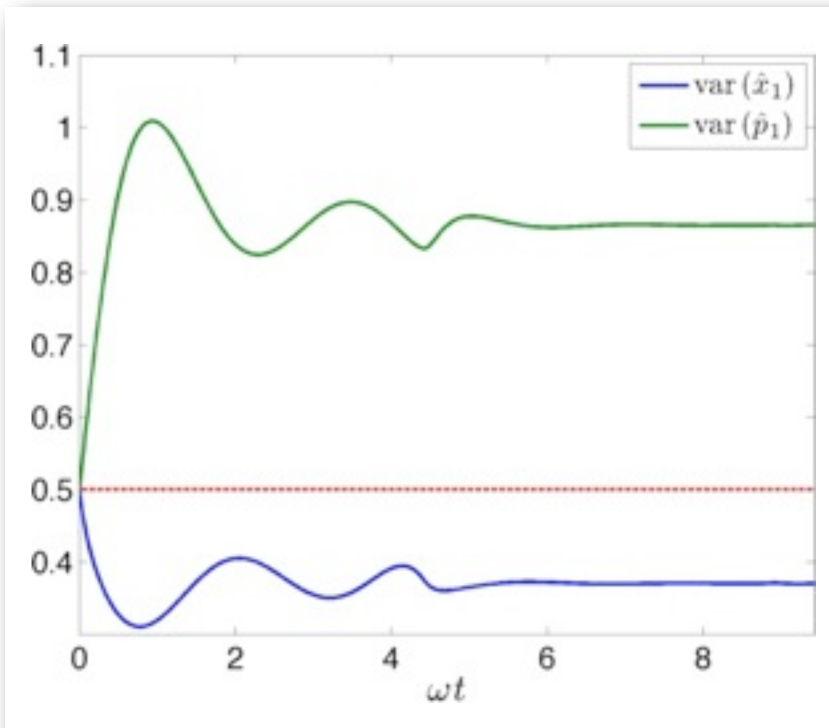
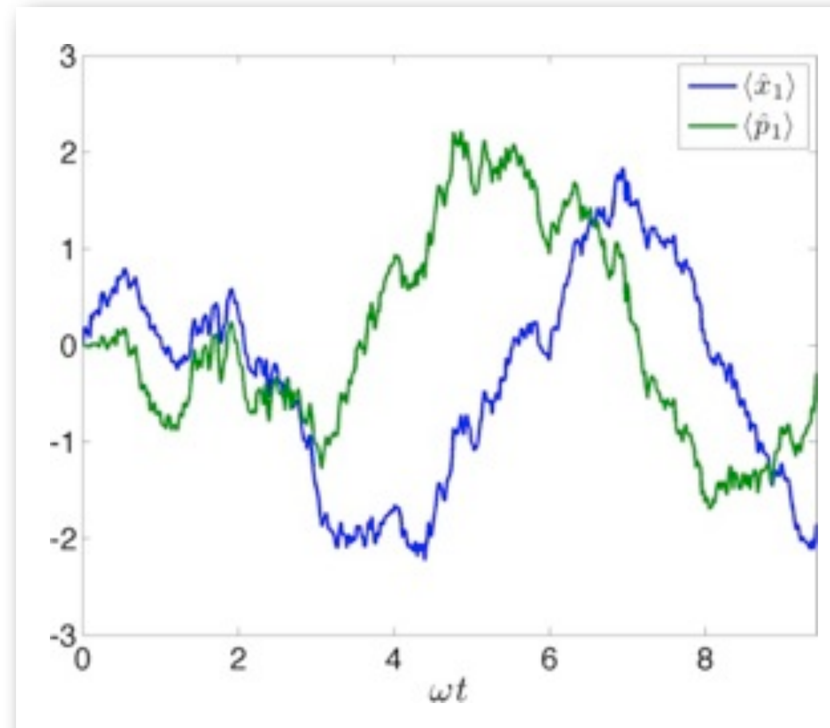
Continuous Local Probing of a Bose-Einstein Condensate

Andrew Wade, Jacob Sherson, Klaus Mølmer





Aug 25th - QNLO 2012 - Sandbjerg Estate

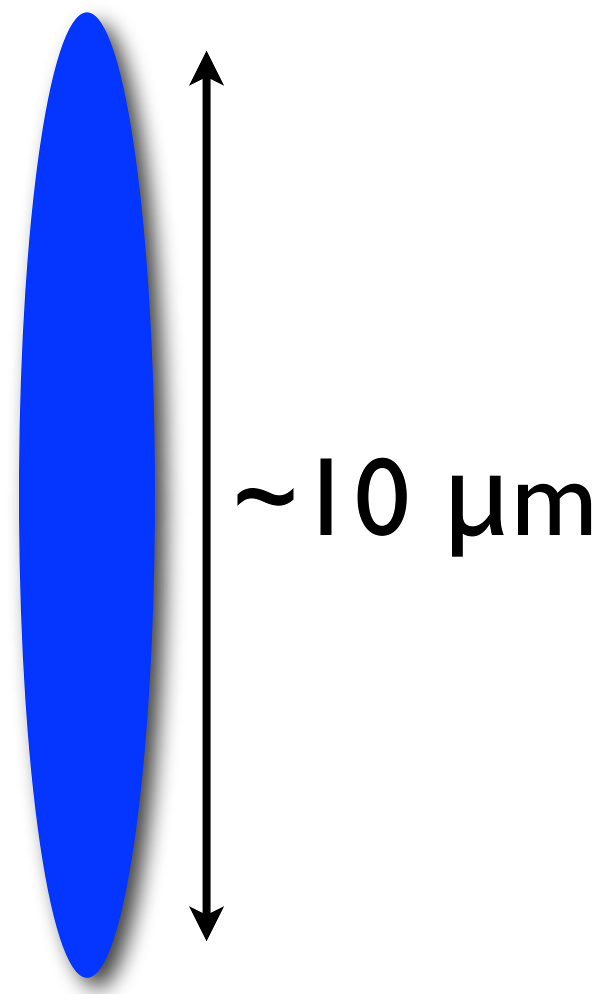
AARHUS UNIVERSITET
INSTITUT FOR FYSIK OG ASTRONOMI

Lundbeck Foundation Theoretical Center for Quantum System Research

The System/Overview

1/ Measurement
2/ BEC
3/ Interaction
4/ Observations

The System/Overview



Ultracold gas (BEC)

1/ Measurement
2/ BEC
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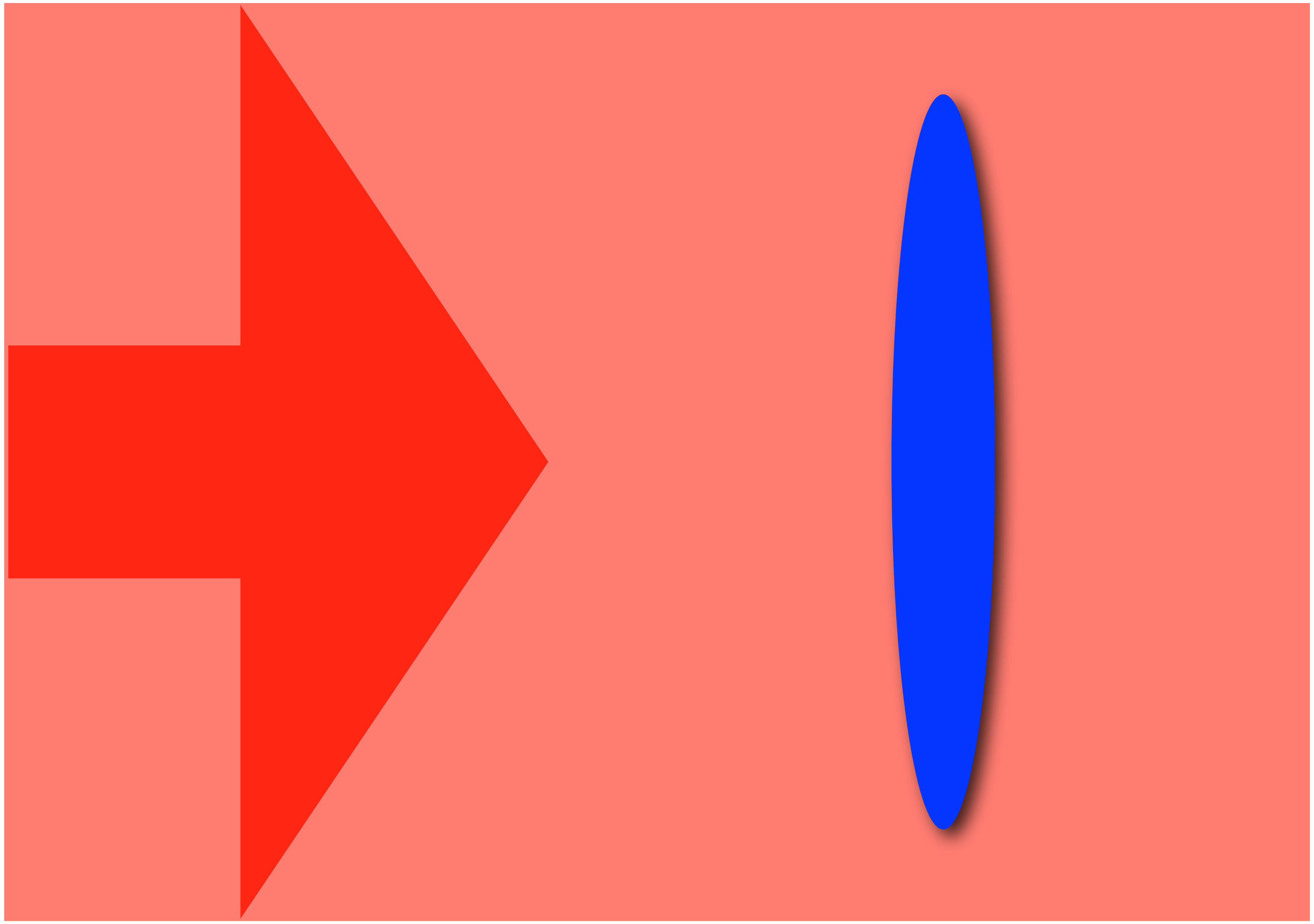
The System/Overview

