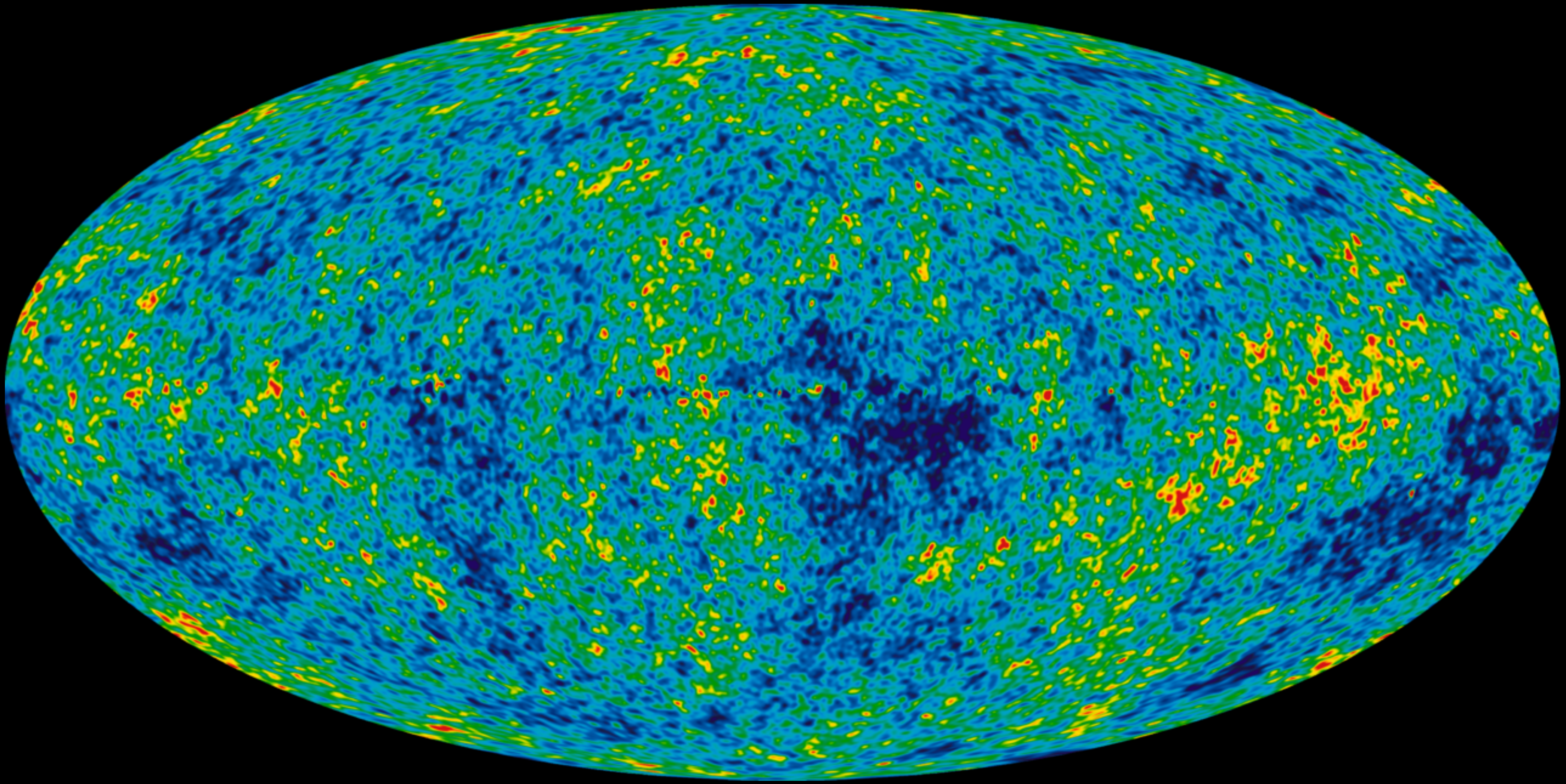
Cavity Optomechanics

Florian Marquardt

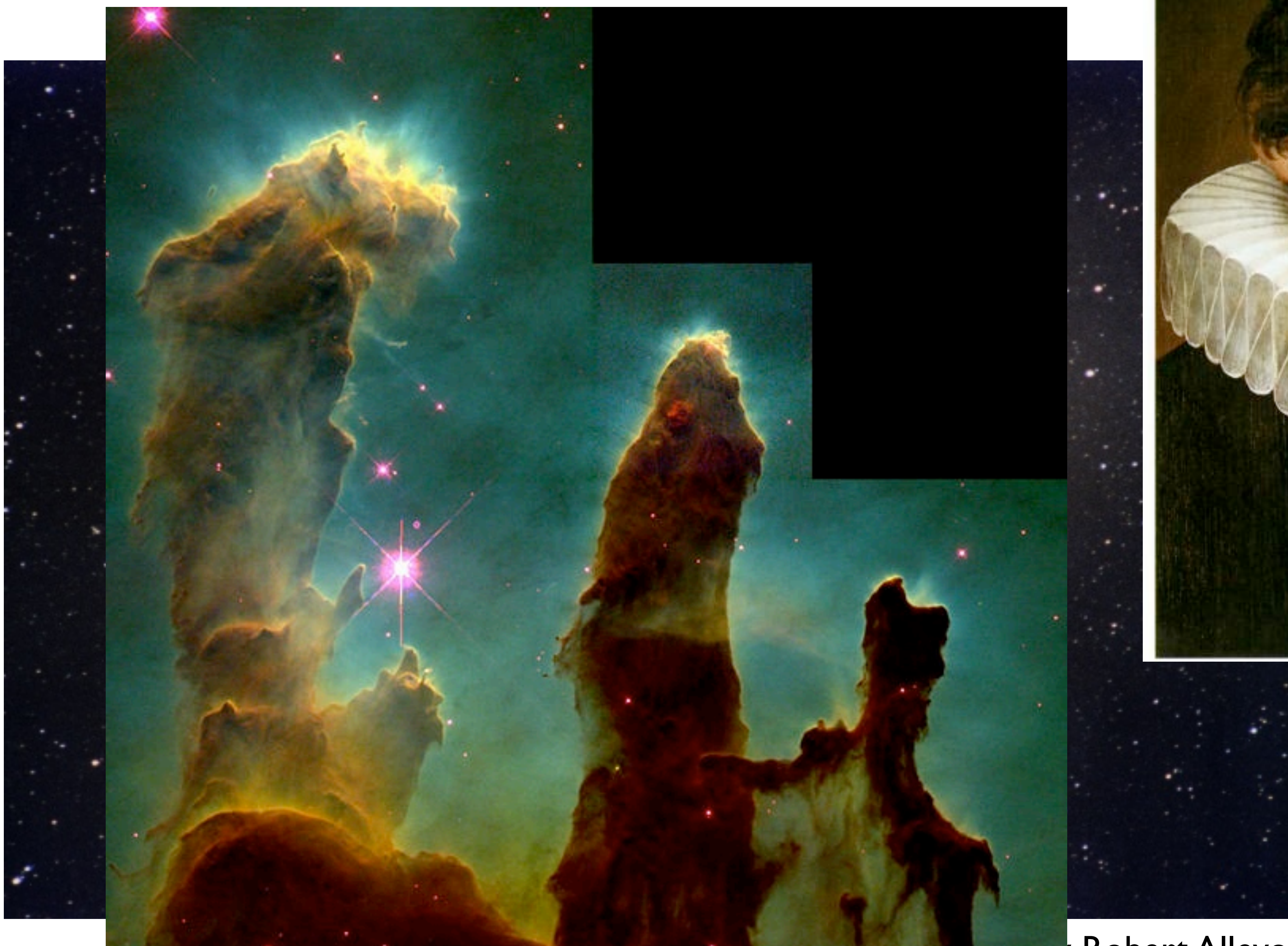
University of Erlangen-Nuremberg,
and Max-Planck-Institute
for the Science of Light

Radiation forces

baryon-photon fluid: sound speed $c/\sqrt{3}$



Radiation pressure



(Comet Hale-Bopp, by Robert Allevo)

Johannes Kepler
De Cometis, 1619

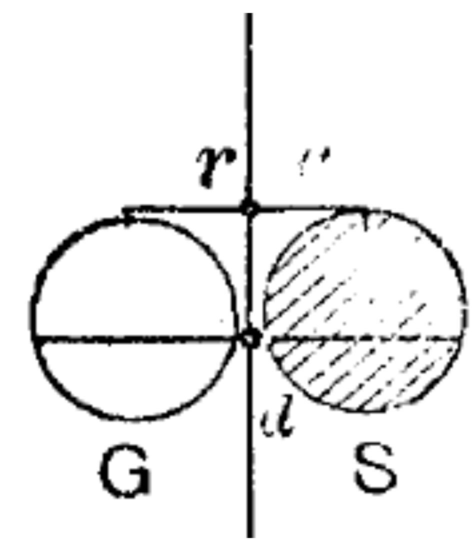
Radiation pressure

Nichols and Hull, 1901
Lebedev, 1901

A PRELIMINARY COMMUNICATION ON THE PRESSURE OF HEAT AND LIGHT RADIATION.

BY E. F. NICHOLS AND G. F. HULL.

MAXWELL,[1] dealing mathematically with the stresses in an electro-magnetic field, reached the conclusion that "in a medium in which waves are propagated there is a pressure normal to the waves and numerically equal to the energy in unit volume."



$$F = \frac{2I}{c}$$

Nichols and Hull, Physical Review **13**, 307 (1901)

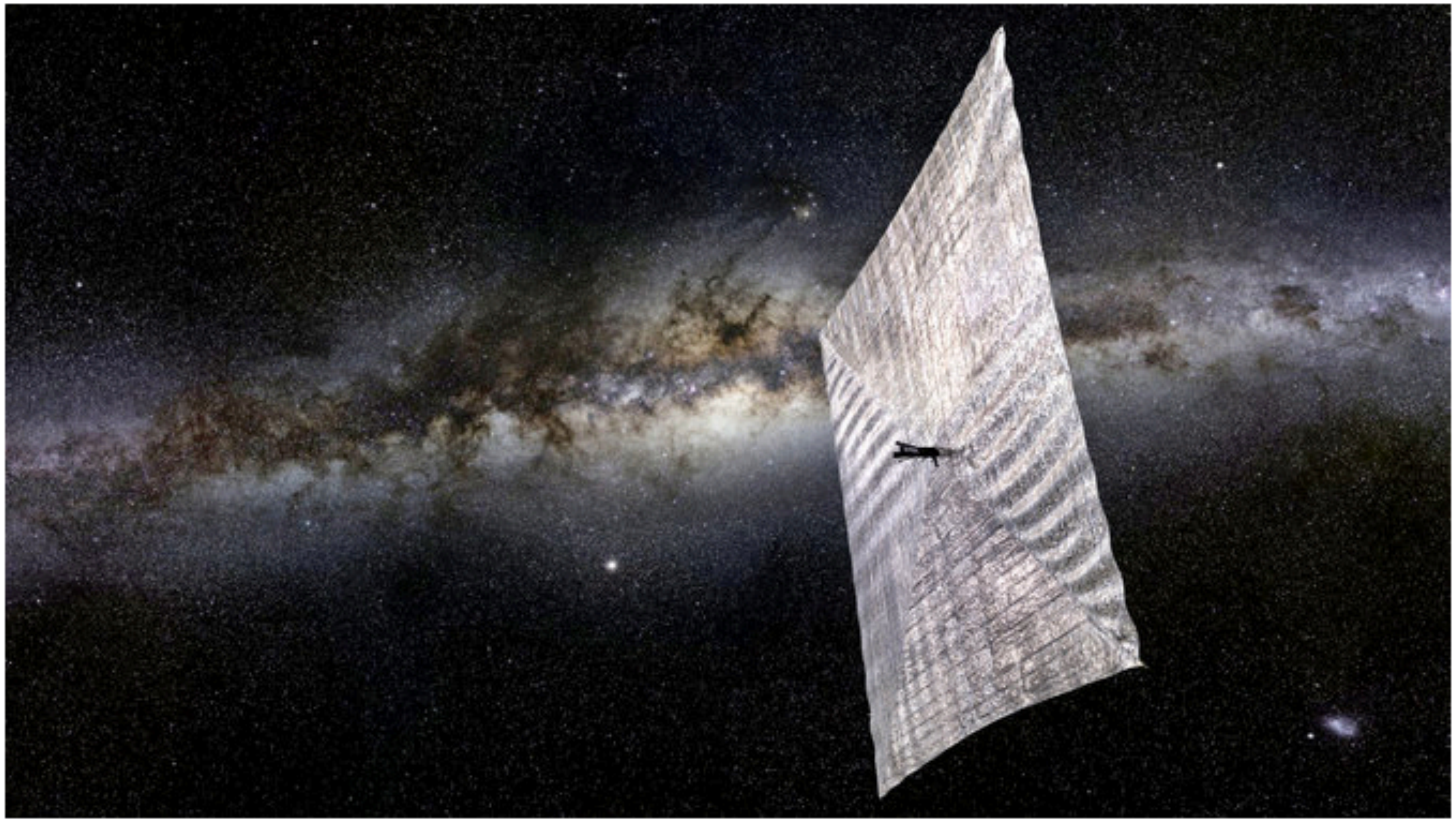
SPACE & COSMOS

The New York Times

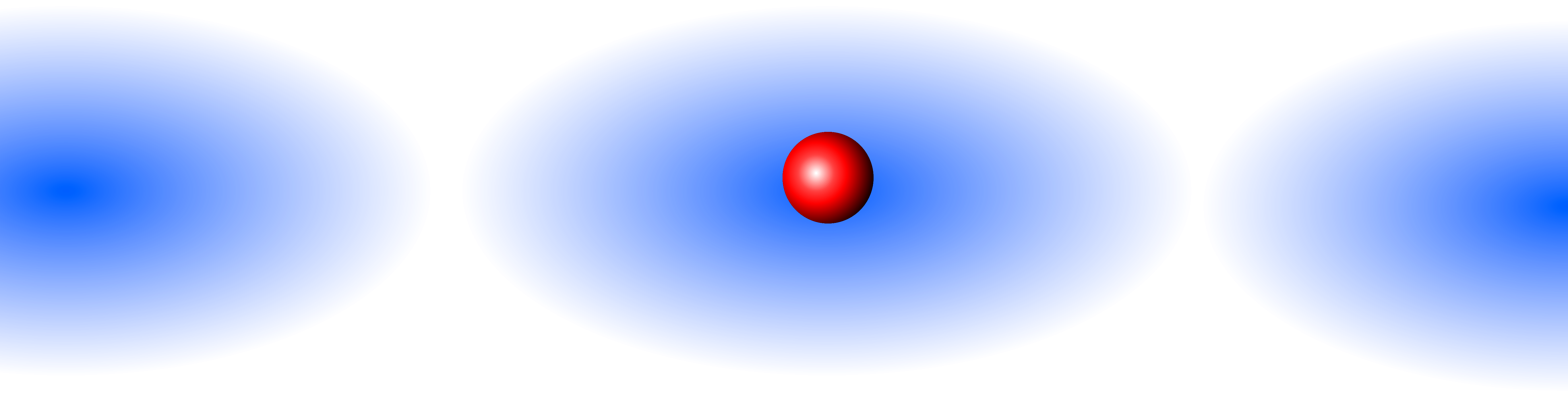
After Silences and Setbacks, the LightSail Spacecraft Is Revived, Deploying Its Solar Sail

By KENNETH CHANG JUNE 7, 2015

The technology, using sunlight to traverse the solar system in the same way mariners once crossed oceans in sailing ships, is not a new idea, but it has not been widely used. While particles of light impart only a smidgen of momentum, the force is continuous and provides propulsion without fuel.



Radiation forces



Trapping and cooling

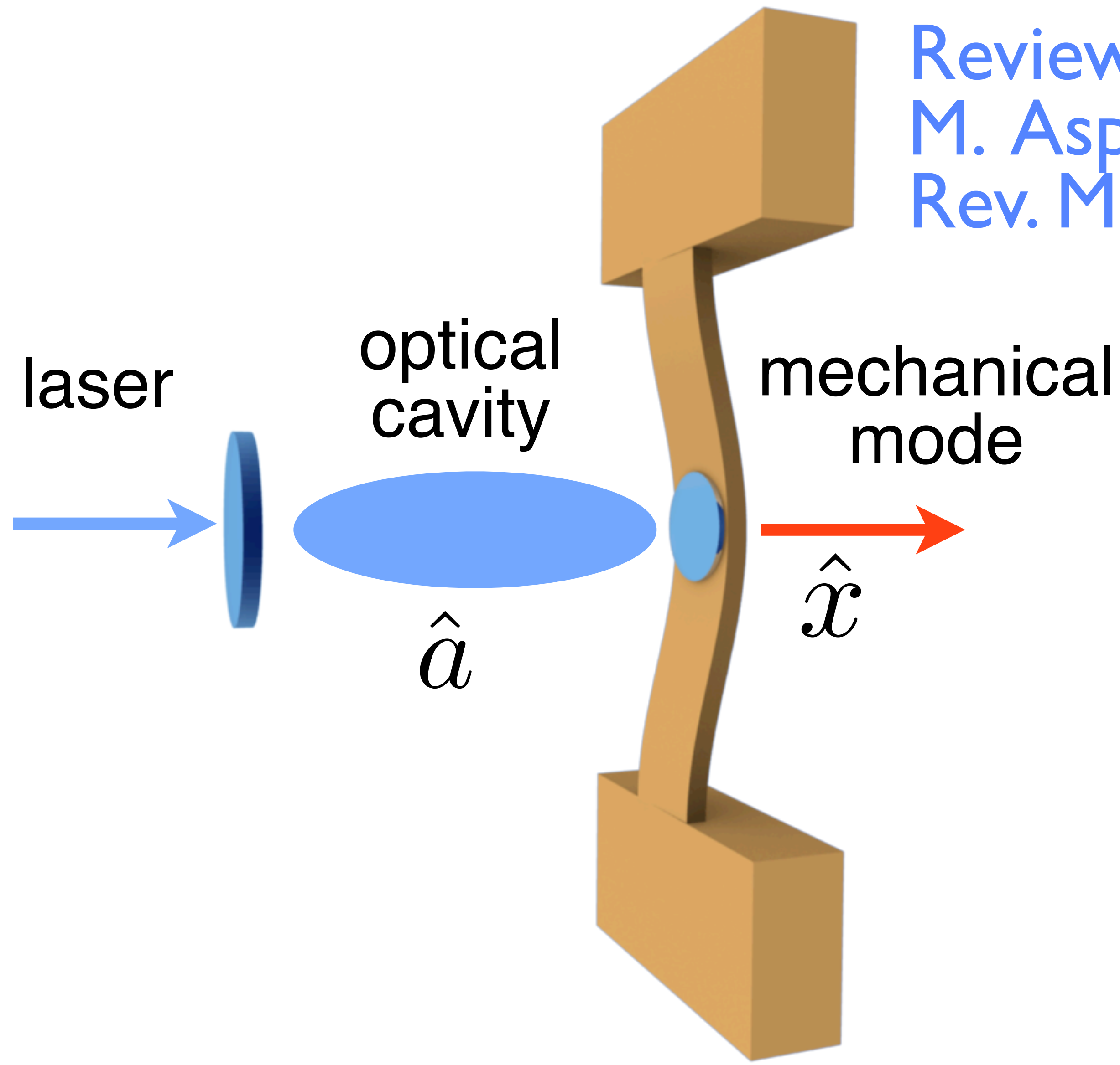
- Optical tweezers
- Optical lattices

...but usually no back-action from motion onto light!

Optomechanical Hamiltonian

Optomechanical Hamiltonian

Review "Cavity Optomechanics":
M. Aspelmeyer, T. Kippenberg, FM
Rev. Mod. Phys. 2014



$$\hat{H} = \hbar\omega_{\mathrm{cav}} \cdot (1 - \hat{x}/L)\hat{a}^\dagger \hat{a} + \hbar\omega_M \hat{b}^\dagger \hat{b} + \ldots$$

$$\hat{x} = x_{\mathrm{ZPF}}(\hat{b} + \hat{b}^\dagger) \qquad x_{\mathrm{ZPF}} = \sqrt{\frac{\hbar}{2m\Omega}}$$

