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Observing the Geometric Phase of a Superconducting Harmonic Oscillator

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FNSNF Swiss National Science Foundation

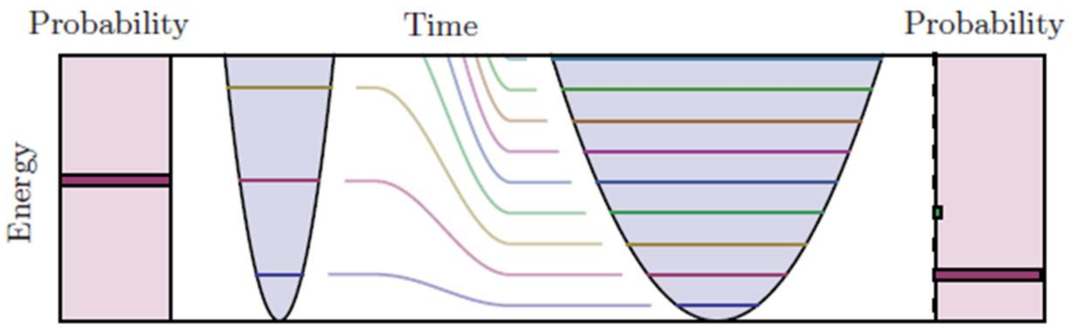
GEOMDISS

Geometric Phase

Original setting – cyclic adiabatic processes

- System stays in the energy eigenstate of an adiabatically changing Hamiltonian...

Example of an adiabatic process



M.V. Berry, Proc. R. Soc. Lond. A **392**, 1984

Geometric Phase

Original setting – cyclic adiabatic processes

– System stays in the energy eigenstate of an adiabatically changing Hamiltonian...

... and accumulates a phase

$$-\frac{1}{\hbar}\int E(t)\,\mathrm{d}t$$

?

M.V. Berry, Proc. R. Soc. Lond. A **392**, 1984

Geometric Phase

Original setting – cyclic adiabatic processes

– System stays in the energy eigenstate of an adiabatically changing Hamiltonian...

... and accumulates a phase

Dynamical Phase | Geometric Phase

$$-\frac{1}{\hbar}\int E(t)\,\mathrm{d}t \quad + \quad \gamma_g$$

Independent of evolution time and system energy

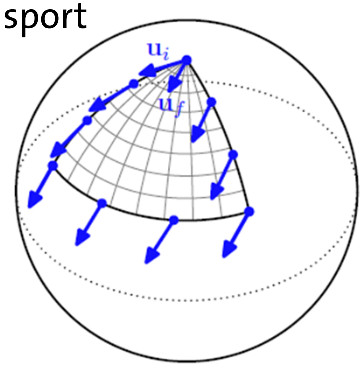
M.V. Berry, Proc. R. Soc. Lond. A **392**, 1984

Geometric Phase

Original setting – cyclic adiabatic processes

- System stays in the energy eigenstate of an adiabatically changing Hamiltonian...

Parallel transport analogy



... and accumulates a phase

Dynamical Phase		Geometric Phase
$-\frac{1}{\hbar}\int E(t)\,\mathrm{d}t$	$+$	γ_g

Independent of evolution time and system energy

M.V. Berry, Proc. R. Soc. Lond. A **392**, 1984

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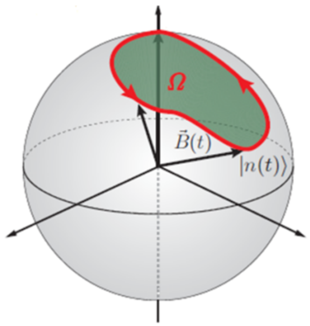
Geometric Phase

- Two-level system (Qubit)

$$\gamma_g = \pm\Omega/2$$

Berry phase



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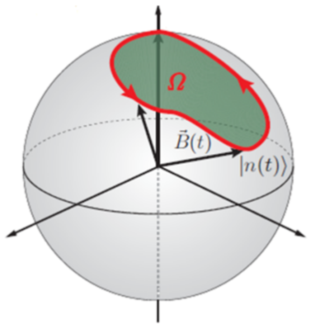
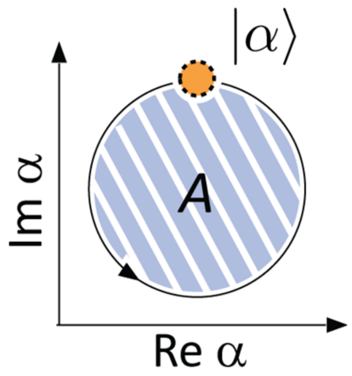


M.V. Berry, Proc. R. Soc. Lond. A **392**, 1984

- Harmonic oscillator

$$\gamma_g = -2A$$



S. Chaturvedi, *et al.*, J. Phys. A: Math. Gen. **20**, 1987
G. Vacanti, *et al.*, arXiv:1108.0701v1 [quant-ph]

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Geometric Phase

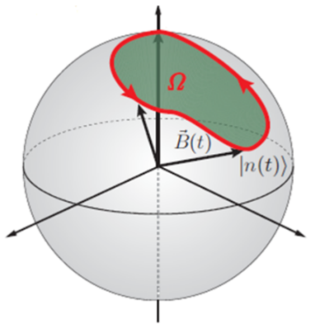
- Two-level system (Qubit)

