Outline of lectures

- Brief introduction to optical communications.
- Basic classical physics of parametric devices.
- Some conventional applications of parametric devices.
- Basic quantum physics of parametric devices.
- Signals, noise and information in parametric links.
- Some novel applications of parametric devices.

Transition from classical to quantum optics

- Each frequency component of the field is a simple harmonic oscillator.
- Classical mode amplitude A $\rightarrow$ quantum mode operator a.
- Boson commutation relation (CR): $[a,a^+] = 1$, where $[a,b] = ab - ba$. F & B
- Quadrature operator $q = (ae^{-i\phi} + a^+e^{i\phi})/2^{1/2}$, number operator $n = a^+a$.
- Numbers (quadratures) are measured by direct (homodyne) detection. F & B
- Number states are eigenstates of the number operator: $a^+a|n\rangle = n|n\rangle$.
- Coherent states (CS) $|\alpha\rangle$ are eigenstates of the amplitude operator:

  $a|\alpha\rangle = \alpha|\alpha\rangle$ or $\langle\alpha|a^+ = \langle\alpha|\alpha^*$.
- Classical mode with amplitude $\alpha \rightarrow$ coherent state (CS) with parameter α.
- For a CS, $\langle q\rangle = \langle\alpha|q|\alpha\rangle = (\alpha e^{-i\phi} + \alpha^* e^{i\phi})/2^{1/2}$ and $\langle n\rangle = \langle\alpha|n|\alpha\rangle = |\alpha|^2$, like the classical mode with amplitude α.
- Where do quantum effects appear? Quadrature and number fluctuations!

[W. Louisell, RaNiQE (1964); R. Loudon, QToL (2000).]

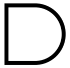
Operators and commutation relations

- Plane-wave function $f(z) = \exp(ikz)$, k = momentum.
- Notice that $-id_z f = kf$: k is the eigenvalue of the operator $-id_z$.
- $(zd_z - d_z z)f = -f$, so $[z,-id_z] = i$, where $[a,b] = ab - ba$.

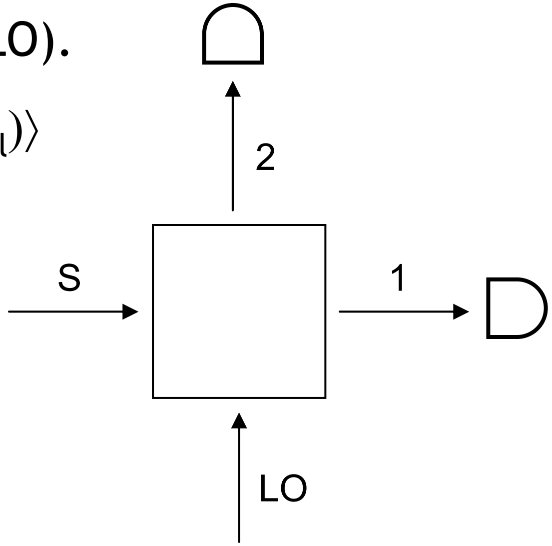
- Let q and p be conjugate operators, so $[q,p] = i$, and define $a = (q + ip)/\sqrt{2}$. Then $[a,a^+] = 1$: boson commutation relation (CR).
- Here, a is a mode-amplitude operator, and q and p are mode-quadrature operators (real and imaginary parts).

- Quantum phenomena, including noise, are consequences of the CR.

Direct and homodyne detection

- Number (quadrature) moments can be measured by direct (homodyne) detection.
- $\langle n_s \rangle$ is measured directly by a photon counter. 
- To measure $\langle q_s \rangle$, one uses a beam splitter to combine a_s and A_l, where l denotes the local oscillator (LO).
- $\langle n_1 \rangle - \langle n_2 \rangle = |A_l| \langle a_s \exp(-i\phi_l) + a_s^{+} \exp(i\phi_l) \rangle$

  $= \sqrt{2} |A_l| \langle q_s(\phi_l) \rangle$.



[R. Loudon, QToL (2000).]

Number states

- The number operator $n = a^+a$ and the eigenvalue equation is $a^+a|n\rangle = n|n\rangle$.

- What effect do a and a^+ have on the number state $|n\rangle$?
- $(a^+a)a|n\rangle = (aa^+ - 1)a|n\rangle = (n - 1)a|n\rangle$ and $\langle n|a^+a|n\rangle = n\langle n|n\rangle = n$. Hence, $a|n\rangle = n^{1/2}|n-1\rangle$.
- $(a^+a)a^+|n\rangle = a^+(a^+a + 1)|n\rangle = (n + 1)a^+|n\rangle$ and $\langle n|aa^+|n\rangle = \langle n|(a^+a + 1)|n\rangle = (n + 1)\langle n|n\rangle = n + 1$. Hence, $a^+|n\rangle = (n + 1)^{1/2}|n+1\rangle$.
- a is the lowering (destruction) operator and a^+ is the raising (creation) operator.

