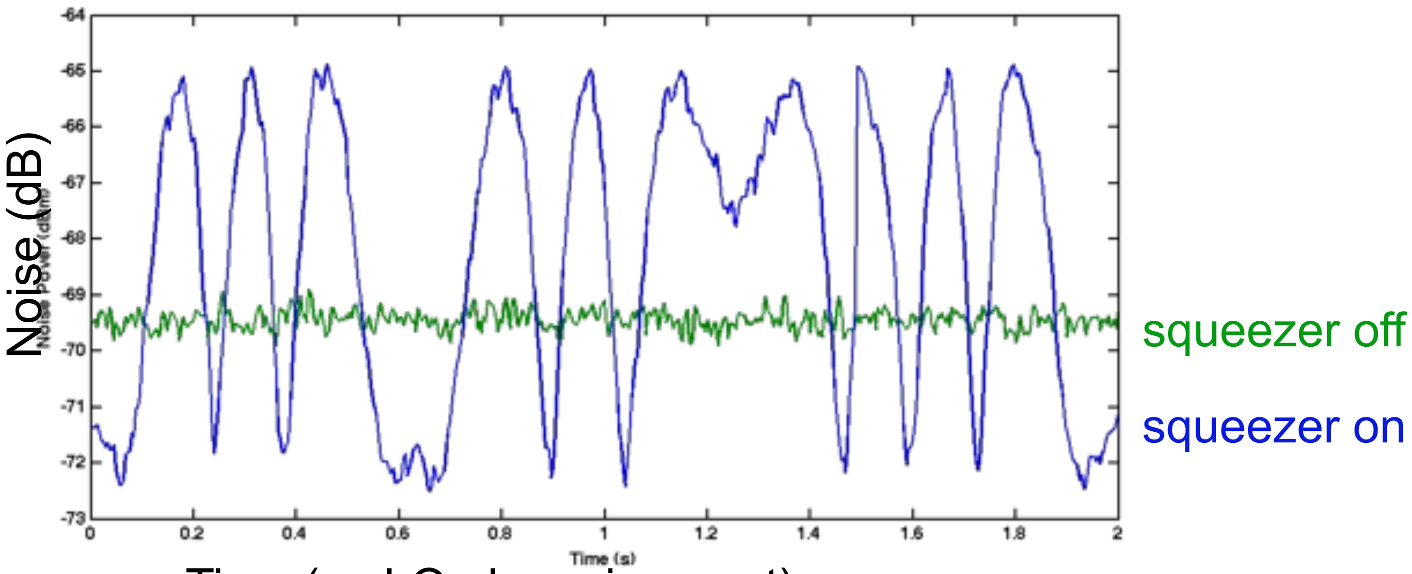




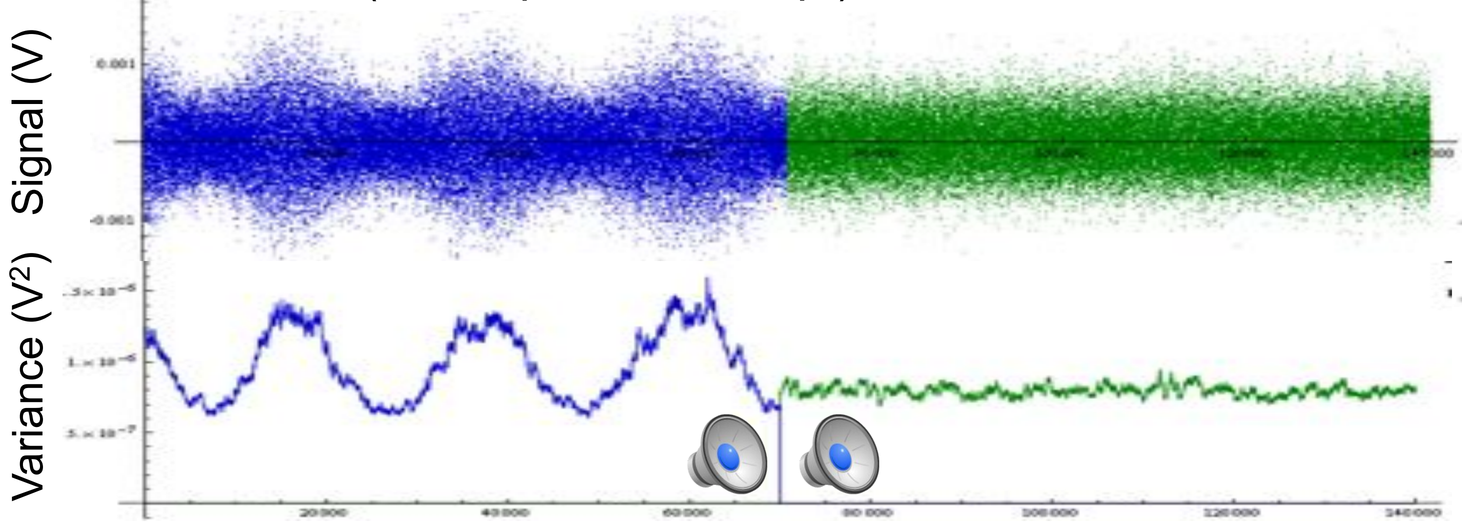
Quantum efficiency 95%
Shot-noise limited by 12 dB
at 2 MHz.

Polarization beyond the shot noise limit

Spectrum Analyzer



Oscilloscope

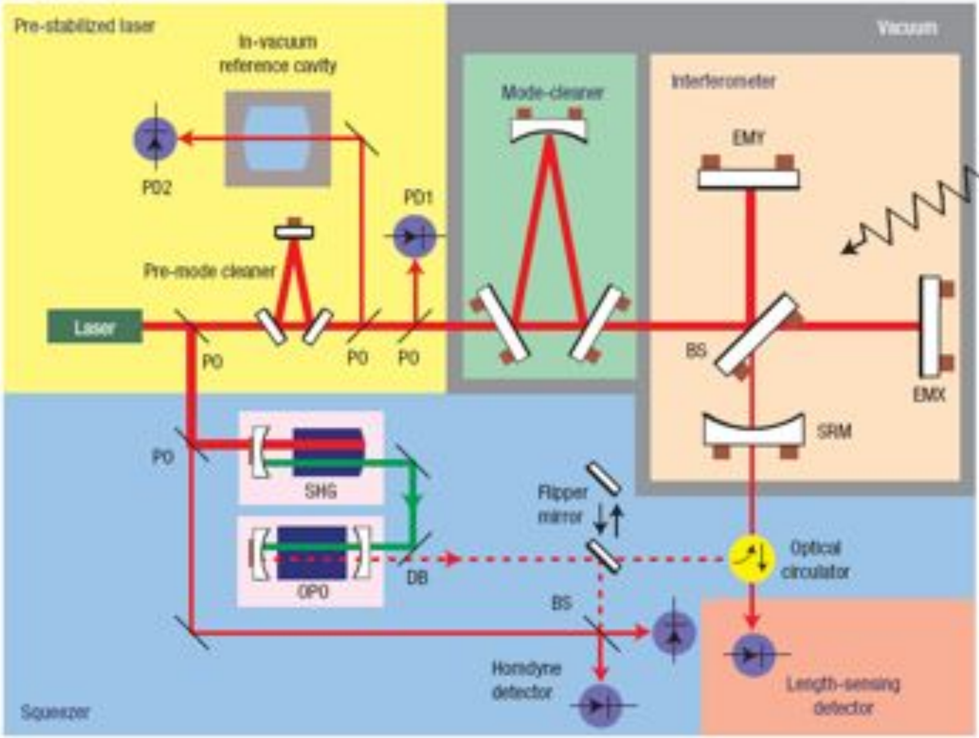


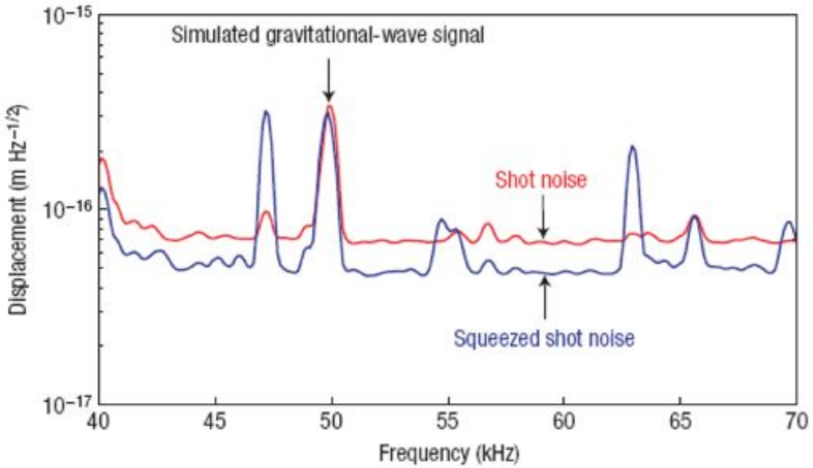
Squeezed-light GW detector

LETTERS

A quantum-enhanced prototype gravitational-wave detector

K. GODA[1], O. MIYAKAWA[2], E. E. MIKHAILOV[3], S. SARAF[4], R. ADHIKARI[2], K. McKENZIE[5], R. WARD[2], S. VASS[2], A. J. WEINSTEIN[2] AND N. MAVALVALA[1]*





Nature Physics **4**, 472 (2008)

related work from ANU, Hanover

GEO 600 sensitivity boost

A gravitational wave observatory operating beyond the quantum shot-noise limit

N.Phys 2011

The LIGO Scientific Collaboration †*

Roman Schnabel

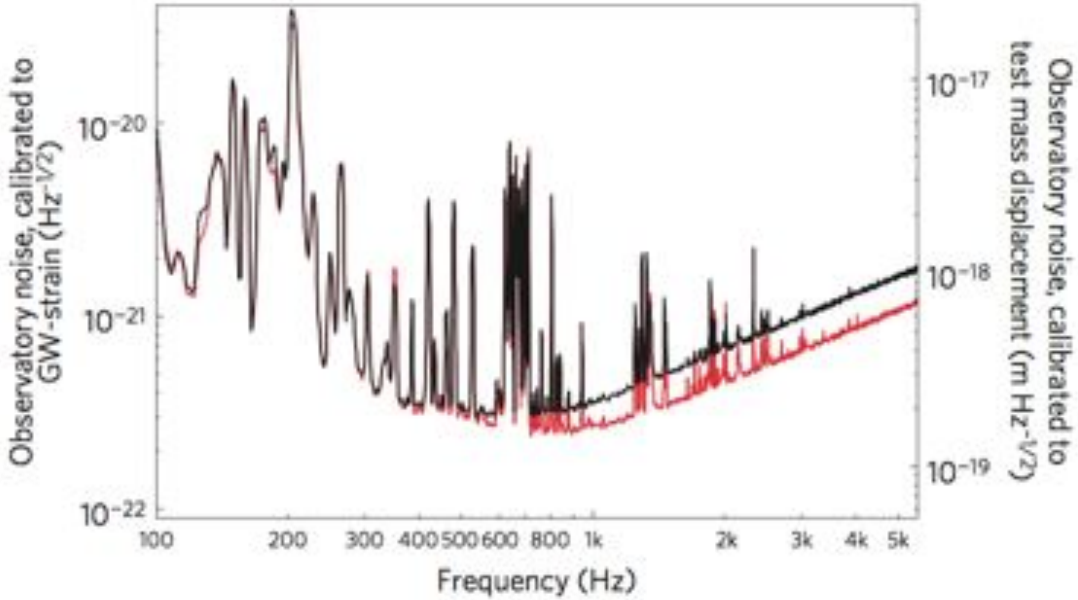


Figure 3 | Nonclassical reduction of the GEO 600 instrumental noise using squeezed vacuum states of light.

LIGO sensitivity boost

Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light

The LIGO Scientific Collaboration*

N.Phot 2013

Nergis Mavalvala

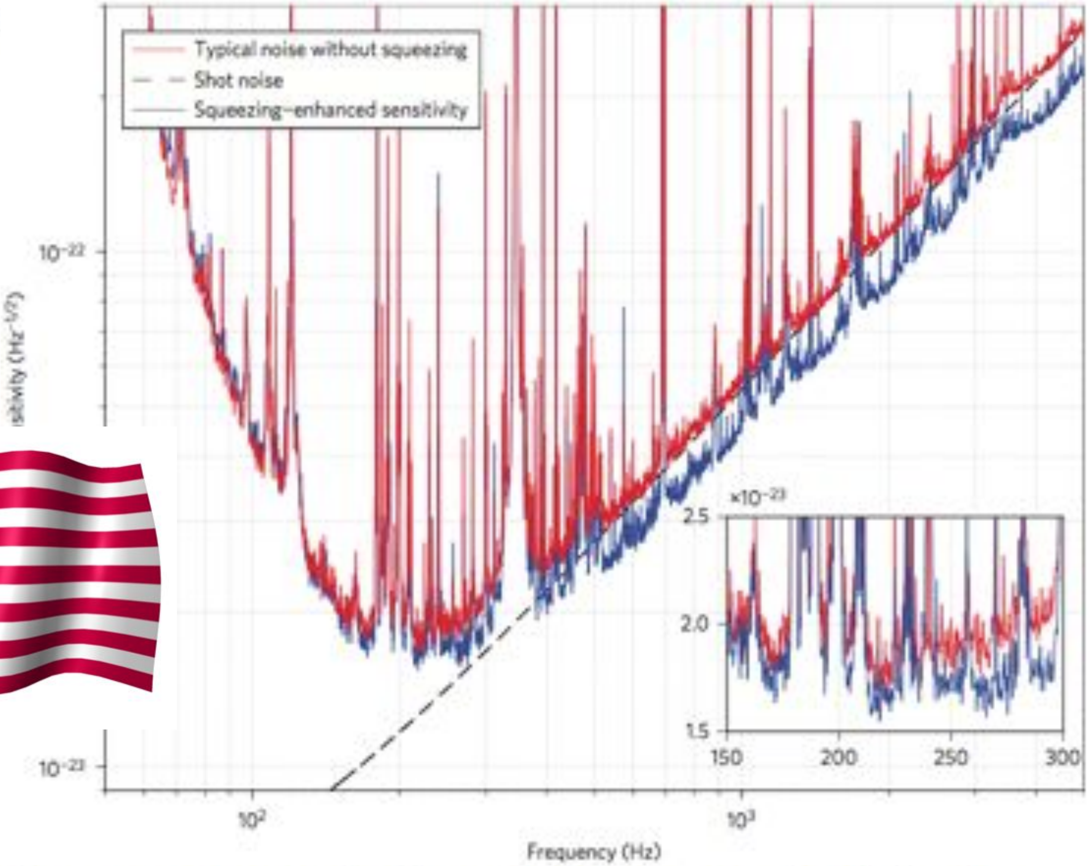


Figure 2 | Strain sensitivity of the H1 detector measured with and without squeezing injection.