Strong coherent state



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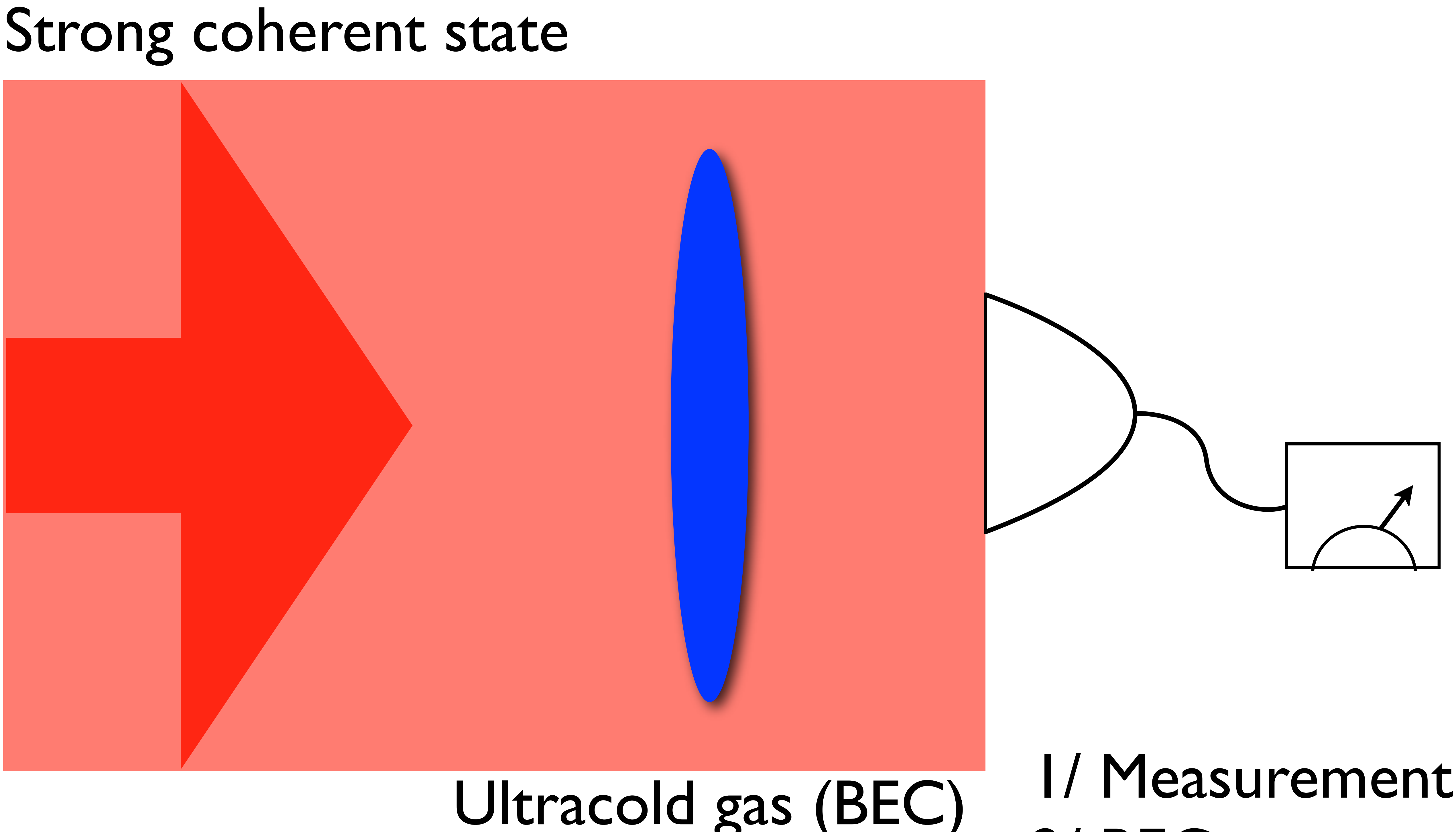

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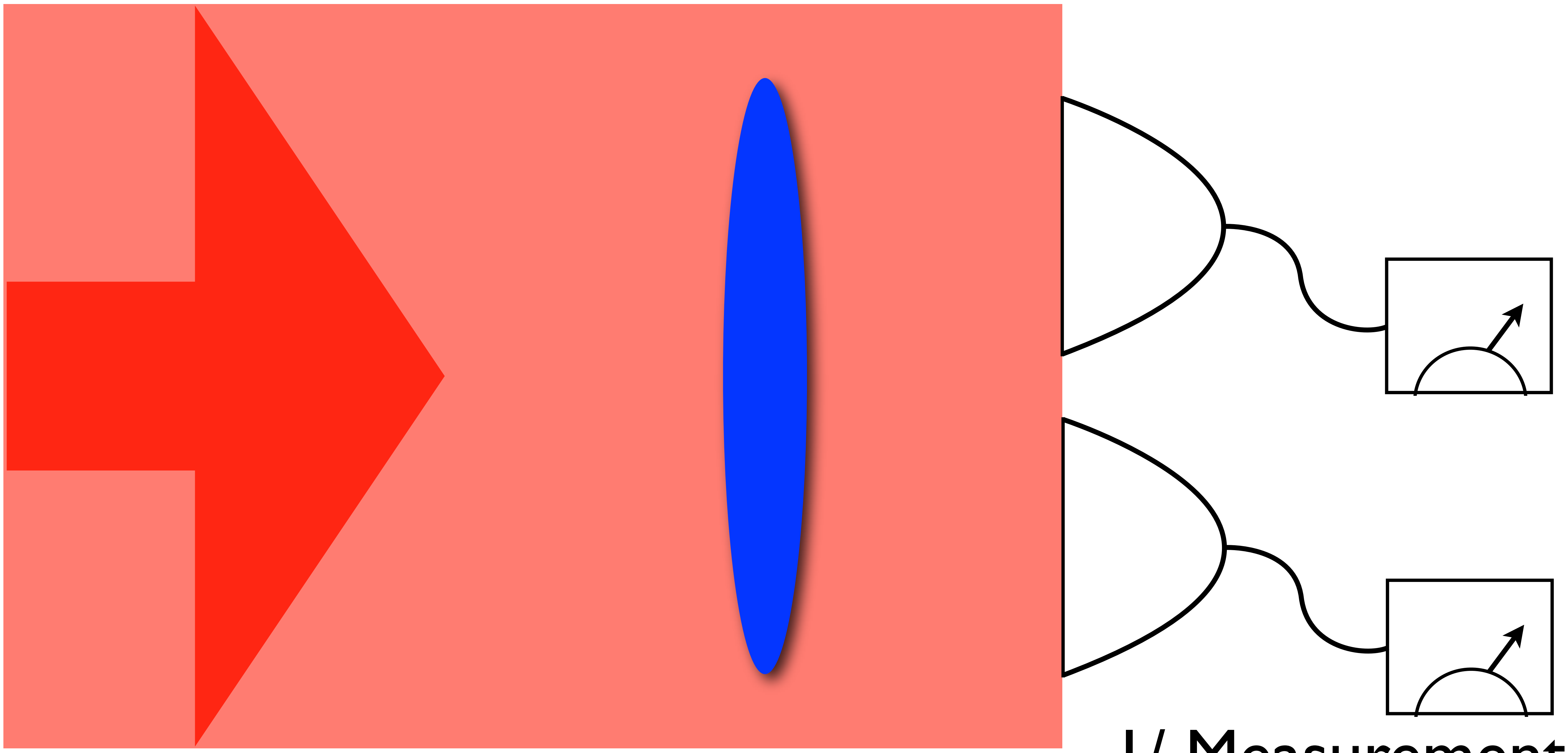