- Number states do not support mean fields: $\langle n|a|n\rangle = n^{1/2}\langle n|n-1\rangle = 0$.

Coherent states and quantum fluctuations

- Coherent states (CS) $|\alpha\rangle$ are eigenstates of the amplitude operator:

  $a|\alpha\rangle = \alpha|\alpha\rangle$ or $\langle\alpha|a^+ = \langle\alpha|\alpha^*$.
- Coherent states do support mean fields: $\langle\alpha|a|\alpha\rangle = \alpha\langle\alpha|\alpha\rangle = \alpha$.
- It is easy to check that $|\alpha\rangle = \exp(-|\alpha|^2/2)\Sigma_n\alpha^n|n\rangle/(n!)^{1/2}$.
- For the vacuum state $|0\rangle$, $a|0\rangle = 0$ and $\langle 0|a^+ = 0$.

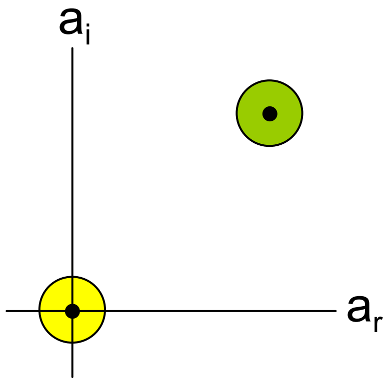
  Hence, $\langle q\rangle = 0$ and $\langle\delta q^2\rangle = \langle q^2\rangle - 0$.

  $\langle q^2\rangle = \langle 0|a^2e^{-i2\phi} + aa^+ + a^+a + (a^+)^2e^{i2\phi}|0\rangle/2$

  $= \langle 0|a^2e^{-i2\phi} + (a^+a + 1) + a^+a + (a^+)^2e^{i2\phi}|0\rangle/2$

  $= 1/2$ (independent of ϕ).

  In a similar way, $\langle n\rangle = 0$ and $\langle\delta n^2\rangle = 0$.



- For the CS $|\alpha\rangle$, $\langle\delta q^2\rangle = 1/2$. For other states, $\langle\delta q^2\rangle \geq 1/2$.
- For this CS, $\langle n\rangle = |\alpha|^2$ and $\langle\delta n^2\rangle = |\alpha|^2 = \langle n\rangle$: Shot noise!

  [W. Louisell, RaNiQE (1964); R. Loudon, QToL (2000).]

Quadrature distributions of coherent states

- We know that the variance $\langle \delta q^2 \rangle = 1/2$. What is the distribution?
- CS $|\alpha\rangle$ is defined by the equation $a|\alpha\rangle = \alpha|\alpha\rangle$, where $a = (q + ip)/\sqrt{2}$.
- $(q + ip)|\alpha\rangle = \sqrt{2}\alpha|\alpha\rangle \leftrightarrow (q' + d/dq')\psi_\alpha = \sqrt{2}\alpha\psi_\alpha$ $(p \leftrightarrow -id/dq')$.
- Vacuum state: $d\psi_\alpha/dq' = -q'\psi_\alpha$, so $\psi_0(q') = \pi^{-1/4} \exp[-(q')^2/2]$.
- In general, $d\psi_\alpha/\psi_\alpha = \sqrt{2}(\alpha - q')dq'$, so $\ln\psi_\alpha = \sqrt{2}[\alpha q' - (q')^2/2]$.
- Coherent state: $\psi_\alpha(q') = \pi^{-1/4} \exp[i\langle p\rangle q' - (q' - \langle q\rangle)^2/2]$.
- The quadrature fluctuations have Gaussian statistics (for any LO phase)!

[W. Louisell, RaNiQE (1964).]

Semi-classical model of coherent states

- In the semi-classical (SC) model, $A \rightarrow \alpha + \delta\alpha$, where α is a number and $\delta\alpha$ is a Gaussian random variable with $\langle\delta\alpha\rangle = 0$, $\langle\delta\alpha^2\rangle = 0$ and $\langle|\delta\alpha|^2\rangle = 1/2$.

- $\langle Q(\phi)\rangle = (\alpha e^{-i\phi} + \alpha^* e^{i\phi})/2^{1/2} = \langle q(\phi)\rangle$ and $\langle\delta Q^2\rangle = 1/2 = \langle\delta q^2\rangle$: Correct!

- $\langle N\rangle = |\alpha|^2 + 1/2 \approx \langle n\rangle$ and $\langle\delta N^2\rangle \approx |\alpha|^2 + 1/4 \approx \langle n\rangle$: Almost correct!

- The SC method (half-photon per mode rule) predicts the quadrature variance exactly and the number variance accurately.

[W. Louisell, Phys. Rev. **124**, 1646 (1961); J. Gordon, Phys. Rev. **129**, 481 (1963).]