GEO 600 sensitivity boost

A gravitational wave observatory operating beyond the quantum shot-noise limit

N.Phys 2011

The LIGO Scientific Collaboration †*

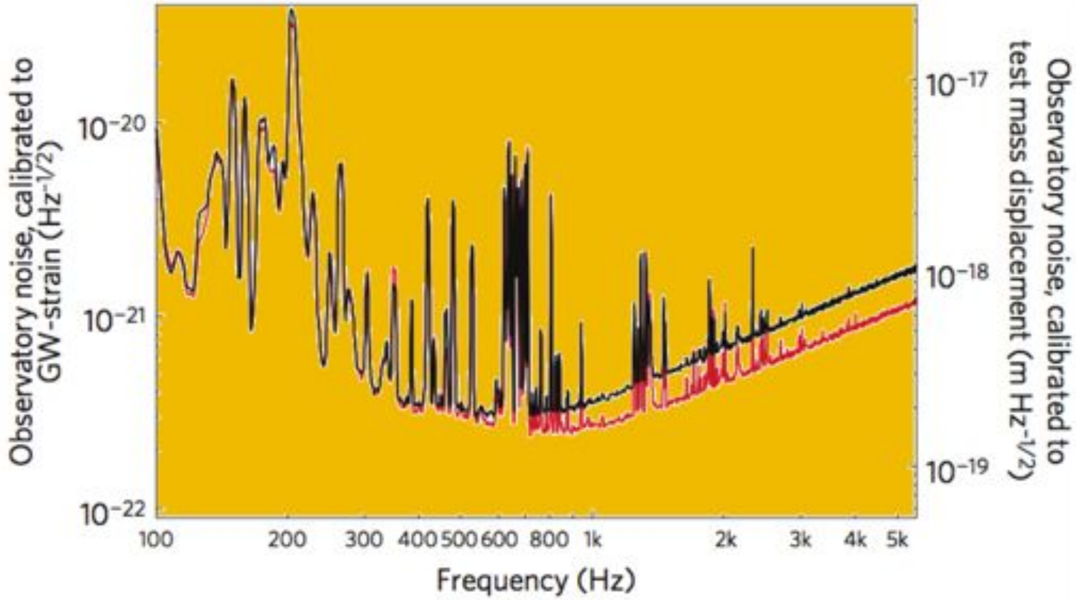
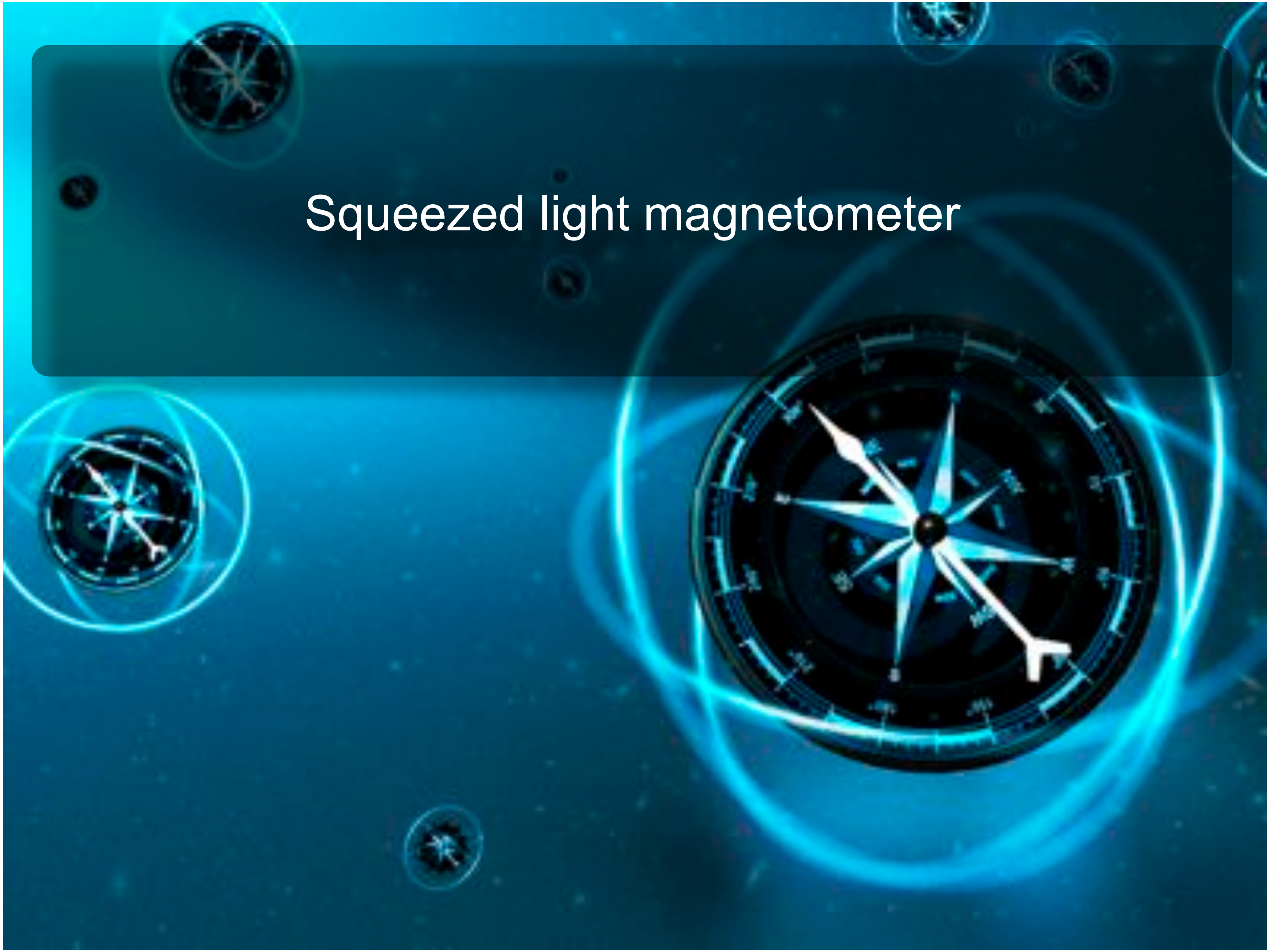


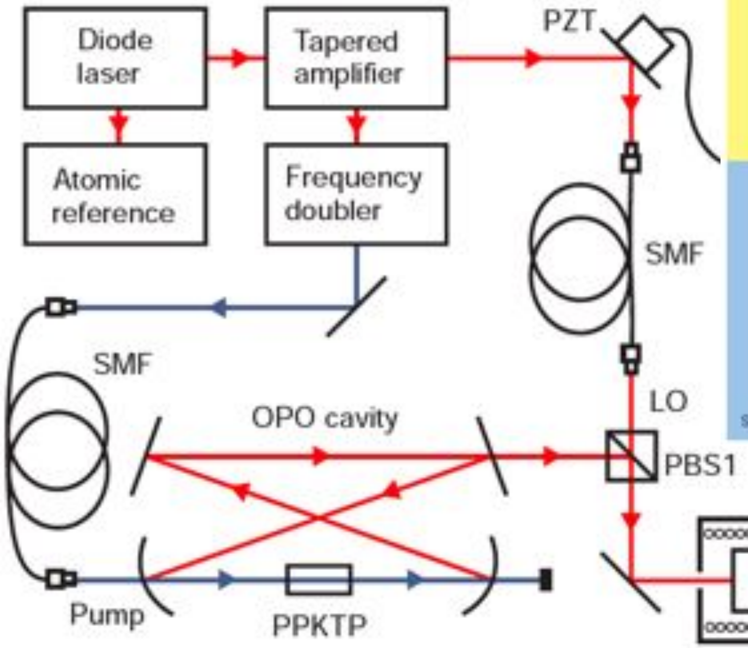
Figure 3 | Nonclassical reduction of the GEO 600 instrumental noise using squeezed vacuum states of light.

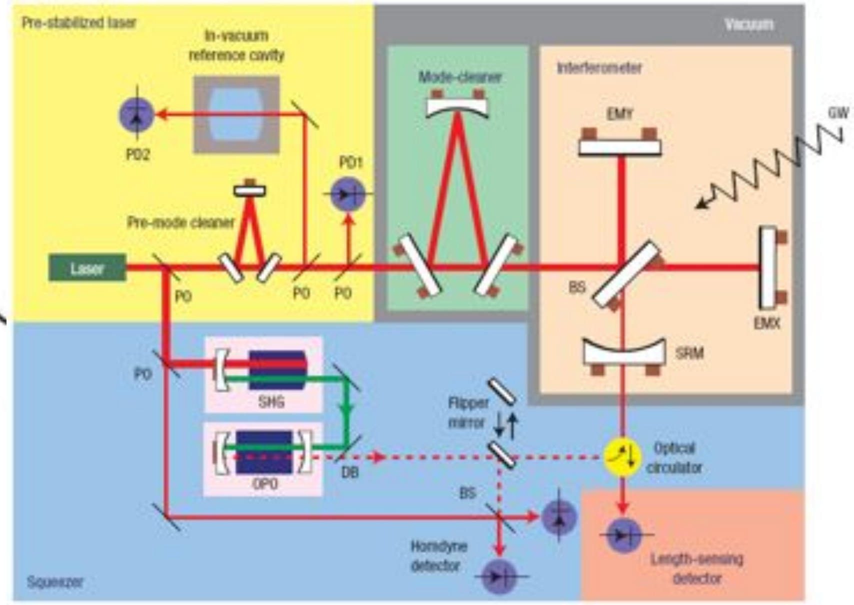
Squeezed light magnetometer



Squeezed-light magnetometer

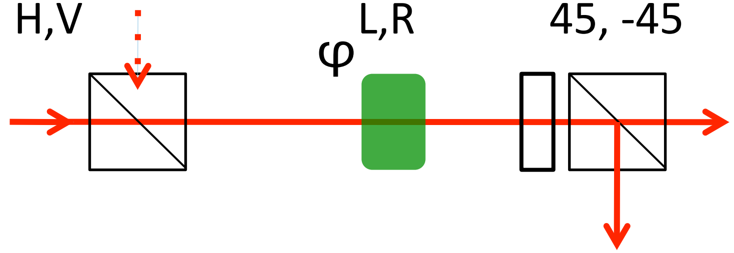
External cavity diode laser
795 nm (Rb D_1 line)





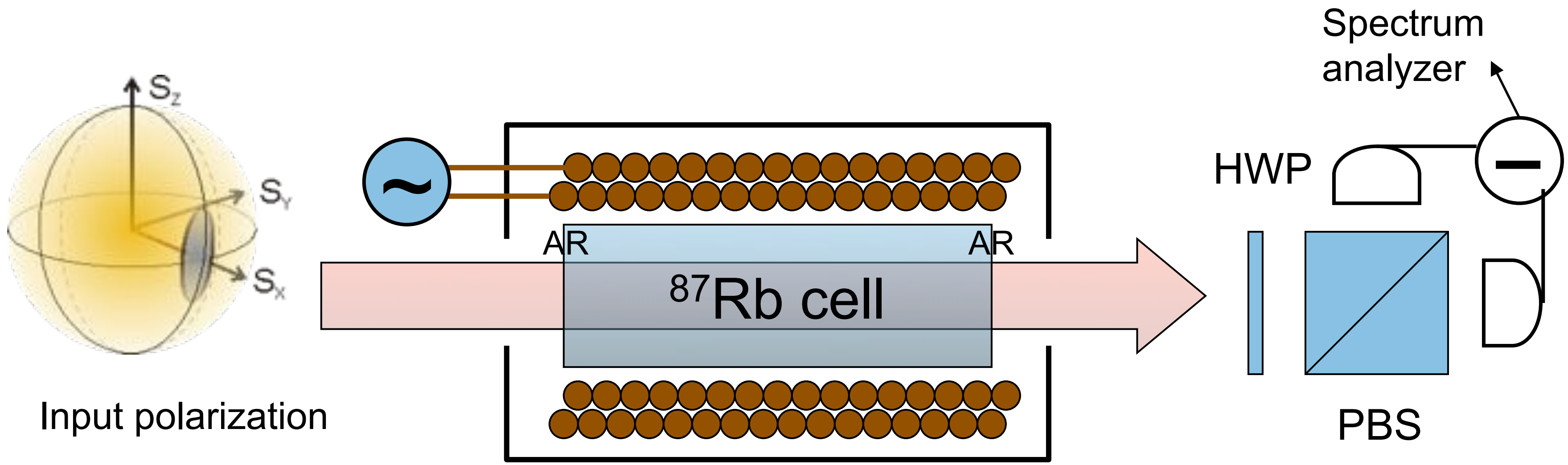
PPKTP OPO
cavity bandwidth 8 MHz
Parametric gain 4.6