Optomechanical Hamiltonian

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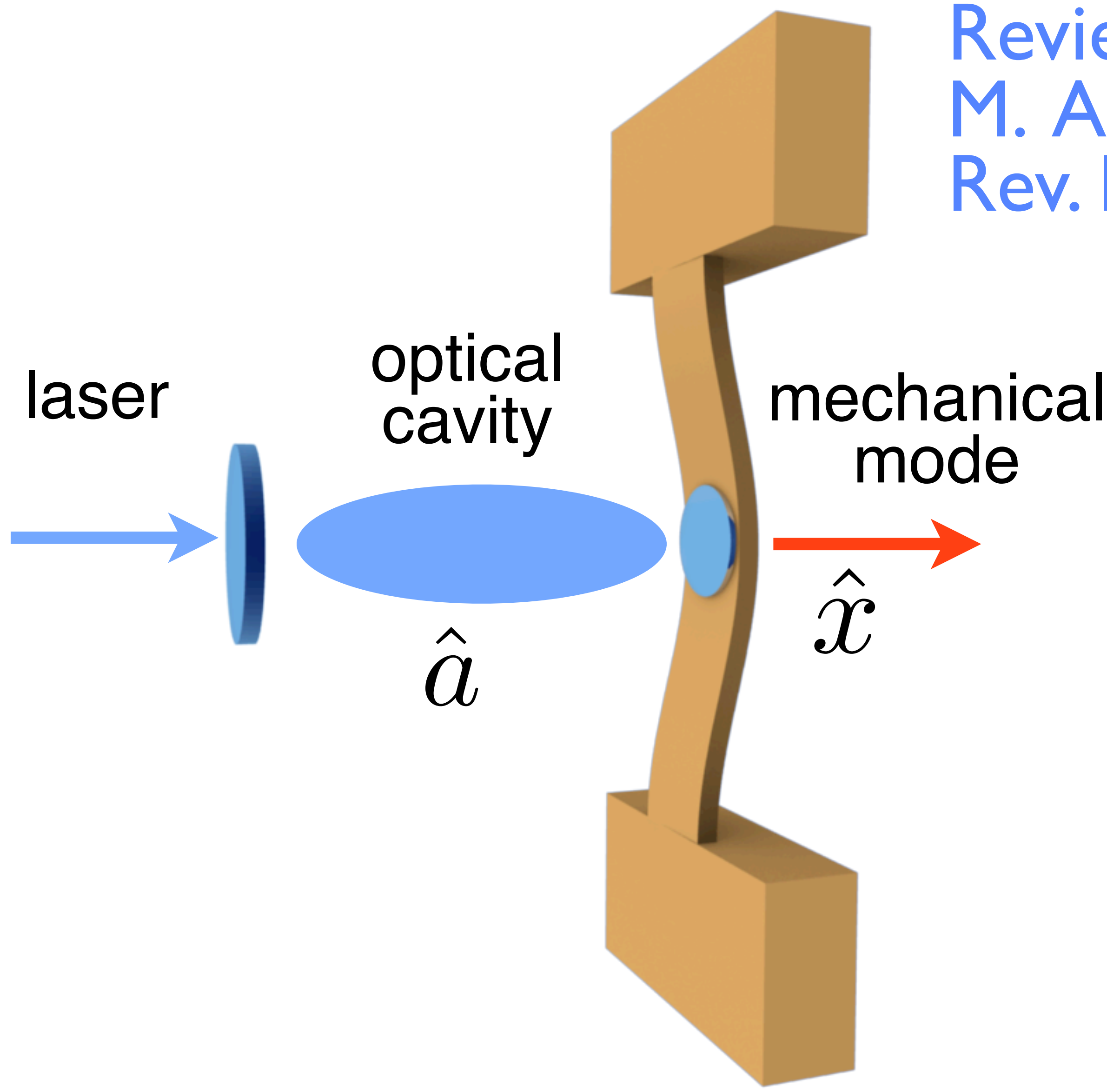


$$g_0 \sim \mathrm{Hz} - \mathrm{MHz}$$

$$\hat{H} = -(\Delta + g_0(\hat{b} + \hat{b}^\dagger))\hat{a}^\dagger\hat{a} + \Omega\hat{b}^\dagger\hat{b} + \ldots$$

laser detuning

$$\Delta = \omega_L - \omega_{\mathrm{cav}}$$

optomech. coupling

$$\hat{x} = x_{\mathrm{ZPF}}(\hat{b} + \hat{b}^\dagger)$$

$$x_{\mathrm{ZPF}} = \sqrt{\hbar/2m\Omega}$$

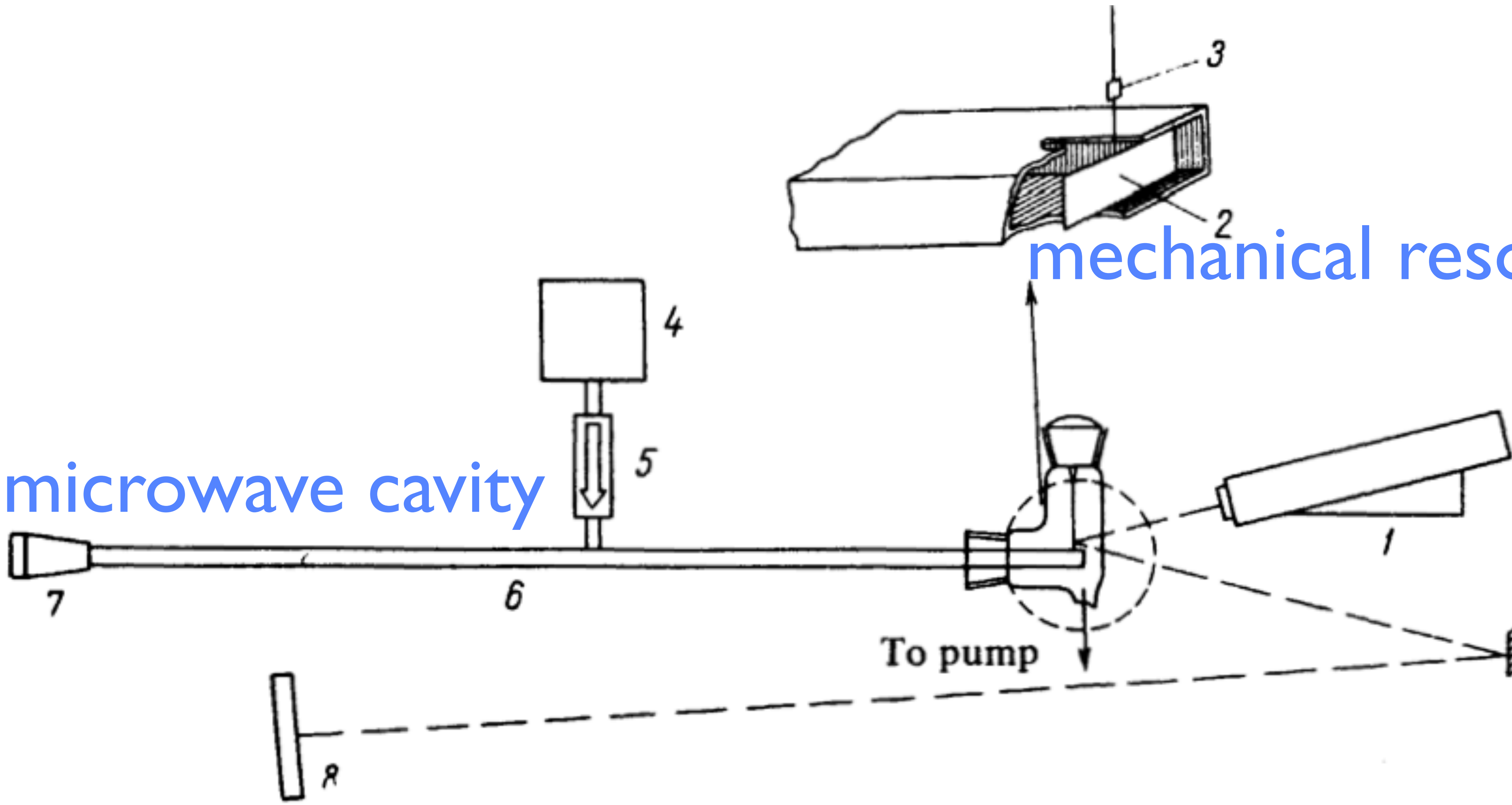
A bit of history

First cavity optomechanics experiments

optomechanical change of mechanical damping rate

Braginsky, Manukin, Tikhonov JETP 1970

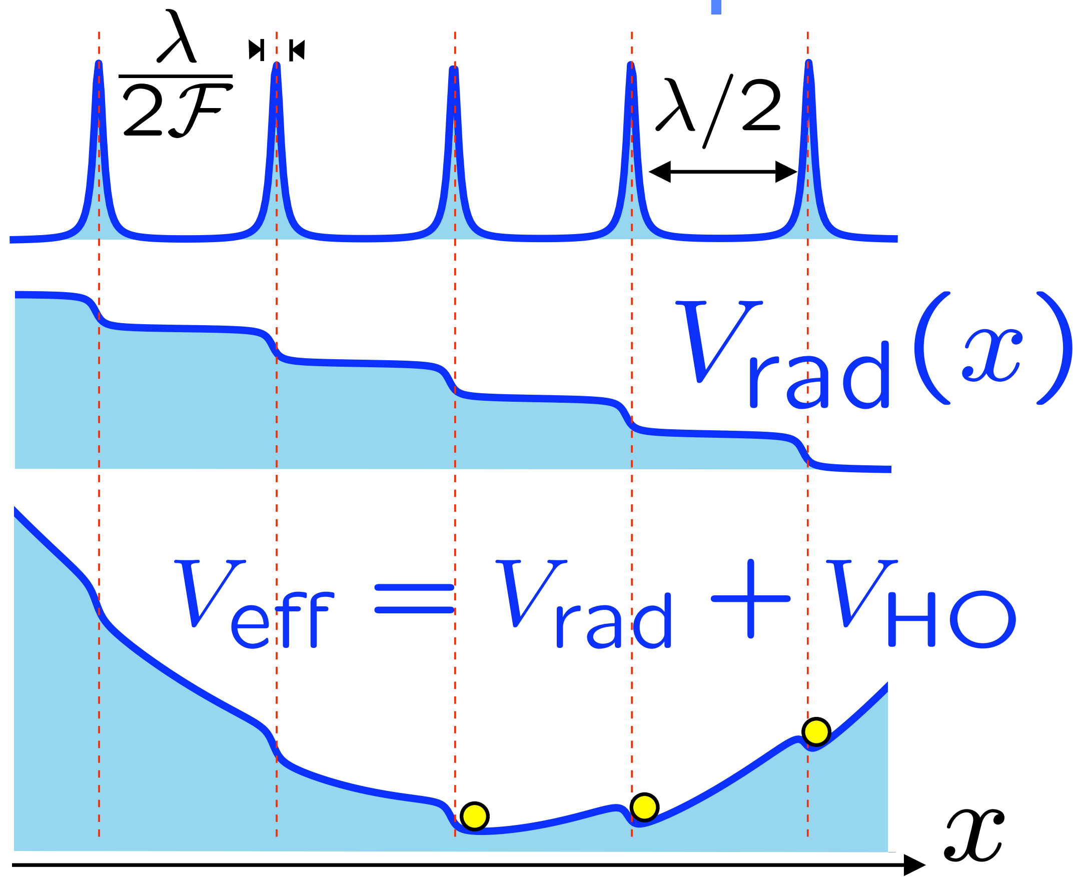
INVESTIGATION OF DISSIPATIVE PONDEROMOTIVE EFFECTS OF ELECTROMAGNETIC RADIATION

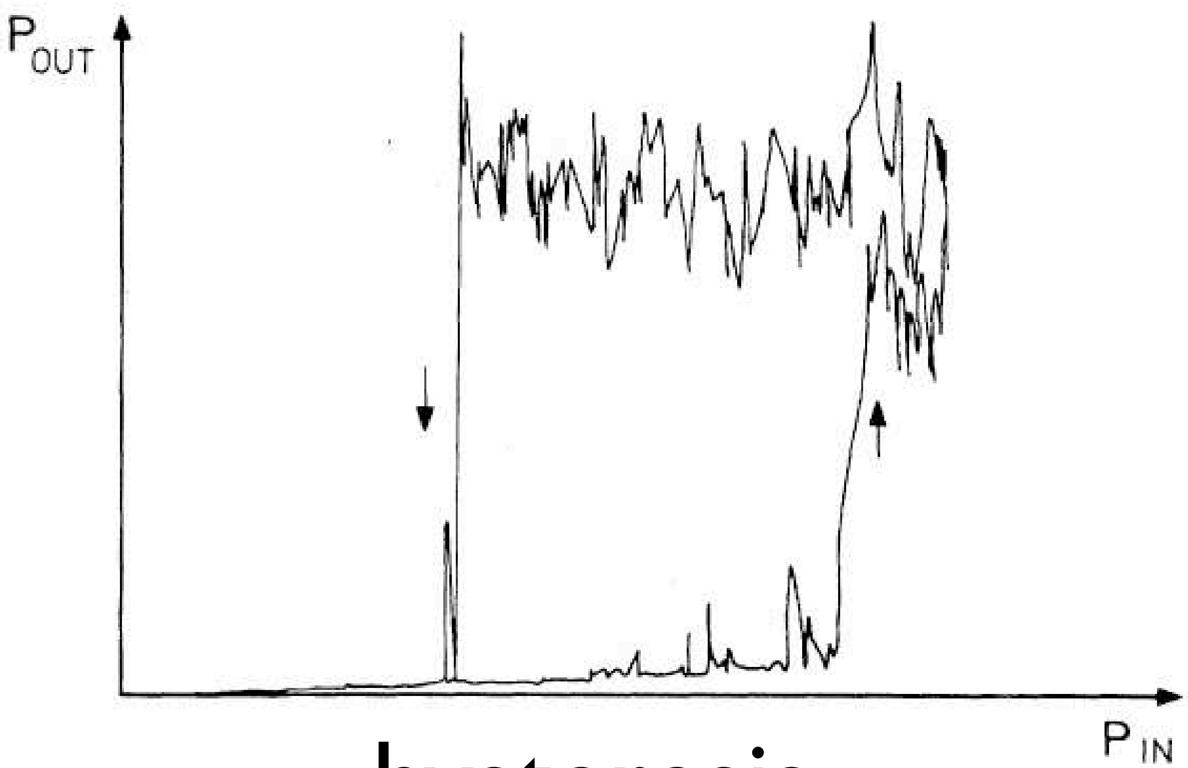


First cavity optomechanics experiments

Static bistability in an optical cavity experiment
Dorsel, McCullen, Meystre, Vignes, Walther PRL 1983

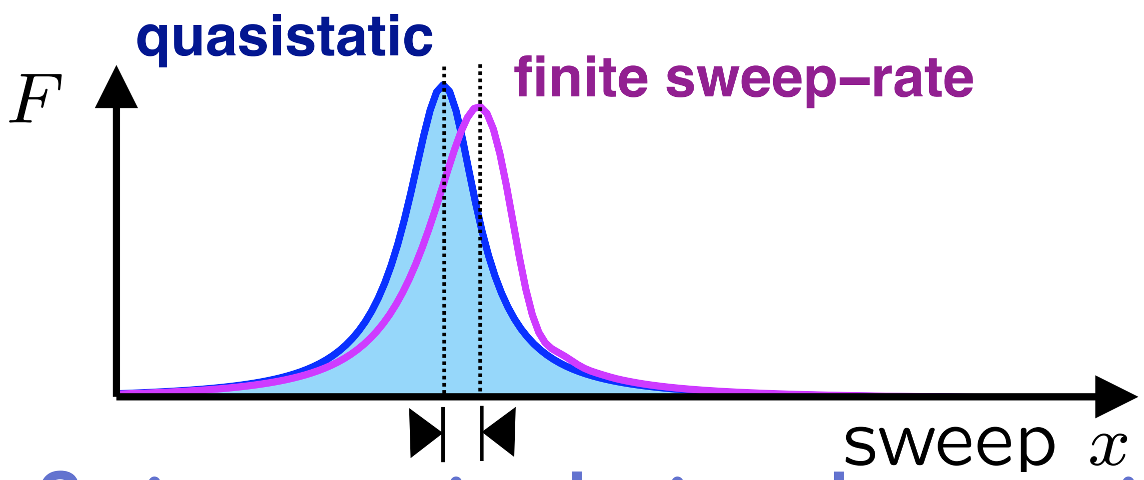
force vs. mirror position



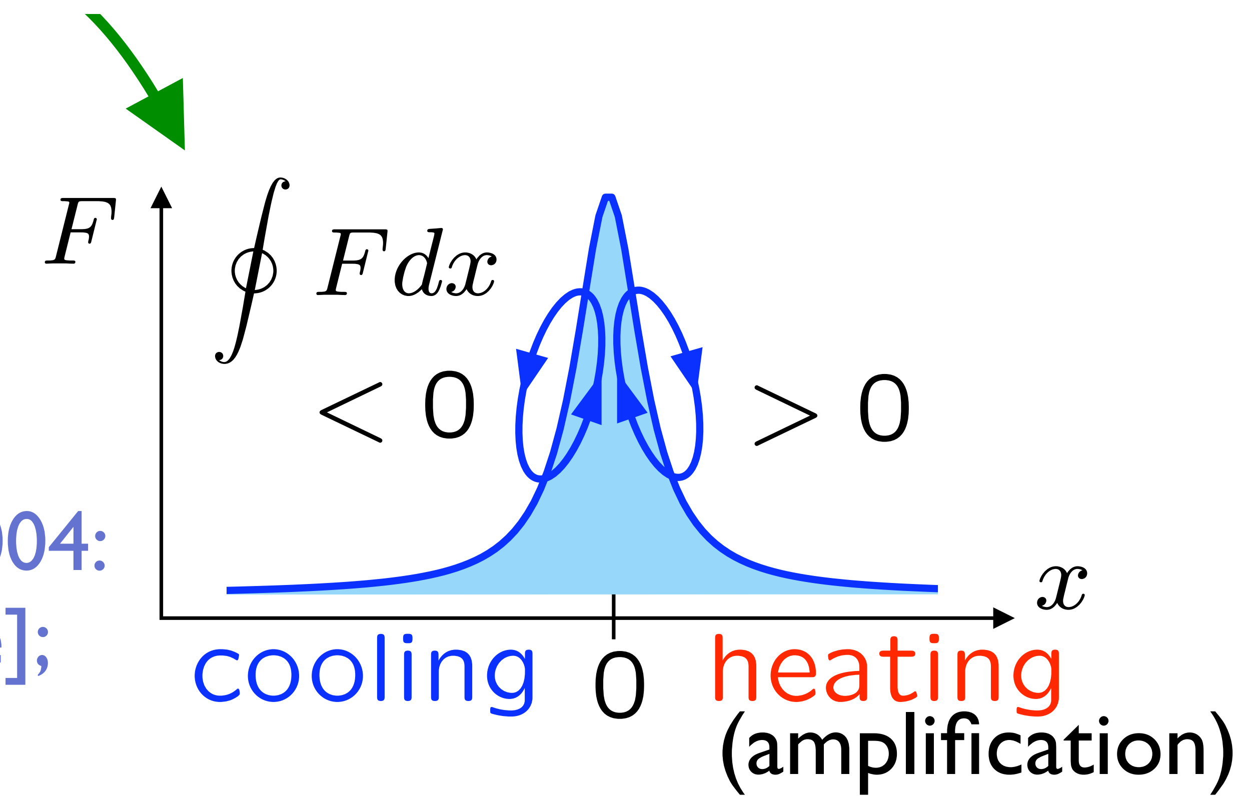


hysteresis

Basic physics: dynamics



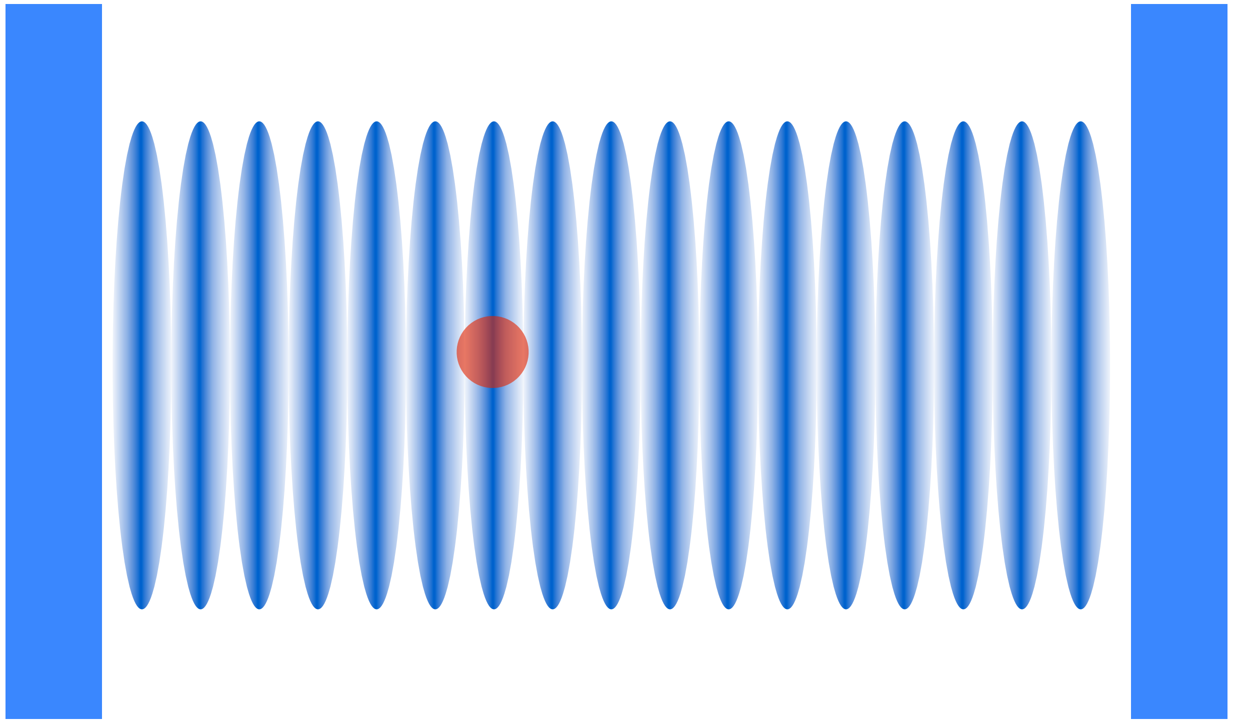
finite optical ringdown time κ^{-1} –
delayed response to cantilever motion



Höhberger-Metzger and Karrai 2004: 300K to 17K [photothermal force]; 2006: radiation pressure cavity cooling [Aspelmeyer, Heidmann/ Cohadon, Kippenberg]

A zoo of devices

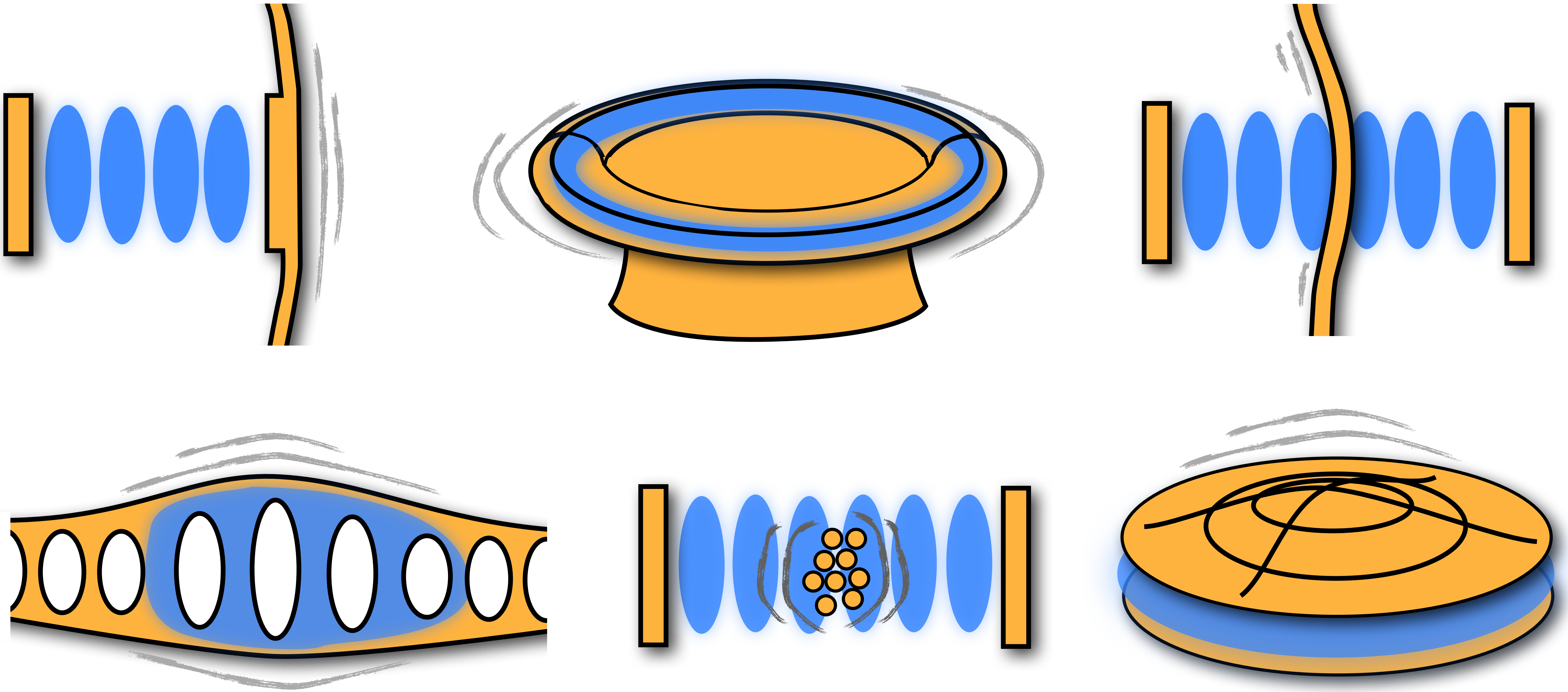
Optomechanical Hamiltonian



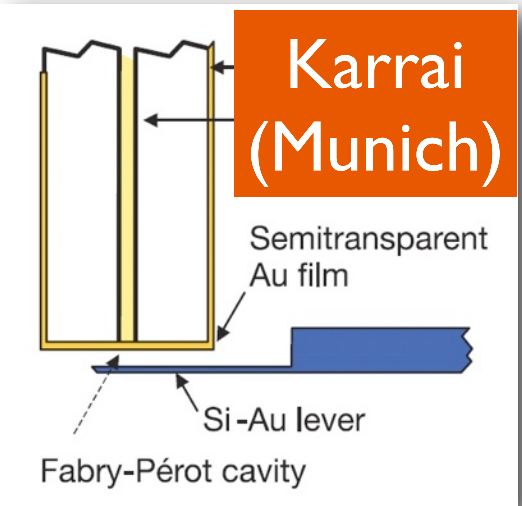
$$\hat{H} = \hbar\omega_{\mathrm{cav}}(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b} + \ldots$$

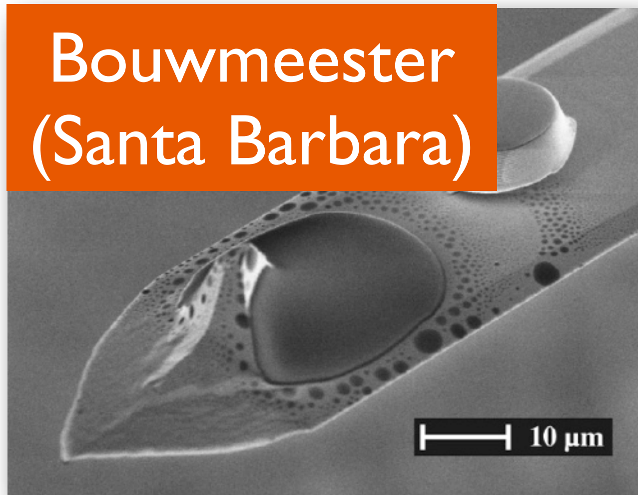
...any dielectric moving inside a cavity
generates an optomechanical interaction!