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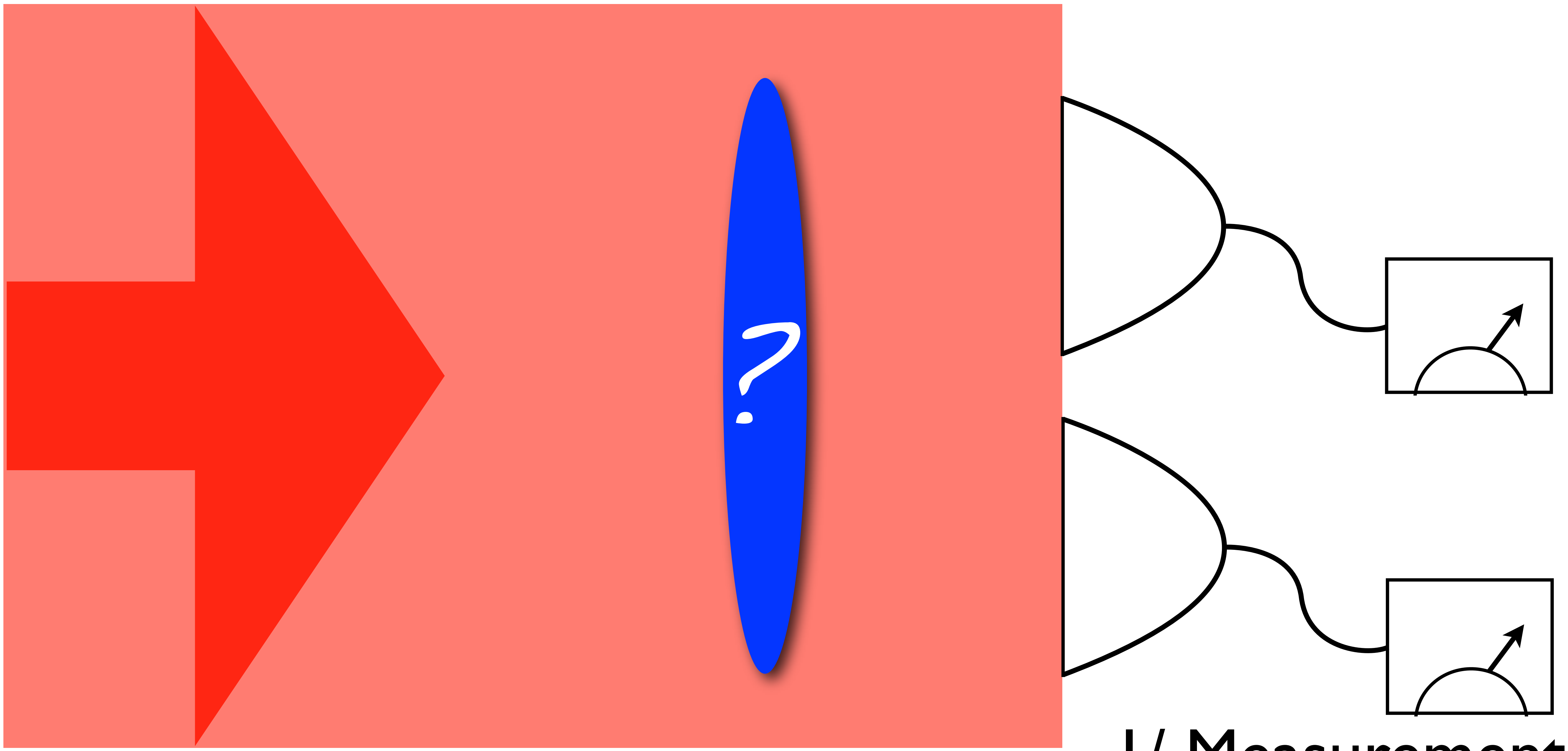

Ultracold gas (BEC)

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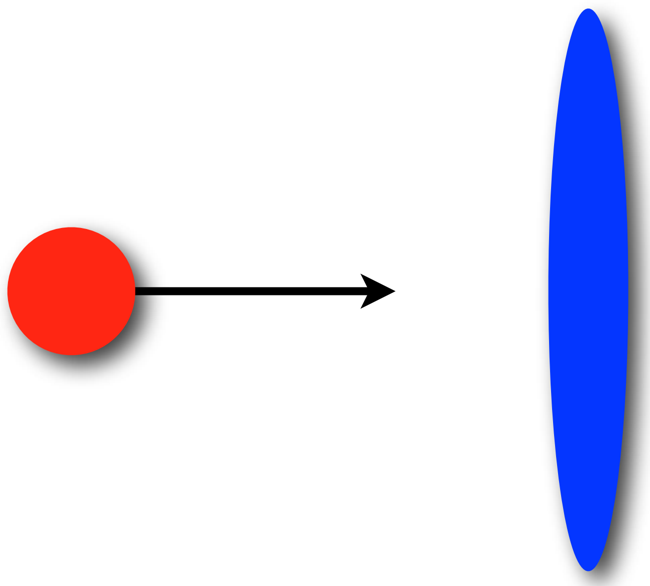
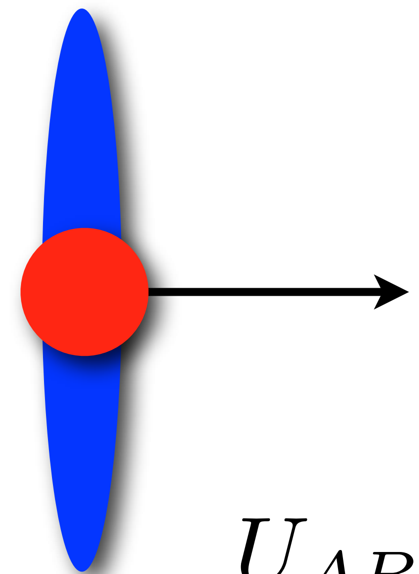
1/ Generalized Measurement