Shot-noise limited
balanced polarimeter

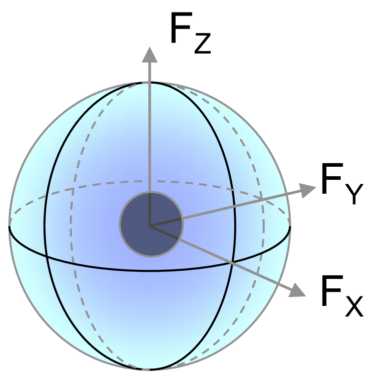


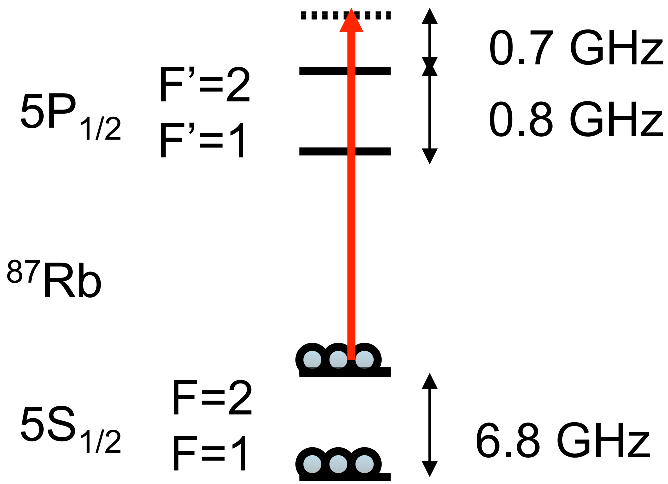
Morgan W. Mitchell

Prototype optical magnetometer (2010)

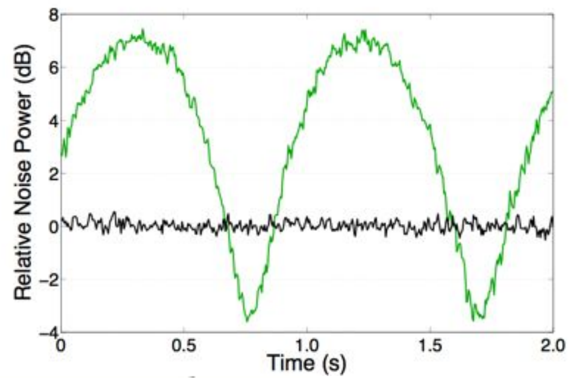


^{87}Rb purity	> 99%
Temperature:	21°
Atomic state:	thermal
Buffer gas:	none
Cell coating:	none
Optical losses:	4%
Probe power:	620 μW
Probe waist:	950 μm





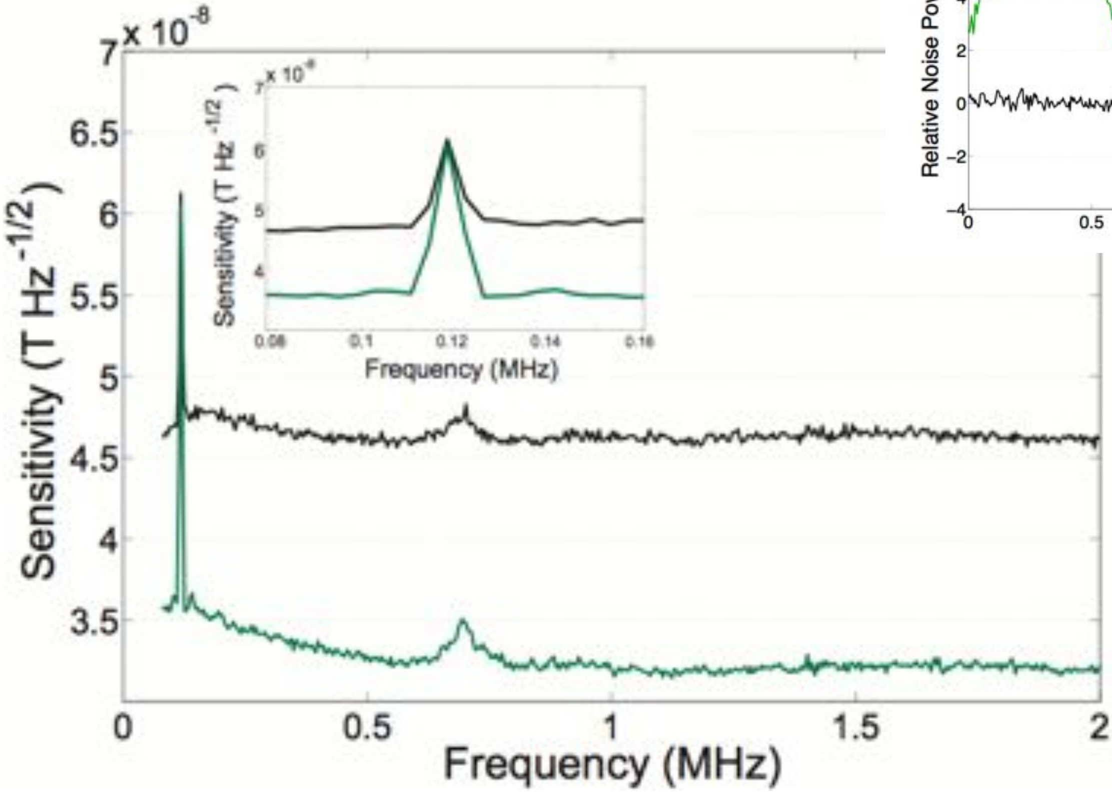
Improved SNR with squeezing



squeezer ON

squeezer OFF

3.6 dB

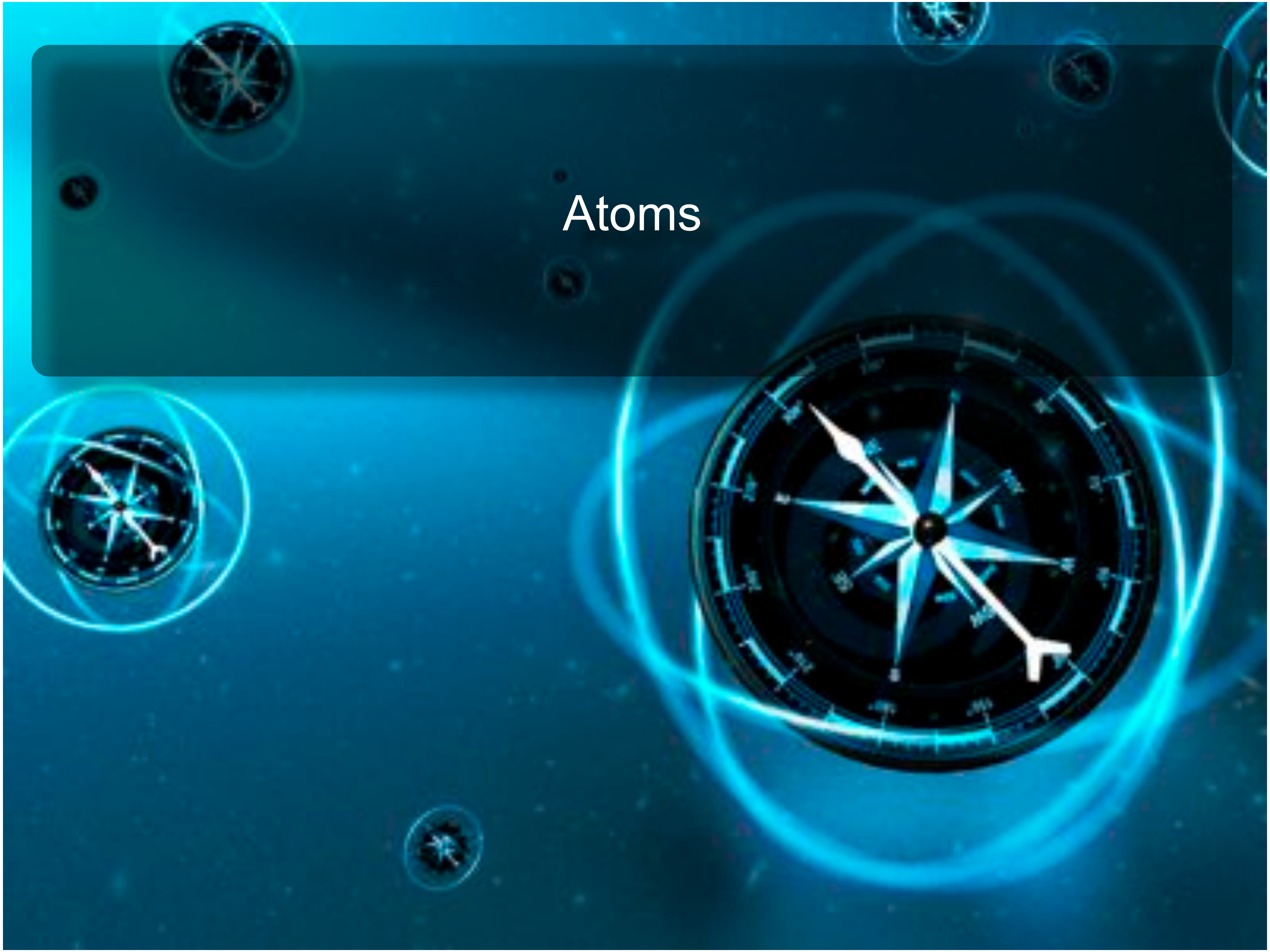


polarized probe

3.2 dB

squeezed probe

Wolfgramm, Cerè, Beduini, Predojević, Koschorreck, MWM Phys. Rev. Lett. 105, 053601 (2010)