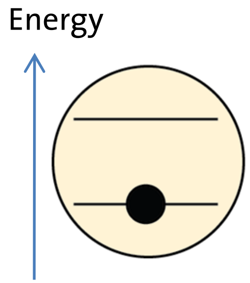
Berry phase

$$\gamma^{(g)} \neq \gamma^{(e)}$$

M.V. Berry, Proc. R. Soc. Lond. A **392**, 1984

$$\gamma_g = \pm\Omega/2$$



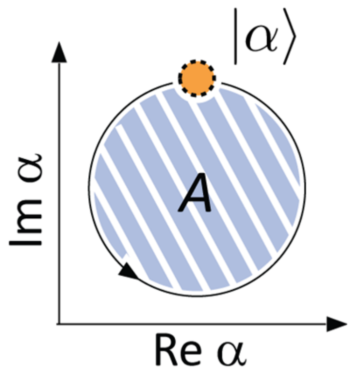


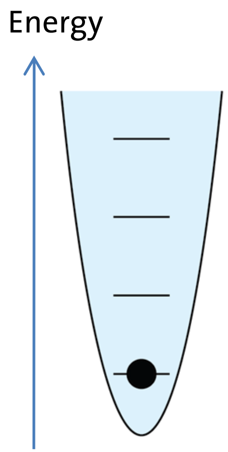
$$|g\rangle \rightarrow e^{i\gamma^{(g)}}|g\rangle$$
$$|e\rangle \rightarrow e^{i\gamma^{(e)}}|e\rangle$$

- Harmonic oscillator

$$\gamma^{(0)} = \gamma^{(1)} = \gamma^{(2)} = \ldots$$

$$\gamma_g = -2A$$





$$\ldots$$
$$|2\rangle \rightarrow e^{i\gamma}|2\rangle$$
$$|1\rangle \rightarrow e^{i\gamma}|1\rangle$$
$$|0\rangle \rightarrow e^{i\gamma}|0\rangle$$

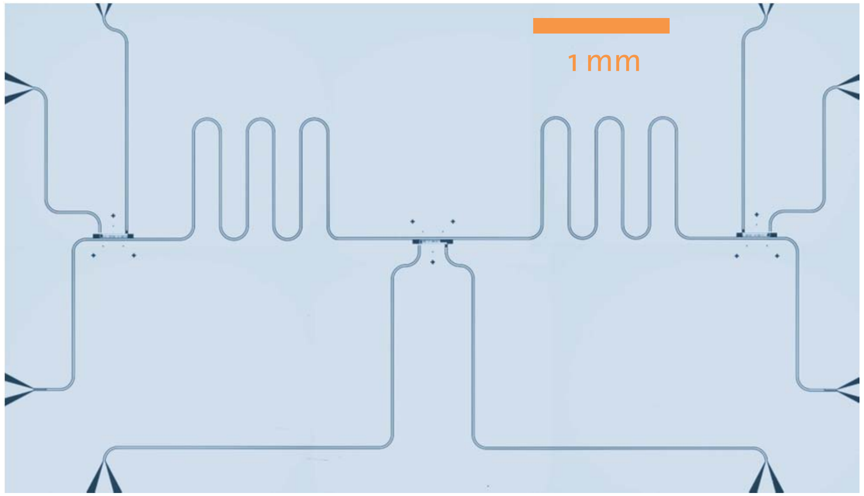
S. Chaturvedi, *et al.*, J. Phys. A: Math. Gen. **20**, 1987
G. Vacanti, *et al.*, arXiv:1108.0701v1 [quant-ph]

Our Circuit QED Setup

On-chip Superconducting Circuit

- Solid state architecture for QIP
- Operates at microwave (GHz) frequencies

At 20mK inside a dilution refrigerator



A. Blais, *et al.*, Phys. Rev. A **69**, 2004
J. Koch, *et al.*, Phys. Rev. A **76**, 2007

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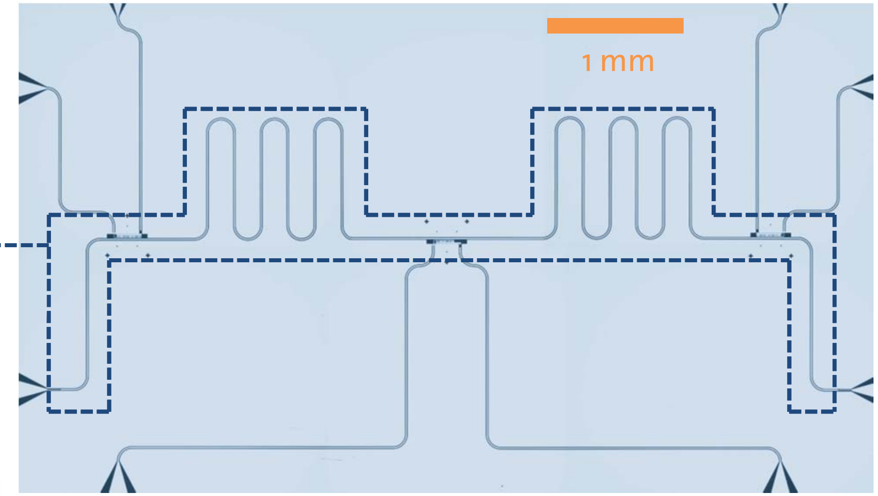
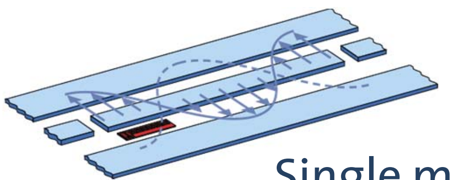