A



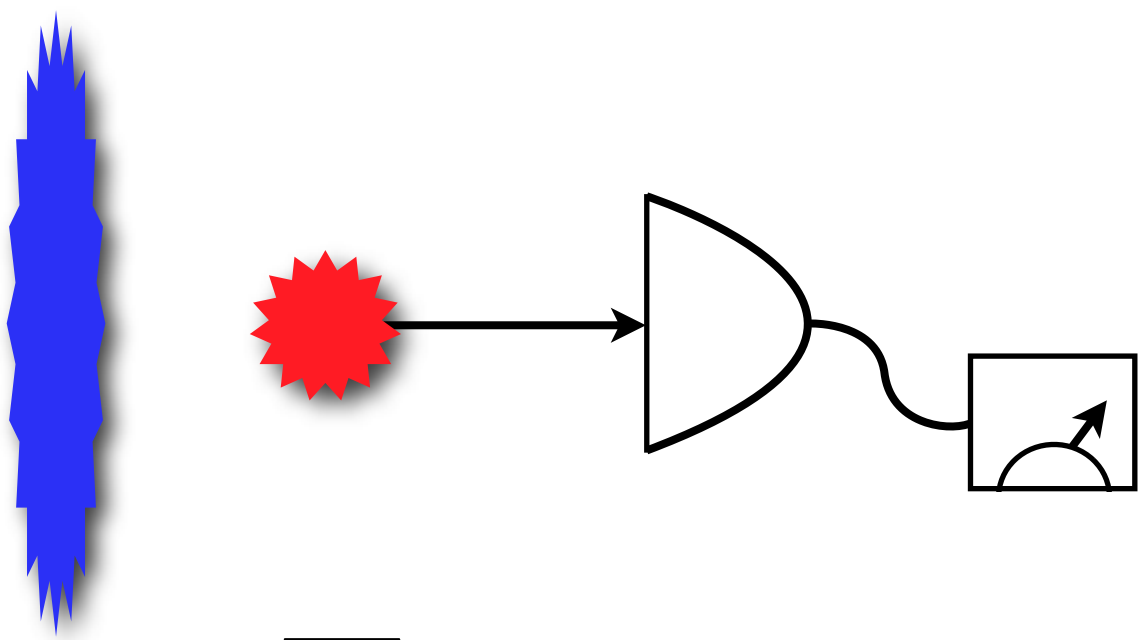
$$\rho_A \otimes \rho_B$$

B



$$U_{AB}\rho_A \otimes \rho_B U^\dagger_{AB}$$

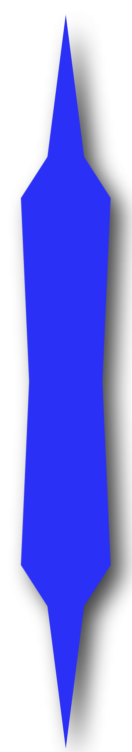
C



$$\rho'_A = \sum_\mu M_\mu \rho_A M^\dagger_\mu$$

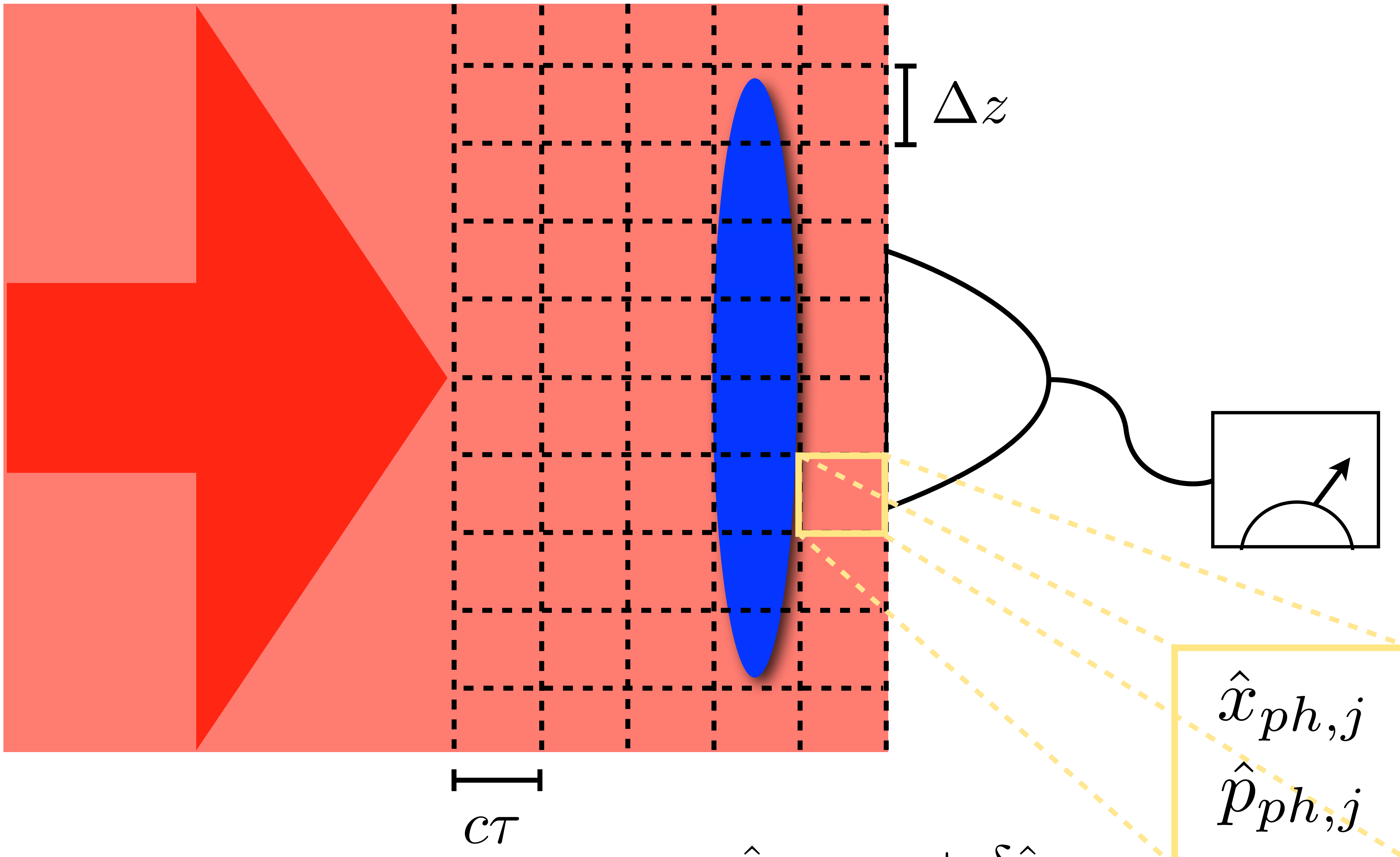
$$M_\mu = \langle \mu | U_{AB} | \psi_B \rangle$$

D



$$\rho''_A = \frac{M_\mu \rho_A M^\dagger_\mu}{\mathrm{Tr}_A \left[\rho_A M^\dagger_\mu M_\mu \right]}$$

1/ Treatment of Light

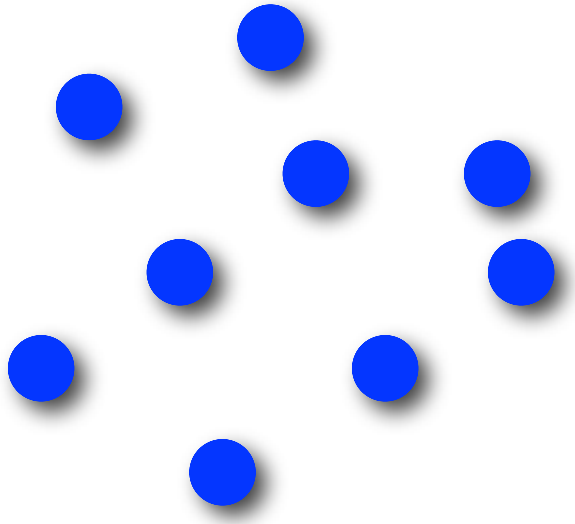


$$\hat{a}_j \approx \alpha + \delta\hat{a}_j$$

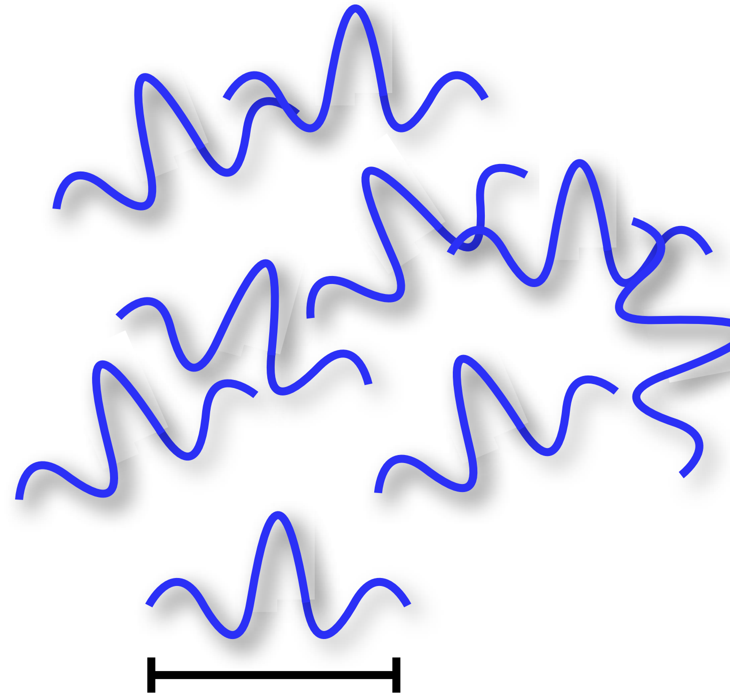
$$\hat{a}_j^{\dagger}\hat{a}_j \approx |\alpha|^2 + \beta\hat{x}_{ph,j}$$