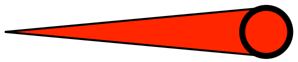

Atoms

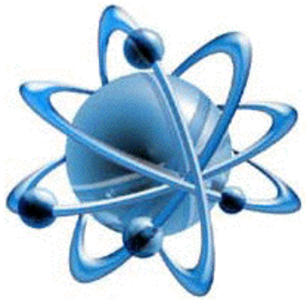
Collective variables description

$|\Psi\rangle = c_H\,|H\rangle + c_V\,|V\rangle$

$|\Phi\rangle = c_g\,|g\rangle + c_e\,|e\rangle$

Single particles

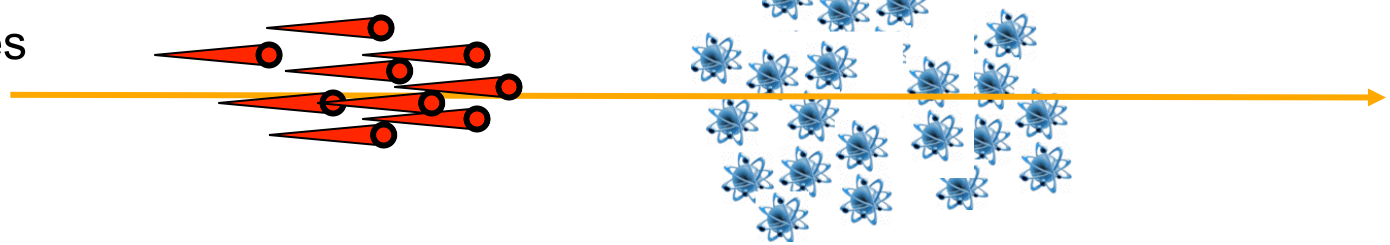




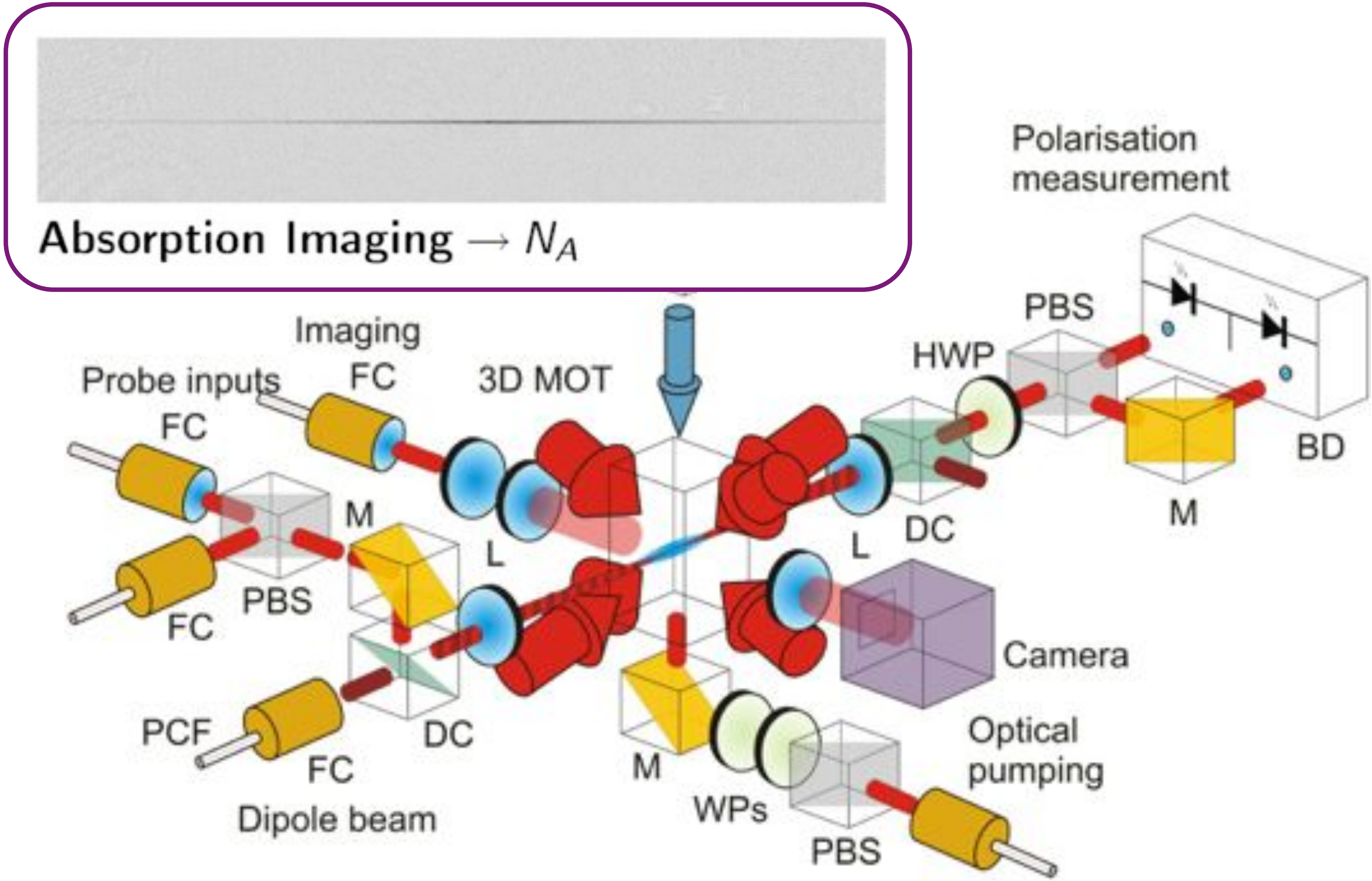
$S = \sum_i s_i$

$J = \sum_i j_i$

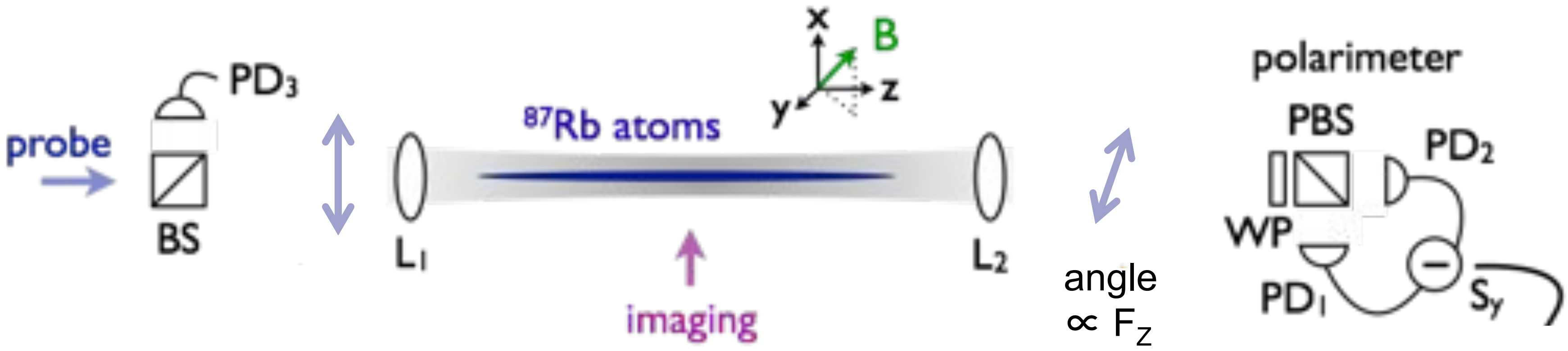
Ensembles



Cold atom magnetometer



cold ^{87}Rb ensemble



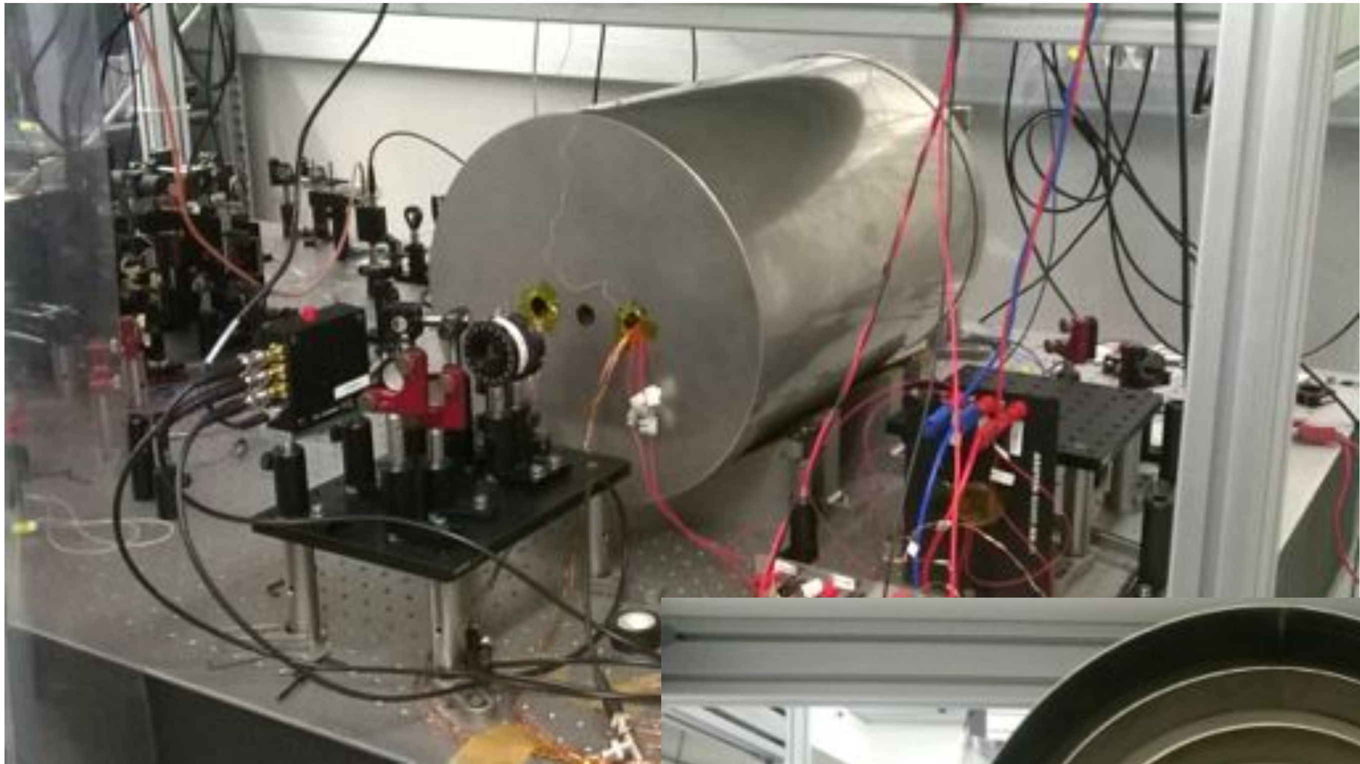
1μs long pulses
linearly polarized
"mode matched" to atoms
0.7 GHz from D_2 line

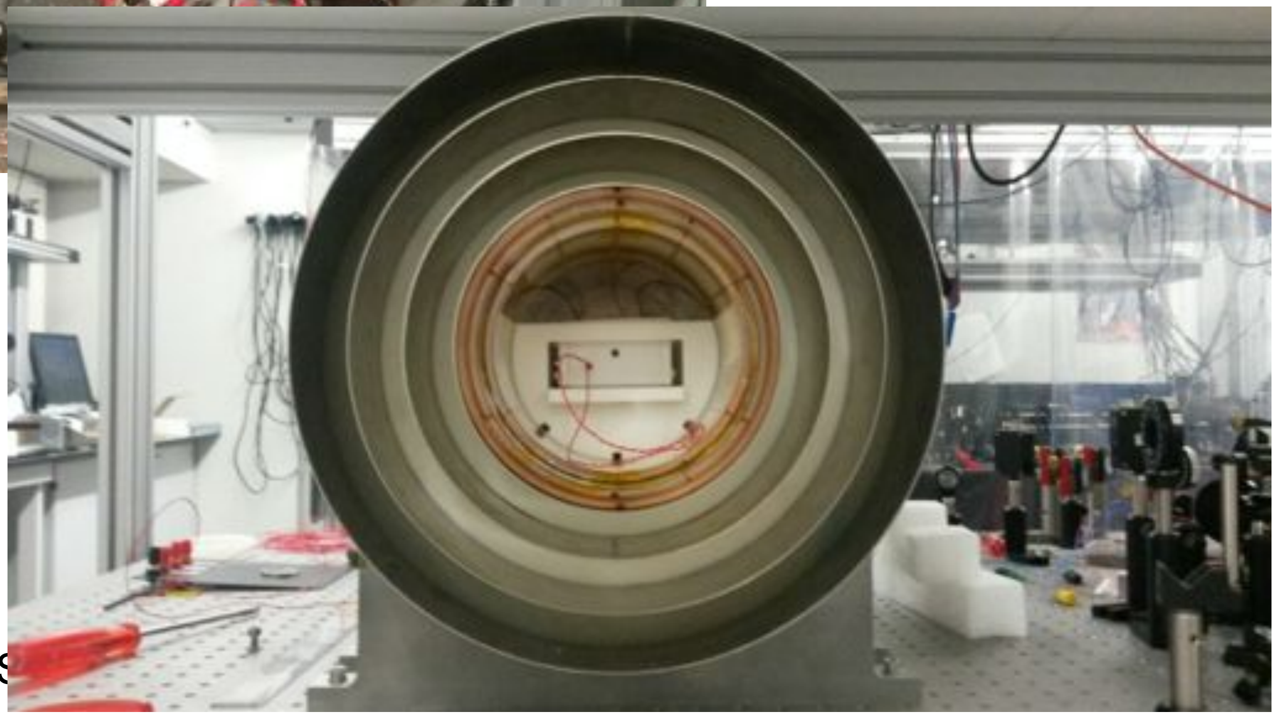
~10^6 ^{87}Rb atoms at 25μK
f=1 ground-state

[1] effective OD > 50
[2] Sensitivity 512 spins, < SQL
[3] QND measurement
[4] spin squeezing

1 Kubasik, et al. PRA 79, 043815 (2009)
2 Koschorreck, et al. PRL (2010)
3 Koschorreck, et al. PRL (2010), Sewell, et al. N. Phot. (2013)
4 Sewell, et al. PRL (2012) PRX (2014)

Hot atoms as a quantum system



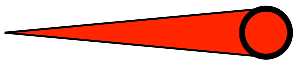


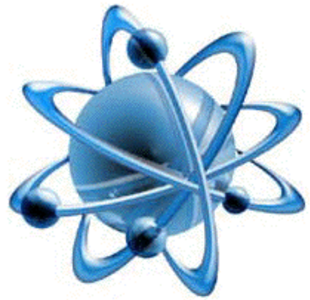
cell @ 80-170 °C
densities 10^{14} cm^{-3}
N_2 buffer gas
OD in the 100s

Collective variables description

$|\Psi\rangle = c_H\,|H\rangle + c_V\,|V\rangle$ $|\Phi\rangle = c_g\,|g\rangle + c_e\,|e\rangle$

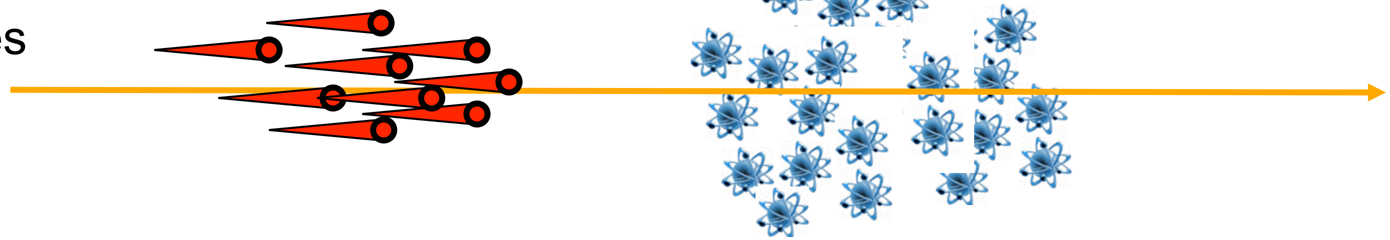
Single particles





$S = \sum_i s_i$ $J = \sum_i j_i$

Ensembles



Commutation relations

$$[\hat{x}, \hat{p}] = i\hbar \longrightarrow \hat{p}\psi(x) = -i\hbar\partial_x\psi(x)$$

$$[\hat{a}, \hat{a}^\dagger] = 1 \longrightarrow \text{harmonic oscillator}$$

$$[\hat{F}_i, \hat{F}_j] = i\hbar\epsilon_{ijk}\hat{F}_k \longrightarrow \text{angular momentum}$$