Transmission
Line Resonator
($\omega_r/2\pi \approx 7$ GHz)



Single mode approximation
$\Rightarrow$ Harmonic oscillator

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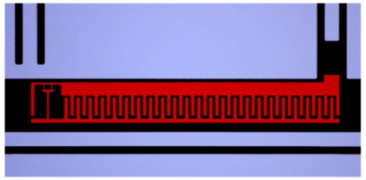
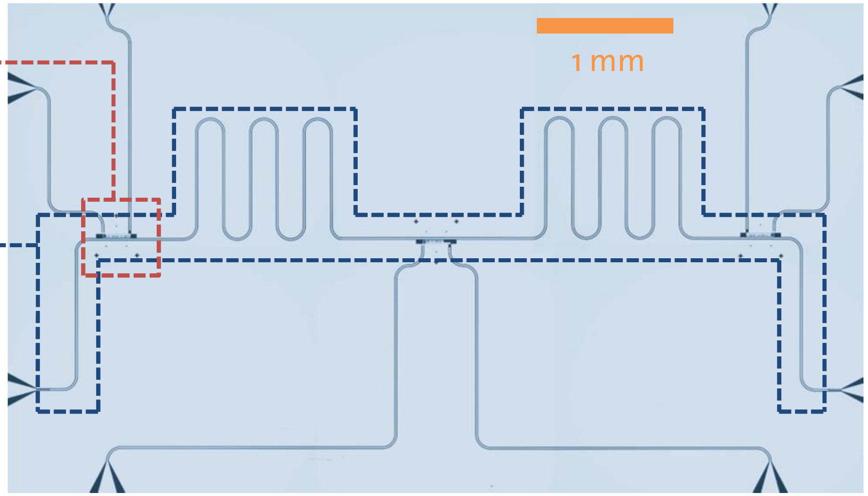
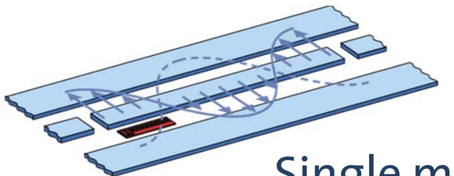
On-chip Superconducting Circuit



Qubit
(artifical atom)
($\omega_q/2\pi \approx 8$ GHz)

Transmission
Line Resonator
($\omega_r/2\pi \approx 7$ GHz)





Single mode approximation
$\Rightarrow$ Harmonic oscillator

Large detuning
$\Rightarrow$ No energy exchange, i.e.,
Dispersive coupling
($\chi/2\pi \approx 1$ MHz)

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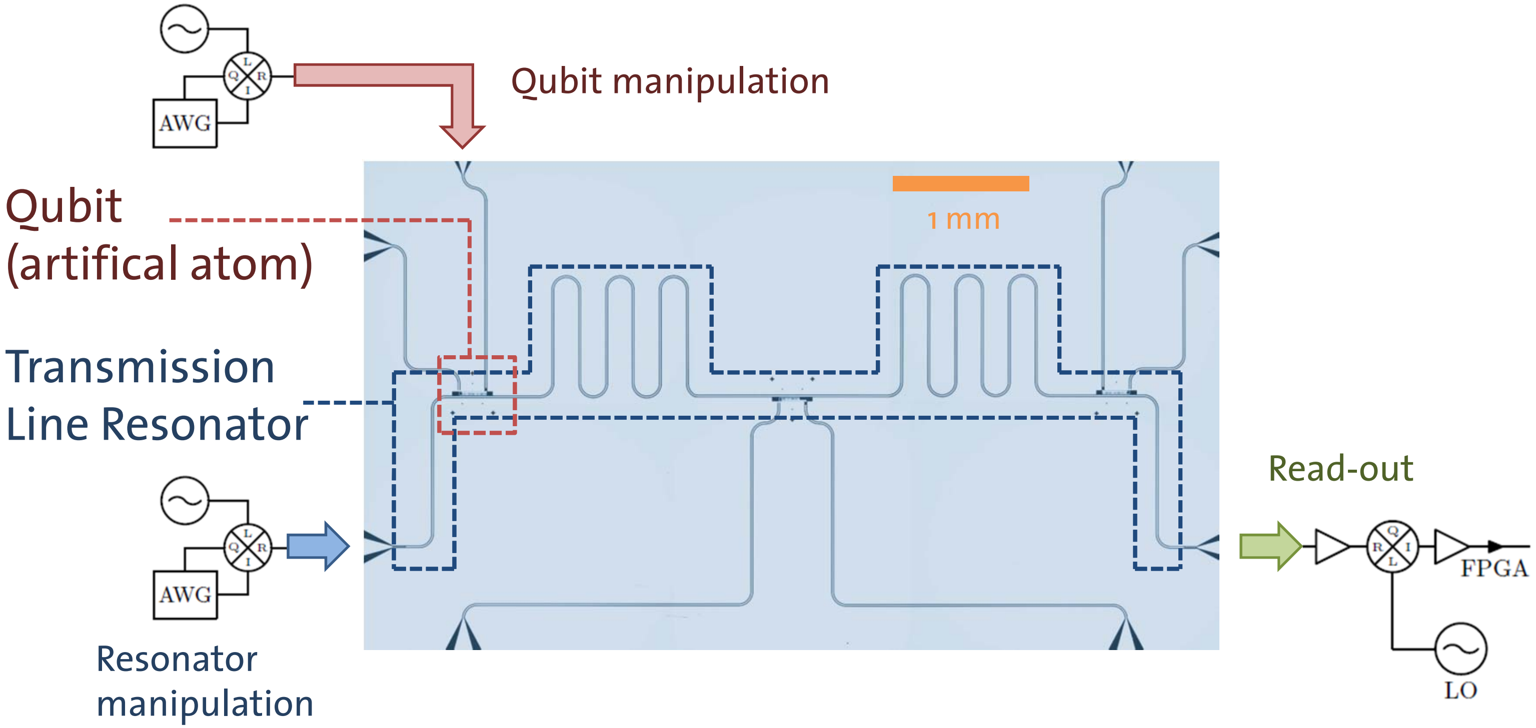
Our Circuit QED Setup