Classical simple harmonic oscillator

- For a simple harmonic oscillator, the Hamiltonian $H = (Q^2 + P^2)/2$, where Q and P are conjugate variables.
- The Hamilton equations are $d_tQ = dH/dP = P$, $d_tP = -dH/dQ = -Q$.
- Periodic evolution: $Q(t) = Q(0)\cos t + P(0)\sin t$, $P(t) = -Q(0)\sin t + P(0)\cos t$.

- Define the complex amplitude $A = (Q + iP)/\sqrt{2}$, so that $Q = (A + A^*)/\sqrt{2}$ and $P = (A - A^*)/\sqrt{2}i$.
- Then the Hamiltonian $H = A^*A$ and the Hamilton equation $d_tA = -idH/dA^*$.

  Check: $d_tA = (P - iQ)/\sqrt{2} = -iA = -idH/dA^*$.
- Periodic evolution: $A(t) = A(0)\exp(-it)$.

- This relationship is always true. Let $H(Q,P) = H(A,A^*)$ be arbitrary. Then

  $\sqrt{2}d_tA = d_tQ + id_tP = dH/dP - idH/dQ$

  $= (idH/dA - idH/dA^*)/\sqrt{2} - i(dH/dA + dH/dA^*)/\sqrt{2} = -i\sqrt{2}dH/dA^*$.

Heisenberg and Schrodinger pictures of QM evolution

- The Hamilton equation is $d_t A = -i\partial H/\partial A^*$, where $H(A,A^*)$ is the Hamiltonian.
- In the Schrodinger picture, operators a_s are constant and the state vector $|\psi\rangle$ evolves according to the Schrodinger equation $d_t|\psi\rangle = -iH_s|\psi\rangle$, where the Hamiltonian $H_s = H(A \to a_s)$.
- Let $d_t U = -iH_s U$, so the unitary operator $U(t) = \exp(-iH_s t)U(0)$. Then $|\psi(t)\rangle = U(t)|\psi(0)\rangle$. Notice that $U^+U = 1 = UU^+$.
- The expectation value $\langle a_s\rangle = \langle\psi(t)|a_s|\psi(t)\rangle = \langle\psi(0)|U^+(t)a_s U(t)|\psi(0)\rangle$.
- In the Heisenberg picture, operators a_h evolve and the state vector $|\psi\rangle$ is constant.
- Let $a_h = U^+(t)a_s U(t)$, so $\langle a_h\rangle = \langle a_s\rangle$ by construction.
- For any moment $m(a)$, $\langle\psi(t)|m(a_s)|\psi(t)\rangle = \langle\psi(0)|m[a_h(t)]|\psi(0)\rangle$.
- The Heisenberg equation is $d_t a_h = -i[a_h,H_h]$, where $H_h = U^+H_s U = H_s(a_s \to a_h)$.
- For spatial evolution, $d_t \to -d_z$ + renormalization.

Quantum simple harmonic oscillator

- In the Heisenberg picture the operators evolve in time.

- $A \rightarrow a$ and $A^* \rightarrow a^+$, where $[a,a^+] = 1$.
- $H(A,A^*) \rightarrow H(a,a^+)$, so $H = a^+a$, and $dH/dA^* \rightarrow [a,H]$.
- $d_t a = -i[a,H] = -i[aa^+a - a^+a^2] = -i[(a^+a + 1)a - a^+a^2] = -ia$.
- Hence, $a(t) = a(0)\exp(-it)$.

- The Heisenberg operators evolve in the same way as the classical variables (quadratic Hamiltonian).

- This quantization procedure is equivalent to the replacement of the Poisson bracket by the commutator.

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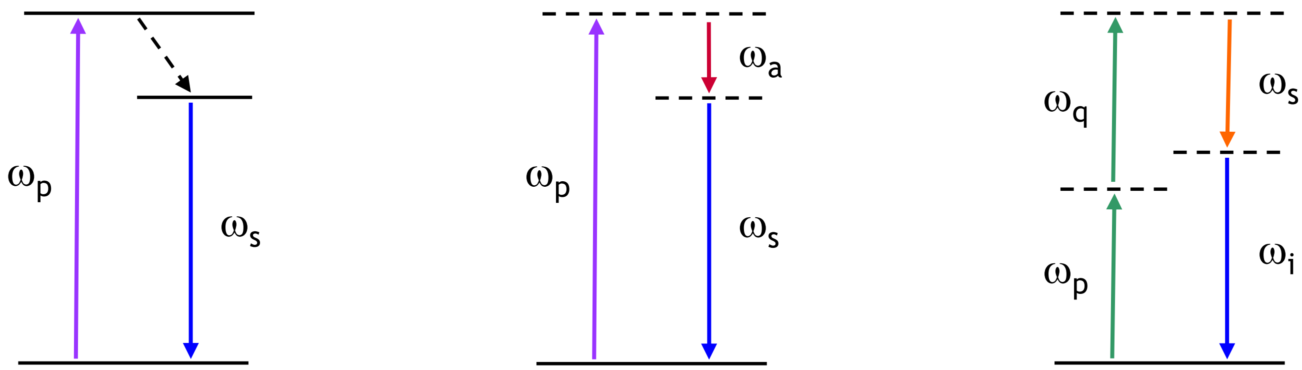
Erbium-doped, Raman and parametric amplification