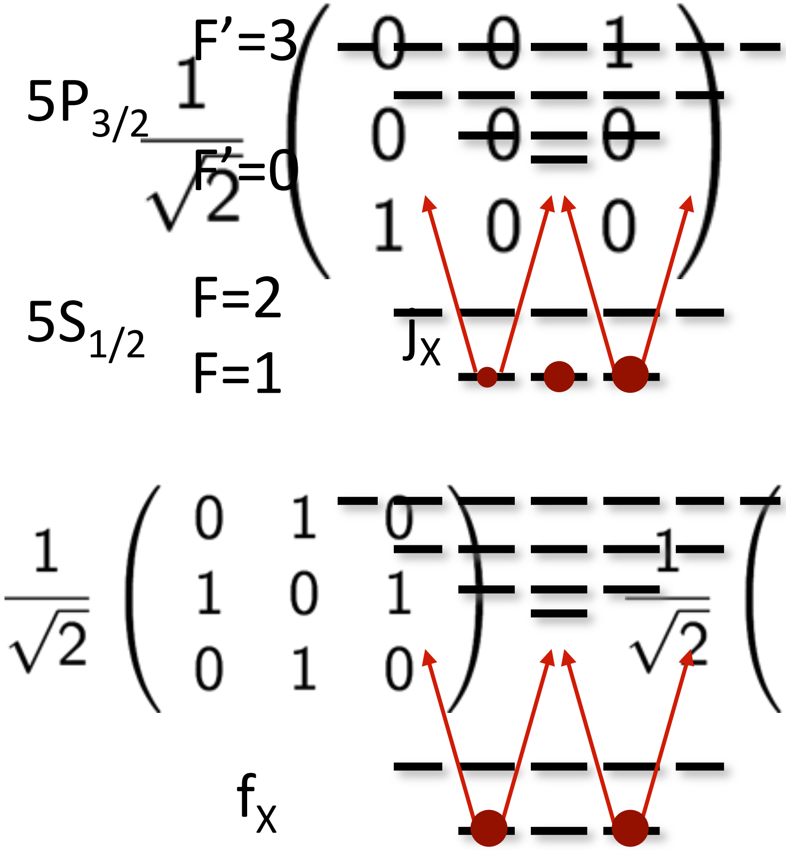
Robertson (-Schrödinger) relation

$$[\hat{x}, \hat{p}] = i\hbar \qquad [\hat{F}_x, \hat{F}_y] = i\hbar\hat{F}_z$$

$$\delta x \delta p \geq \frac{1}{2}\hbar \qquad \delta F_x \delta F_y \geq \frac{1}{2}?|\langle F_z \rangle|$$

$$\delta A \delta B \geq \frac{1}{2}|\langle [A, B] \rangle|$$

Macroscopic quantum variables



$$\mathbf{F}_i = \sum_i \mathbf{f}^{(i)}$$

"spin orientation"

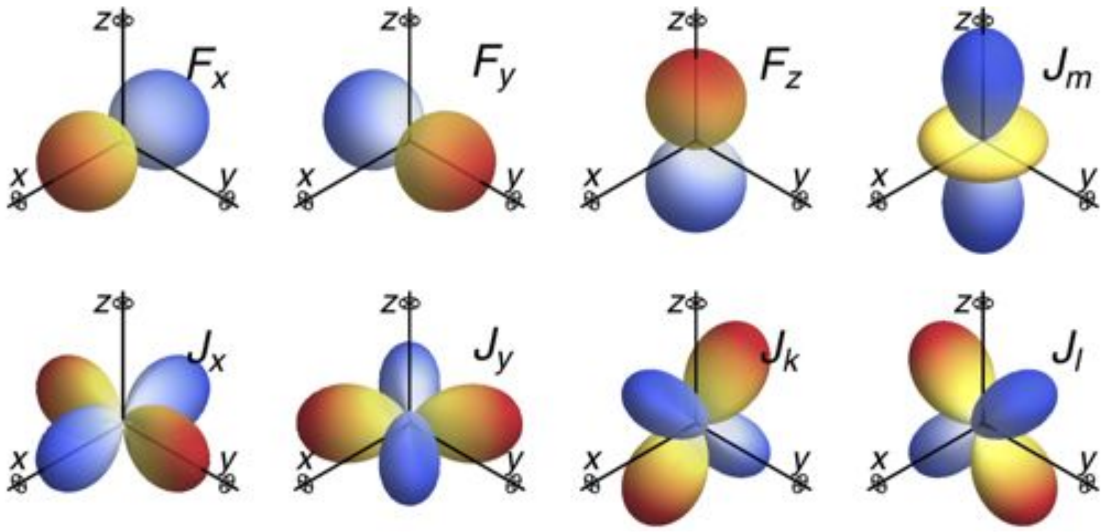
$$j_X = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad j_Y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\mathbf{j} = \sum_i \mathbf{j}^{(i)}$$

"spin alignment"

$$f_X = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad f_Y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad f_Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Macroscopic quantum variables



Wigner distribution representation

$$[f_x, f_y] = if_z \qquad [F_x, F_y] = iF_z$$

$$[j_x, j_y] = 2if_z \qquad [J_x, J_y] = 2iF_z$$

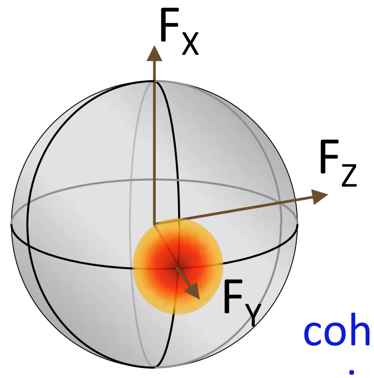
G. Colangelo et al. NJP 2013

“Phase space” for atomic spin ensembles

$[F_x, F_y] = i\, F_z$

and cycl. permutations

$\delta F_x\, \delta F_z \geq \frac{1}{2}\, |\langle F_y \rangle|$



coherent
spin state

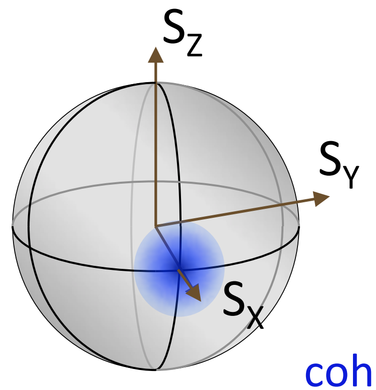
$$\delta F_z = \sqrt{\frac{N}{2}}$$

standard quantum limit
(F=1)

Stokes operators

$$S_X = (n_H - n_V)/2$$
$$S_Y = (n_{\swarrow\!\nearrow} - n_{\nwarrow\!\searrow})/2$$
$$S_Z = (n_L - n_R)/2$$

$$[S_x, S_y] = iS_z$$

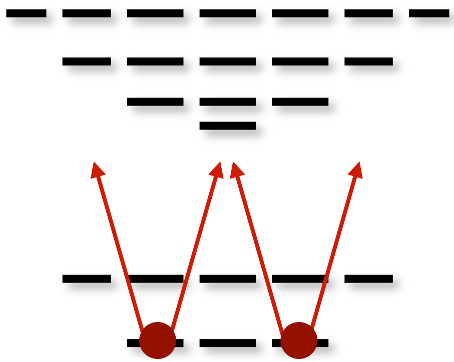


coherent
polarization state

$$\delta S_y = \frac{1}{2}\sqrt{N}$$

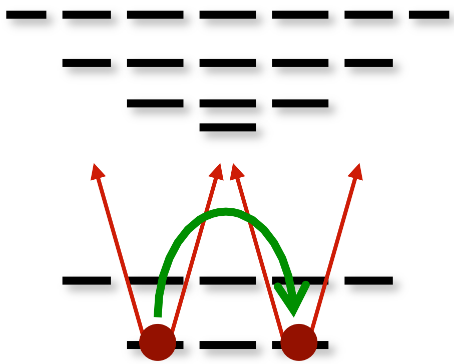
standard quantum limit

Coupling of atoms and light



Faraday rotation

$$S_z F_z$$



Alignment-to-orientation conversion

$$S_x J_x + S_y J_y$$

Coupling of atoms and light

$$H_{\text{eff}} = \alpha^{(1)} S_z F_z + \alpha^{(2)} (S_x J_x + S_y J_y)$$

QND AOC

$$\frac{d}{dt} S_y = \frac{1}{i} [S_y, H_{\text{QND}}]$$

$$= \frac{1}{i} [S_y, \alpha^{(1)} S_z F_z] = \alpha^{(1)} S_x F_z$$

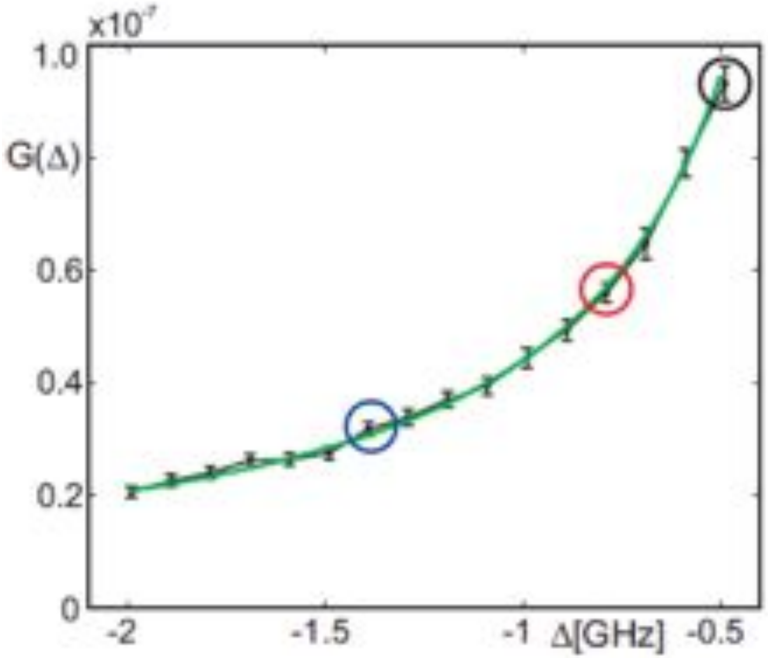
$$\frac{d}{dt} F_z = \frac{1}{i} [F_z, H_{\text{QND}}] = 0$$

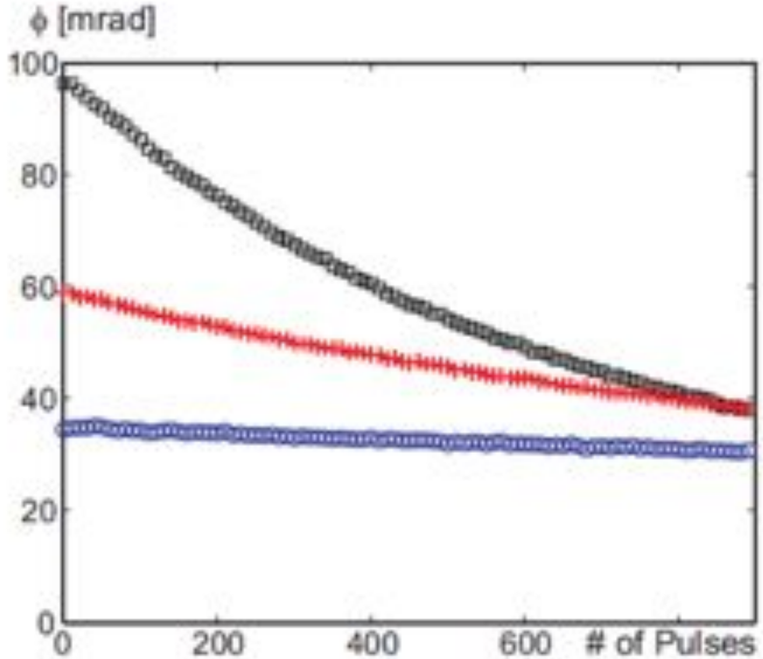
non-destructive Faraday rotation probing

$$N_- - N_+ = N_A$$

$$G(\Delta) = \frac{\phi}{N_A}$$

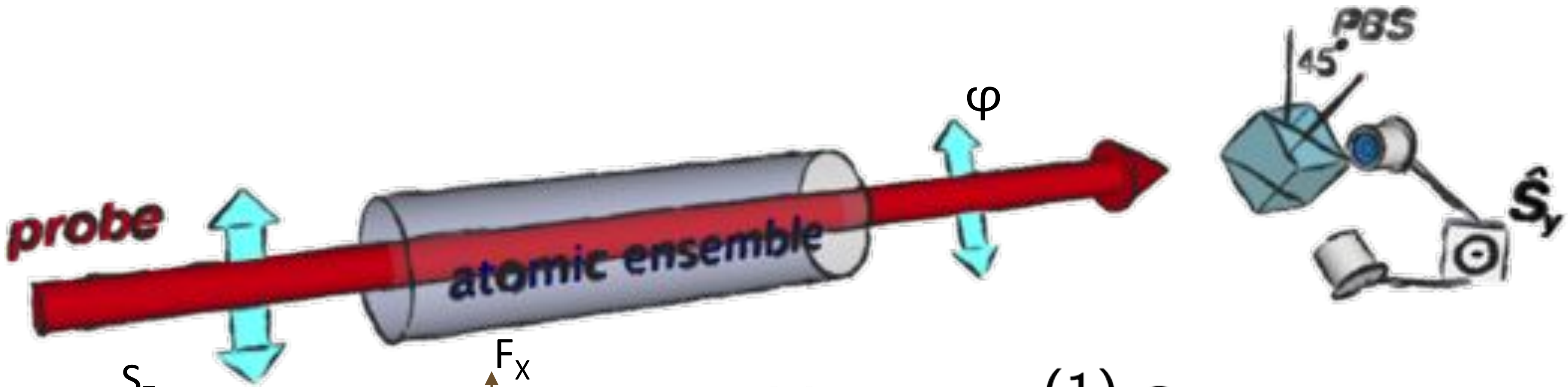
Absorption Imaging $\rightarrow N_A$

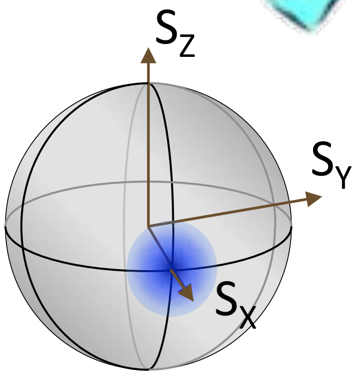


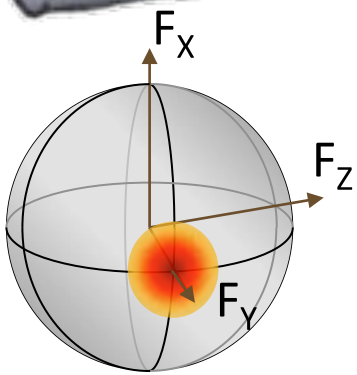


Kubasik, Koschorreck, Napolitano, de Echaniz, Crepaz, Eschner, Polzik, MWM, *PRA* **79**, 043815 (2009)