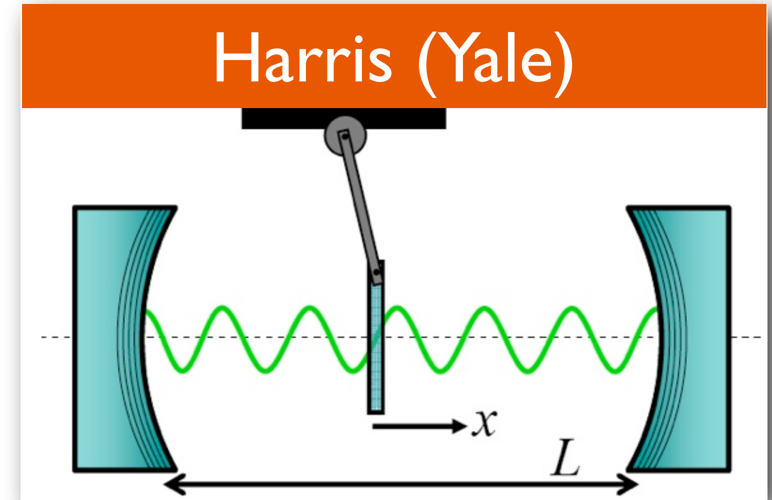
The zoo of optomechanical (and analogous) systems

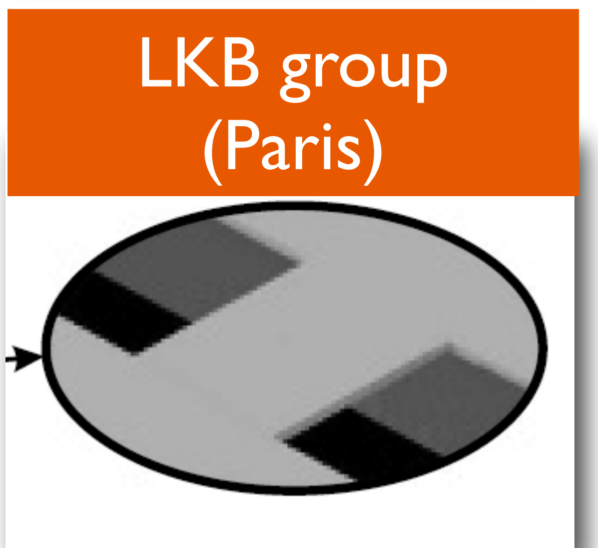


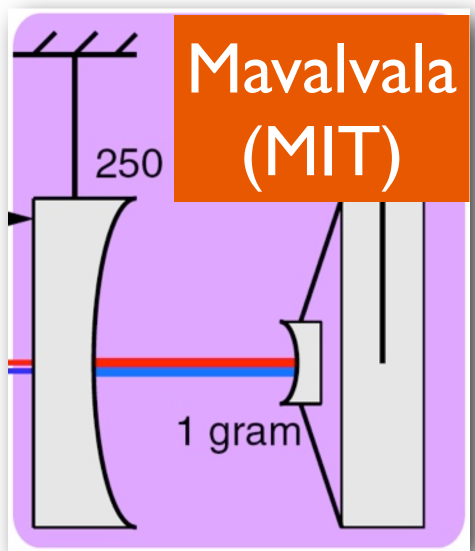
The zoo of optomechanical (and analogous) systems

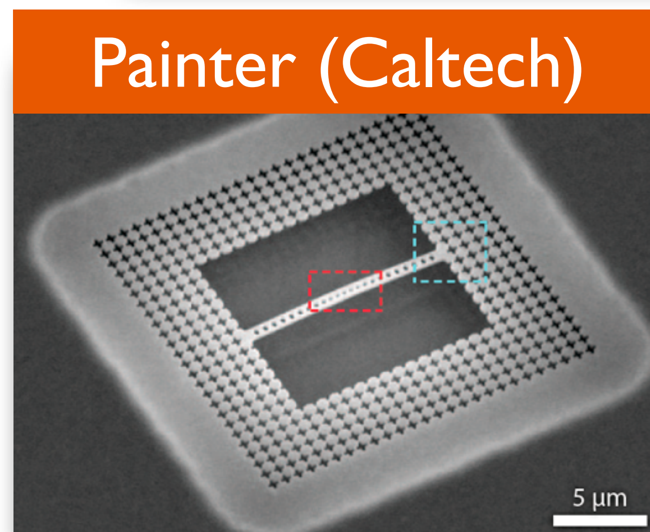


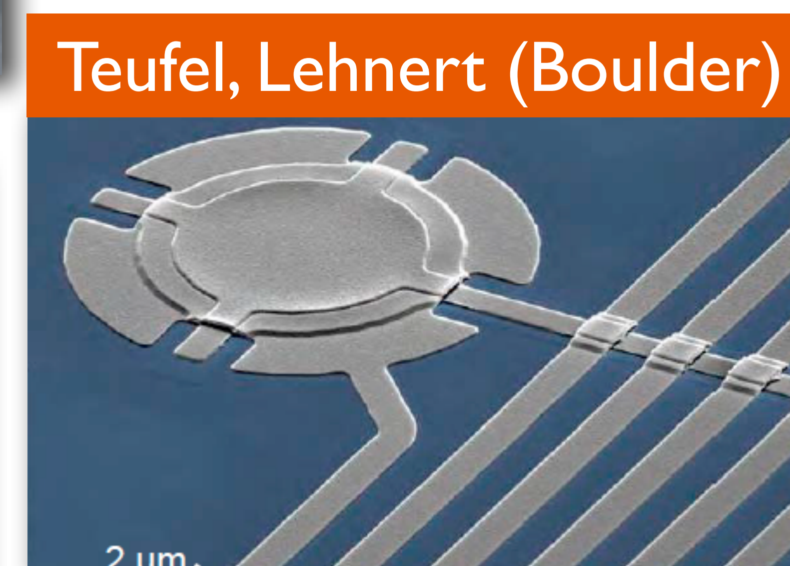


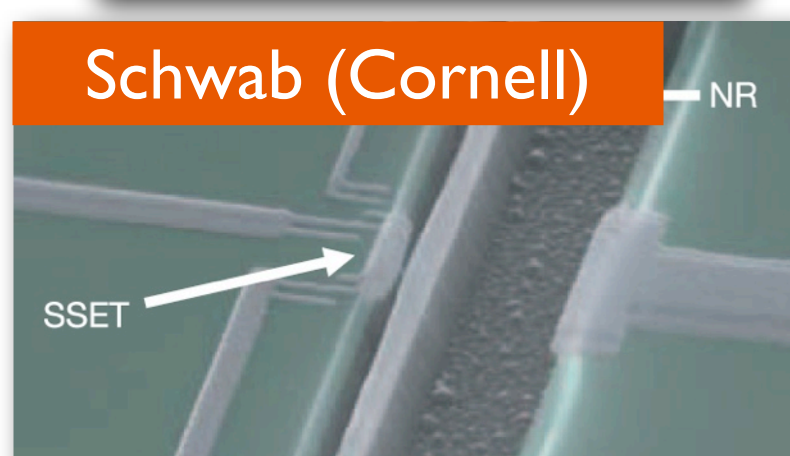


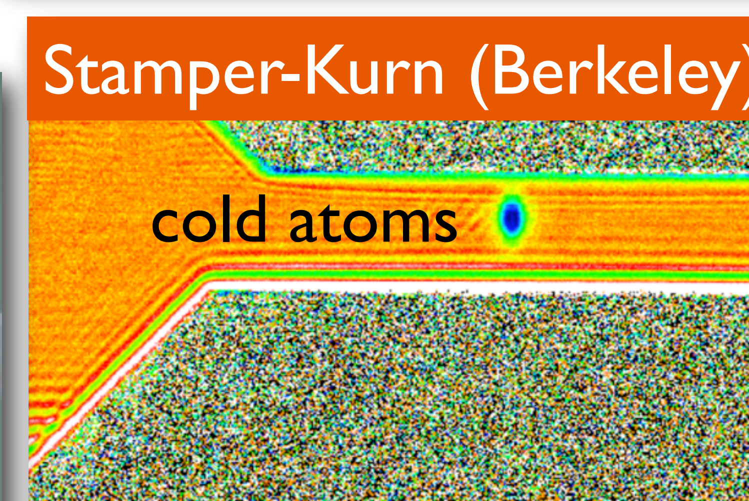






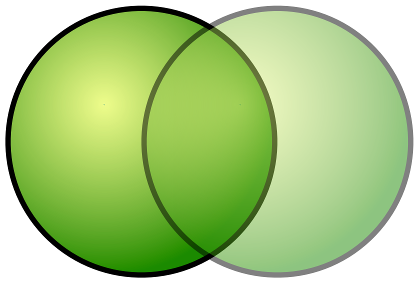




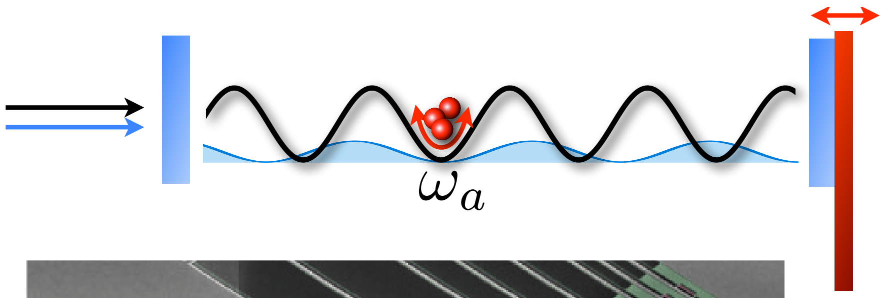


Why?

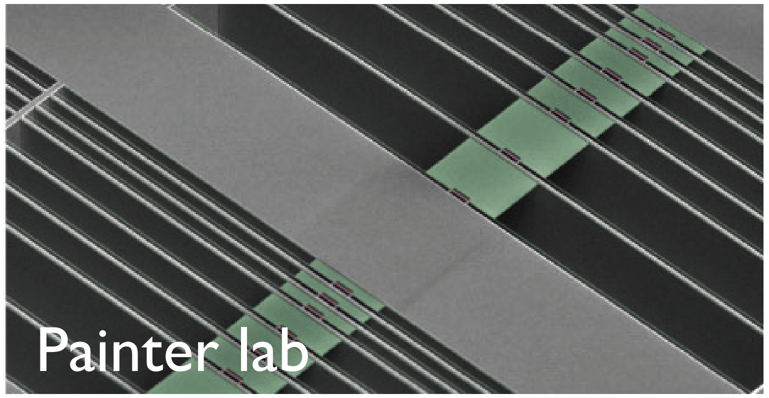
Optomechanics: general outlook



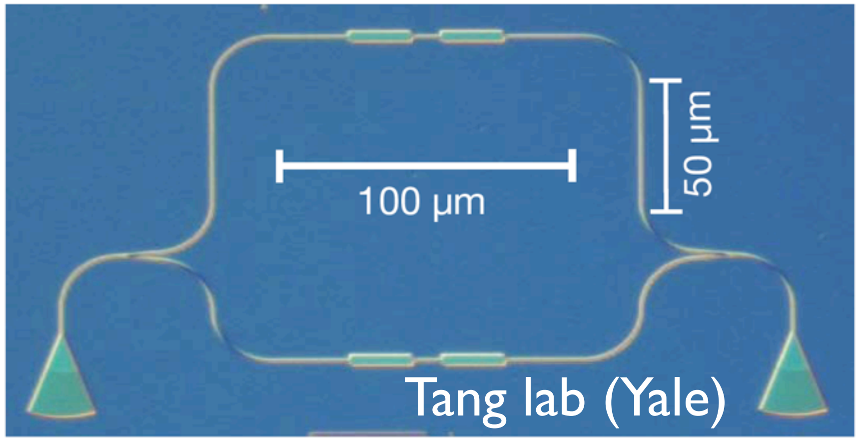
Fundamental tests of quantum mechanics in a new regime:
entanglement with ‘macroscopic’ objects, unconventional decoherence?
[e.g.: gravitationally induced?]



Mechanics as a ‘bus’ for connecting hybrid components: superconducting qubits, spins, photons, cold atoms, ...

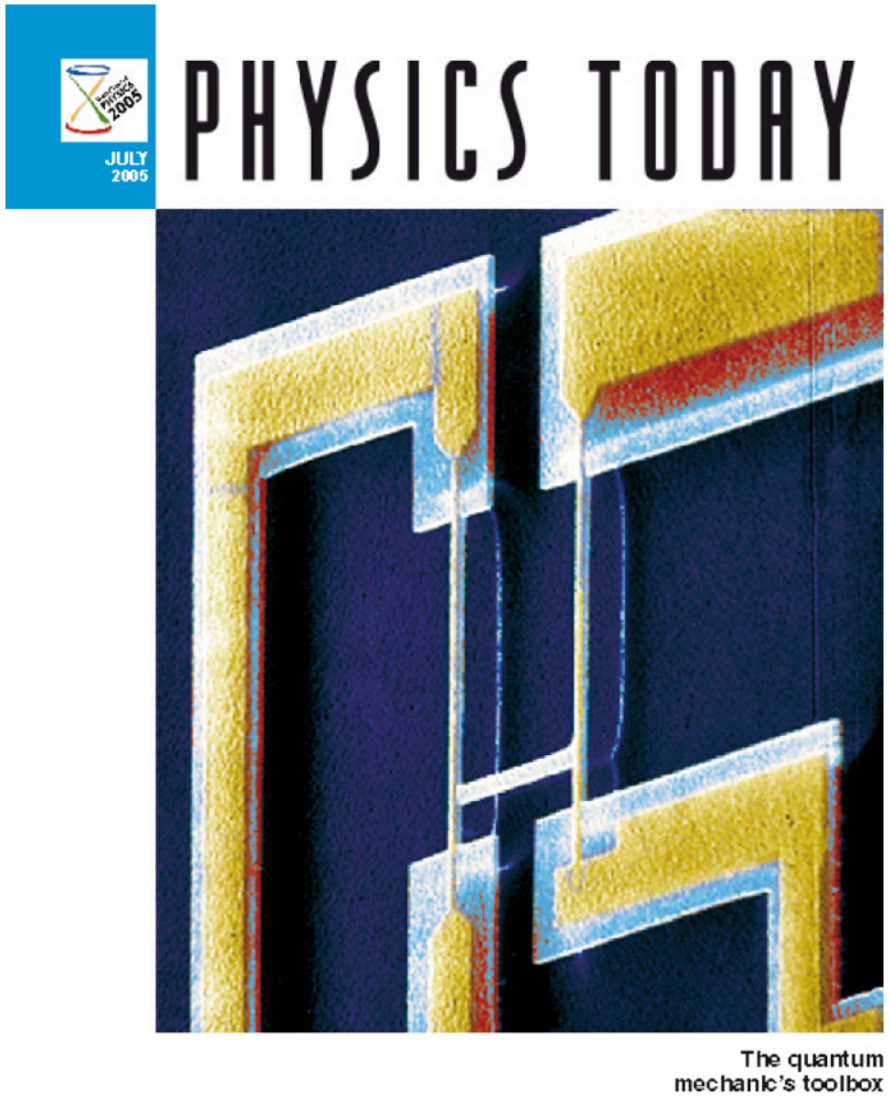


Precision measurements
small displacements, masses, forces, and accelerations



Optomechanical circuits & arrays
Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

Towards the quantum regime of mechanical motion



Putting Mechanics into Quantum Mechanics

Nanoelectromechanical structures are starting to approach the ultimate quantum mechanical limits for detecting and exciting motion at the nanoscale. Nonclassical states of a mechanical resonator are also on the horizon.

Keith C. Schwab and Michael L. Roukes

Everything moves! In a world dominated by electronic devices and instruments it is easy to forget that all measurements involve motion, whether it be the motion of electrons through a transistor, Cooper pairs or quasiparticles through a superconducting quantum interference device (SQUID), photons through an optical interferometer—or the simple displacement of a mechanical element.

achieved to read out those devices, now bring us to the realm of quantum mechanical systems.

The quantum realm

What conditions are required to observe the quantum properties of a mechanical structure, and what can we learn when we encounter them? Such questions have received

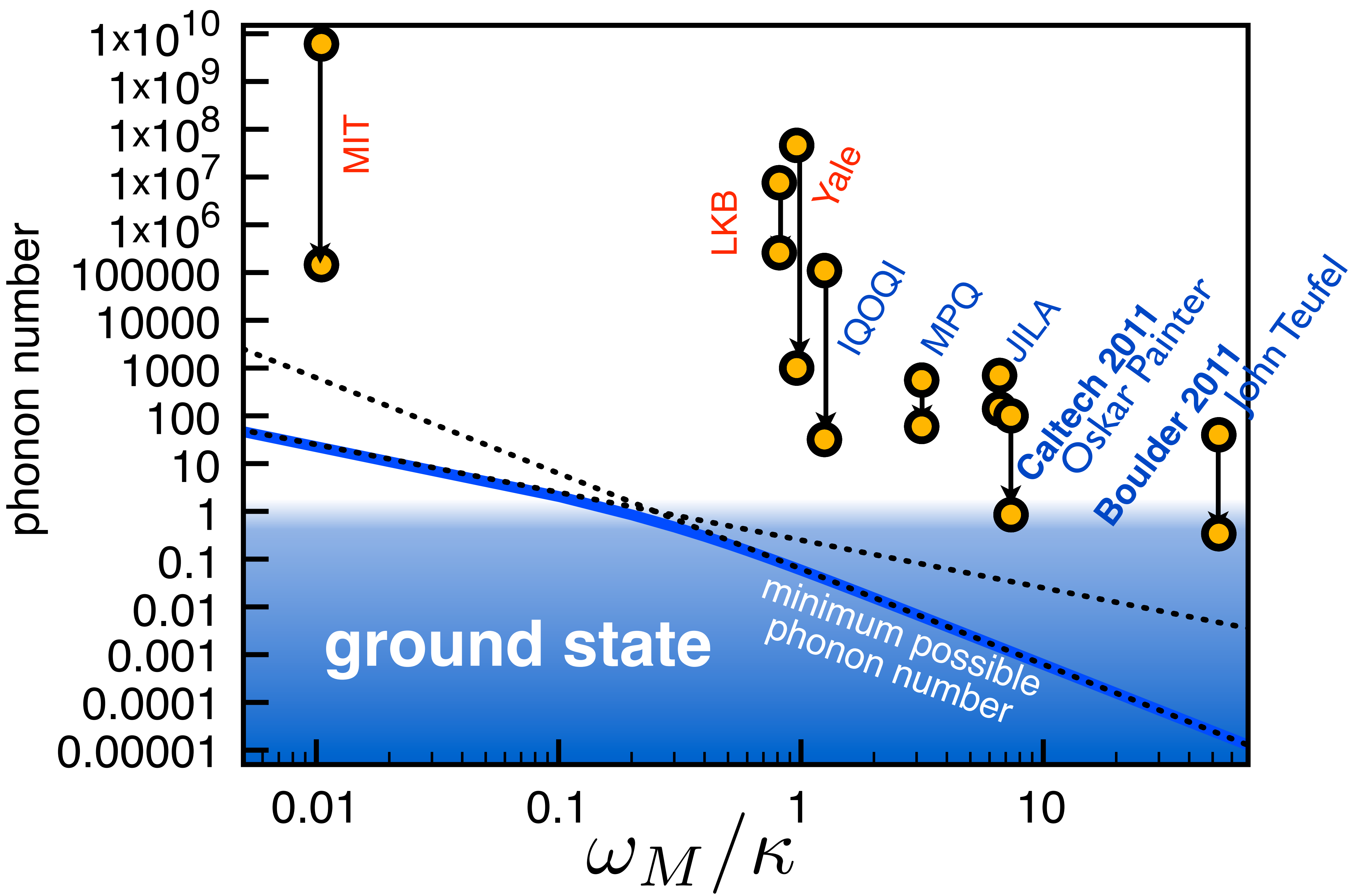
Schwab and Roukes, Physics Today 2005

- nano-electro-mechanical systems

Superconducting qubit coupled to nanoresonator: Cleland & Martinis 2010

- optomechanical systems

Laser-cooling towards the ground state

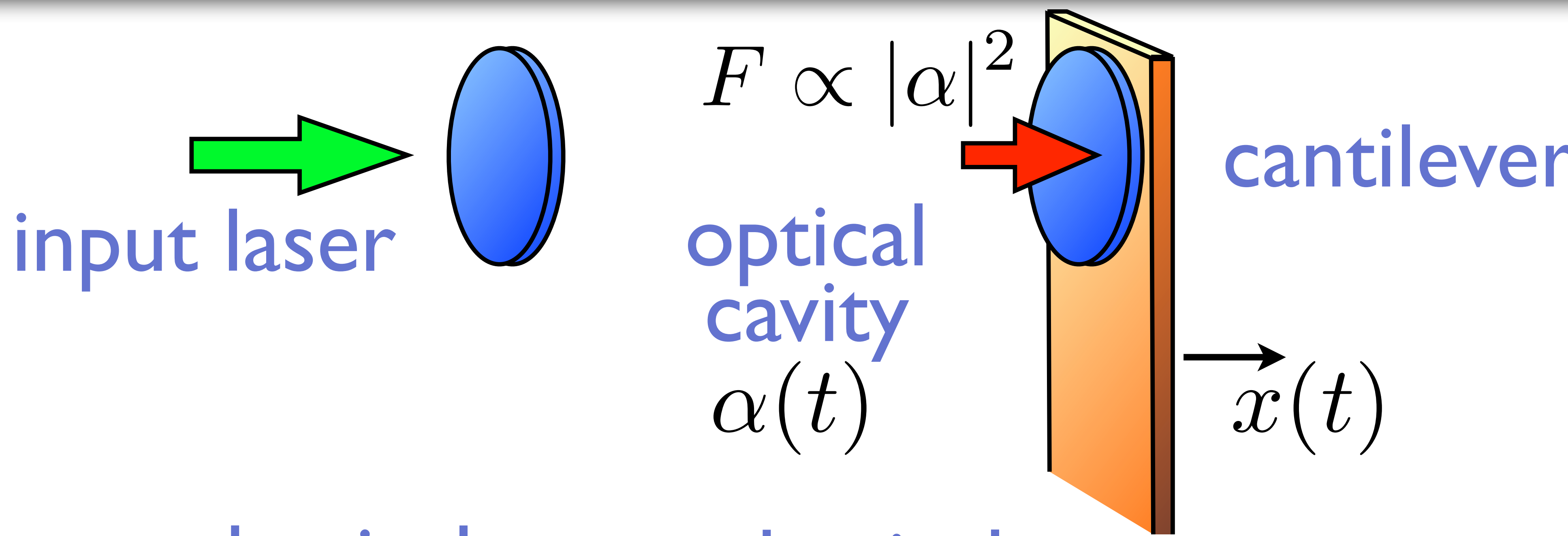


analogy to (cavity-assisted)
laser cooling of atoms

FM et al., PRL **93**, 093902 (2007)
Wilson-Rae et al., PRL **99**, 093901 (2007)