On-chip Superconducting Circuit



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Our Circuit QED Setup

Hamiltonian of the system

$$H = (\omega_r + 2\chi\sigma_{ee} - \omega)a^\dagger a + (\varepsilon^* a + \varepsilon a^\dagger)/2 + \Omega\sigma_x/2$$

A. Blais, *et al.*, Phys. Rev. A **69**, 2004
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Our Circuit QED Setup

Hamiltonian of the system

Resonator manipulation

Qubit manipulation

$$H = (\omega_r + 2\chi\sigma_{ee} - \omega)a^\dagger a + (\varepsilon^* a + \varepsilon a^\dagger)/2 + \Omega\sigma_x/2$$

External drive pulses

A. Blais, *et al.*, Phys. Rev. A **69**, 2004
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Our Circuit QED Setup

Hamiltonian of the system

Projector onto the qubit excited state

Resonator manipulation

Qubit manipulation

$$H = (\omega_r + 2\chi\sigma_{ee} - \omega)a^\dagger a + (\varepsilon^* a + \varepsilon a^\dagger)/2 + \Omega\sigma_x/2$$

Dispersive coupling term
(χ / $2\pi \approx 1$ MHz)

External drive pulses

A. Blais, *et al.*, Phys. Rev. A **69**, 2004
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OUR CIRCUIT QED SETUP

Hamiltonian of the system

Resonator manipulation

Qubit manipulation

$$H = (\omega_r + 2\chi\sigma_{ee} - \omega)a^\dagger a + (\varepsilon^* a + \varepsilon a^\dagger)/2 + \Omega\sigma_x/2$$

Dispersive coupling term
(χ / $2\pi \approx 1$ MHz)

Without qubit drive:

Resonator = separate system with qubit-dependent resonance frequency

ω_r
for the qubit in the ground state

$\omega_r + 2\chi$
for the qubit in the excited state

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Our Circuit QED Setup

Hamiltonian of the system

Resonator manipulation

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$$H = (\omega_r + 2\chi\sigma_{ee} - \omega)a^\dagger a + (\varepsilon^* a + \varepsilon a^\dagger)/2 + \Omega\sigma_x/2$$

Dispersive coupling term
(χ / $2\pi \approx 1$ MHz)

Without qubit drive:

Resonator = separate system with qubit-dependent resonance frequency

ω_r
for the qubit in the ground state

$\omega_r + 2\chi$
for the qubit in the excited state

Adiabatic variation of ε
(time scale slower than $1/\delta \approx 25$ ns)

→ System follows ground state of the Hamiltonian

A. Blais, *et al.*, Phys. Rev. A **69**, 2004
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Our Circuit QED Setup

Hamiltonian of the system

Resonator manipulation

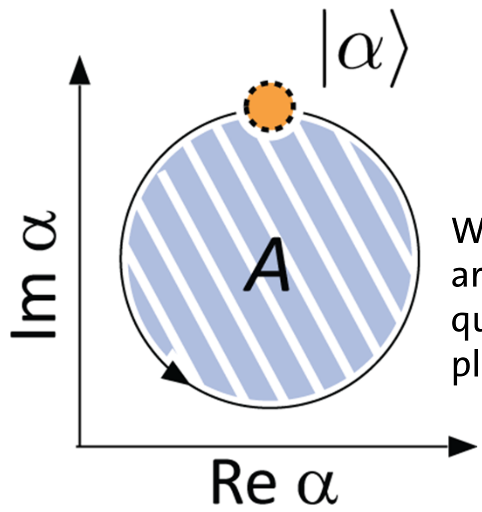
Qubit manipulation

$$H = (\omega_r + 2\chi\sigma_{ee} - \omega)a^\dagger a + (\varepsilon^* a + \varepsilon a^\dagger)/2 + \Omega\sigma_x/2$$

Dispersive coupling term
(χ / $2\pi \approx$ 1 MHz)

Ground state = coherent state $|\alpha\rangle$, where

$$\alpha \propto \varepsilon$$



We can steer α around the quadrature plane at will.