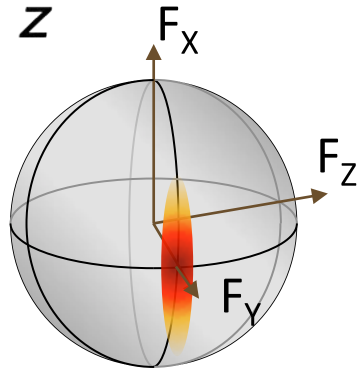
Faraday rotation spin measurement







$$H_{\mathrm{eff}} = \alpha^{(1)} S_z F_z$$

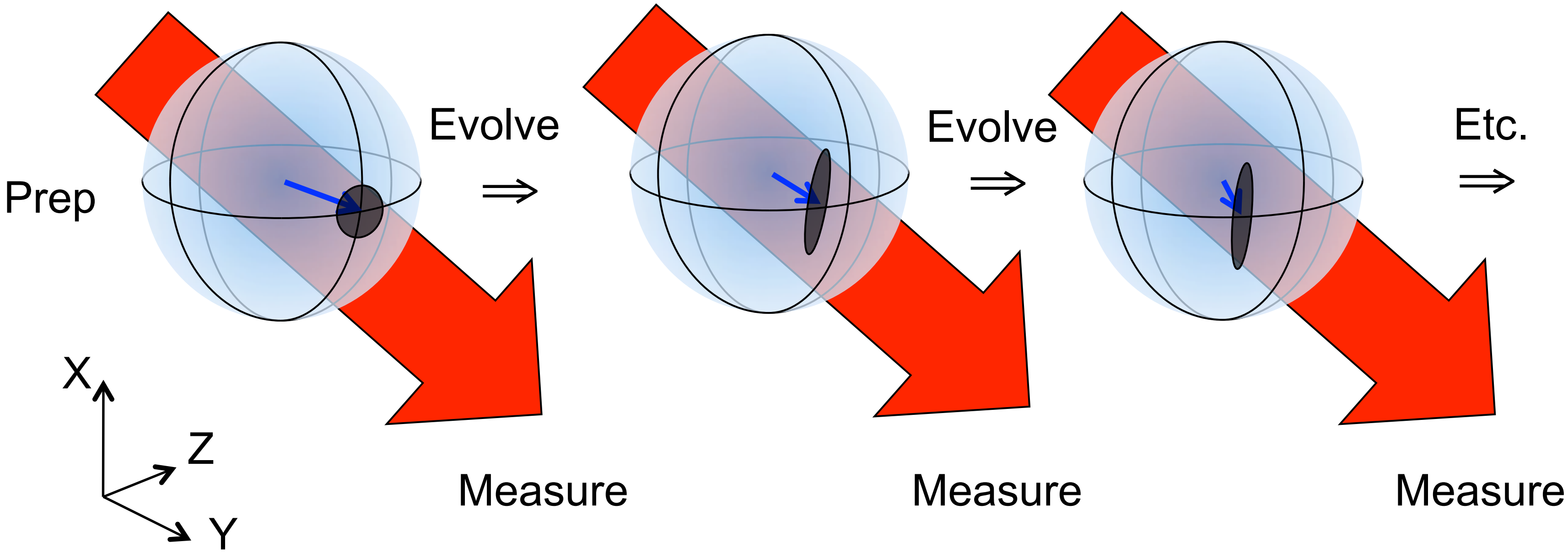


$$F_z^{(\mathrm{out})} = F_z^{(\mathrm{in})}$$

$$S_y^{(\mathrm{out})} = S_y^{(\mathrm{in})} + \tau \alpha^{(1)} S_x^{(\mathrm{in})} F_z^{(in)}$$

shot noise signal

Measurement-induced squeezing



Kuzmich, Mabuchi, Polzik, Vuletic, Takahashi, Thompson

Proposal
Clocks
Magnetometer
Other

F=1/2

F=4 ^{133}Cs

J=1/2

J=1/2

I=1/2 ^{171}Yb

J=1/2

To boldly go where others have gone before

9 APRIL 2004 VOL 304 SCIENCE www.sciencemag.org

REPORTS

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

PRL **94,** 203002 (2005) PHYSICAL REVIEW LETTERS week ending 27 MAY 2005

Suppression of Spin Projection Noise in Broadband Atomic Magnetometry

JM Geremia,* John K. Stockton, and Hideo Mabuchi

Physics and Control & Dynamical Systems, California Institute of Technology, Pasadena California 91125, USA

(Received 2 September 2003; revised manuscript received 15 February 2005; published 24 May 2005)

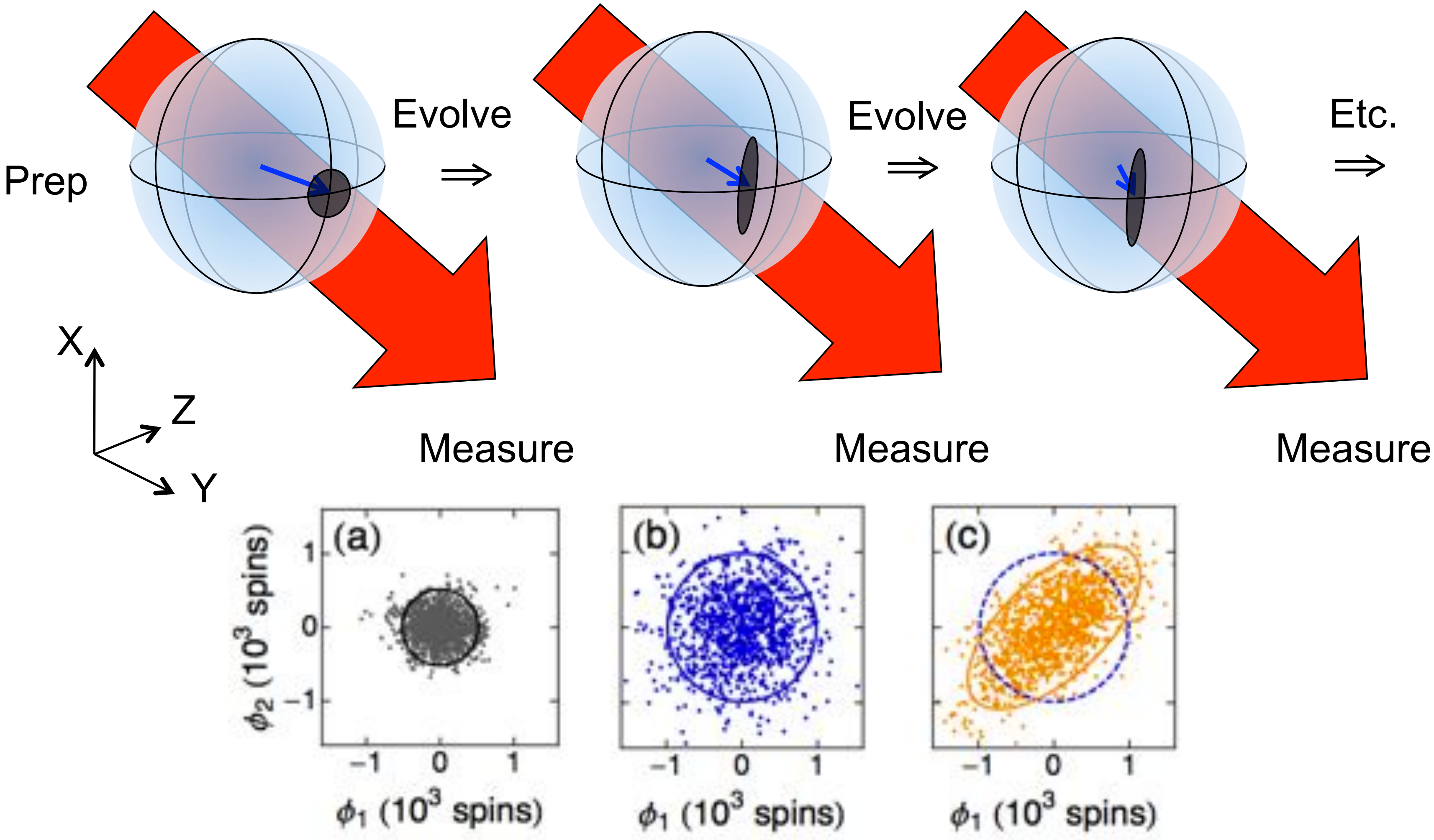
PRL **101,** 039902 (2008) PHYSICAL REVIEW LETTERS week ending 18 JULY 2008

Erratum: Suppression of Spin Projection Noise in Broadband Atomic Magnetometry [Phys. Rev. Lett. 94, 203002 (2005)]

J. M. Geremia, John K. Stockton, and Hideo Mabuchi

(Received 11 June 2008; published 17 July 2008)

Measurement-induced squeezing

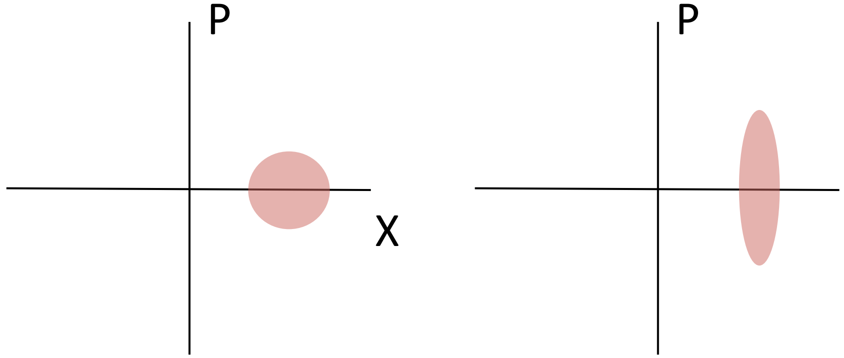


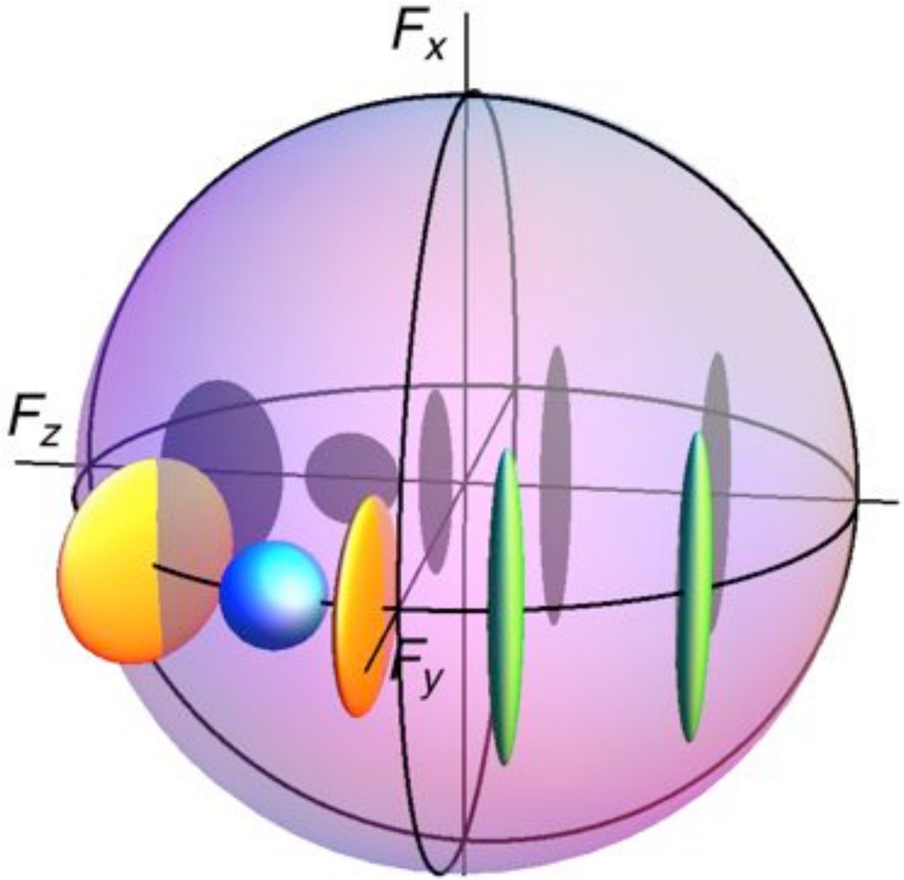
Sewell et al. PRL **109**, 253605 (2012)
Sewell et al. PRX **4**, 021045 (2014)