2/ Bose-Einstein Condensation

300K

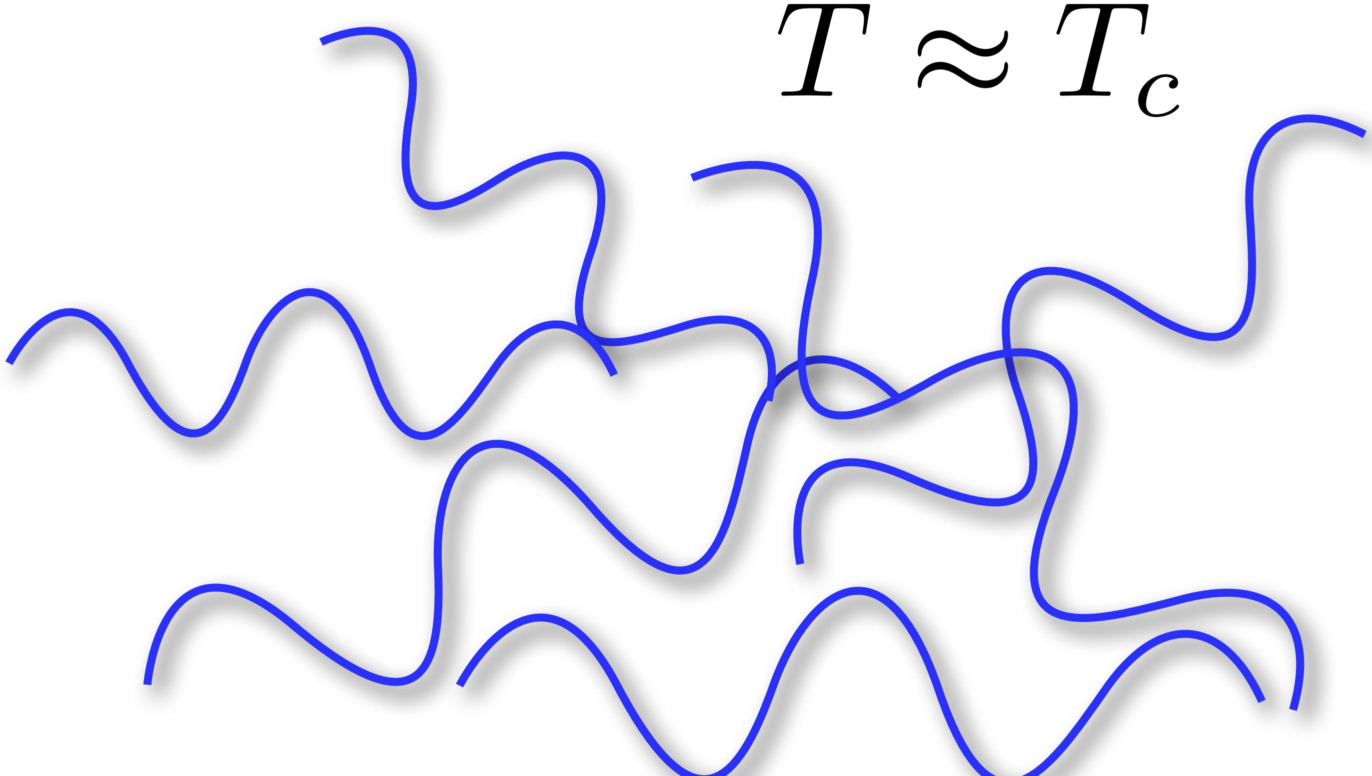


Maxwell-Boltzmann



λ_{dB}

$T \approx T_c$



$T \ll T_c$

Second quantization

$$\hat{H} = \int d\mathbf{r}\, \hat{\psi}^{\dagger}(\mathbf{r}) \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + \frac{g}{2}\hat{\psi}^{\dagger}(\mathbf{r})\hat{\psi}(\mathbf{r}) \right] \hat{\psi}(\mathbf{r})$$

$\left[\hat{a}, \hat{a}^{\dagger}\right] = 1$	$\left[\hat{\psi}(\mathbf{r}), \hat{\psi}^{\dagger}(\mathbf{r}')\right] = \delta(\mathbf{r} - \mathbf{r}')$
$\|n\rangle = \frac{1}{\sqrt{n!}} \left(\hat{a}^{\dagger}\right)^n \|0\rangle$	$\|\phi_n\rangle = \frac{1}{\sqrt{n!}} \left[\int d\mathbf{r}\phi(\mathbf{r})\hat{\psi}^{\dagger}(\mathbf{r})\right]^n \|0\rangle$
$\langle \hat{a}^{\dagger}\hat{a} \rangle = n$	$\langle \hat{\psi}^{\dagger}(\mathbf{r})\hat{\psi}(\mathbf{r}) \rangle = n(\mathbf{r})$
$\hat{a} \approx \alpha$	$\hat{\psi}(\mathbf{r}) \approx \psi(\mathbf{r})$

Gross-Pitaevskii Equation

$$i\hbar\frac{d\psi\left(\mathbf{r},t\right)}{dt}=\left[-\frac{\hbar^2}{2m}\nabla^2+V\left(\mathbf{r}\right)+\frac{g}{2}|\psi\left(\mathbf{r},t\right)|^2\right]\psi\left(\mathbf{r},t\right)$$

Fiber optics: Self-Phase Modulation

Four-Wave Mixing

Second Harmonic Generation

Bogoliubov Approximation

$$\hat{\psi}(\mathbf{r}) \approx \psi(\mathbf{r}) + \delta\hat{\psi}(\mathbf{r})$$

$$\hat{a}_j \approx \alpha + \delta\hat{a}_j$$

Hamiltonian to second order in $\delta\hat{\psi}(\mathbf{r})$

First order vanishes for GPE solution

$$\delta\hat{\psi}(\mathbf{r}) = \sum_{j\neq 0}\left[u_j(\mathbf{r})\,\hat{\alpha}_j - v_j^*(\mathbf{r})\,\hat{\alpha}_j^\dagger\right] \qquad \left[\hat{\alpha}_j, \hat{\alpha}_j^\dagger\right] = 1$$

Bogoliubov Transformation diagonalizes Hamiltonian

Well what did that mean?

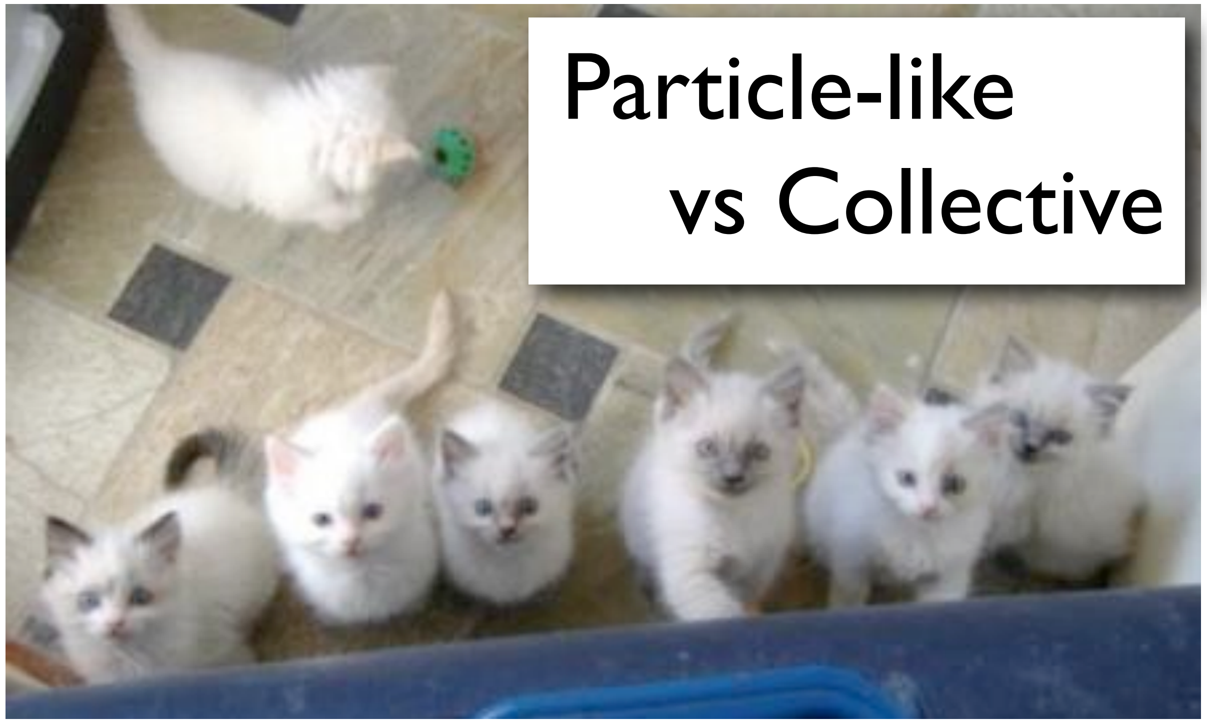
$$\hat{K} = E_0 + \sum_{j \neq 0} E_j \hat{\alpha}_j^\dagger \hat{\alpha}_j$$

We have reformulated the problem in terms of noninteracting quasi-particles.