- In erbium-doped fiber amplifers (EDFAs), each light mode interacts with an excited electron.
- In Raman fiber amplifiers (RFAs), each light mode interacts with a vibration mode (optical phonon).
- In parametric fiber amplifiers (PAs), each signal light mode interacts with another light mode (idler). This interaction is enabled by one or two pump waves, and is called four-wave mixing (FWM).
- In linear QM, each process involves two modes (all modes are equal).

[P. Becker, EDFA (Academic, 1999), C. Headley, RAiFOCS (Elsevier, 2004); M. Marhic, FOPA (Cambridge, 2007).]

Classical and quantal models of parametric processes

- Classical MI or PC (2-mode amplification).
- $H_c = \delta(|A_s|^2 + |A_i|^2) + \gamma A_s^* A_i^* + \gamma^* A_s A_i$.
- H_c is quadratic in the mode amplitudes.
- $d_z A_j = i\partial H_c/\partial A_j^*$ ($d_t \to -d_z$).
- $d_z A_s = i\delta A_s + i\gamma A_i^*$.
- CMEs are linear in the mode amplitudes.
- $A_s(z) = \mu(z)A_s(0) + \nu(z)A_i^*(0)$.
- $|\mu|^2 - |\nu|^2 = 1$ (conserves action flux).
- SC theory of noise: $\langle|\delta\alpha_j|^2\rangle = 1/2$.
- Linear combinations of GRVs are GRVs.

- Quantal MI or PC (2-mode squeezing).
- $H_h = \delta(a_s^+ a_s + a_i^+ a_i) + \gamma a_s^+ a_i^+ + \gamma^* a_s a_i$.
- $[a_j, a_k^+] = \delta_{jk}$.
- $H_h = H_c(A_j \to a_j)$.
- $d_z a_j = i[a_j, H_h]$.
- $d_z a_s = i\delta a_s + i\gamma a_i^+$.
- CMEs are linear in the mode operators.
- $a_s(z) = \mu(z)a_s(0) + \nu(z)a_i^+(0)$.
- $|\mu|^2 - |\nu|^2 = 1$ (conserves probability).
- QM transfer functions same as CM.
- QM theory of detection: $\langle 0|a_j^+ a_j|0\rangle = 0$, $\langle 0|a_j a_j^+|0\rangle = 1$.
- Output quadrature distributions are G.

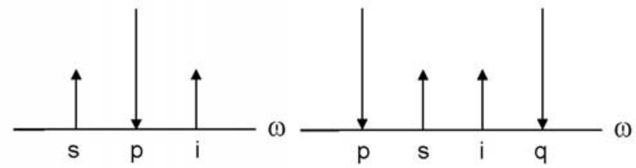
[W. Louisell, RaNiQE (1964); R. Loudon, QToL (2000).]

Noise-figure calculations

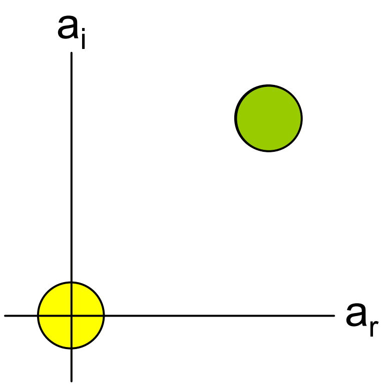
- The output quadrature and number moments can be calculated.
  - Rewrite the output operators in terms of the input operators.
  - Use the BCR ($a_j a_j^+ = a_j^+ a_j + 1$) to rewrite the operator products in normal form (a^+ before a). Extra terms appear ($\langle q^2 \rangle$ calculation).
  - For coherent-state inputs $\langle (a_j^+)^m a_k^n \rangle = (\alpha_j^*)^m \alpha_k^n$, like classical inputs ($\alpha_j = \langle a_j \rangle$).
  - The extra terms are the quantum noise terms.
- For direct detection the signal-to-noise ratio $S = \langle n \rangle^2 / \langle \delta n^2 \rangle$.
- For homodyne detection the signal-to-noise ratio $S = \langle q \rangle^2 / \langle \delta q^2 \rangle$.
- The noise figure $F = S(0)/S(z)$ is a figure of demerit.