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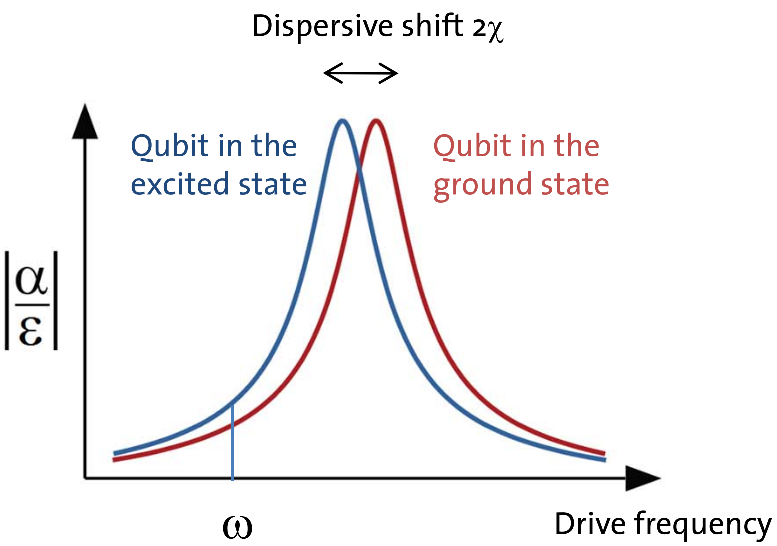
J. Koch, *et al.*, Phys. Rev. A **76**, 2007

Our Circuit QED Setup



Different field amplitudes

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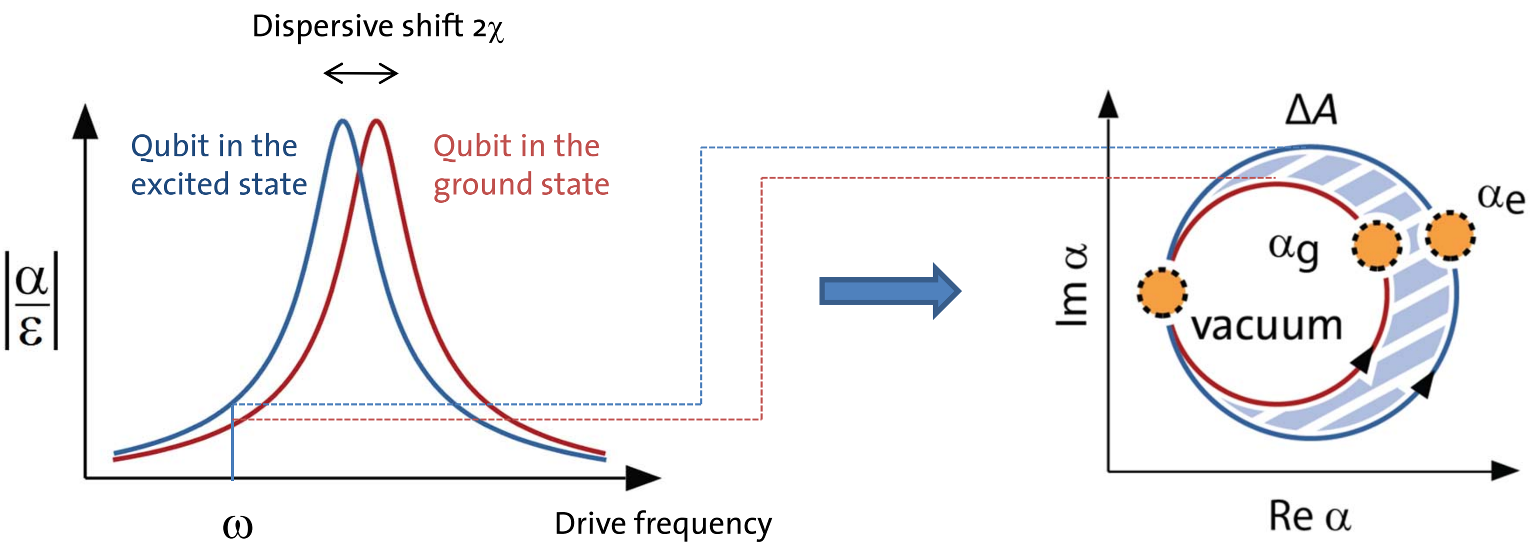
Our Circuit QED Setup