Quasi-particles represent collective excitations, unless

$g = 0$ or $E_j \gg 0$

Quasi-particles = Real particles



Uniform gas ($V(\mathbf{r}) = 0$): low energy excitations are phonons

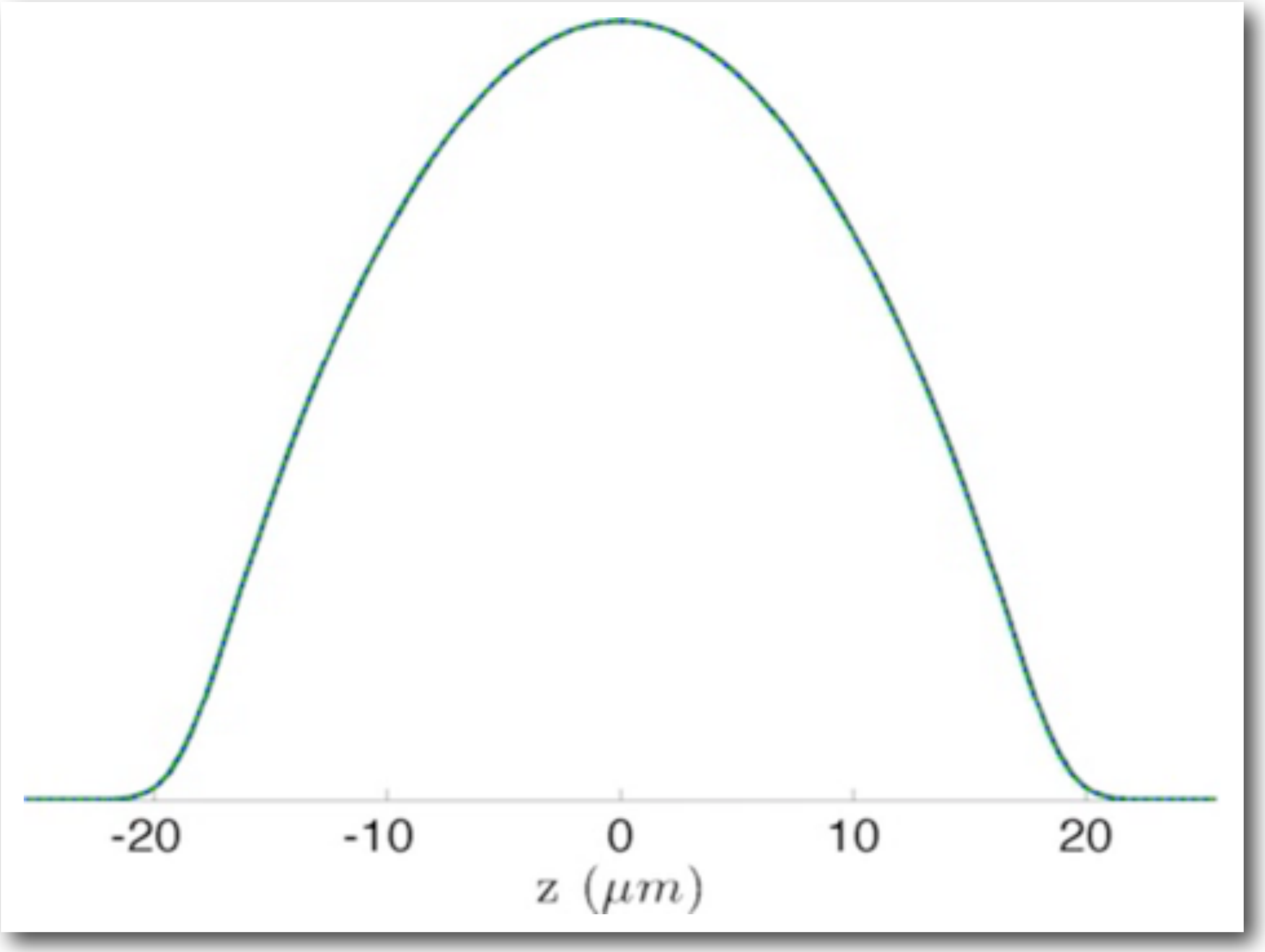
$$E(p) = cp \qquad c = \sqrt{\frac{gn}{m}}$$

Harmonic trap:

$$V(\mathbf{r}) = \frac{m}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

$$n(\mathbf{r}, t) \approx \psi(\mathbf{r})^2 + \psi(\mathbf{r})\langle \delta\hat{\psi}(\mathbf{r}) + \delta\hat{\psi}^\dagger(\mathbf{r})\rangle$$
$$= \psi(\mathbf{r})^2 + 2\psi(\mathbf{r}) \sum_{j \neq 0} f_j^-(\mathbf{r}) \langle \hat{x}_j \rangle$$

$$\hat{x}_j = \frac{1}{\sqrt{2}}\left(\hat{\alpha}_j + \hat{\alpha}_j^\dagger\right)$$



Uniform gas ($V(\mathbf{r})=0$): low energy excitations are phonons

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$$=\psi(\mathbf{r})^2+2\psi(\mathbf{r})\sum_{j\neq 0}f_j^-(\mathbf{r})\langle\hat{x}_j\rangle$$

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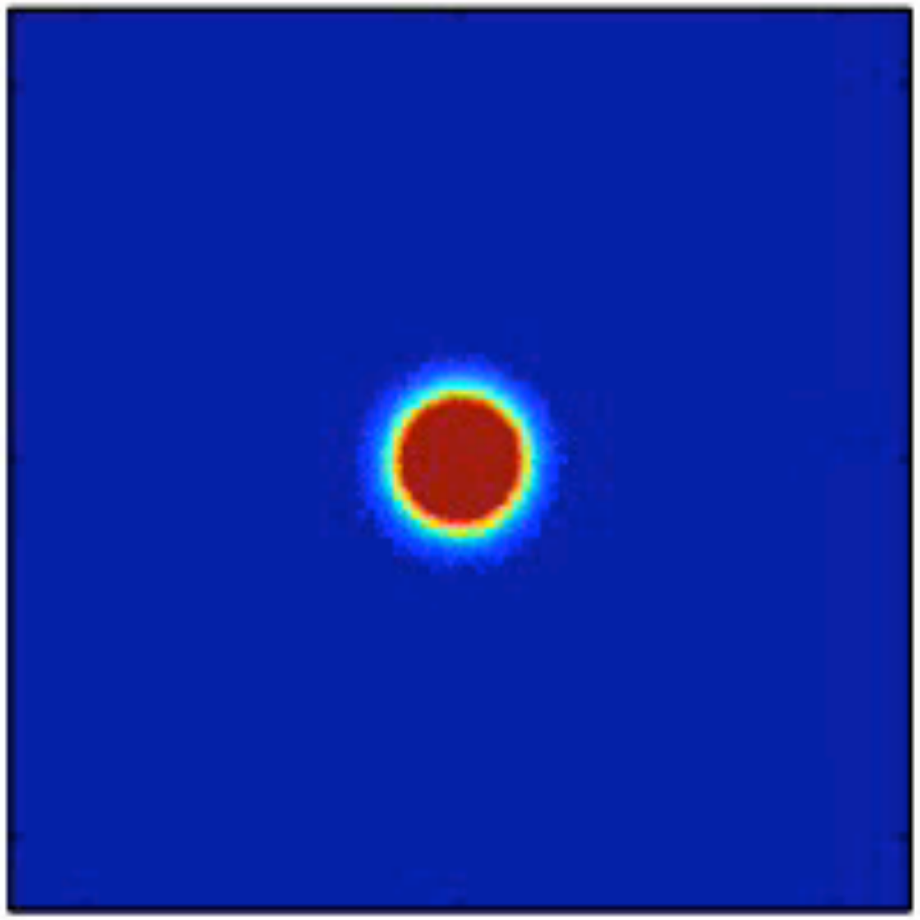
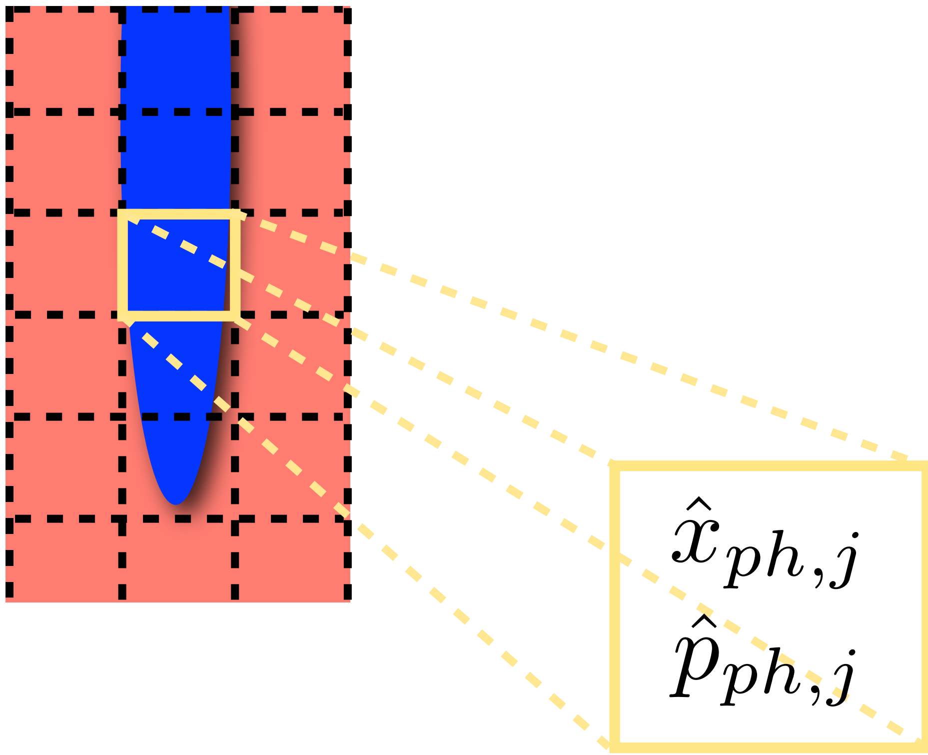