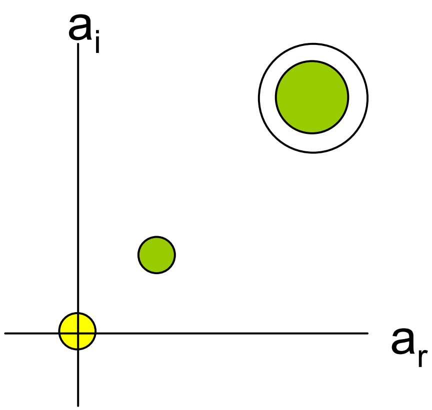
[W. Louisell, RaNiQE (1964); R. Loudon, QToL (2000).]

Noise figures of MI and PC



- IO relations: $a_s(z) = \mu(z)a_s(0) + \nu(z)a_i^+(0)$, $a_i(z) = \mu(z)a_i(0) + \nu(z)a_s^+(0)$.
- MI and PC are PI if $\langle a_i(0)\rangle = 0$. Both signal quadratures are amplified.

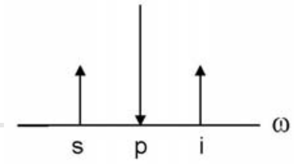




- For homodyne detection $S = \langle q\rangle^2/\langle \delta q^2\rangle$. The noise figure $F = S(0)/S(z)$.
- $F = 1 + (G-1)/G$, where $G = |\mu|^2$ is the PI gain.
- In the high-gain regime, F = 2 (3 dB). The signal amplitude is amplified, as are its fluctuations and the idler fluctuations.
- The SC model predicts the NFs accurately!

[W. Louisell, PR **124**, 1646 (1961); C. McKinstrie, OE **12**, 5037 (2004) and **13**, 4986 (2005).]

Experimental results



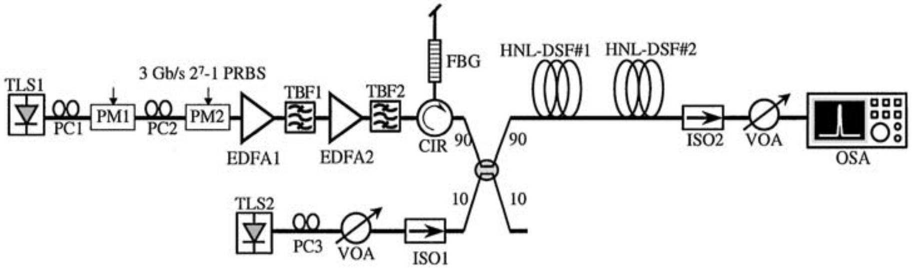


Fig. 1. Parametric wavelength converter.

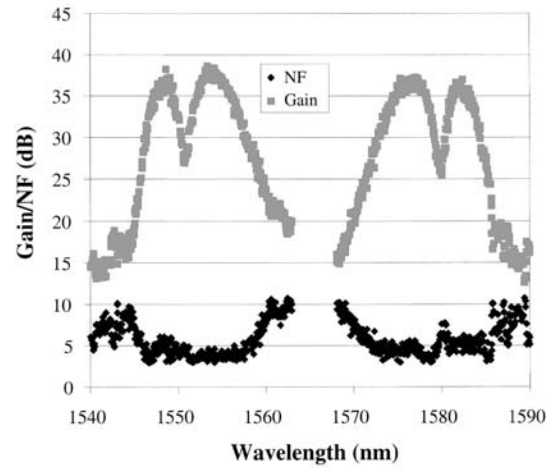
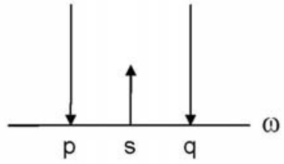


Fig. 2. Conversion gain spectrum and NF of a two-segment OPA at 600-mW pump power with $\lambda_p = 1565.5$ nm.

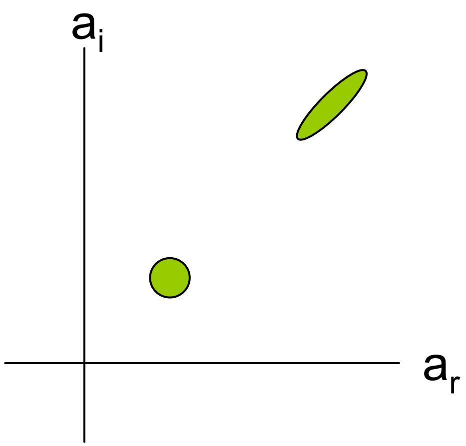
- Many groups have measured NFs of 3 – 4 dB.
- Extra noise due to spontaneous Raman scattering and pump-noise (induced by the erbium booster amplifiers).
- Extra 1 dB is not a show stopper.

[J. Blows, OL **27**, 491 (2002); K. Wong, OL **28**, 692 (2003), R. Tang, OL **29**, 2372 (2004), P. Kylemark, JLT **22**, 409 (2004) & **23**, 2192 (2005); Z. Tong, OE **18**, 2884 (2010).]