Classical dynamics

Equations of motion



mechanical frequency · mechanical damping

$$\ddot{x} = -\omega_M^2 (x - x_0) - \Gamma \dot{x} + F/m$$

radiation pressure

equilibrium position

$$F = \frac{\hbar \omega_R}{L} |\alpha|^2$$

$$\dot{\alpha} = i\omega_R \frac{x}{L} \alpha - \frac{\kappa}{2} (\alpha - \alpha_{\text{in}})$$

laser amplitude

detuning from resonance · cavity decay rate

Linearized optomechanics

$$\alpha(t) = \bar{\alpha} + \delta\alpha(t)$$
$$x(t) = \bar{x} + \delta x(t)$$

$$\Rightarrow \ldots \Rightarrow$$

(solve for arbitrary $F_{\text{ext}}(\omega)$)

$$\delta x(\omega) = \underbrace{\frac{1}{m(\omega_M^2 - \omega^2) - im\omega\Gamma + \Sigma(\omega)}}_{\chi_{xx}^{\text{eff}}(\omega)} F_{\text{ext}}(\omega)$$

$$\delta\omega_M^2 = \frac{1}{m}\text{Re}\Sigma(\omega_M)$$

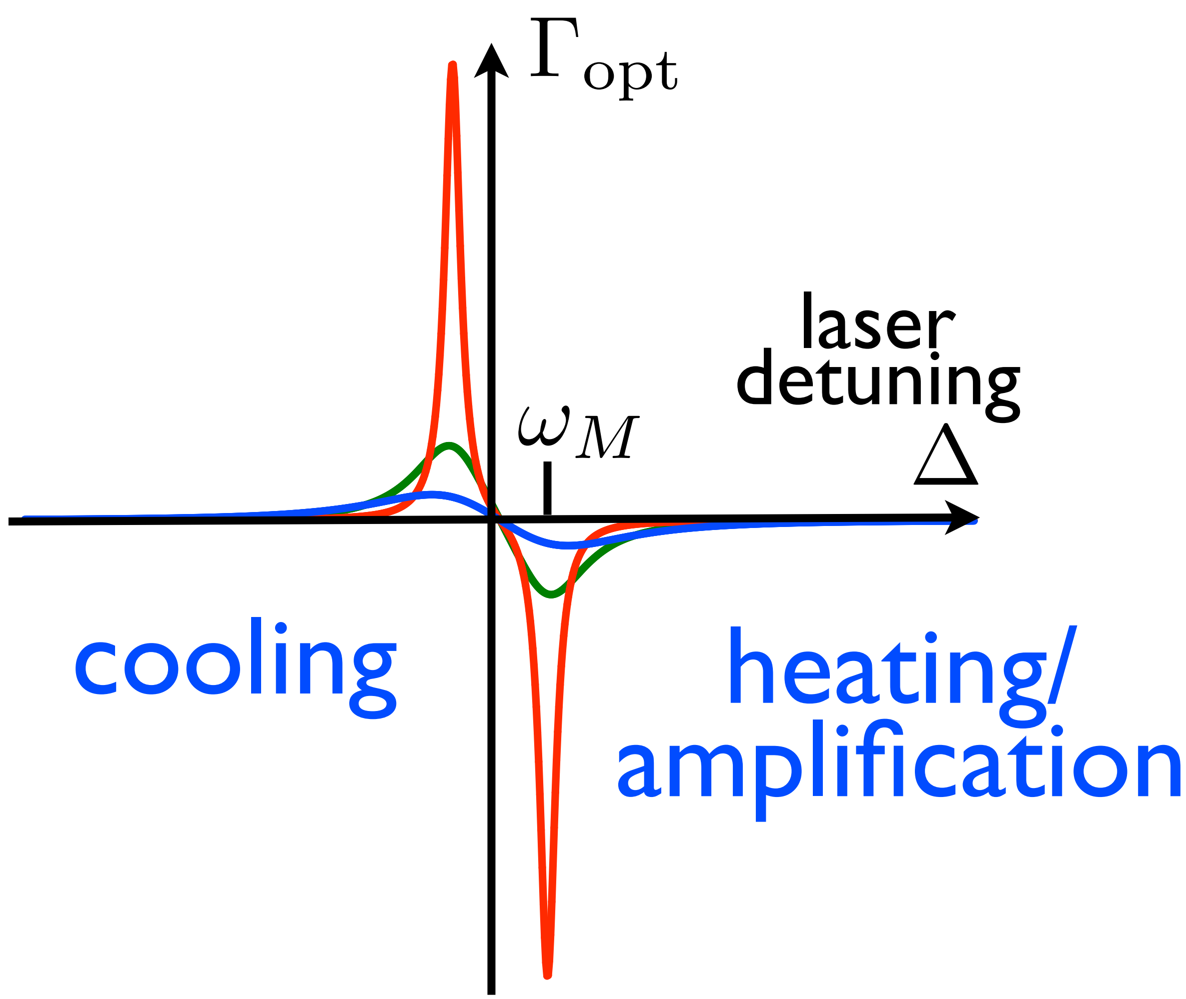
Optomechanical frequency shift ("optical spring")

$$\Gamma_{\text{opt}} = -\frac{1}{m\omega_M}\text{Im}\Sigma(\omega_M)$$

Effective optomechanical damping rate

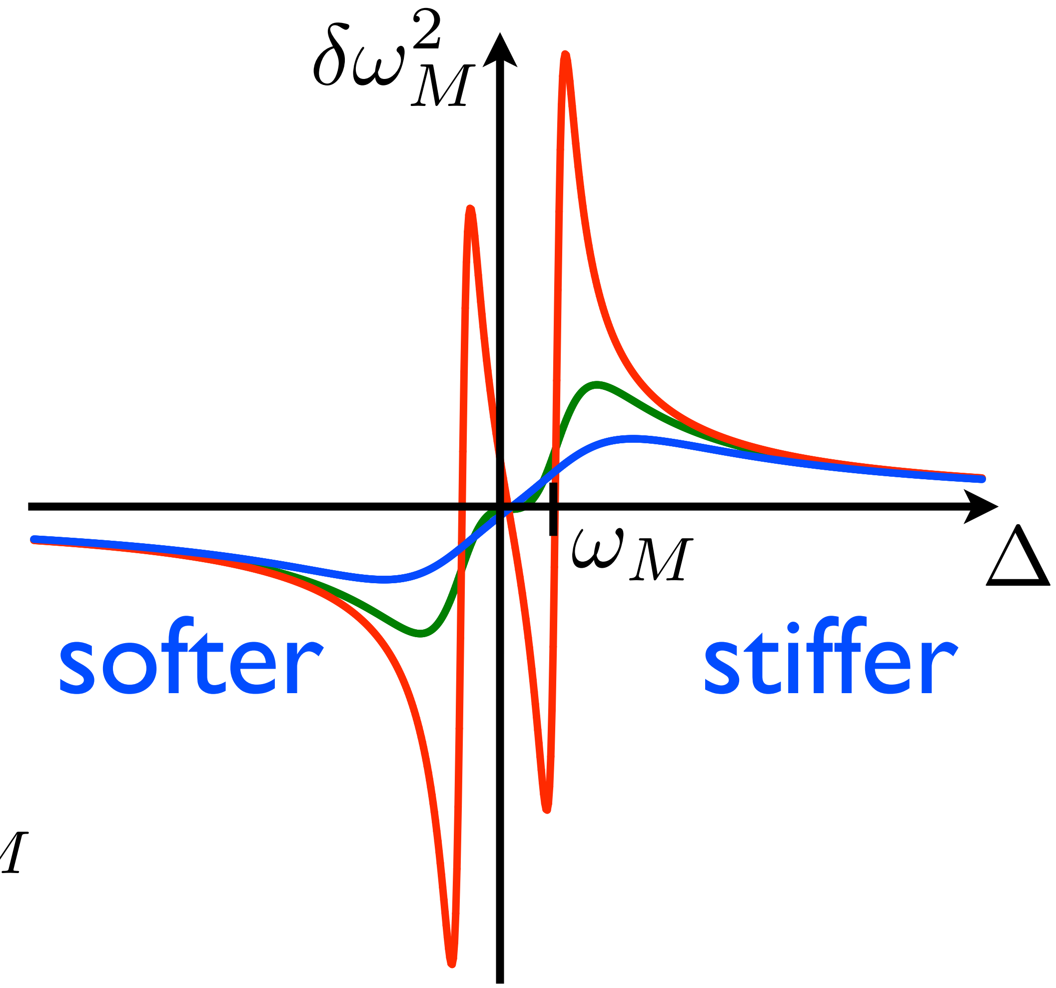
Linearized dynamics

Effective optomechanical damping rate



$(\Delta = \omega_R x_0 / L)$

Optomechanical frequency shift (“optical spring”)



κ/ω_M
2
1
0.2

Quantum picture

Optomechanical Hamiltonian

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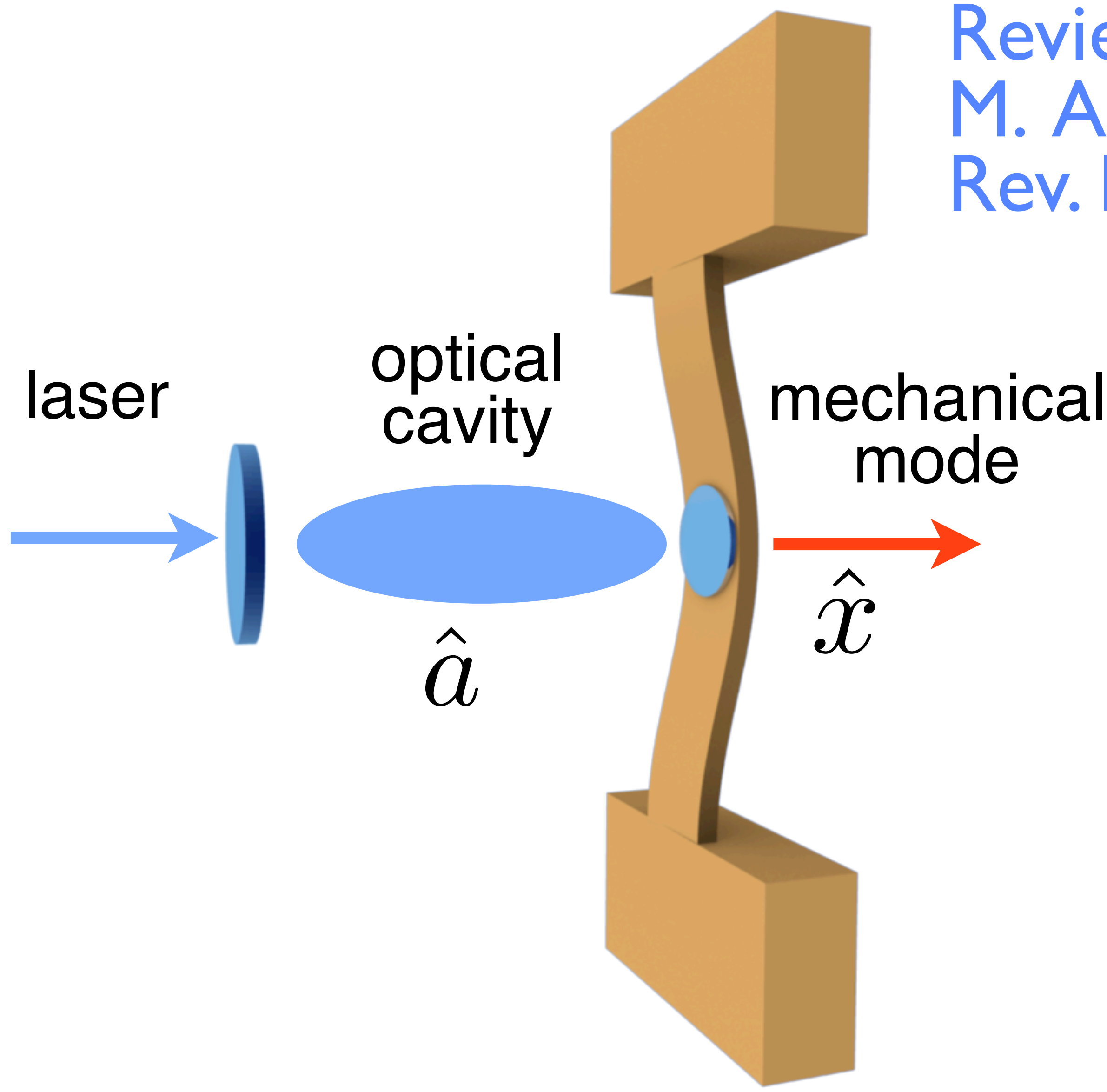


$$g_0 \sim \mathrm{Hz} - \mathrm{MHz}$$

$$\hat{H} = -(\Delta + g_0(\hat{b} + \hat{b}^\dagger))\hat{a}^\dagger\hat{a} + \Omega\hat{b}^\dagger\hat{b} + \ldots$$

laser detuning

$$\Delta = \omega_L - \omega_{\mathrm{cav}}$$

optomech. coupling

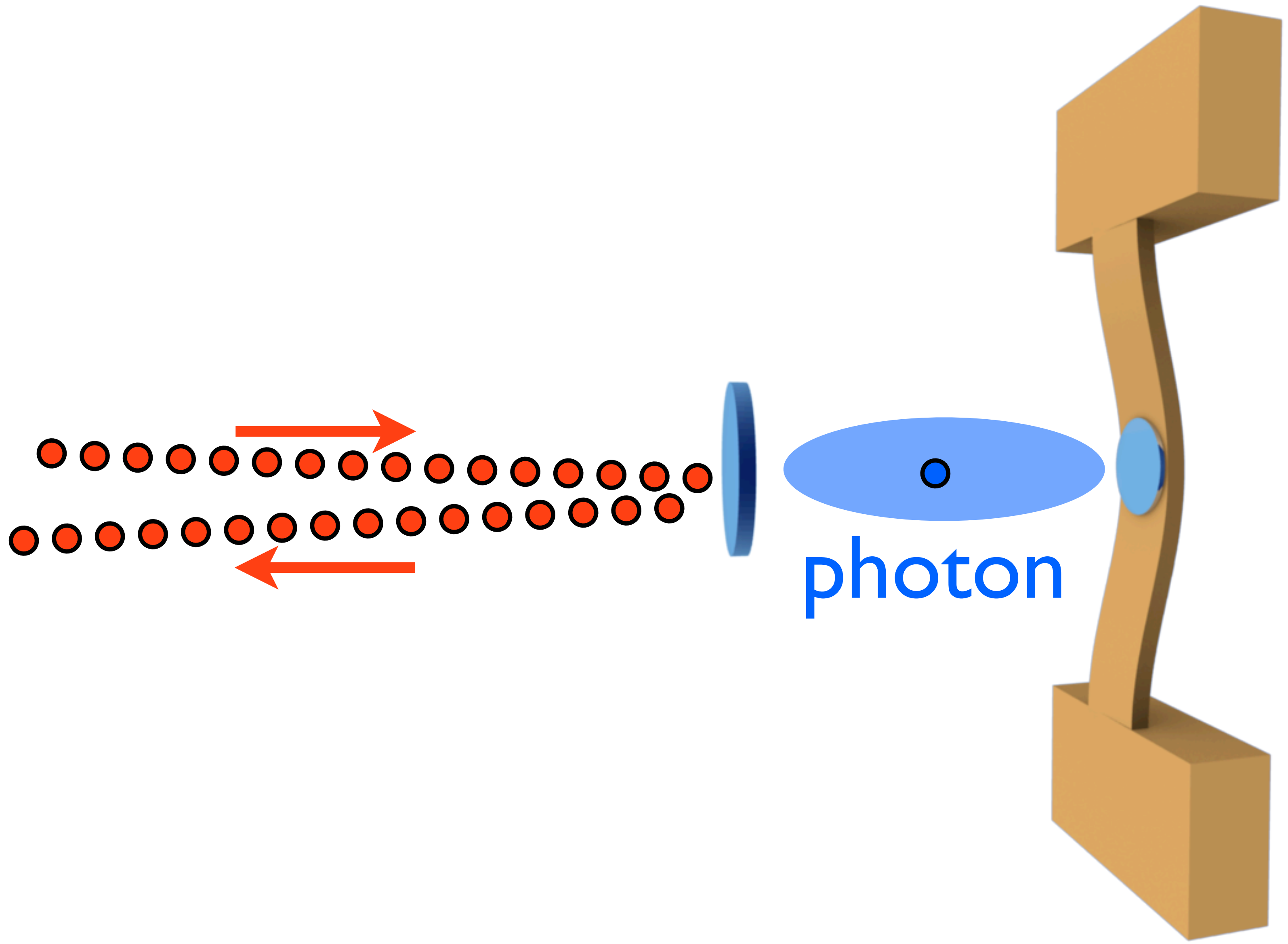
$$\hat{x} = x_{\mathrm{ZPF}}(\hat{b} + \hat{b}^\dagger)$$

$$x_{\mathrm{ZPF}} = \sqrt{\hbar/2m\Omega}$$

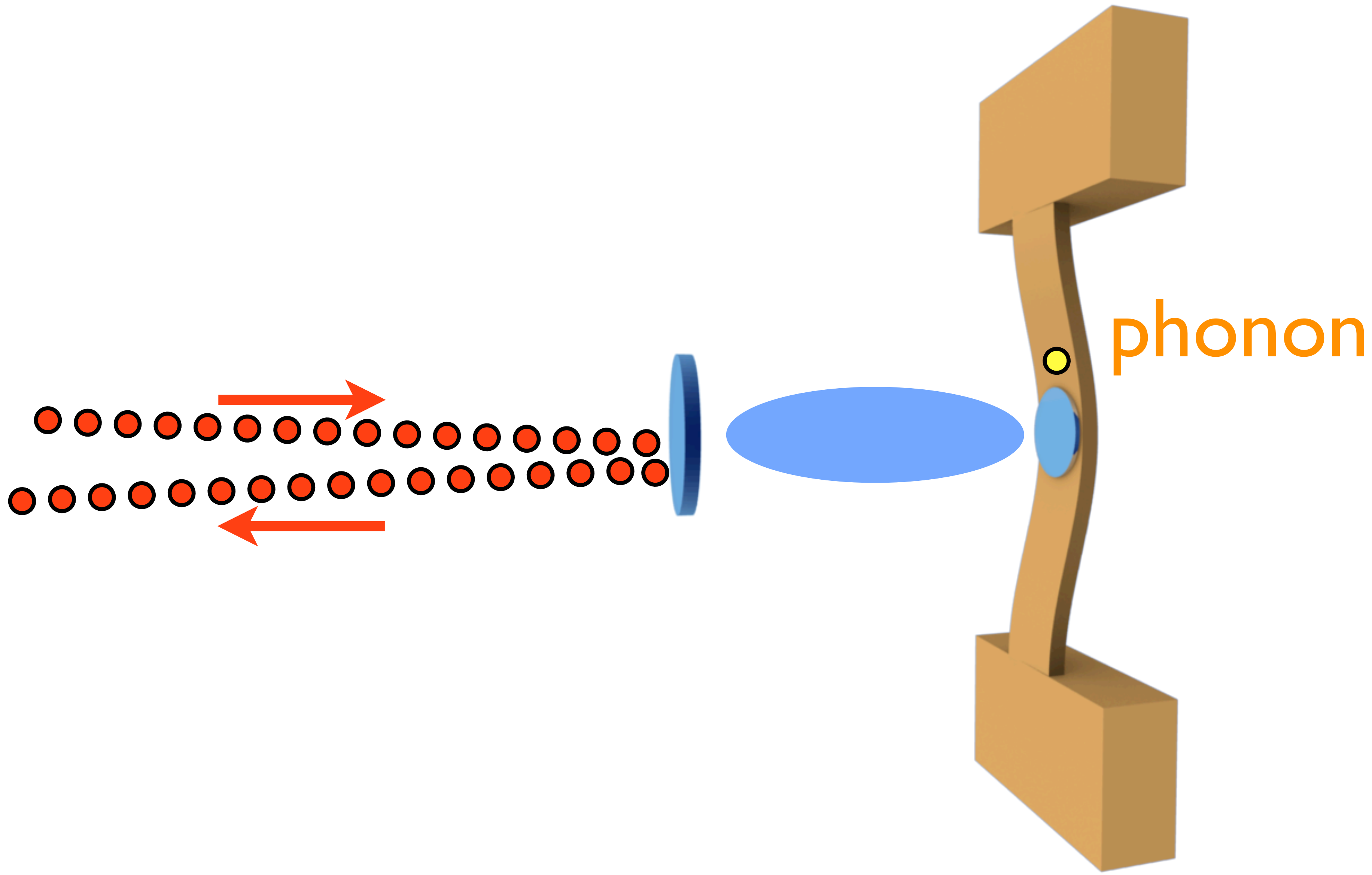
Optomechanical Interaction: Nonlinear

$$\hat{a}^\dagger \hat{a}(\hat{b}^\dagger + \hat{b})$$

Converting photons into phonons



Converting photons into phonons



“Linearized” Optomechanical Hamiltonian

$$\hbar g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger)$$

$$\hat{a} = \alpha + \delta\hat{a}$$

$$\hbar g_0 (\alpha \delta\hat{a}^\dagger + \alpha^* \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

“laser-enhanced optomechanical coupling”: $g = g_0 \alpha$

$g_0 \sim \mathrm{Hz} - \mathrm{MHz}$

bare optomechanical coupling
(geometry, etc.: fixed!)