3/ The Interaction

large detuning



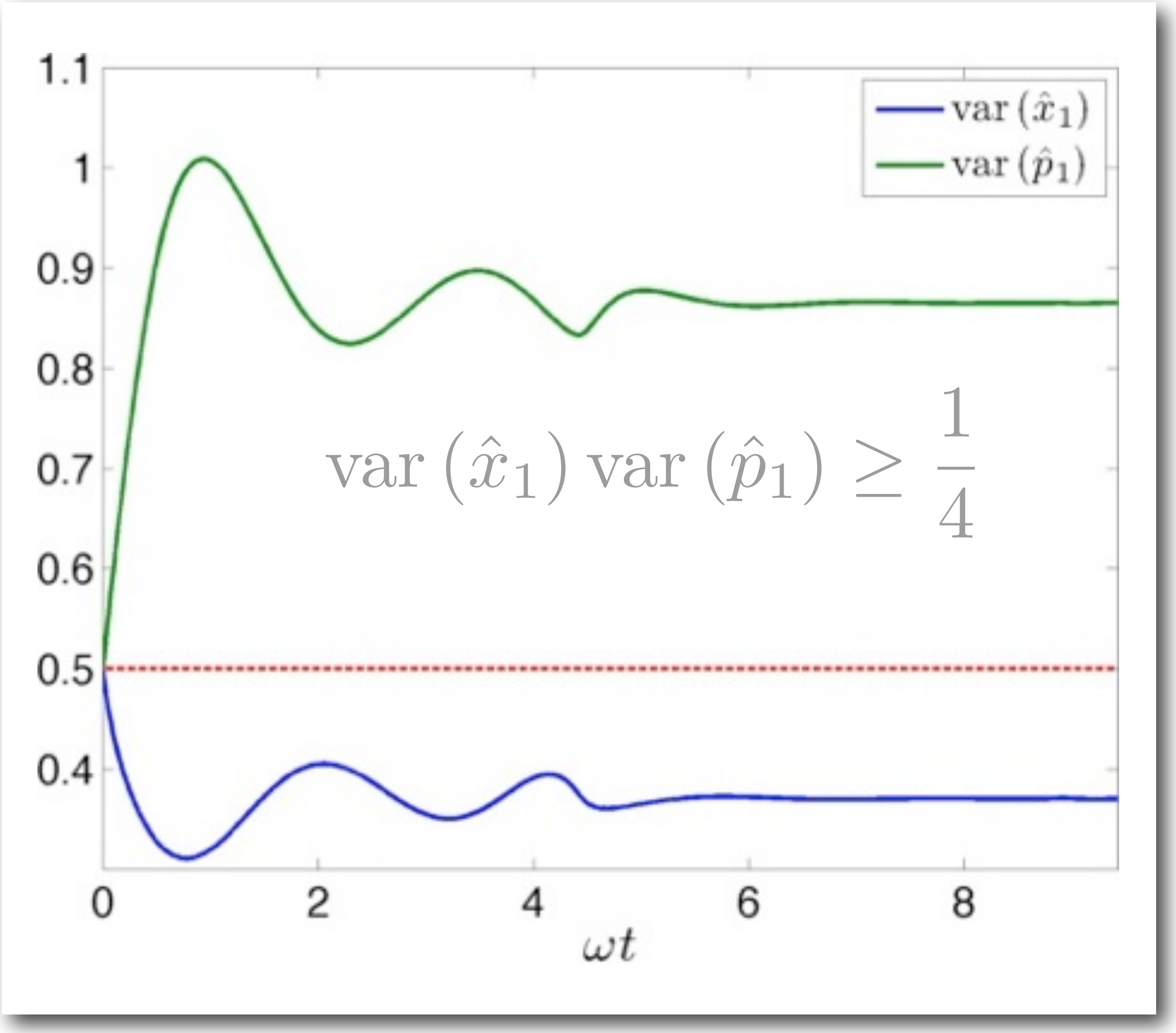
$$\hat{H}_I \propto \sum_j \hat{a}_j^\dagger \hat{a}_j \int_{D_j} \hat{\psi}^\dagger(z)\,\hat{\psi}(z)\,dz$$

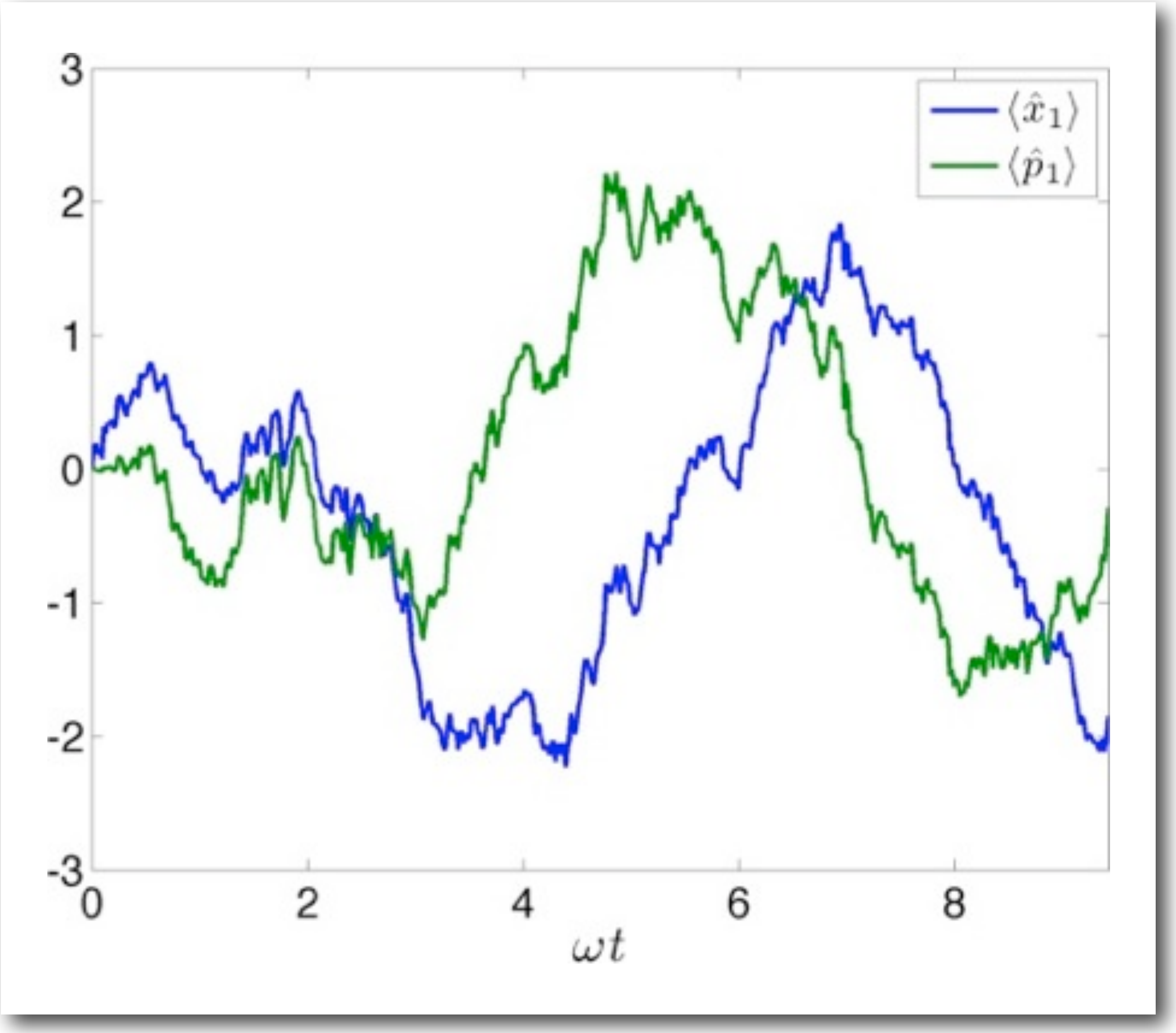
$$\hat{a}_j^\dagger \hat{a}_j \approx |\alpha|^2 + \beta \hat{x}_{ph,j} \qquad \hat{\psi}^\dagger(z)\,\hat{\psi}(z) \approx \psi(\mathbf{r})^2 + 2\,\psi(\mathbf{r}) \sum_{j\neq 0} f_j^-(\mathbf{r})\,\hat{x}_j$$