Noise figure of inverse MI



- IO relation: $a_s(z) = \mu(z)a_s(0) + \nu(z)a_s^+(0)$.
- Inverse MI is always PS.
- If $q(\phi)$ is amplified, $q(\phi+\pi/2)$ is de-amplified.



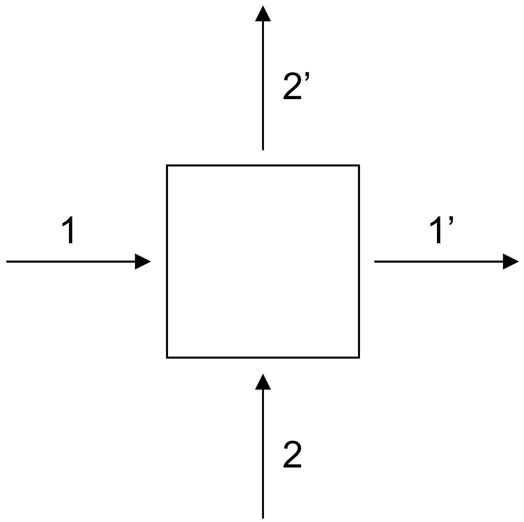
- For homodyne detection $S = \langle q^2\rangle/\langle\delta q^2\rangle$.
- The noise figure $F = S(0)/S(z)$.

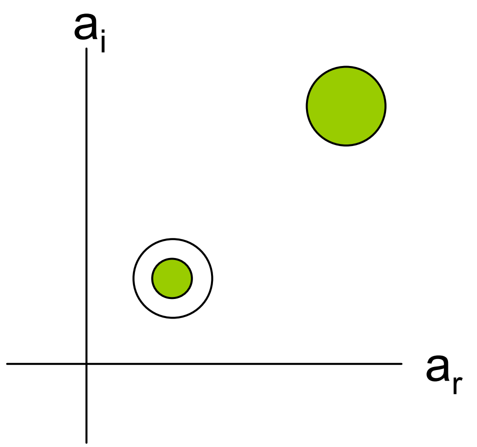
- The noise figures depend on the pump, signal and LO phases.
- For an in-phase (out-of-phase) signal, F = 1 (0 dB). The signal amplitude and its fluctuations are amplified (de-amplified) by the same amount.
- The SC model predicts the NFs accurately!

[R. Loudon, QToL (2000), C. McKinstrie, OE **13**, 4986 (2005) and OC **257**, 146 (2006).]

Noise figure of beam splitting and loss

- Beam splitting (BS) is a real process and a model for fiber loss.

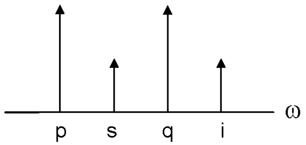


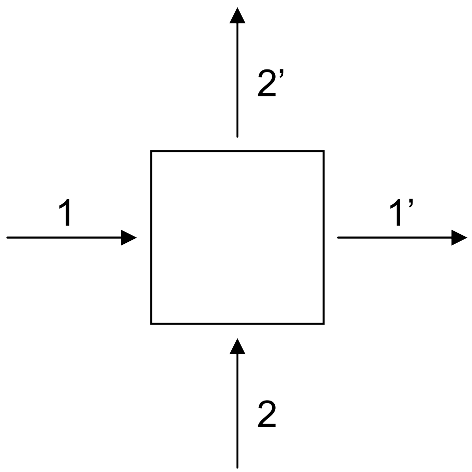


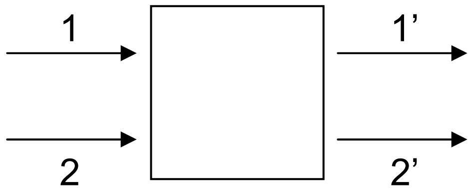
- The IO relations are $a_1' = ta_1 + ira_2$ and $a_2' = ira_1' + ta_2$ (transmission, reflection).
- The identity $|t|^2 + |r|^2 = 1$ reflects power conservation.
- BS adds quantum uncertainty (but not photons) to maintain $\langle \delta q^2 \rangle \geq 1/2$ (HUP).
- For both types of detection, $F = 1/|t|^2 = L$ (weaker signal, same q-noise).
- SC model: $A_2 \rightarrow 0 + \delta\alpha_2$, where $|\delta\alpha_2|^2 = 1/2$: Remember to add vacuum noise!

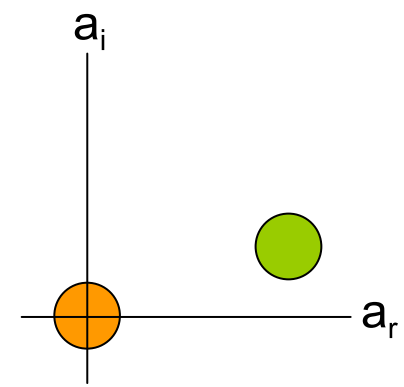
[R. Loudon, QToL (2000); C. McKinstrie, OE **12**, 5037 (2004) and **13**, 4986 (2005).]