Spin squeezing is different than light squeezing

$$[X, P] = i$$

$$\delta X \delta P \geq \frac{1}{2}$$



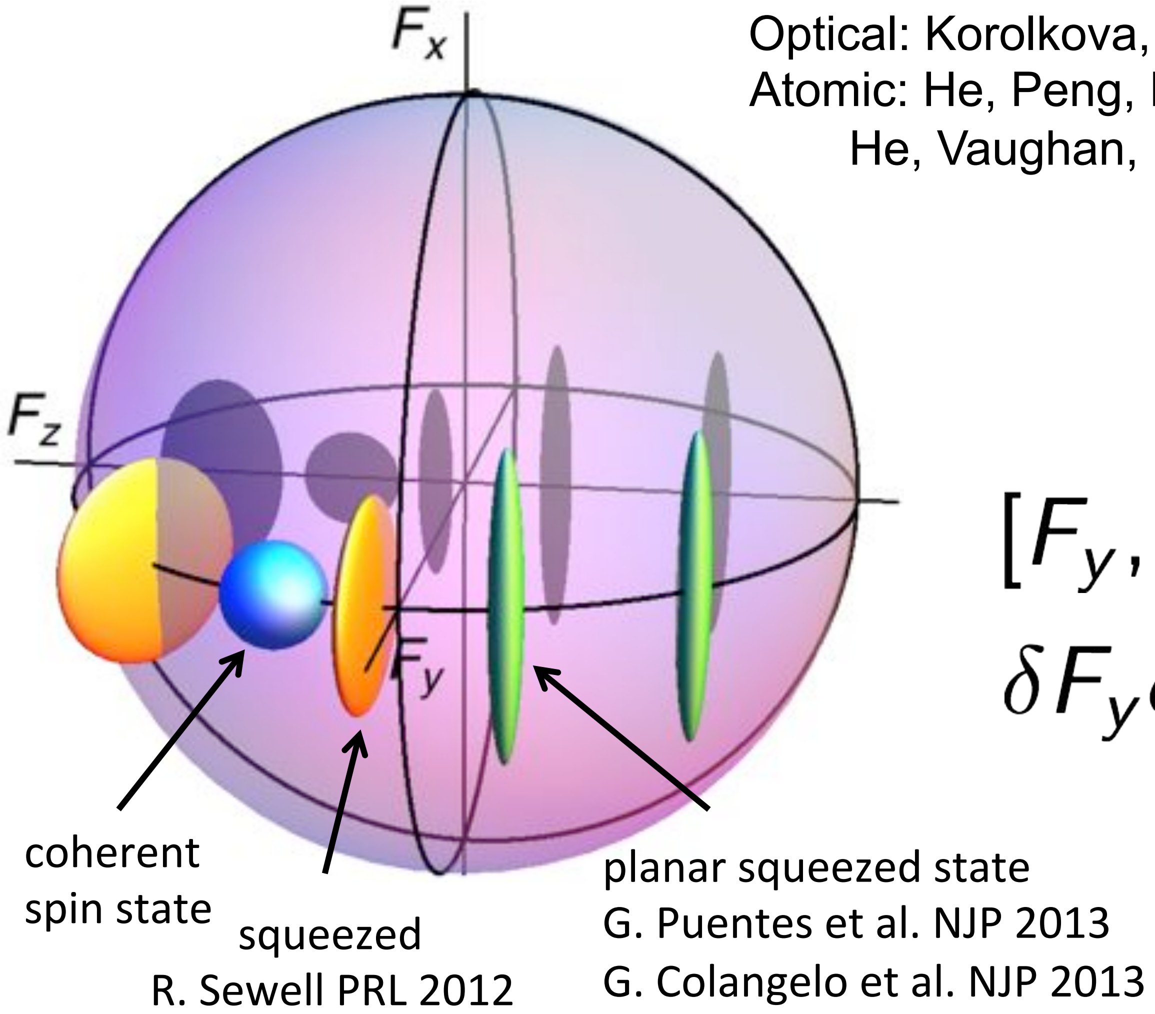


$$[F_y, F_z] = iF_x$$

$$\delta F_y \delta F_z \geq \frac{1}{2} |\langle F_x \rangle|$$

Planar squeezed states

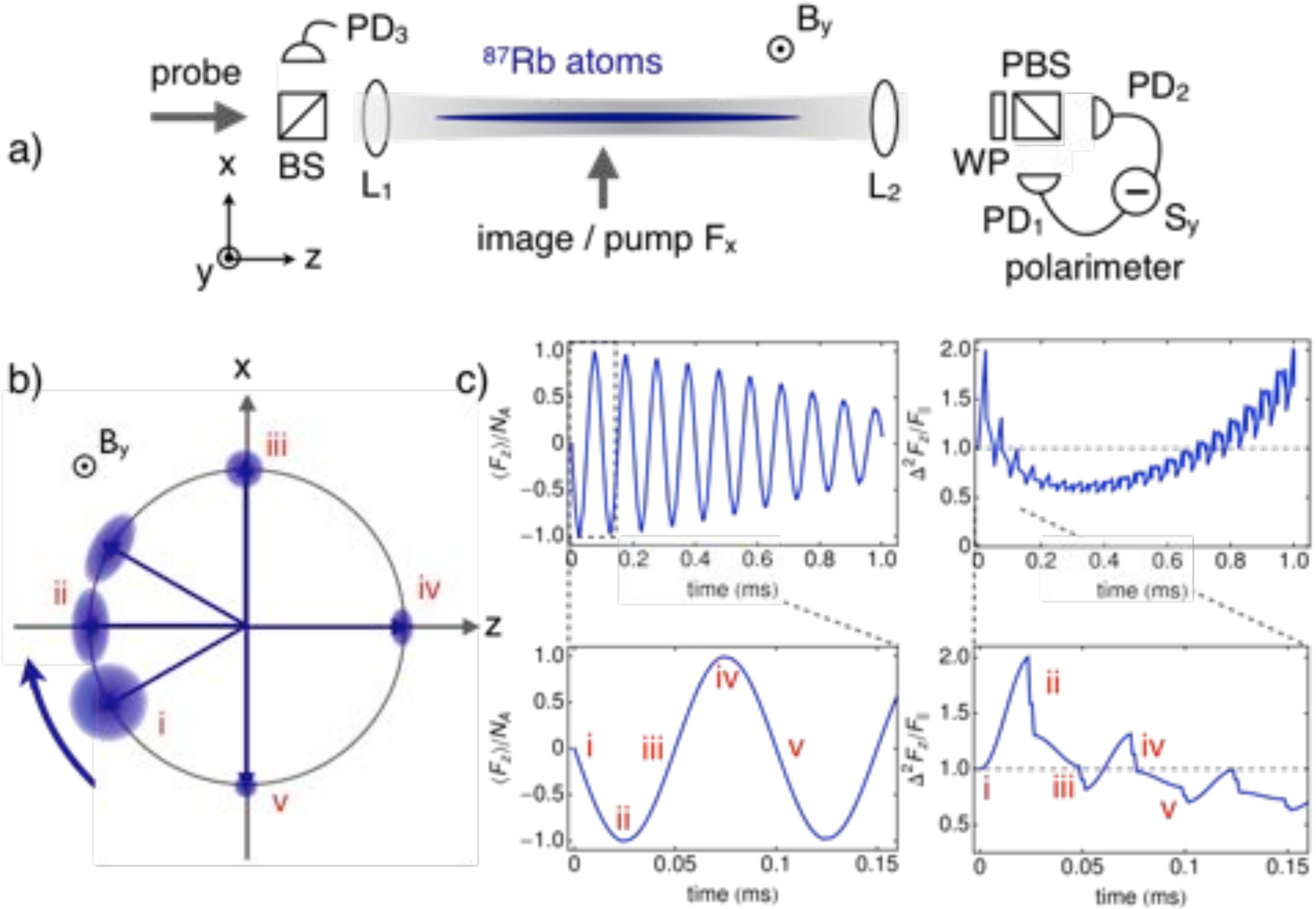
Optical: Korolkova, Leuch, Schnabel, Bachor, Lam
Atomic: He, Peng, Drummond and Reid PRA 2011
He, Vaughan, Drummond and Reid NJP 2012



$$[F_y, F_z] = iF_x$$

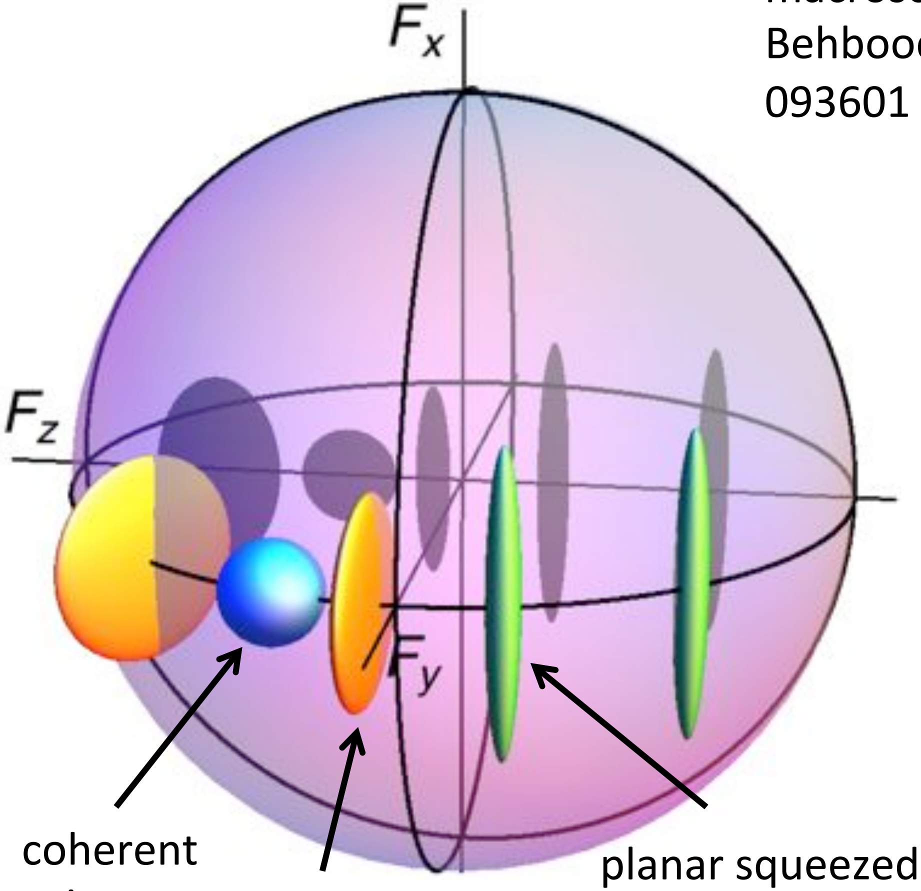
$$\delta F_y \delta F_z \geq \frac{1}{2}|\langle F_x \rangle|$$

Planar squeezed states



G. Puentes et al. NJP 2013 G. Colangelo et al. NJP 2013

Beyond planar squeezing

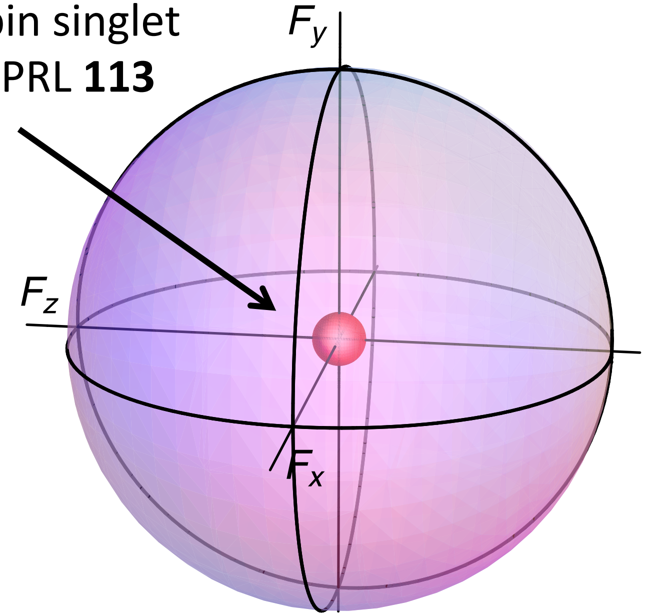


coherent spin state

squeezed
R. Sewell PRL 2012

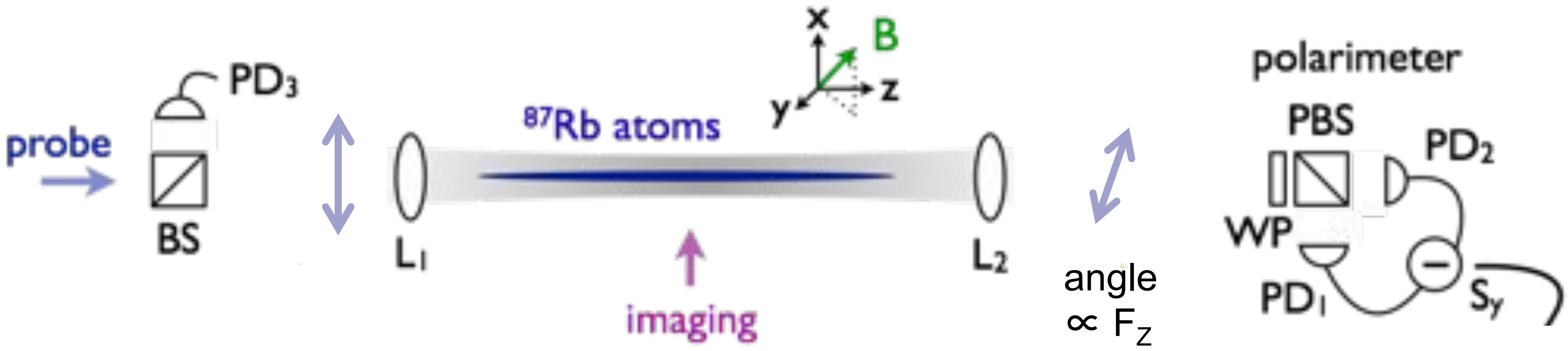
planar squeezed state
G. Puentes et al. NJP 2013
G. Colangelo et al. NJP 2013

macroscopic spin singlet
Behbood et al. PRL **113** 093601 (2014)