Different field amplitudes

Different enclosed areas

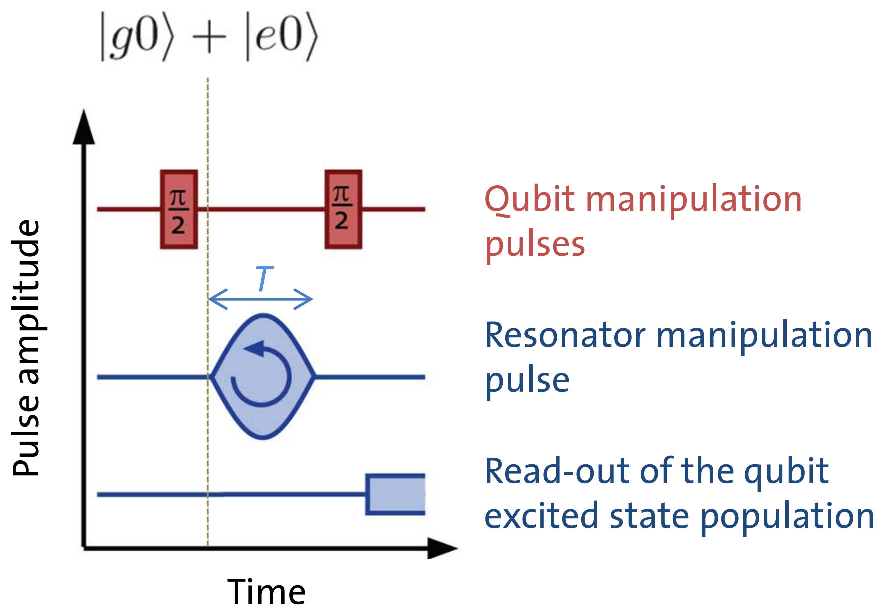
$$|g0\rangle \rightarrow \mathrm{e}^{i\gamma^{(g)}}|g0\rangle$$

$$|e0\rangle \rightarrow \mathrm{e}^{i\gamma^{(e)}}|e0\rangle$$

Different accumulated phases

Measurement Procedure

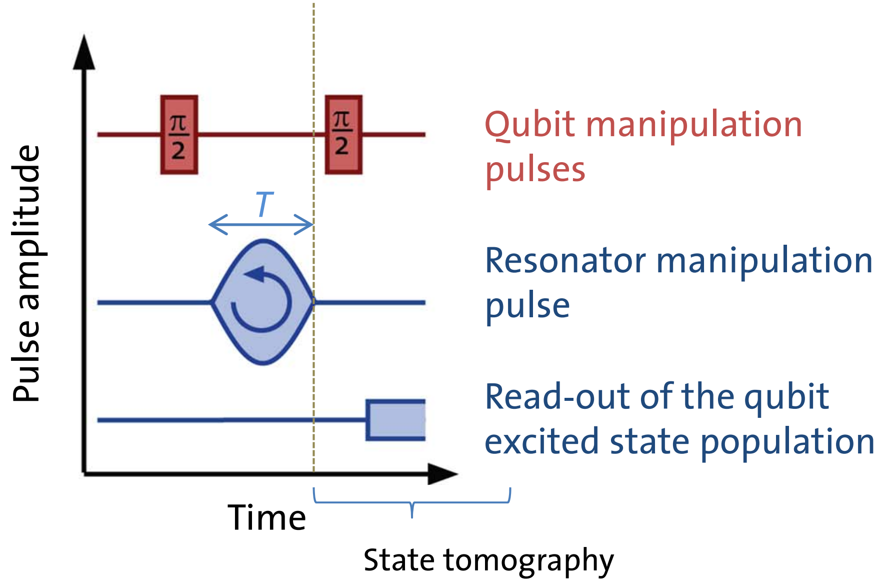
Ramsey Interferometry



Measurement Procedure

Ramsey Interferometry

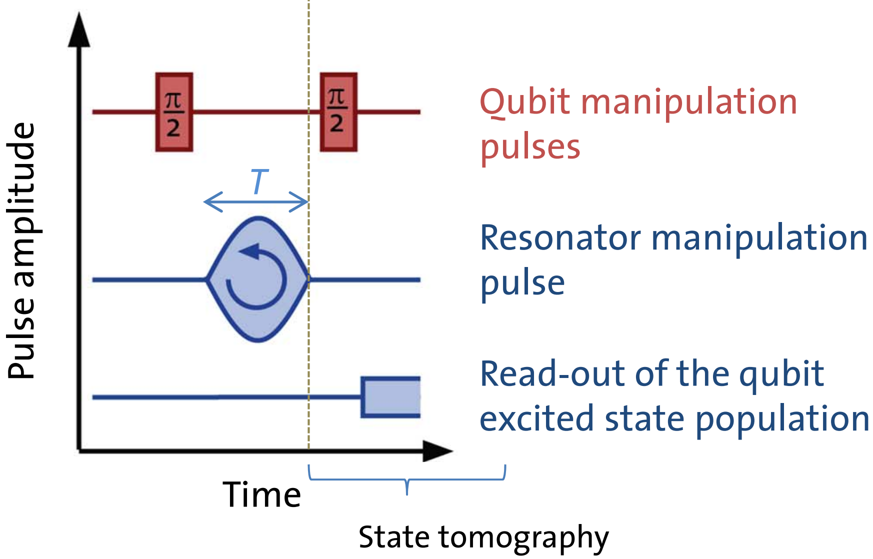
$$e^{i\gamma^{(g)}}|g0\rangle + e^{i\gamma^{(e)}}|e0\rangle$$