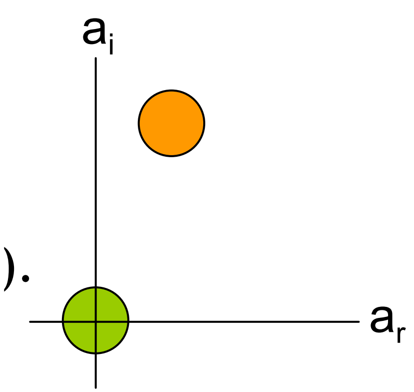
BS converts signals without adding excess noise









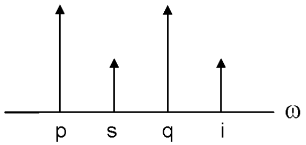


- Frequency conversion by BS is modeled by the same input-output relations as a beam-splitter:

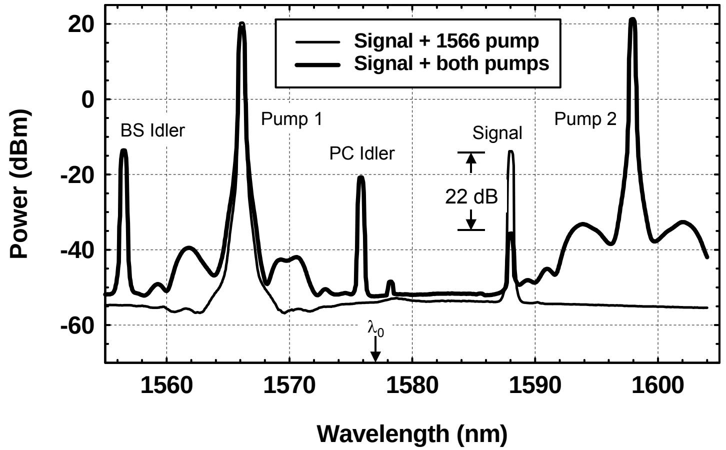
  $a_1(z) = \tau(z)a_1(0) + \rho(z)a_2(0)$, $a_2(z) = -\rho^*(z)a_1(0) + \tau^*(z)a_2(0)$.
- For complete conversion, $|\rho(z)| = 1$, so $a_2(z) \approx a_1(0)$!
- BS converts signals (generates idlers) without adding excess noise.
- SNR of the BS idler was measured to be 3-dB higher than the PC idler.

[C. McKinstrie, Opt. Express **13**, 9131 (2005); A. Gnauck, Opt. Express **14**, 8989 (2006).]

Low-noise FC was demonstrated



- The experiment involved a highly-nonlinear fiber (HNF). Pumps at 1566 and 1598 nm were used to convert a signal at 1588 nm to a BS idler at 1556 nm (32-nm shift), with an efficiency of 0.99.



	Gain (dB)	OSNR w/o polarizer	OSNR w polarizer
BS	0	45.0	47.0
PC	0	42.2	42.7
PC	10	42.8	42.9
EDFA	20	41.5	44.7

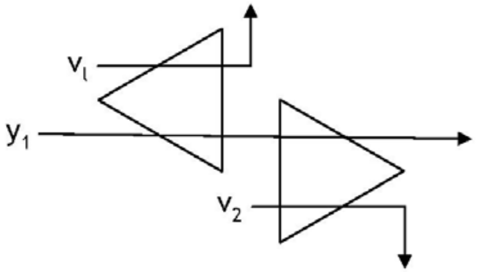
- The output signal-to-noise ratio (SNR) of the BS idler was about 3 dB higher than the SNRs of a PC idler and an erbium-amplified signal.

[A. Gnauck, Opt. Express **14**, 8989 (2006).]

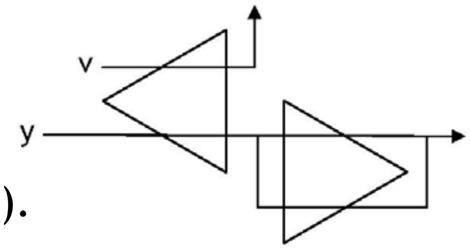
Noise figures of single- and multiple-stage links

- Transmission links are sequences of attenuators followed by amplifiers.
- L = attenuator (fiber) loss, G = PI amplifier gain.

- For one stage of loss and PI gain, $F = (2G - 1)L/G$.
- If the stage is balanced ($G = L$), then $F = 2L - 1$.
- In the high-loss regime, $F \approx 2L$.
- For an s-stage link, $F = 1 + 2s(L - 1) \approx 2sL$.
- Half the noise figure comes from loss and half comes from gain.