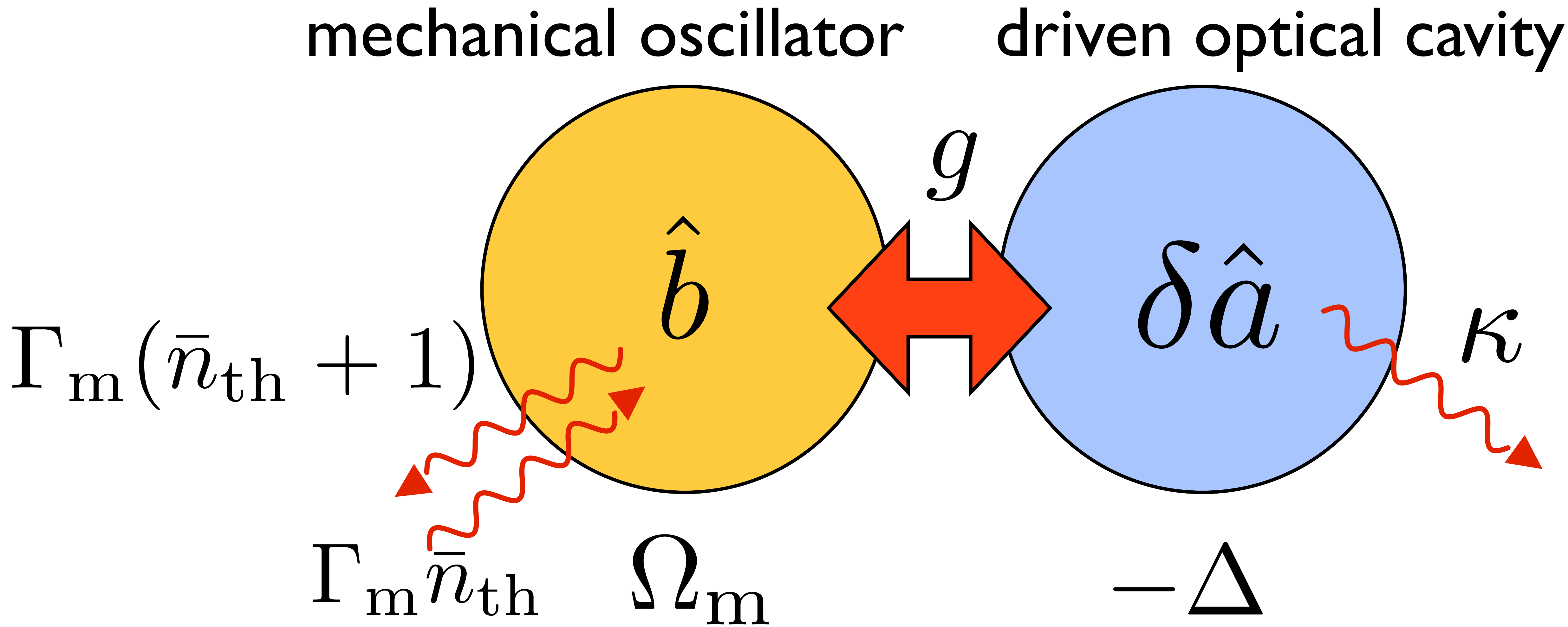
α

laser-driven cavity amplitude

tuneable! **phase**!

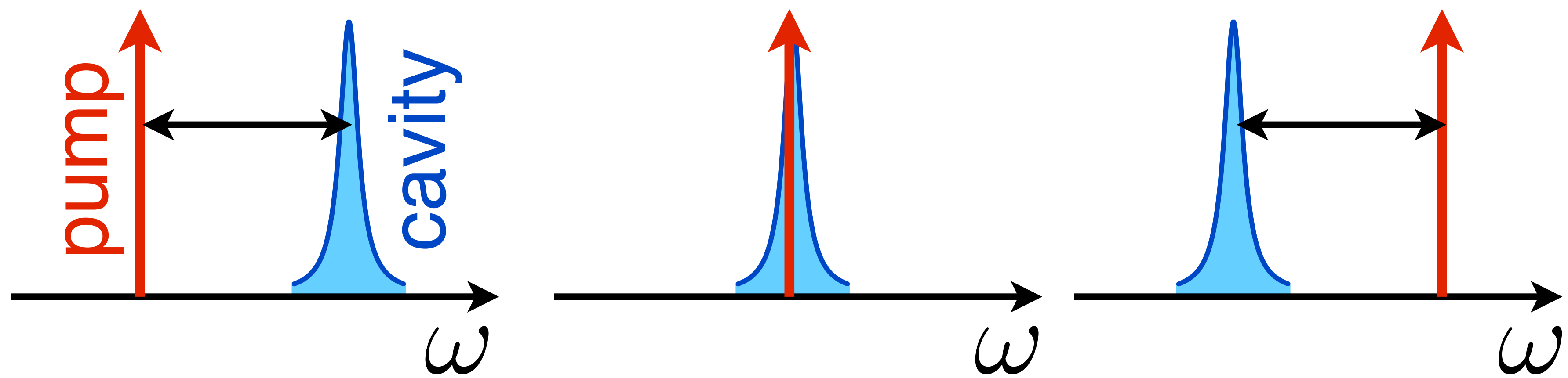
Mechanics & Optics

After linearization: two linearly coupled harmonic oscillators!



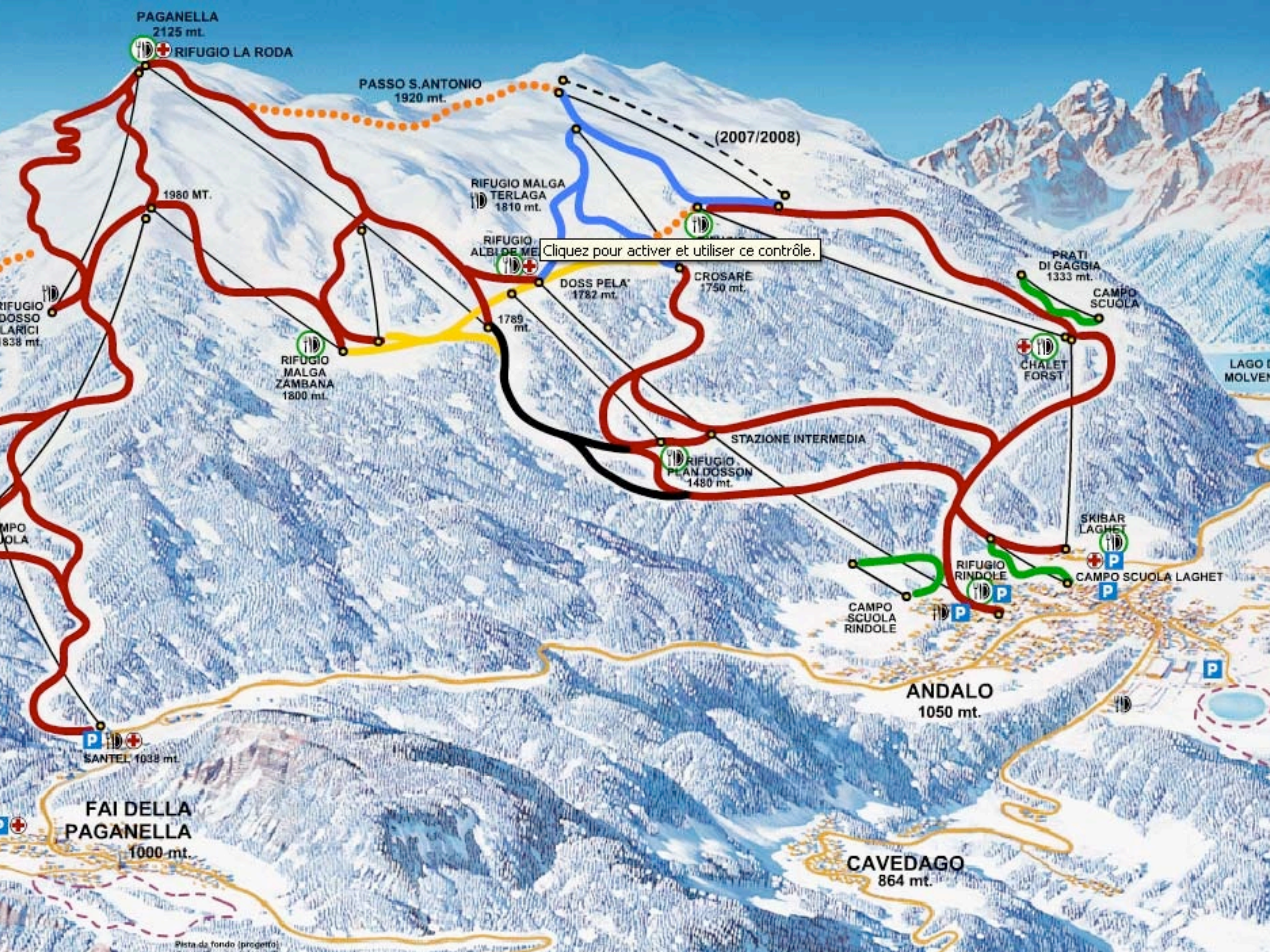
Different regimes

red-detuned | blue-detuned



$\Delta = -\Omega_{\mathrm{m}}$	$\Delta = 0$	$\Delta = +\Omega_{\mathrm{m}}$
beam-splitter (cooling)	QND	squeezer (entanglement)
$\delta\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\delta\hat{a}$	$\hat{x}_a\hat{x}_b$	$\delta\hat{a}^\dagger\hat{b}^\dagger + \hat{b}\delta\hat{a}$

“The slopes of Optomechanics”


PAGANELLA
2125 mt.
RIFUGIO LA RODA
PASSO S.ANTONIO
1920 mt.
(2007/2008)
1980 MT.
RIFUGIO MALGA
TERLAGA
1810 mt.
RIFUGIO
ALBI DE ME
Cliquez pour activer et utiliser ce contrôle.
DOSS PELA'
1782 mt.
CROSARE
1750 mt.
PRATI
DI GAGGIA
1333 mt.
CAMPO
SCUOLA
RIFUGIO
DOSSO
LARICI
1838 mt.
RIFUGIO
MALGA
ZAMBANA
1800 mt.
1789
mt.
CHALET
FORST
LAGO D
MOLVEN
STAZIONE INTERMEDIA
RIFUGIO
PLAN DOSSON
1480 mt.
SKIBAR
LAGHET
RIFUGIO
RINDOLE
CAMPO SCUOLA LAGHET
CAMPO
SCUOLA
RINDOLE
ANDALO
1050 mt.
SANTEL 1038 mt.
FAI DELLA
PAGANELLA
1000 mt.
CAVEDAGO
864 mt.
Pista da fondo (progetto)

Linear Optomechanics

- Displacement detection
- Optical Spring
- Cooling & Amplification
- Two-tone drive: "Optomechanically induced transparency"
- State transfer, pulsed operation
- Wavelength conversion
- Radiation Pressure Shot Noise
- Squeezing of Light
- Squeezing of Mechanics
- Entanglement
- Precision measurements

Optomechanical Circuits

- Bandstructure in arrays
- Synchronization/patterns in arrays
- Transport & pulses in arrays

Nonlinear Optomechanics

- Self-induced mechanical oscillations
- Synchronization of oscillations
- Chaos

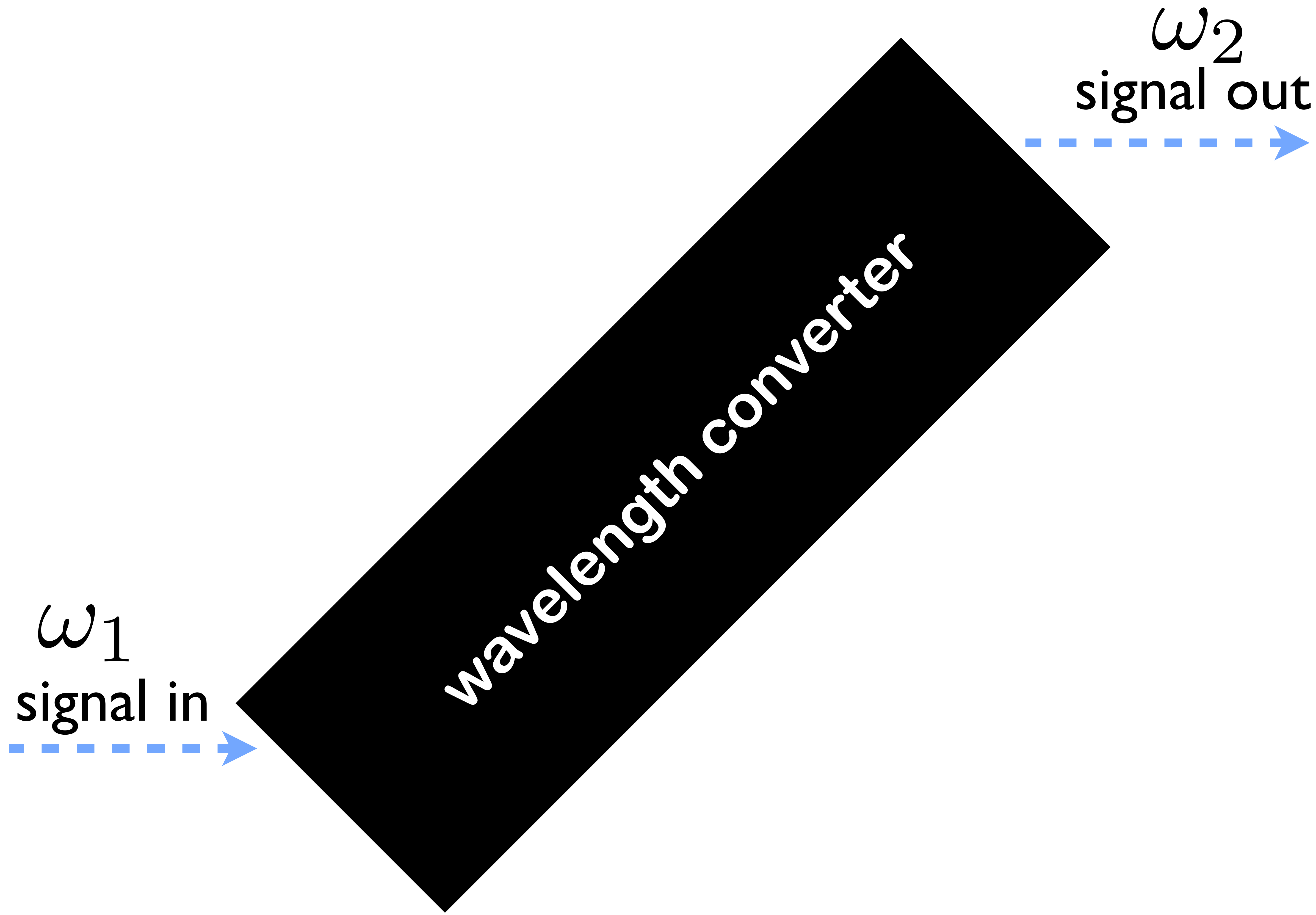
Nonlinear Quantum Optomechanics

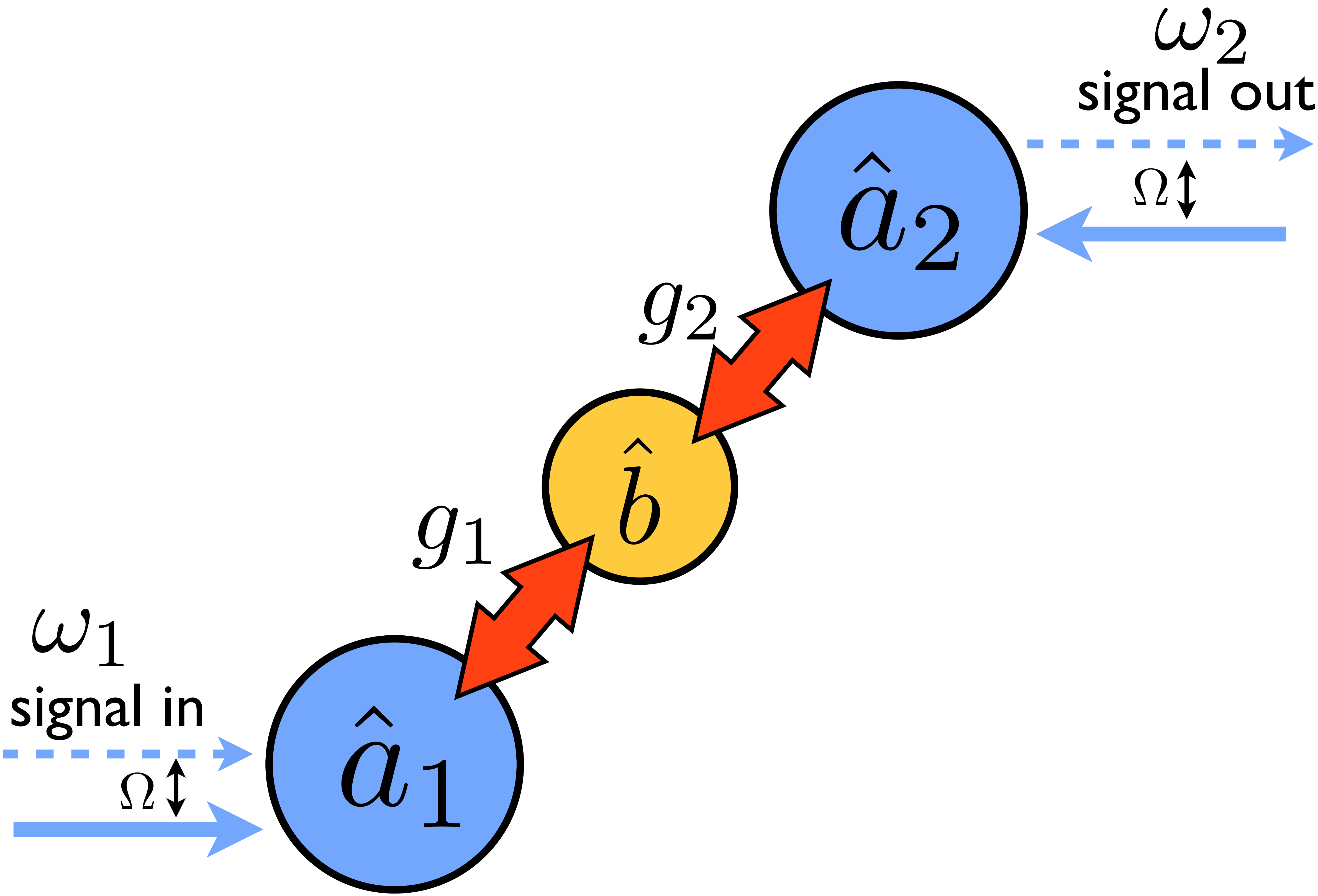
- Phonon number detection
- Phonon shot noise
- Photon blockade
- Optomechanical "which-way" experiment
- Nonclassical mechanical q. states
- Nonlinear OMIT
- Noncl. via Conditional Detection
- Single-photon sources
- Coupling to other two-level systems

Note:

Yesterday's red is today's green!

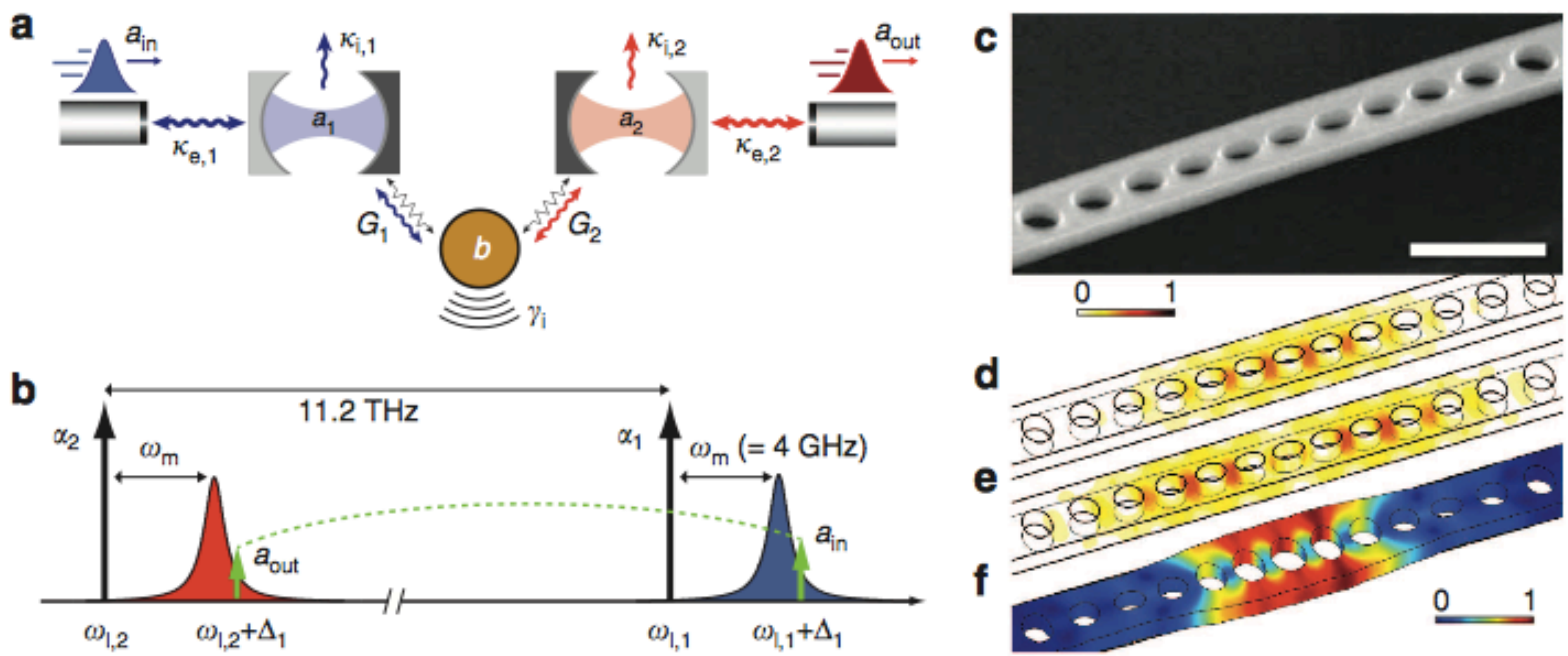
Optomechanical wavelength conversion


ω_2
signal out
wavelength converter
ω_1
signal in



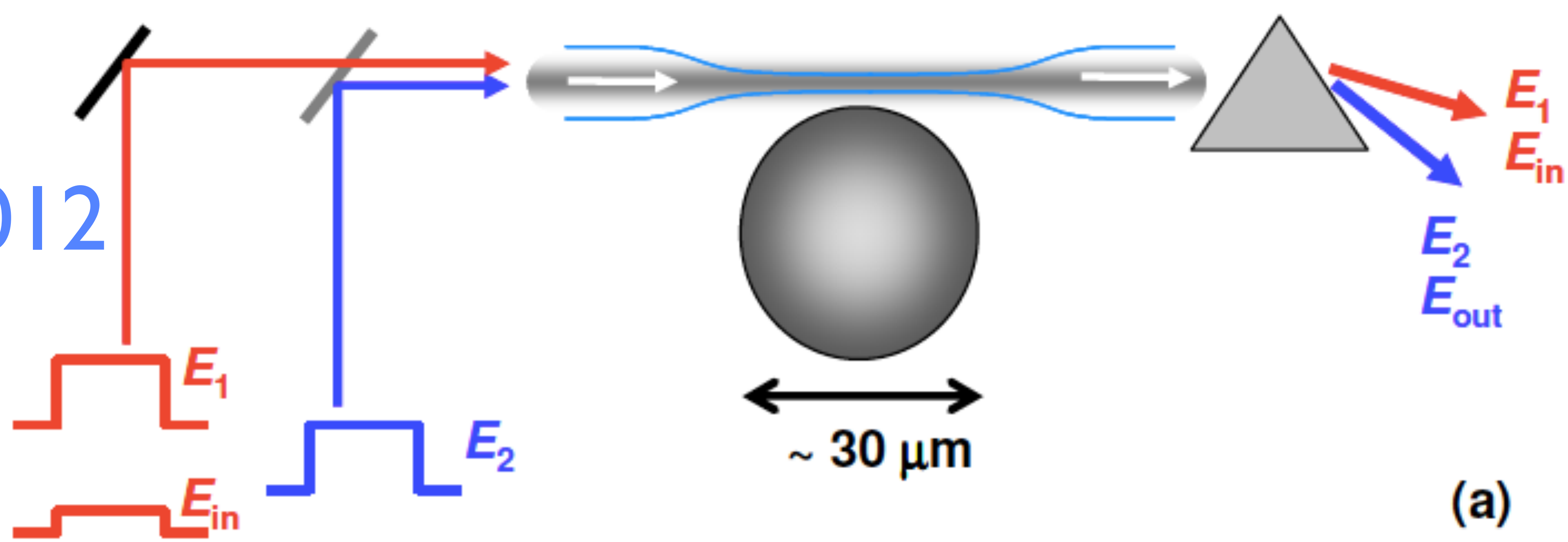
ω_2
signal out
Ω
$\hat{a}_2$
g_2
$\hat{b}$
g_1
ω_1
signal in
Ω
$\hat{a}_1$

optics to optics:

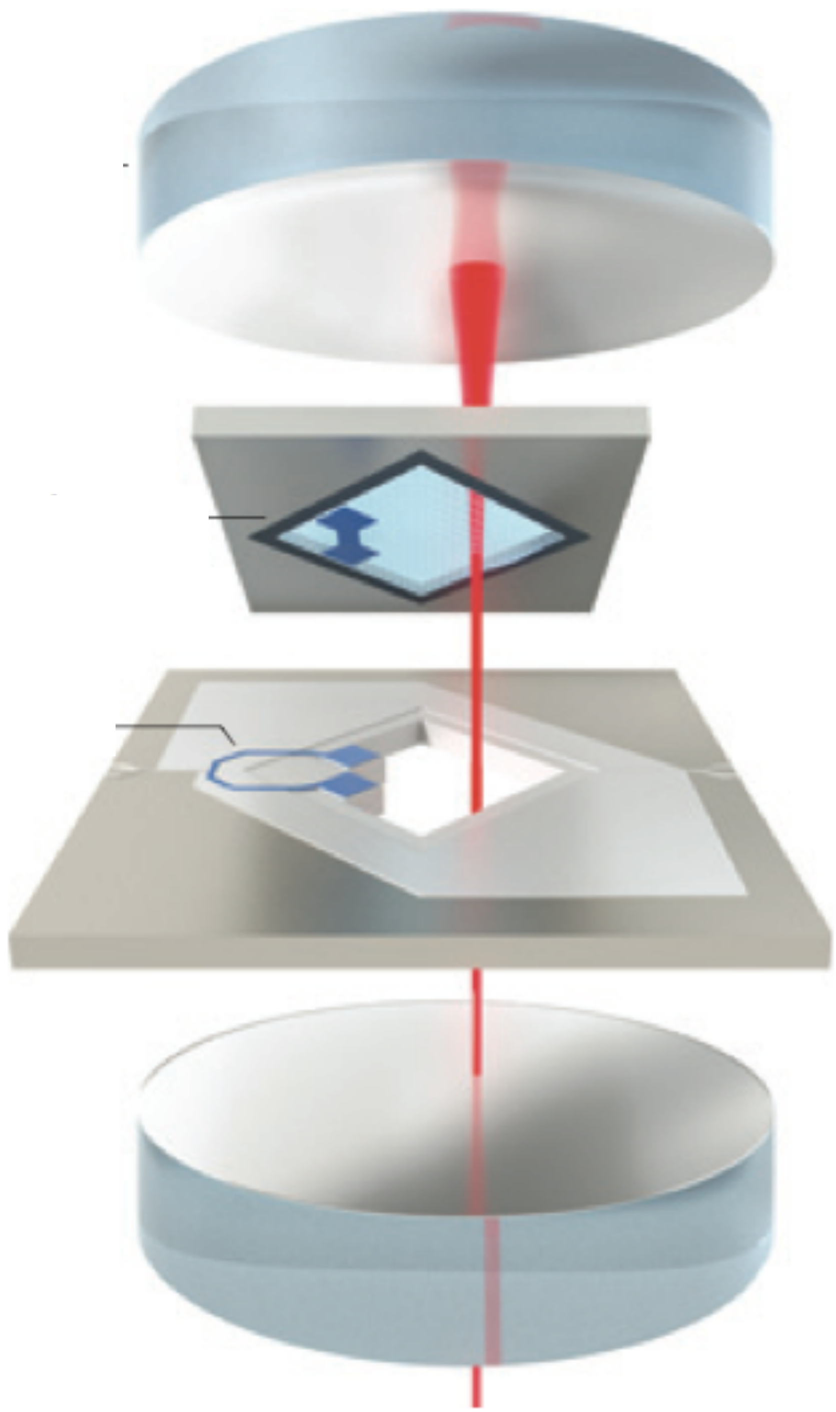


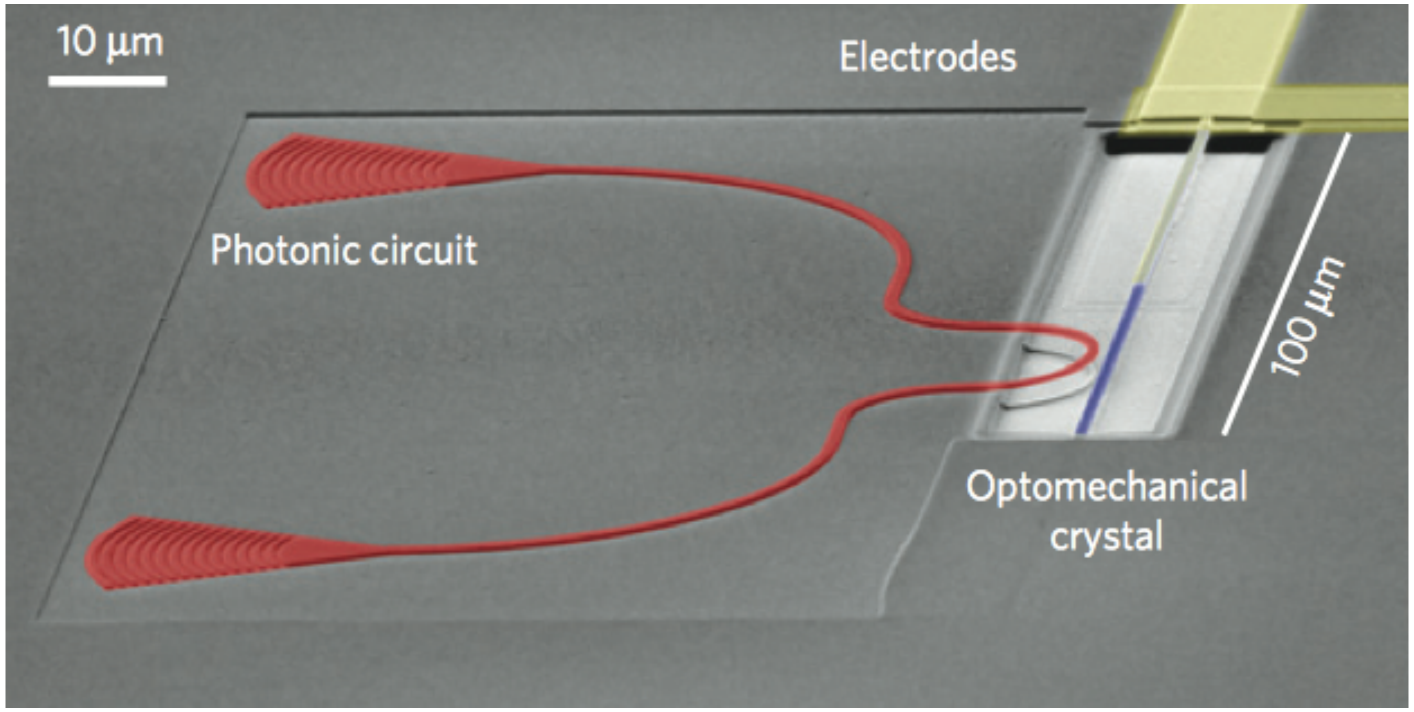
Painter 2012

Wang 2012

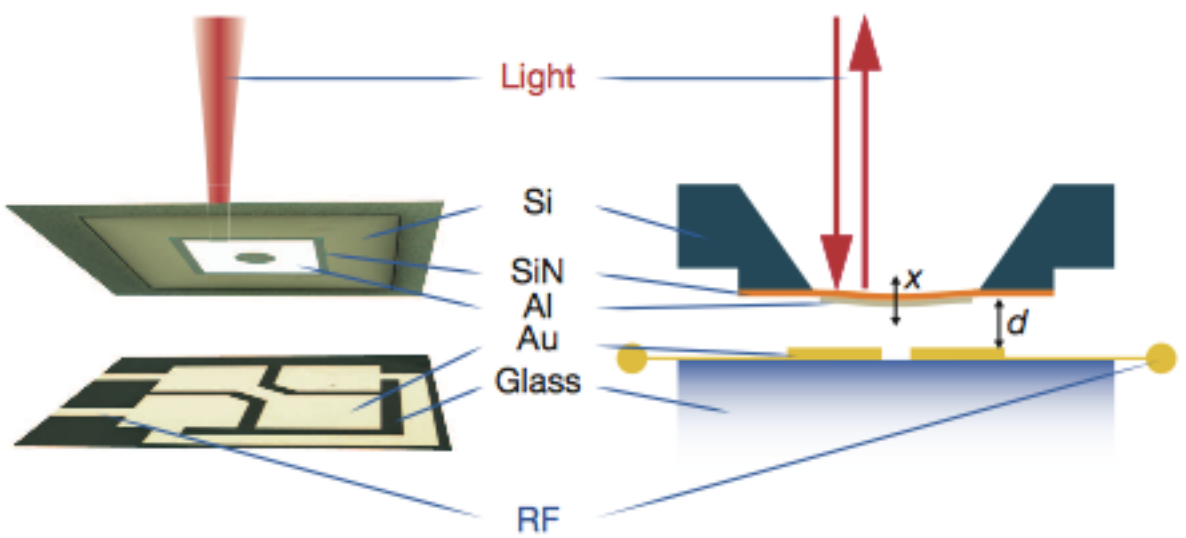


microwave/RF to optics:





Cleland 2013



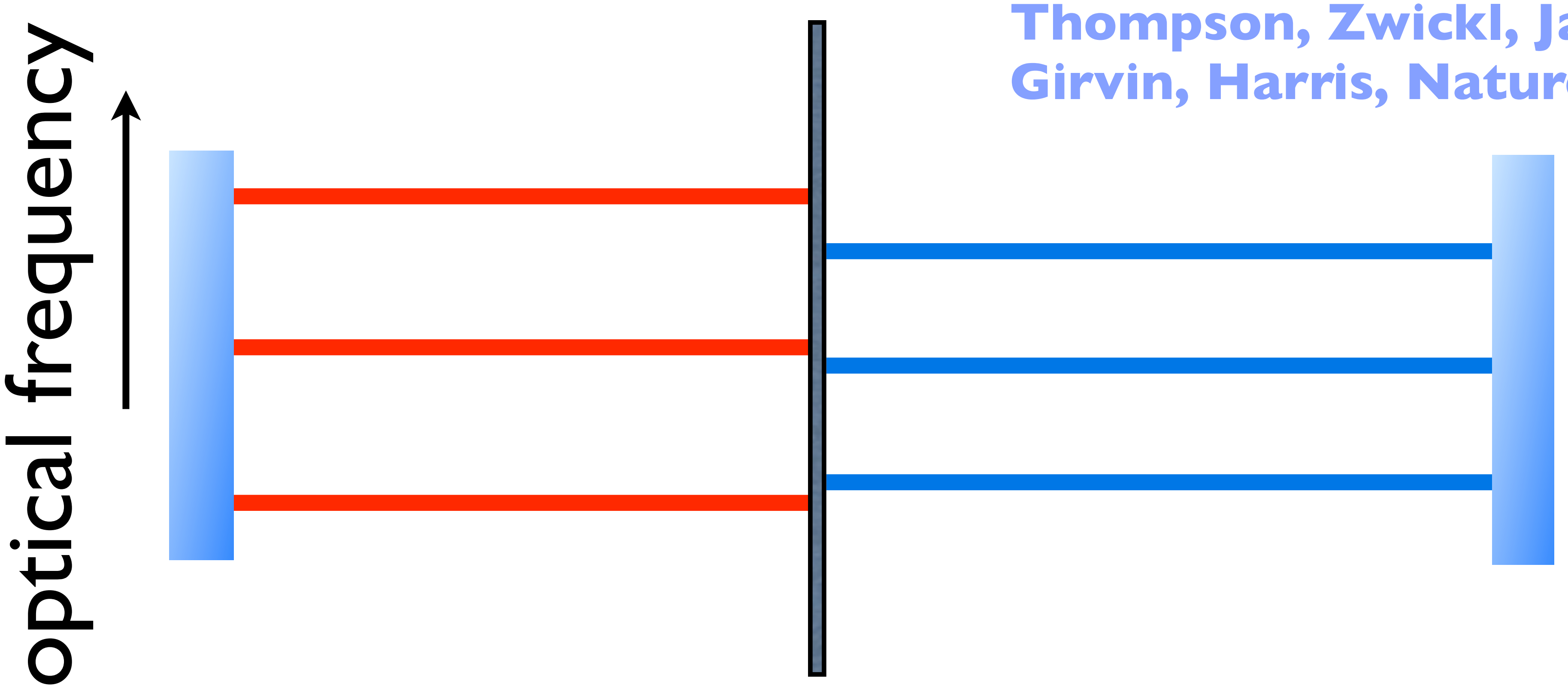
Schliesser, Polzik 2014

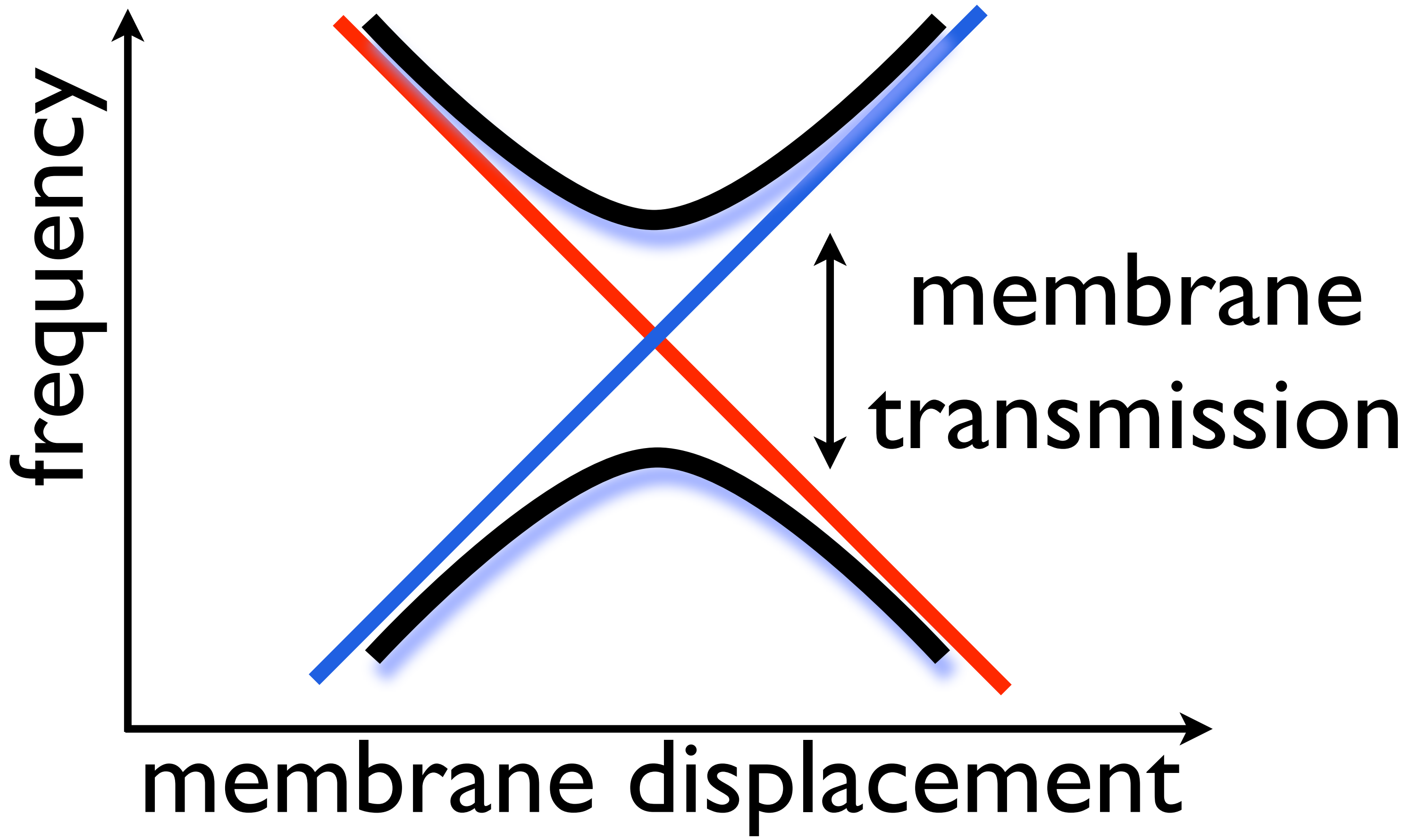
Lehnert, Regal 2014

Detecting the phonon number

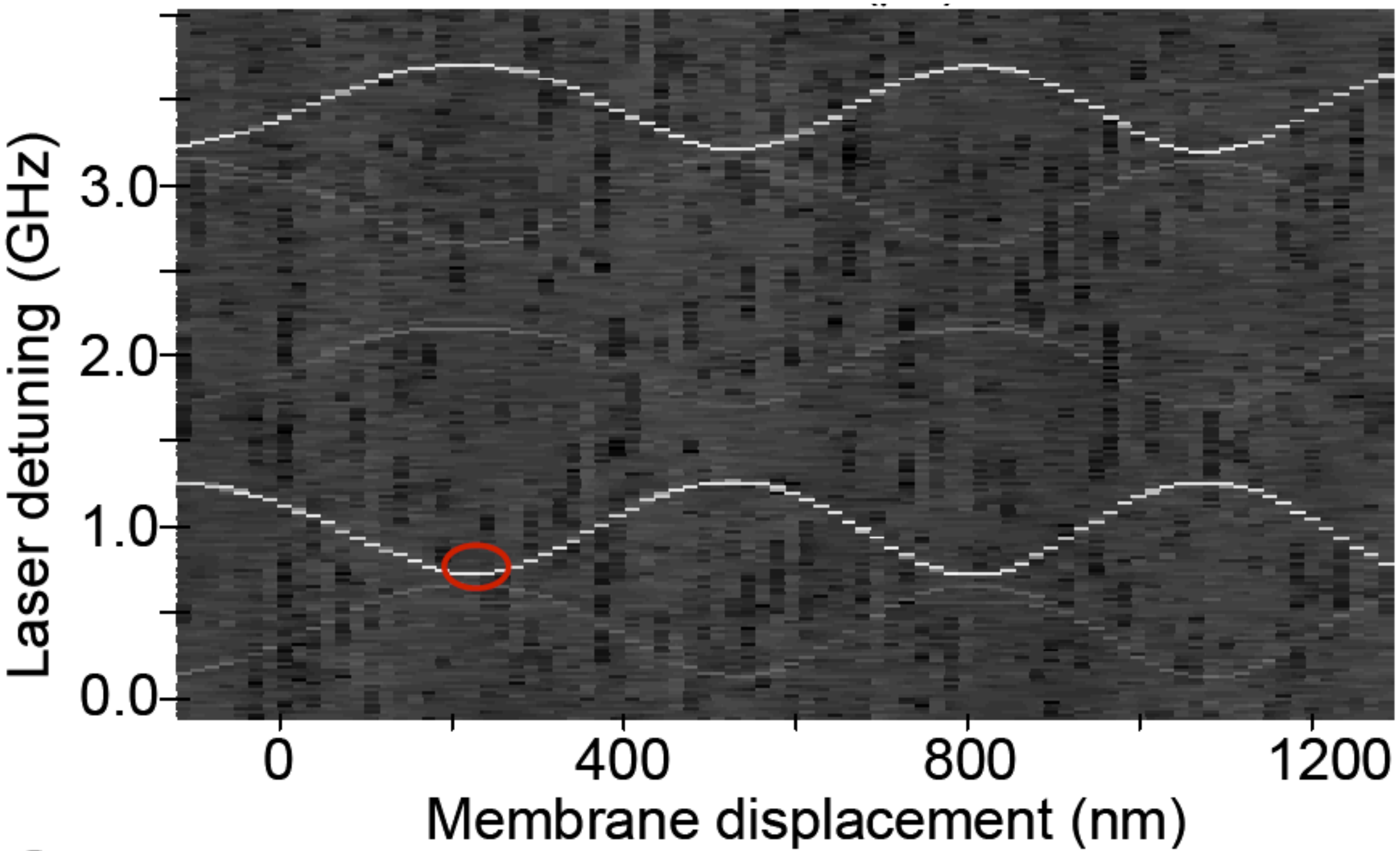
“Membrane in the middle” setup

Thompson, Zwickl, Jayich, Marquardt, Girvin, Harris, Nature 72, 452 (2008)





Experiment (Harris group, Yale)



Mechanical frequency:

$\omega_M = 2\pi \cdot 134$ kHz

Mechanical quality factor:

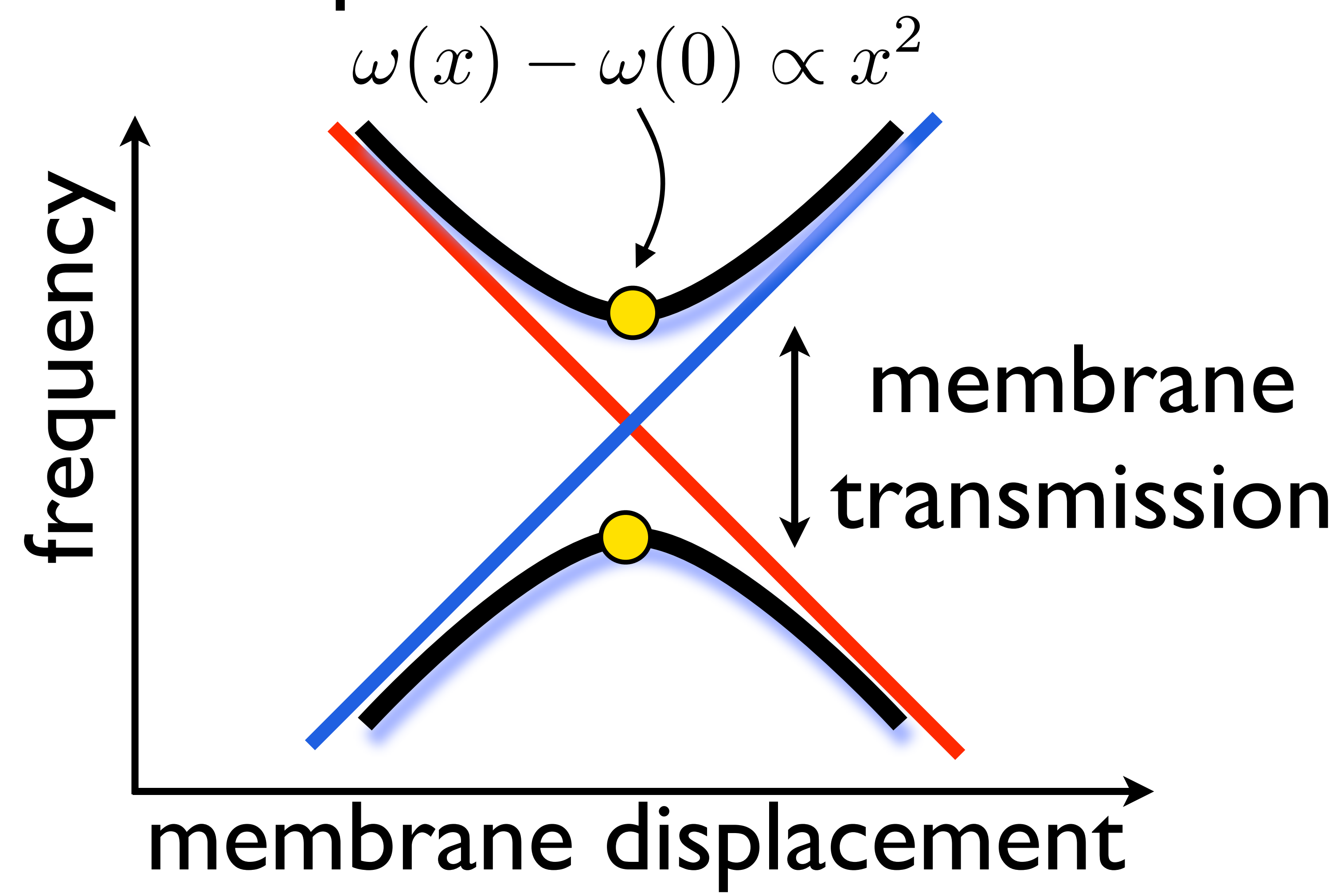
$Q = 10^6 \div 10^7$

Optomechanical cooling from **300K** to **7mK**

Thompson, Zwickl, Jayich, Marquardt, Girvin, Harris, Nature 72, 452 (2008)

Towards Fock state detection of a macroscopic object

Detection of displacement x: *not* what we need!