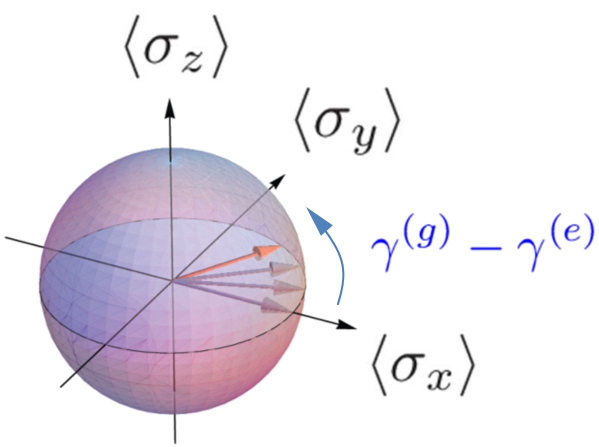
Measurement Procedure

Ramsey Interferometry

$$\mathrm{e}^{\mathrm{i}\gamma^{(g)}}|g0\rangle + \mathrm{e}^{\mathrm{i}\gamma^{(e)}}|e0\rangle$$

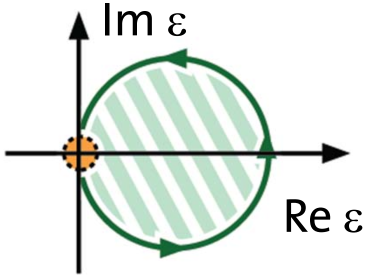


Qubit Bloch sphere

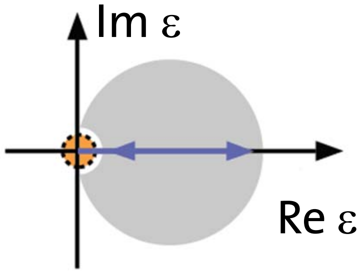


Experimental Results

Area-enclosing trajectory



Straight trajectory



amplitude modulation same as above

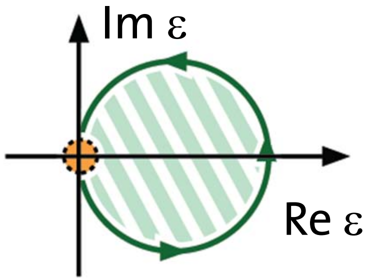
Equal dynamical phase

but

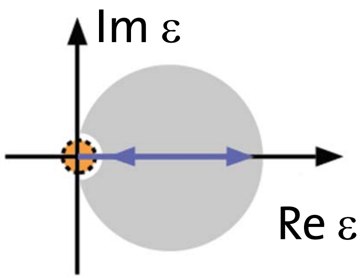
Geometric phase = 0

Experimental Results

Area-enclosing trajectory



Straight trajectory



amplitude modulation same as above

Equal dynamical phase

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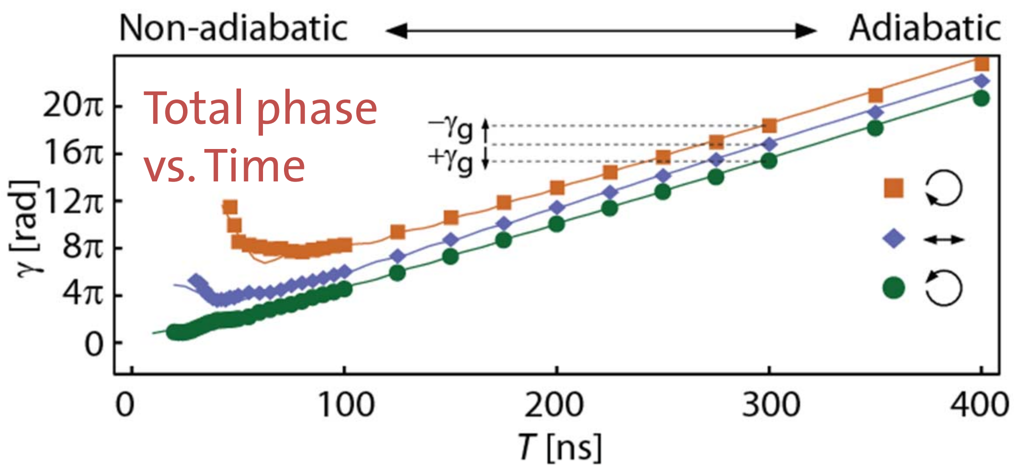
Geometric phase = 0



In the adiabatic limit: ($T \gg 1/\delta$)

- Dynamical phase linear in T

- Geometric phase = difference between phases for area-enclosing and straight paths

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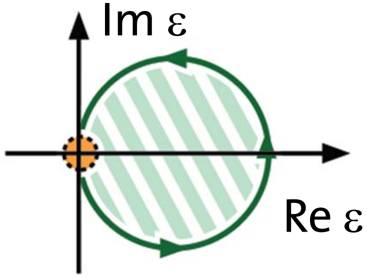
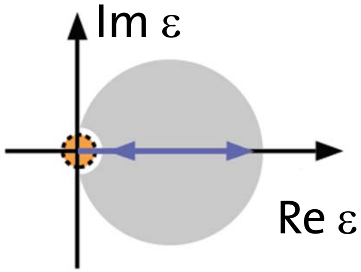
Experimental Results