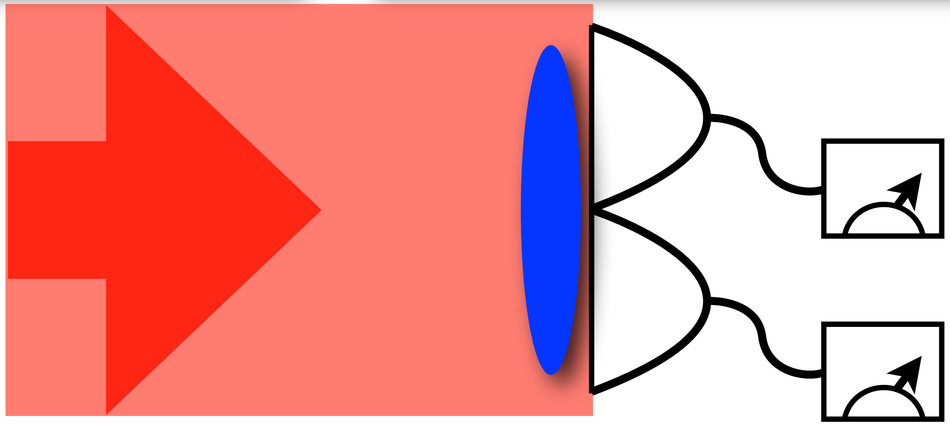
$$\hat{H}_I \propto \sum_j \hat{x}_{ph,j}(\hbar G_{j,0} + \sum_{k\neq 0} \hbar G_{j,k}\hat{x}_k)$$

4/ Toy Example

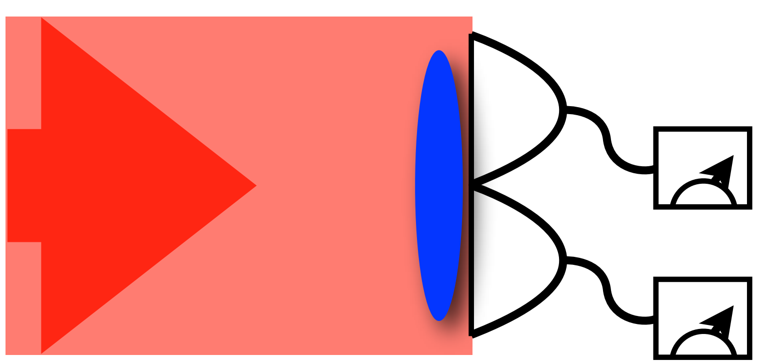
$$\hat{H}_I \propto \sum_j \hat{x}_{ph,j}(\hbar G_{j,0} + \sum_{k\neq 0} \hbar G_{j,k}\hat{x}_k)$$

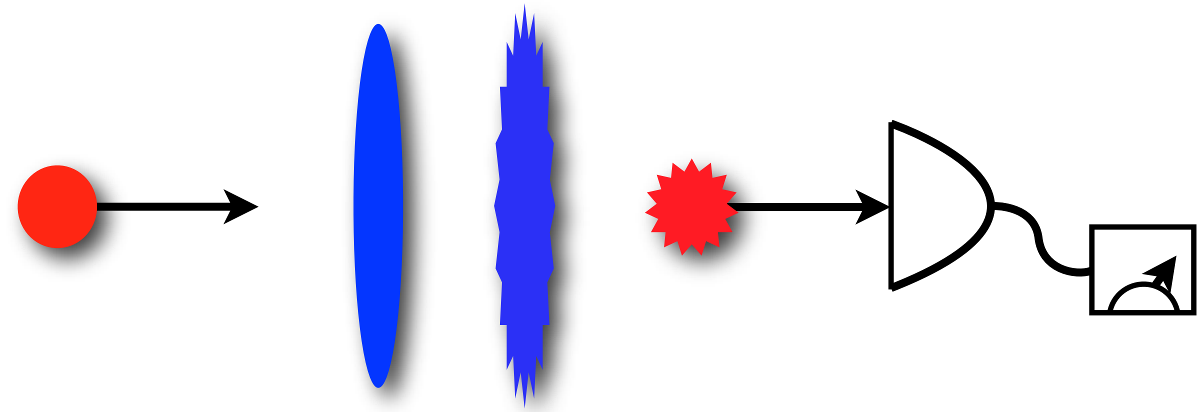


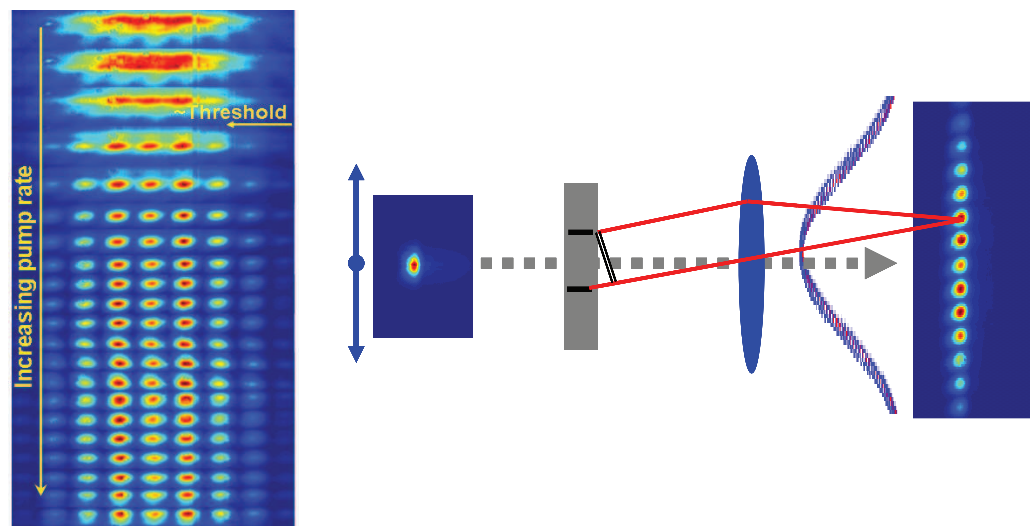


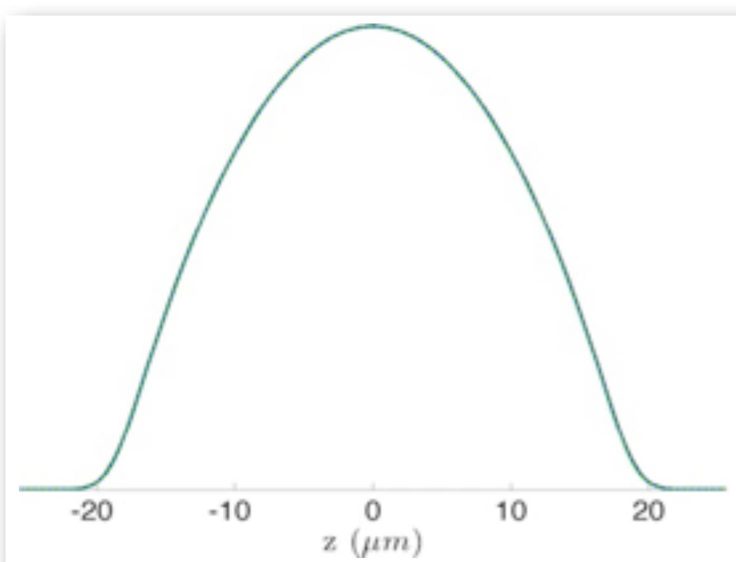


The End









$$\hat{H}_I^T \propto \sum_j \hat{x}_{ph,j}(\hbar G_{j,0} + \sum_{k\neq 0} \hbar G_{j,k}\hat{x}_k)$$