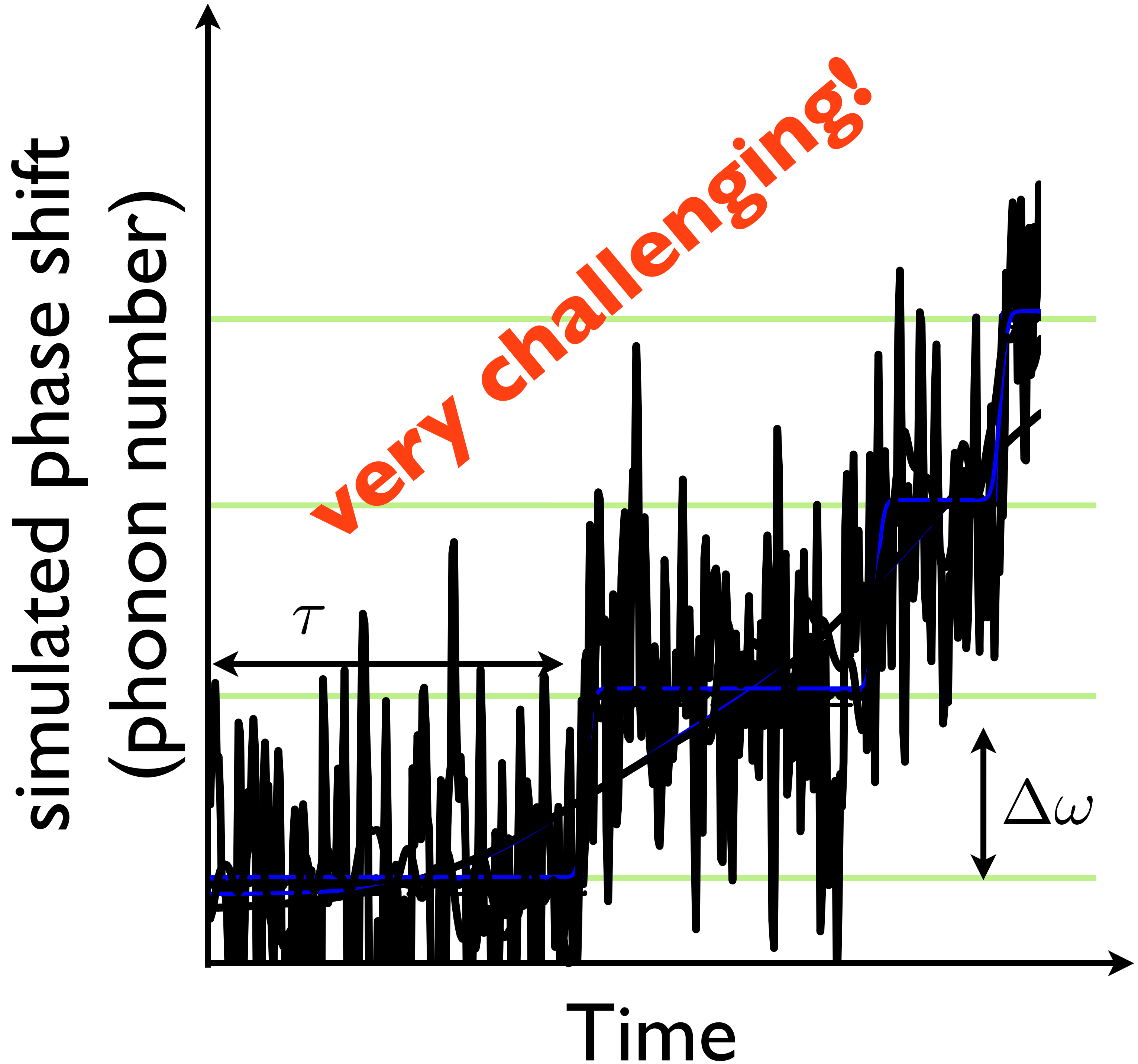
phase shift of measurement beam:

$$\overline{\hat{\theta}} \propto \overline{\hat{x}(t)^2} \propto \overline{(\hat{b}(t) + \hat{b}^\dagger(t))^2} \approx 2\hat{b}^\dagger\hat{b} + 1$$

(Time-average over cavity ring-down time)

QND measurement of phonon number!

Towards Fock state detection of a macroscopic object



Signal-to-noise ratio: $\dfrac{\tau \Delta\omega^2}{S_\omega}$

Optical freq. shift per phonon:

$$\Delta\omega = x_{\mathrm{ZPF}}^2 \omega''$$

Noise power of freq. measurement:

$$S_\omega = \frac{\kappa}{16\bar{n}_{\mathrm{cavity}}}$$

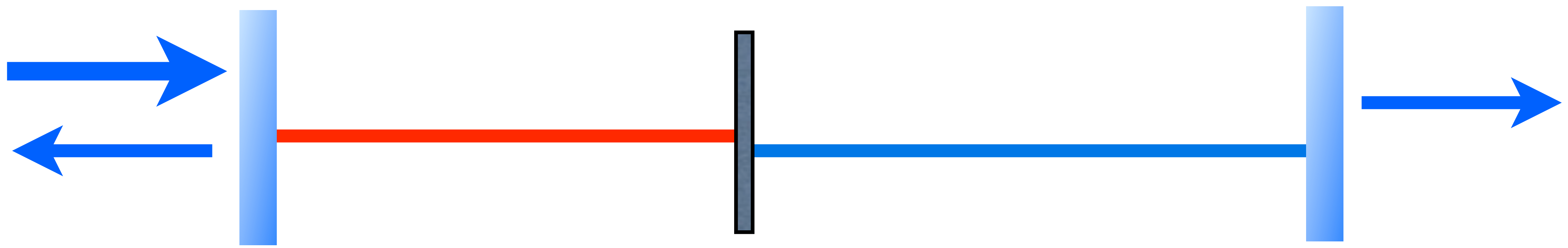
Ground state lifetime:

$$\frac{1}{\tau} = \Gamma \bar{n}_{\mathrm{thermal}}$$

Towards Fock state detection of a macroscopic object

Ideal single-sided cavity: Can observe **only** phase of reflected light, i.e. x^2: good

Two-sided cavity: Can **also** observe transmitted vs. reflected intensity: **linear** in x!



- need to go back to two-mode Hamiltonian!
- transitions between Fock states!

Single-sided cavity, but with losses: same story

Detailed analysis (Yanbei Chen's group, PRL 2009) shows: need

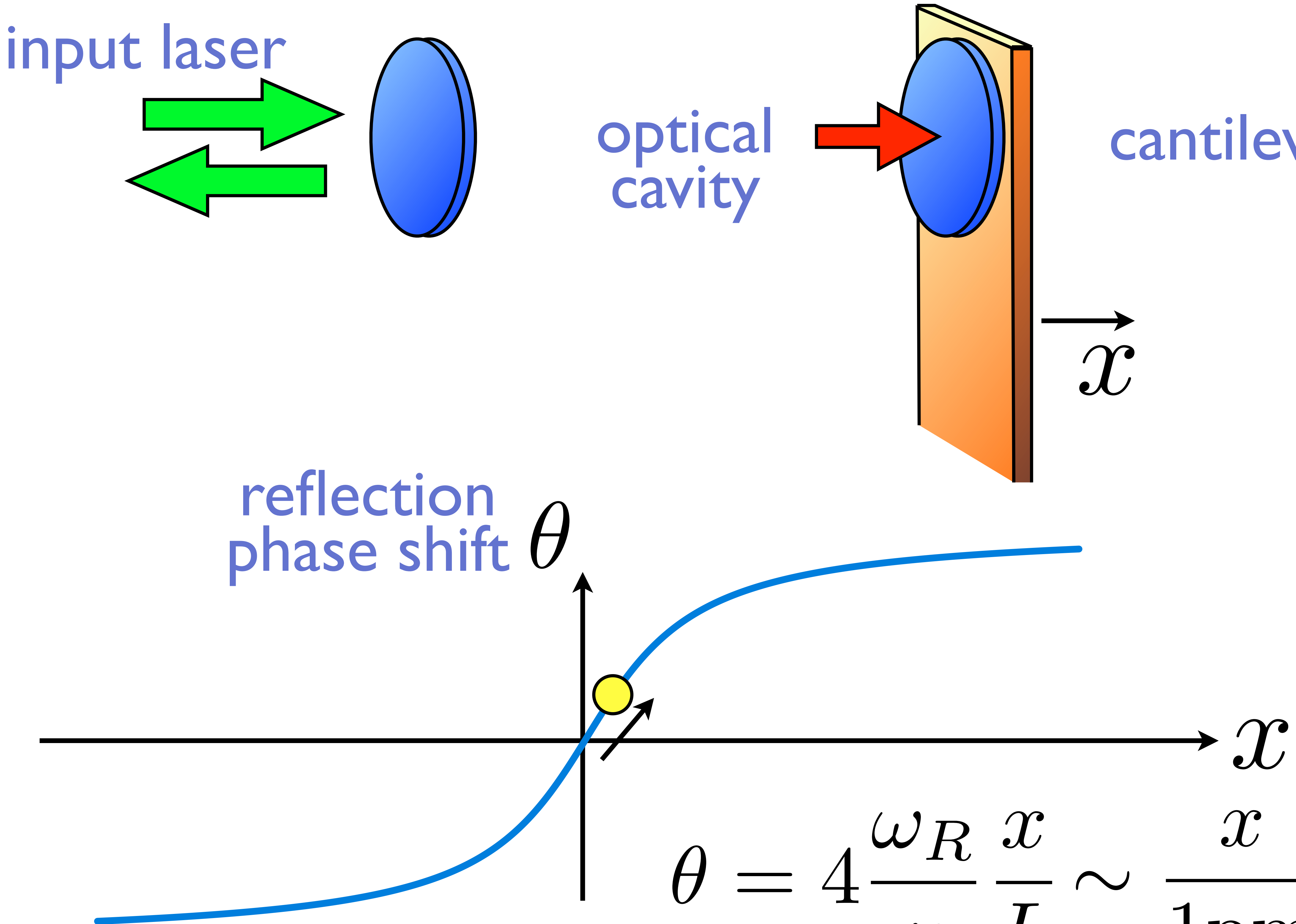
$$g_0 > \kappa_{\text{abs}}$$

absorptive part of photon decay (or 2nd mirror)

Miao, H., S. Danilishin, T. Corbitt, and Y. Chen, 2009, PRL 103, 100402

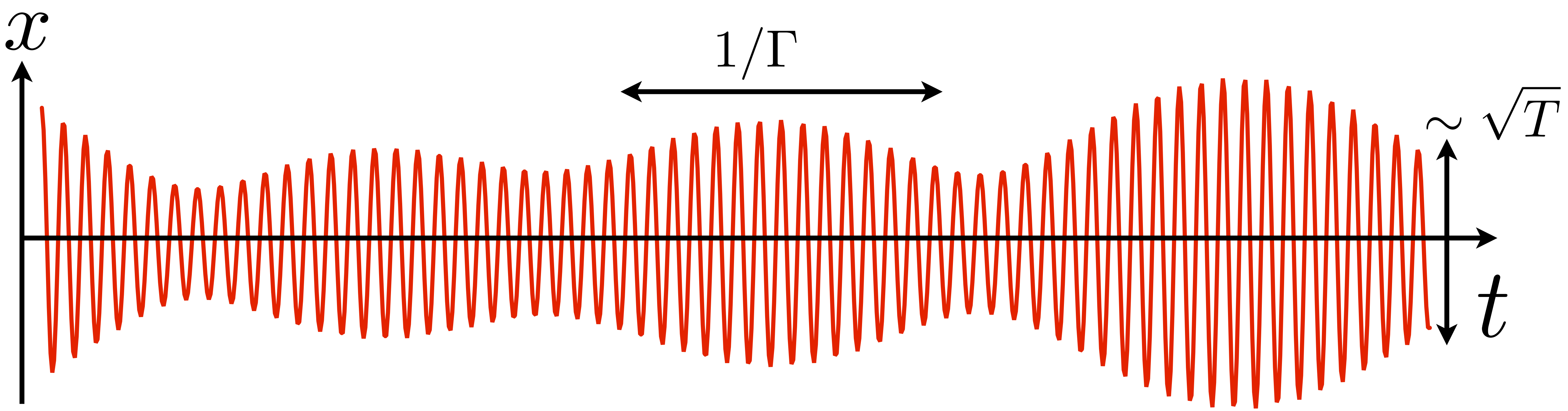
Sensing mechanical motion at the ultimate precision limit

Optical displacement detection



$$\theta = 4\frac{\omega_R}{\kappa}\frac{x}{L} \sim \frac{x}{1\mathrm{pm}}$$

Thermal fluctuations of a harmonic oscillator



Classical equipartition theorem:

$$\frac{m\omega_M^2}{2}\left\langle x^2\right\rangle = \frac{k_B T}{2} \quad \Rightarrow \quad \left\langle x^2\right\rangle = \frac{k_B T}{m\omega_M^2}$$

extract temperature!

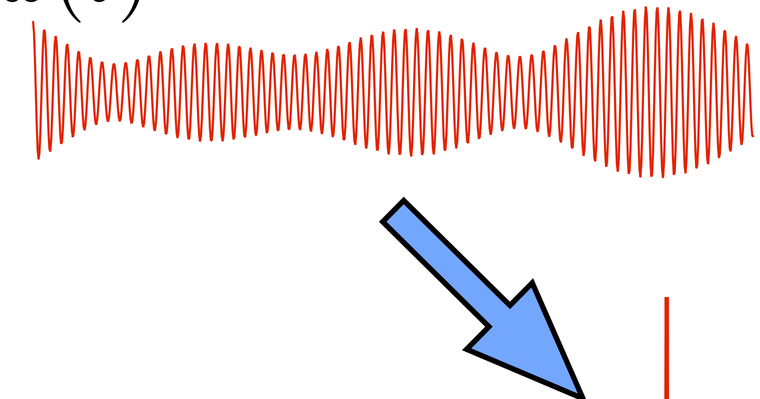
Possibilities:

- Direct time-resolved detection
- Analyze **fluctuation spectrum of x**

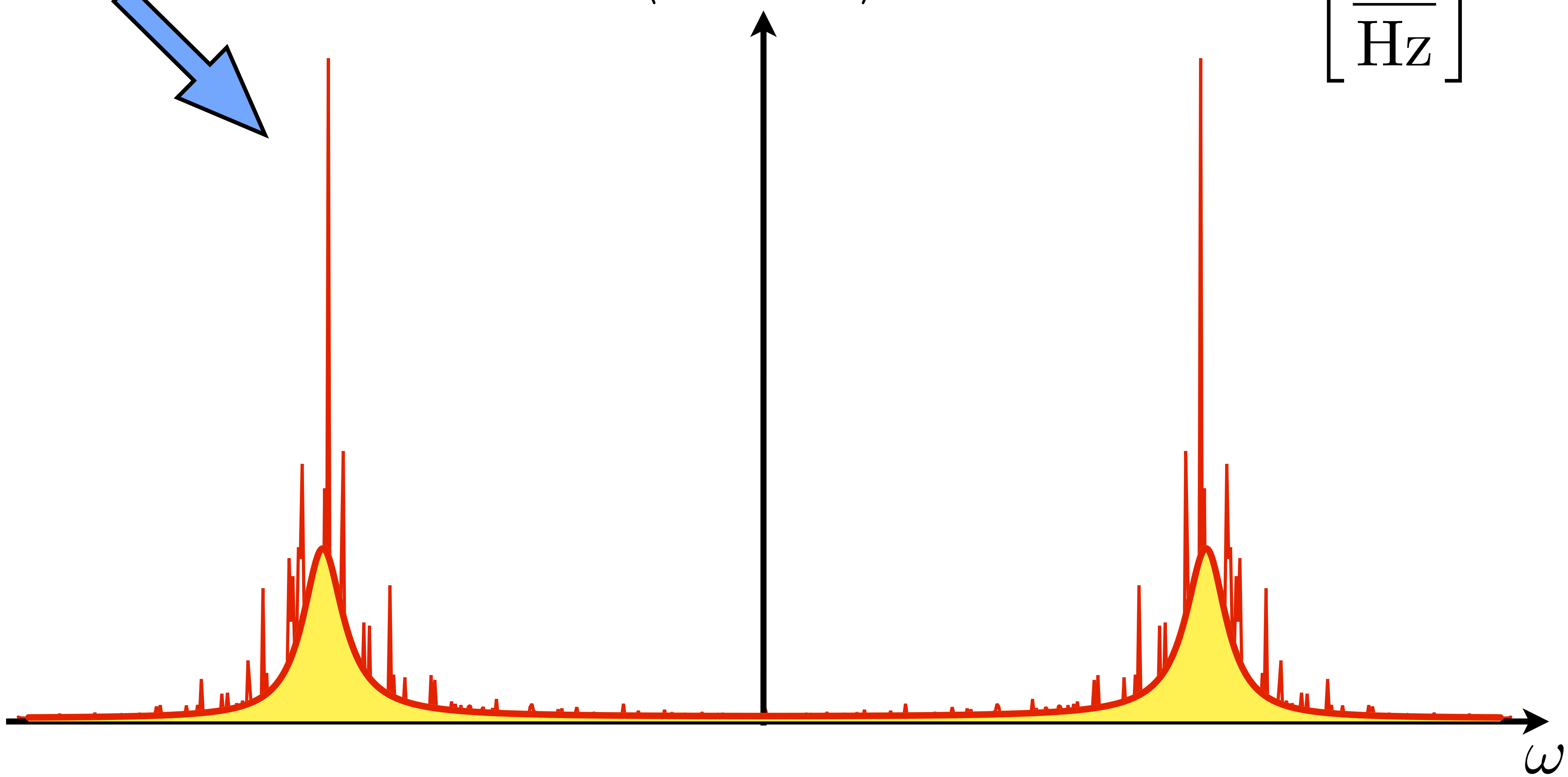
Fluctuation spectrum

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau dt e^{i\omega t} x(t)$$

$x(t)$



$$\langle |\tilde{x}(\omega)|^2 \rangle \equiv S_{xx}(\omega) \qquad \left[\frac{\mathrm{m}^2}{\mathrm{Hz}}\right]$$



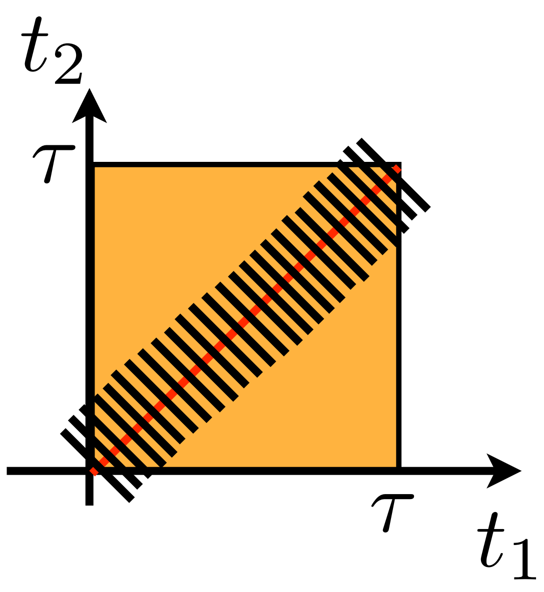
Fluctuation spectrum

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau dt e^{i\omega t} x(t)$$

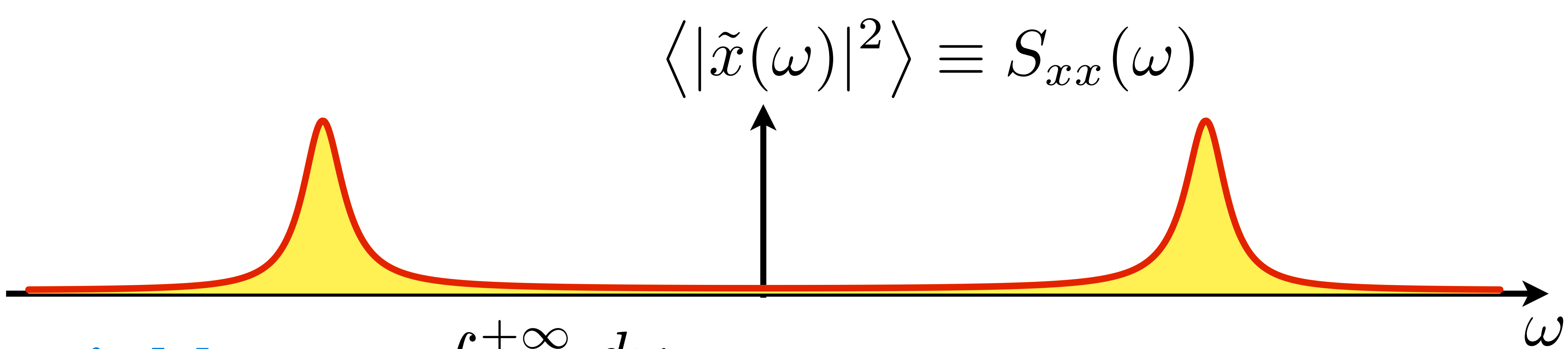
$$S_{xx}(\omega) \equiv \left\langle |\tilde{x}(\omega)|^2 \right\rangle =$$

$$\frac{1}{\tau} \int_0^\tau dt_1 \int_0^\tau dt_2 e^{i\omega(t_2 - t_1)} \langle x(t_2) x(t_1) \rangle$$

$$\approx \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle x(t) x(0) \rangle$$