Area-enclosing trajectory



Straight trajectory



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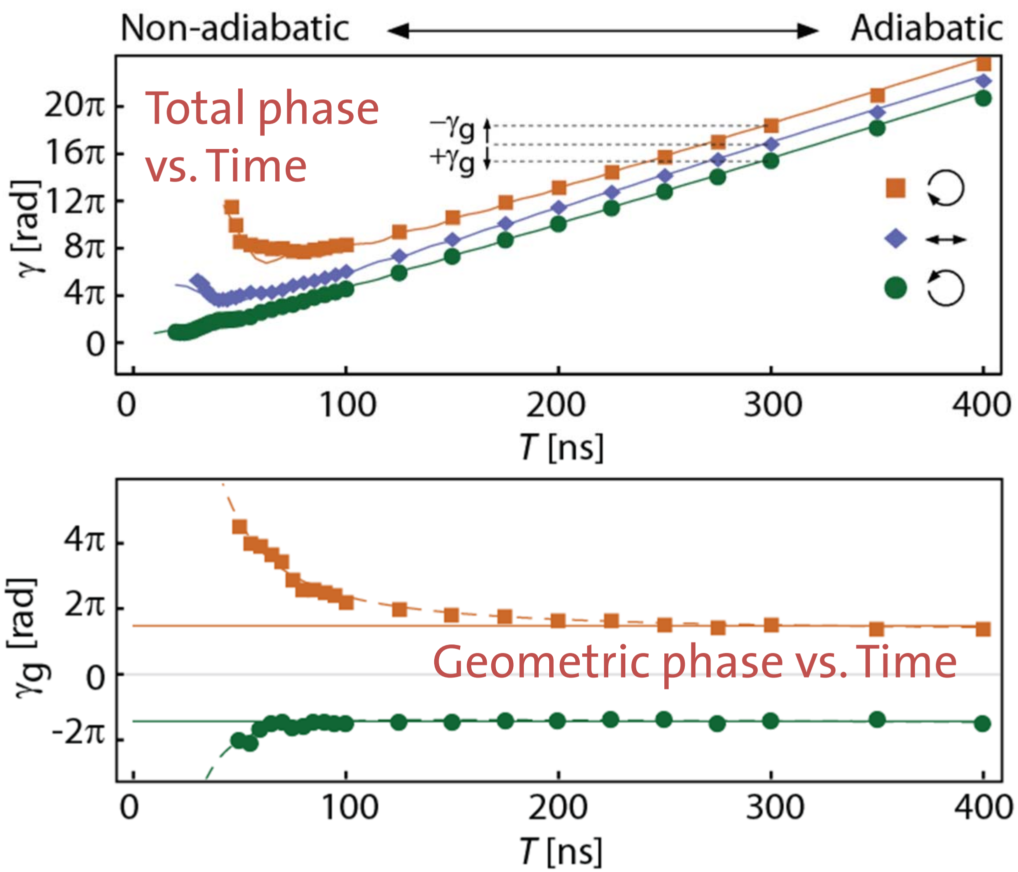
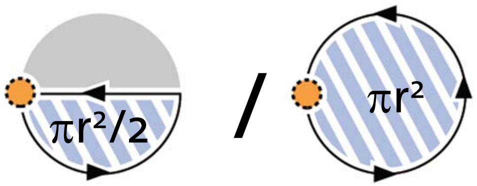
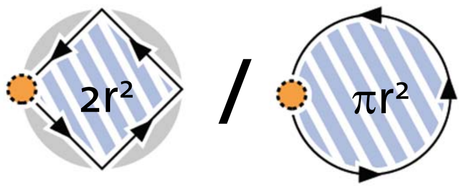
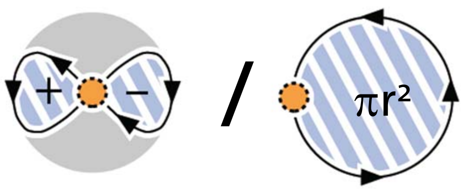


Experimental Results

Geometric phase for different shapes

	Theory	Experiment
	$= 0.5$	0.493 ± 0.016
	$= 2/\pi \approx 0.637$	0.647 ± 0.016
	$= 0$	0.00 ± 0.07

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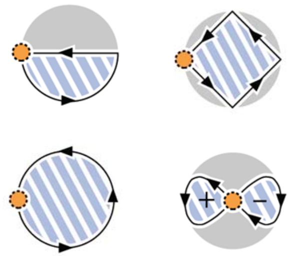
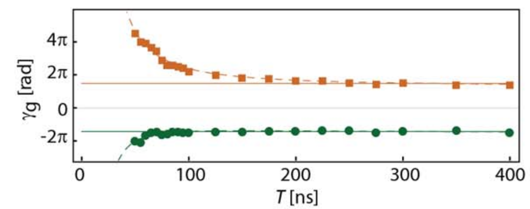
Conclusions

- Microwave circuitry gives us good control over the coherent state of the resonator
  - Scaling of the geometric phase with area
  - Independence of evolution time



Outlook



- Use the resonator as a tool to study
  - Effects of noise, Geometric phase in open systems
  - Possibility to measure two-qubit geometric phase
  - Gates using non-adiabatic geometric phase

Phys. Rev. Lett. 108, 170401 (2012)