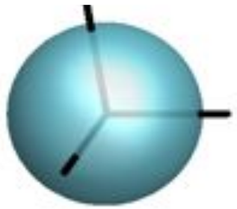


$$\delta F_x \delta F_y \geq 0$$
$$\delta F_y \delta F_z \geq 0$$
$$\delta F_z \delta F_x \geq 0$$

G. Toth, MWM, NJP **12** 053007 (2010)
Phys. Rev. A **87**, 021601(R) (2013)



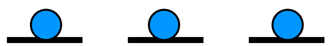
initial "thermal"
spin state



(wait) $vQND_2$

measure F_Z (F_Z) measure F_Z (F_X) measure F_Z (F_Y)

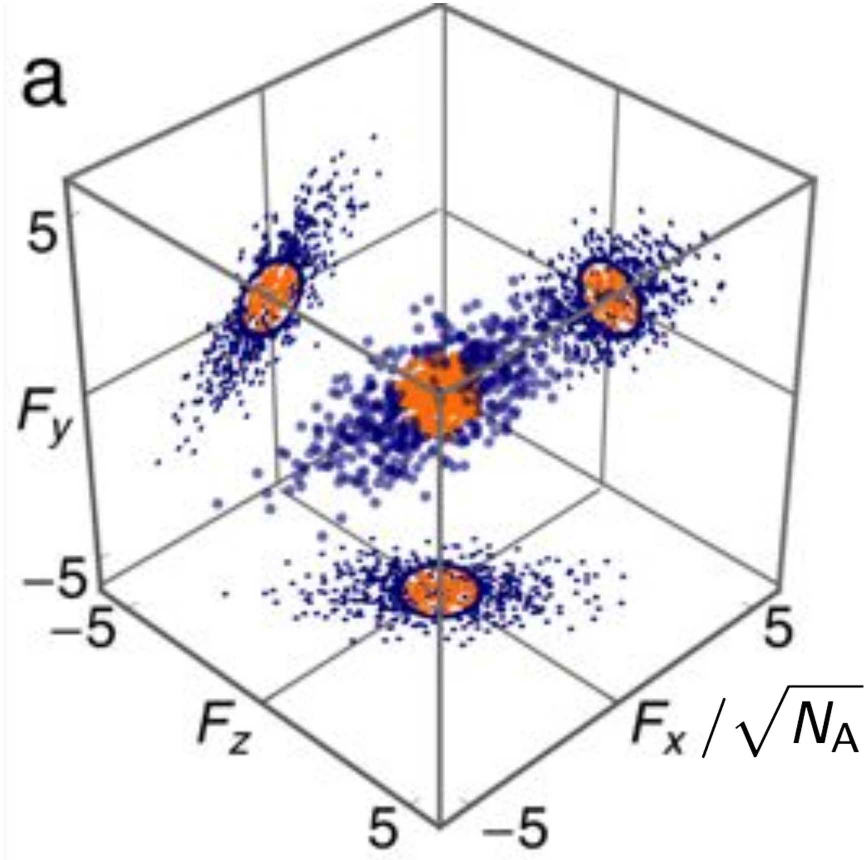
vector QND spin measurement



-1 0 +1 m_F

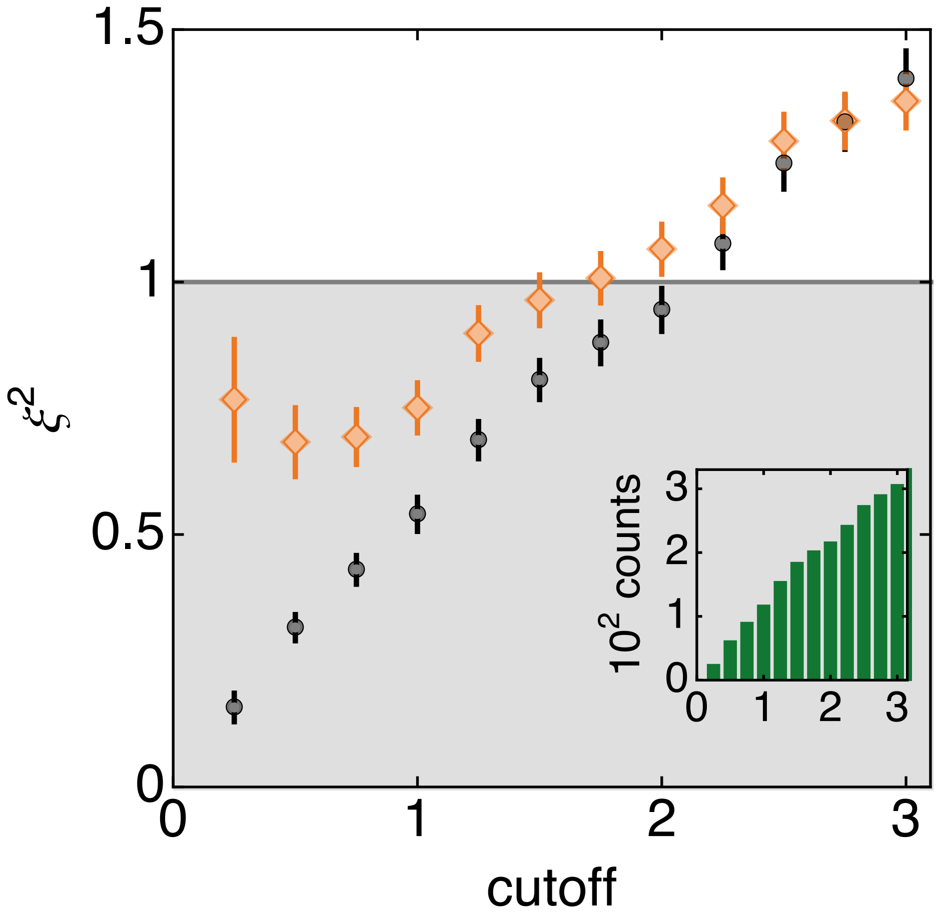
G. Toth, MWM, NJP **12** 053007 (2010)

Vector non-demolition measurements

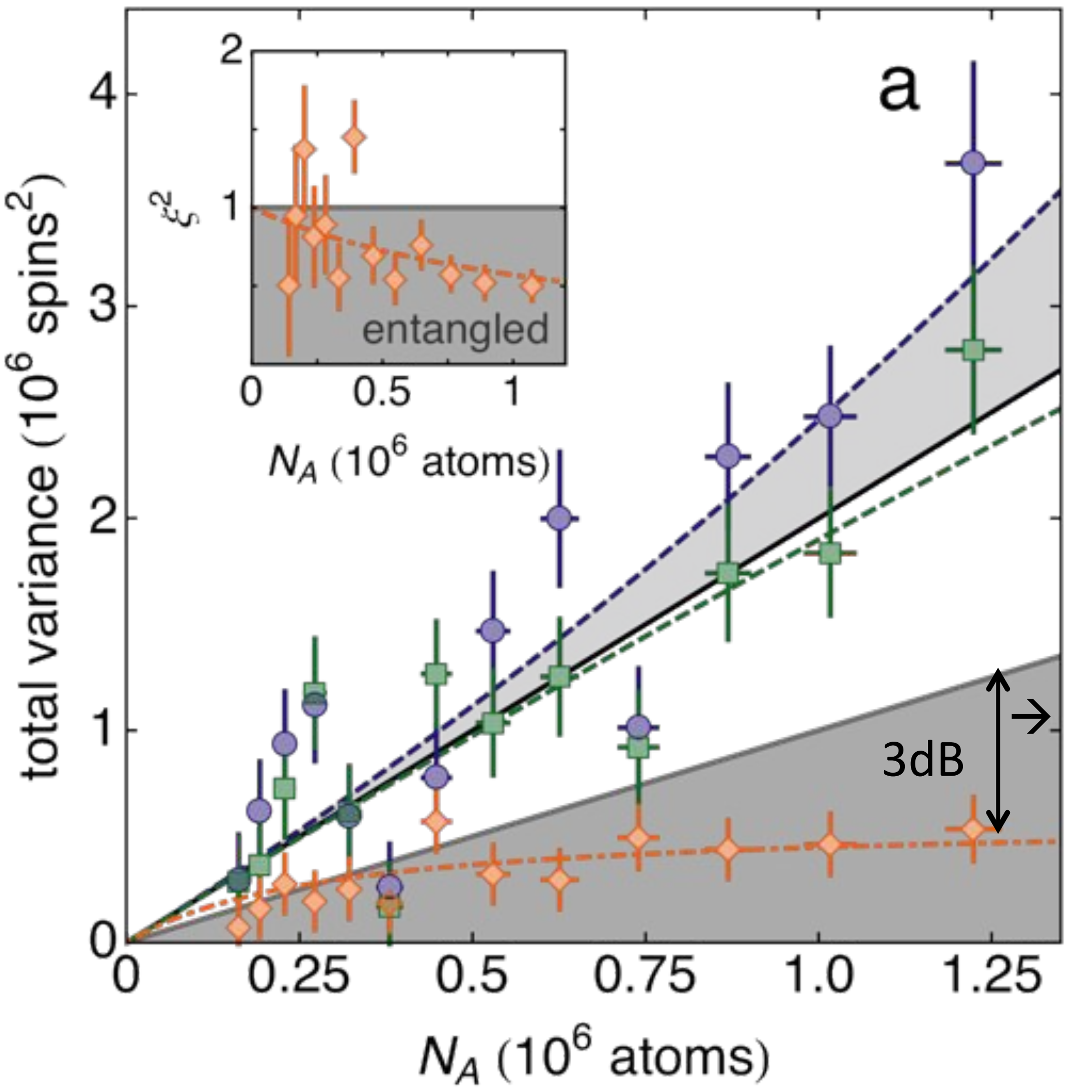


first vector measurement

Squeezing by selection



Unconditional squeezing



$|\Delta \mathbf{F}|^2$ (1st measurement)

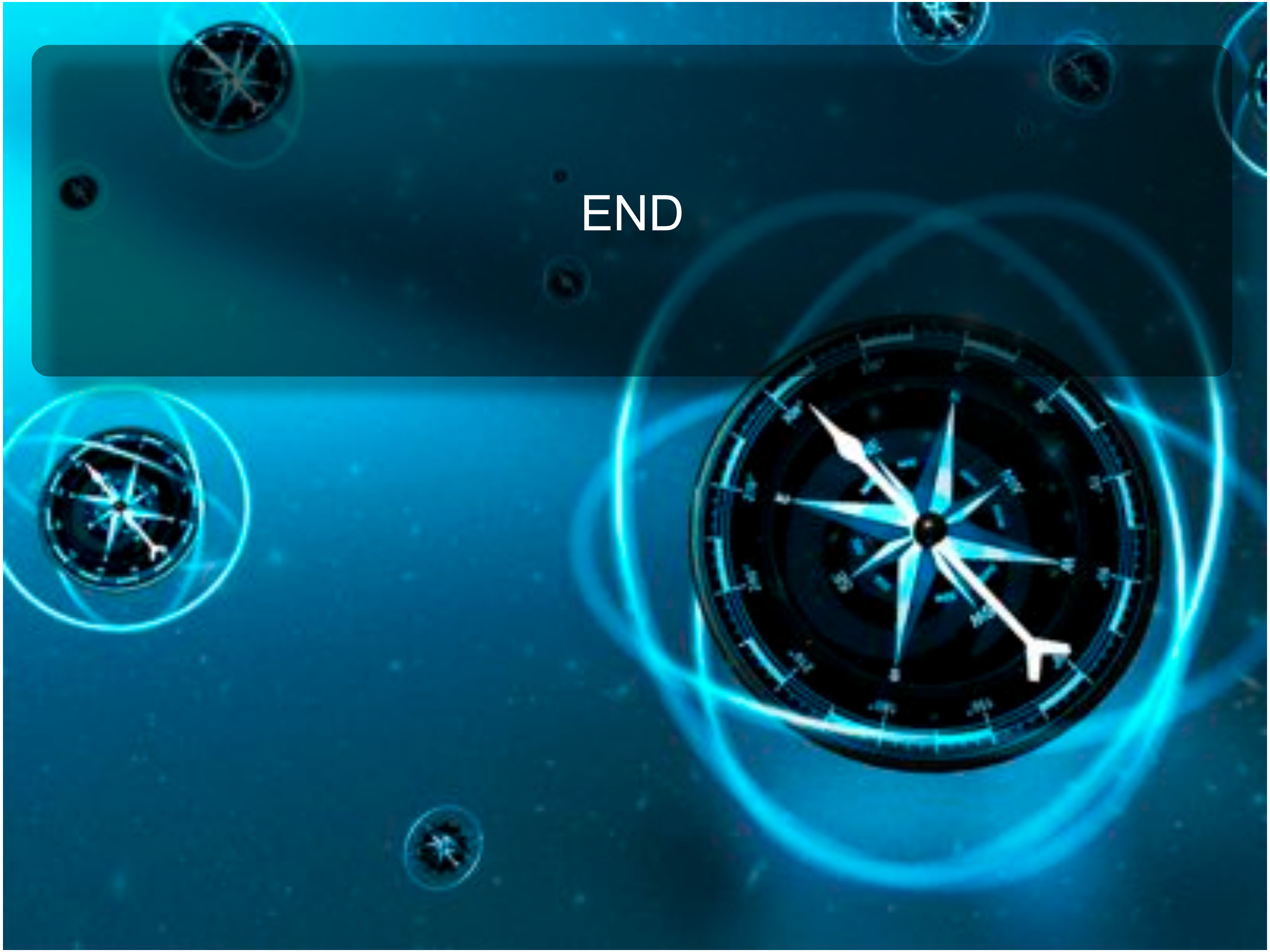
$|\Delta \mathbf{F}|^2$ (2nd measurement)

standard quantum limit

→ at least 550 000 atoms are entangled (maybe pairwise)

conditional variance

Behbood et al. PRL 2014


END