"Wiener-Khinchin theorem"



area yields variance of x:

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{xx}(\omega) = \langle x^2 \rangle$$

Fluctuation-dissipation theorem

General relation between noise spectrum and linear response susceptibility

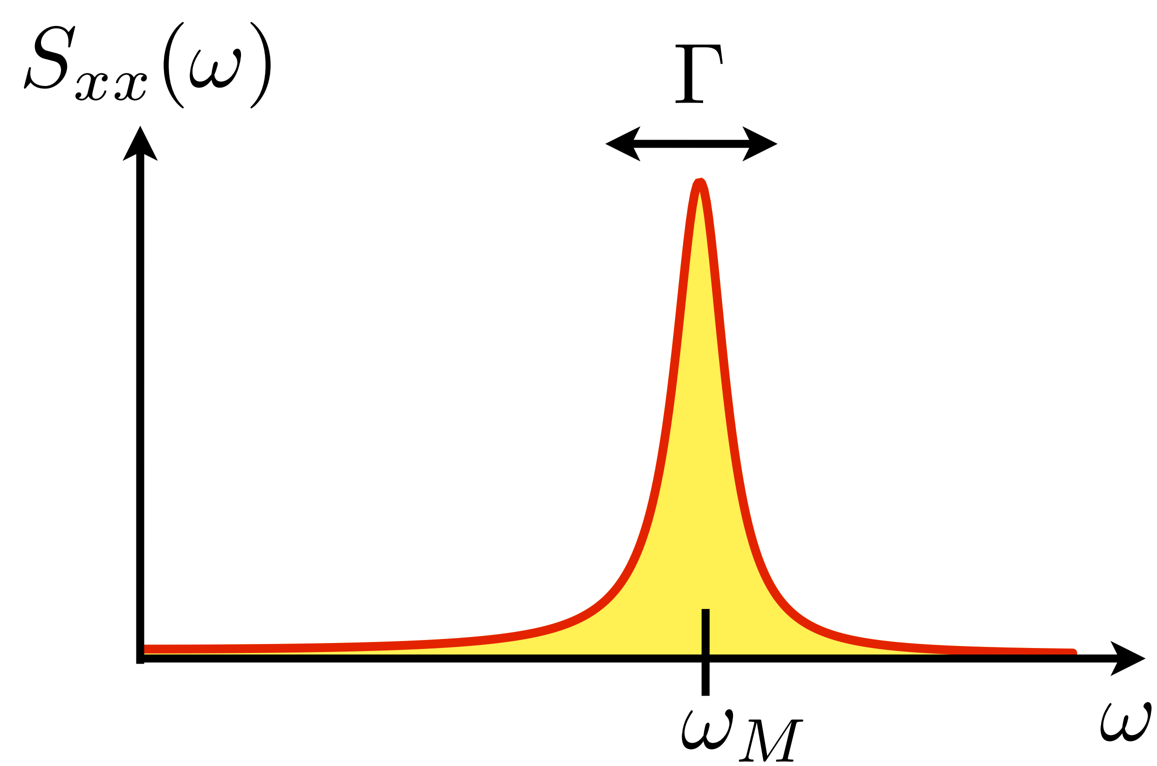
$$\langle \delta x \rangle (\omega) = \underbrace{\chi_{xx}(\omega)}_{\text{susceptibility}} F(\omega)$$

$$\boxed{S_{xx}(\omega) = \frac{2k_B T}{\omega} \mathrm{Im}\chi_{xx}(\omega)}$$ (classical limit)

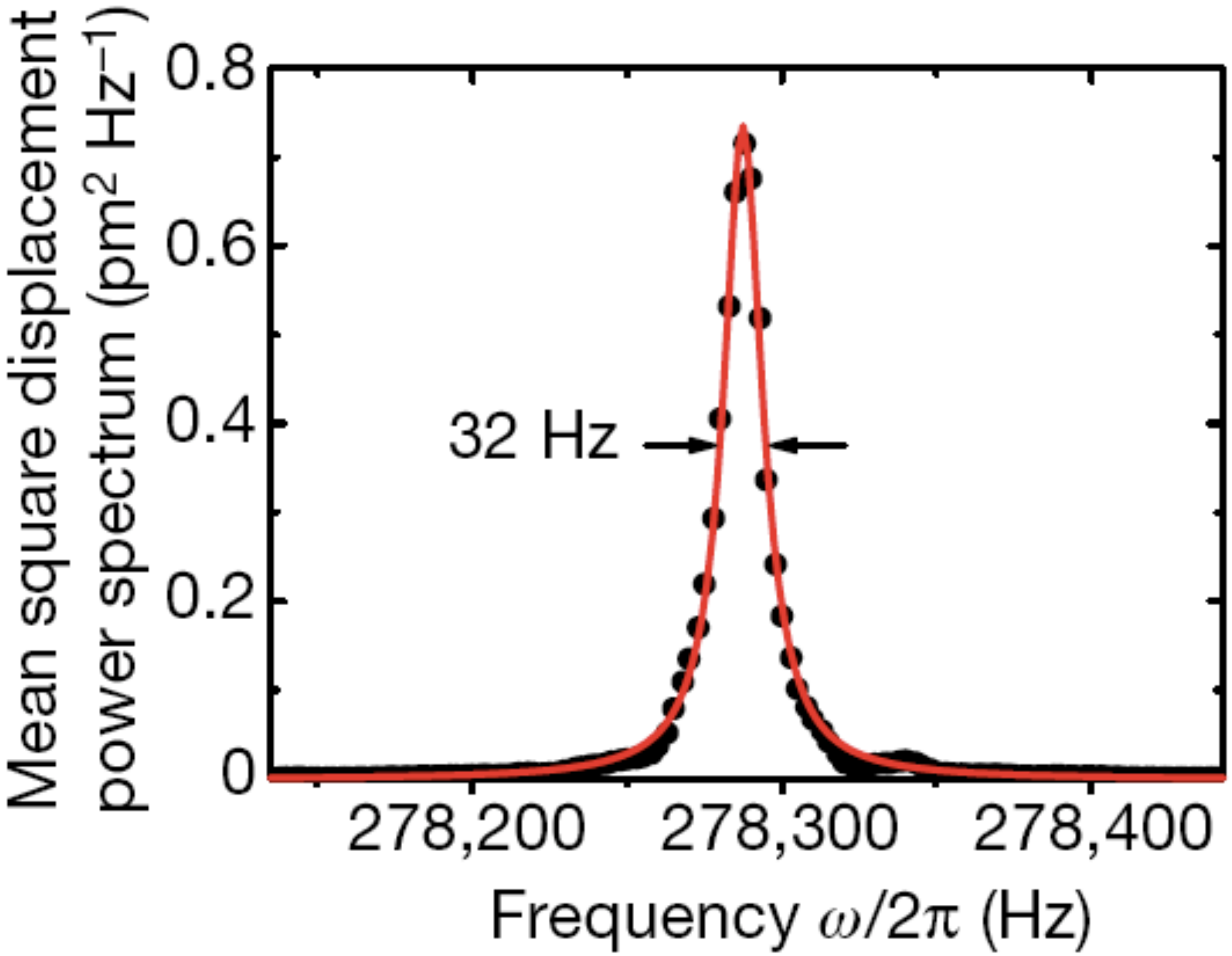
for the damped oscillator:

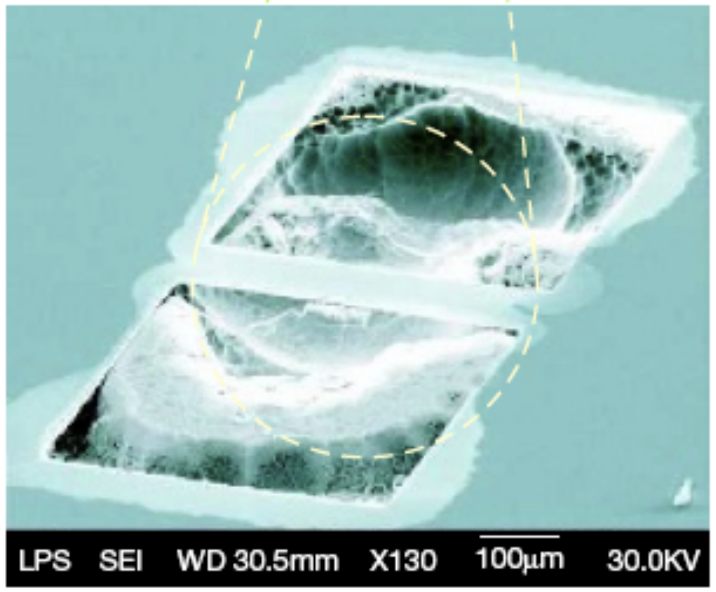
$$m\ddot{x} + m\omega_M^2 x + m\Gamma\dot{x} = F$$

$$x(\omega) = \underbrace{\frac{1}{m(\omega_M^2 - \omega^2) - im\Gamma\omega}}_{\chi_{xx}(\omega)} F(\omega)$$



Displacement spectrum

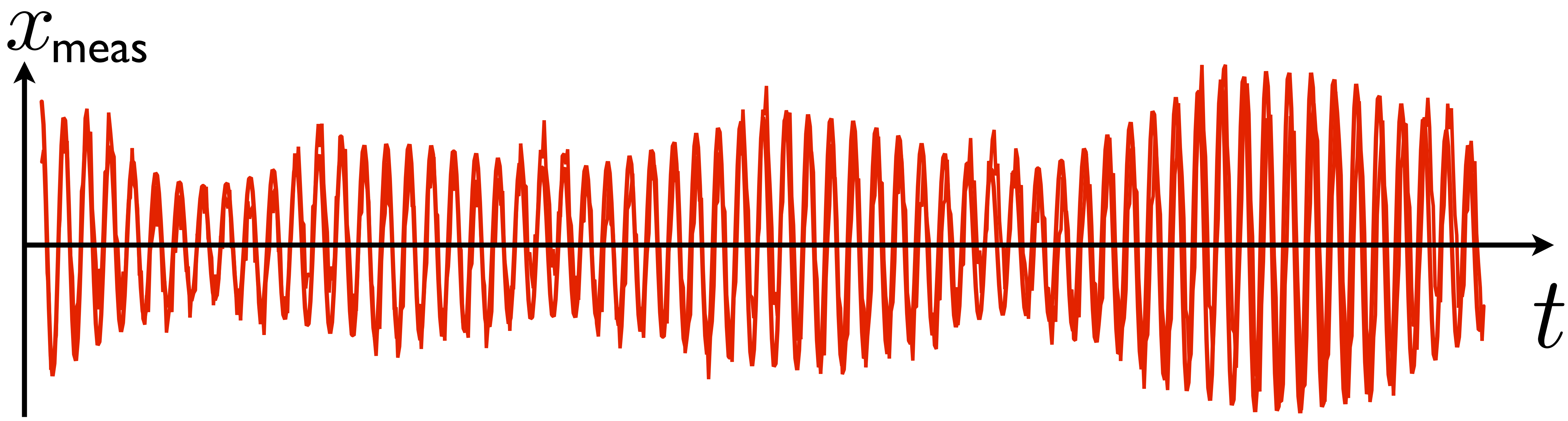




T=300 K

Experimental curve:
Gigan et al., Nature 2006

Measurement noise



$$x_{\text{meas}}(t) = x(t) + x_{\text{noise}}(t)$$

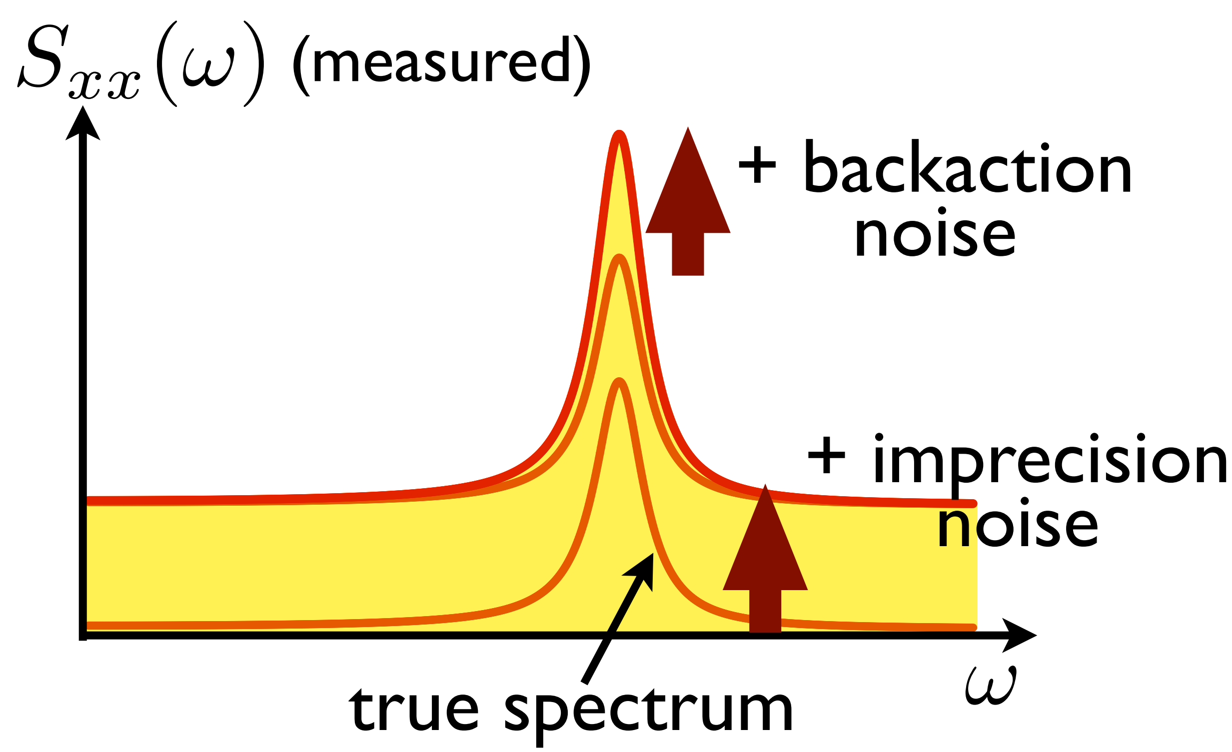
Two contributions to $x_{\text{noise}}(t)$

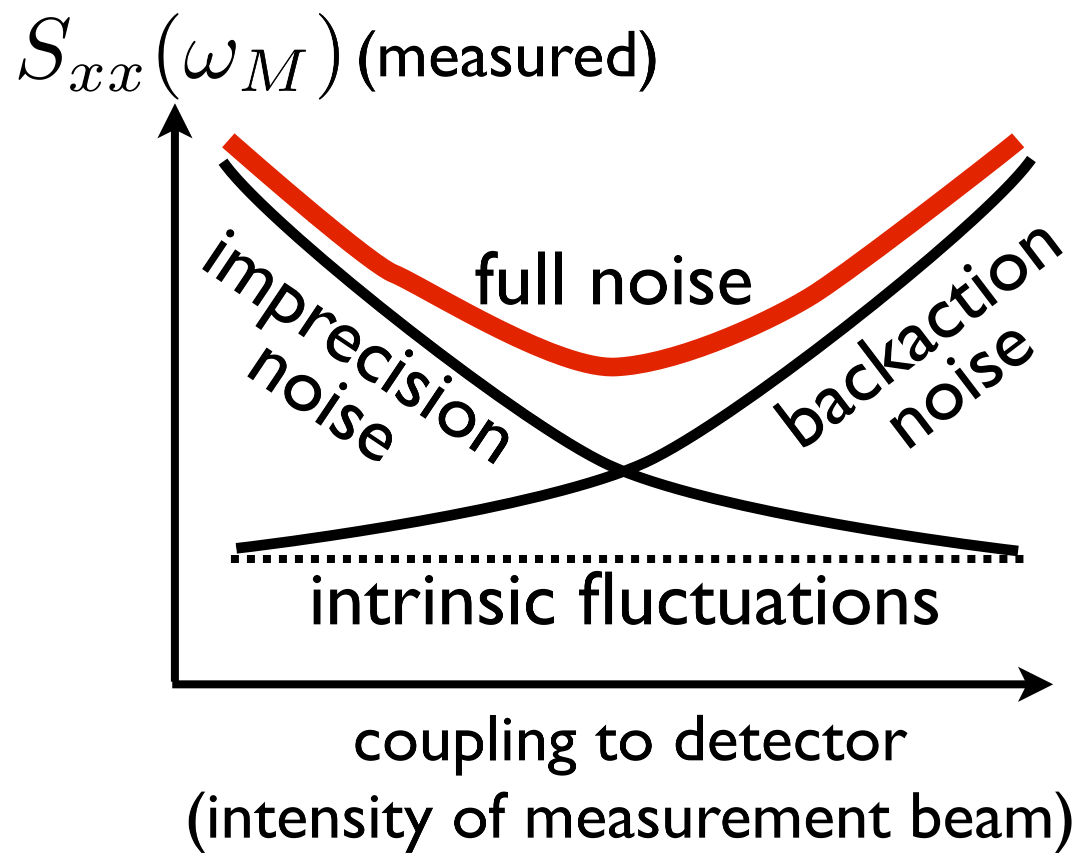
1. measurement imprecision
2. measurement back-action: fluctuating force on system

phase noise of laser beam (shot noise limit!)

noisy radiation pressure force

"Standard Quantum Limit"





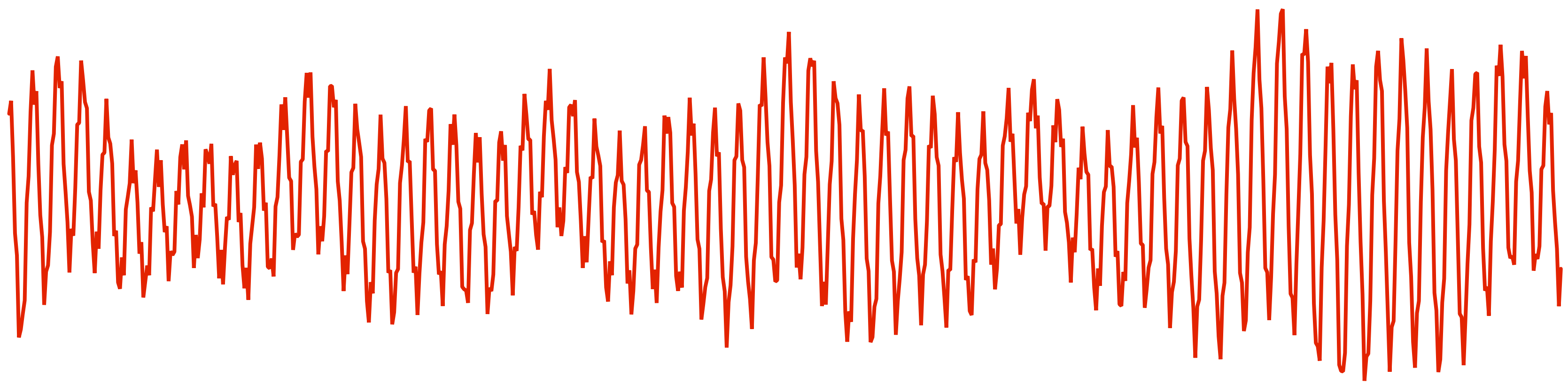
Best case allowed by quantum mechanics:

$$S_{xx}^{(\mathrm{meas})}(\omega) \geq 2 \cdot S_{xx}^{T=0}(\omega)$$

"Standard quantum limit (SQL) of displacement detection"

...as if adding the zero-point fluctuations a second time: "adding half a photon"

Notes on the SQL



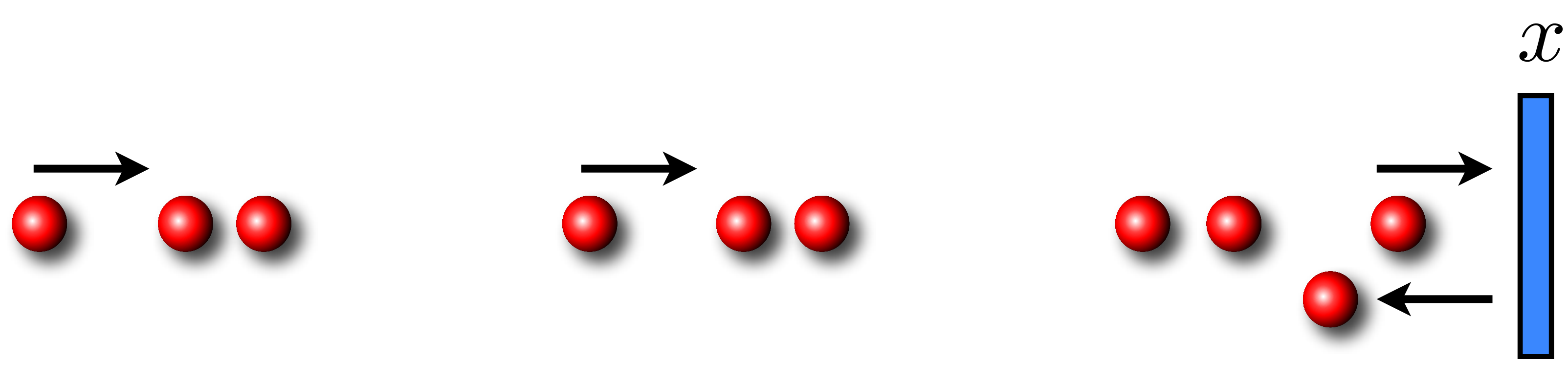
- "weak measurement": integrating the signal over time to suppress the noise
- trying to detect slowly varying "quadratures of motion": $\hat{x}(t) = \hat{X}_1 \cos(\omega_M t) + \hat{X}_2 \sin(\omega_M t)$

  $\left[\hat{X}_1, \hat{X}_2\right] = 2x_{\mathrm{ZPF}}^2$ Heisenberg is the reason for SQL!

  no limit for **instantaneous** measurement of x(t)!
- SQL means: detect $\hat{X}_{1,2}$ down to x_{ZPF} on a time scale $1/\Gamma$

Impressive: $x_{\mathrm{ZPF}} \sim 10^{-15} m$!

Enforcing the SQL (Heisenberg) in a weak optical measurement



reflection phase shift: $\theta = 2kx$
(here: free space)

N photons arrive in time t

fluctuations: $\delta N = \sqrt{\mathrm{Var} N} = \sqrt{\bar{N}}$

Poisson distribution for a coherent laser beam

1. Uncertainty in phase estimation:

$$\delta N \cdot \delta\theta \geq \frac{1}{2} \quad \Rightarrow \quad \delta\theta \geq \frac{1}{2\sqrt{\bar{N}}} \quad \Rightarrow \quad \delta x = \frac{\delta\theta}{2k} \sim \frac{1}{2\sqrt{\bar{N}}2k}$$

2. Fluctuating force: momentum transfer $\Delta p = 2\hbar k \cdot N$

$$\delta p = \sqrt{\mathrm{Var}\Delta p} = 2\hbar k \sqrt{\bar{N}}$$

Uncertainty product: $\delta x \delta p \geq \frac{\hbar}{2}$ Heisenberg