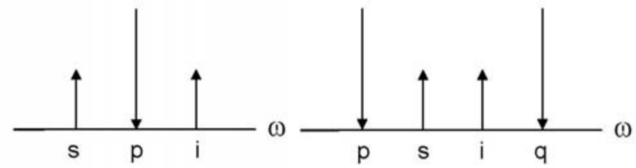
- For a balanced PS link and an in-phase signal, $F = L$.
- For an s-stage link, $F = 1 + s(L-1) \approx sL$.
- The PS noise figure is 3-dB lower than the PI link (not 3s-dB).
- Loss also reduces squeezing, because it adds noise isotropically.



- Can one do better? Yes, by using two-mode PS amplification ($F \approx sL/2$)!

[R. Loudon, JQE **21**, 766 (1985); C. McKinstrie, OE **13**, 4986 (2005); Z. Tong, OE **18**, 15426 (2010).]

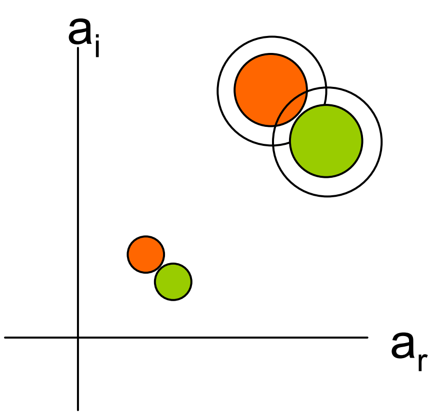
Two-mode PS amplification



- 2-mode PS amplification is produced by MI or PC if both inputs are nonzero.

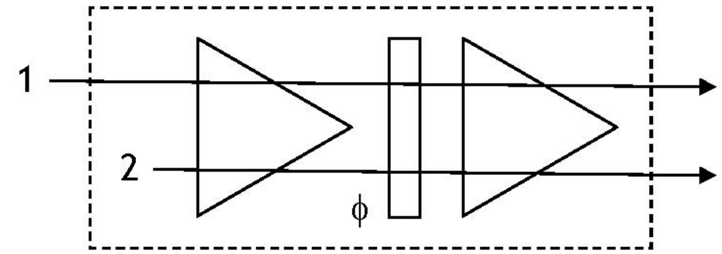
  $a_s(z) = \mu(z)a_s(0) + \nu(z)a_i^+(0)$, $a_i(z) = \mu(z)a_i(0) + \nu(z)a_s^+(0)$.

- For equal inputs $\alpha(0)$, output $\alpha(z) = \mu\alpha(0) + \nu\alpha^*(0) \rightarrow 2G^{1/2}\,|\alpha(0)|$.
- Power is amplified by 4G, noise is amplified by 2G, so the noise figure (NF) of a 2-mode PSA is 0.5 (-3 dB)! [NF is 1 (0 dB) based on total input power.]
- Result assumes pre-existing idler (non-standard).
- Use (standard) 1-input MI (PC) to generate the idler, a signal processor to control the sideband phases and 2-input MI (PC) to provide PS amplification.



[C. McKinstrie, Opt. Express **13**, 4986 (2005); M. Vasilyev, Opt. Express **13**, 7563 (2005); R. Tang, Opt. Express **13**, 10483 (2005); J. Kakande, Opt. Express **18**, 4130 (2010).]

Cascaded parametric amplification (1)



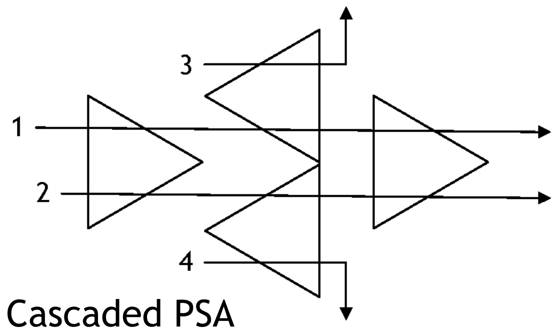
- An ideal CPA involves 2 (signal and idler) modes.
- Phase controller (ϕ) ensures that the mode amplitudes add constructively.
- The signal NF is $(2H_0 - 1)/H_0 = 1 + (H_0 - 1)/H_0$, where

  $$H_0 = G_2G_1 + (G_2 - 1)(G_1 - 1) + 2[G_2(G_2 - 1)G_1(G_1 - 1)]^{1/2}$$

  is the composite in-phase gain, G_1 and G_2 are the individual PI gains.
- $1 < F < 2$, just like a standard PI amplifier. Why?
- The first PA makes the CPA operate in a PI manner. 🙂
- PI operation ensures that $F > 1$ (0 dB). 🙁

[Z. Tong, Opt. Express **18**, 14820 (2010); C. McKinstrie, Opt. Express **18**, 19792 (2010).]

Cascaded parametric amplification (2)



Cascaded PSA

- The simplest CP link involves 4 modes (signal, idler and 2 loss modes).
- General formulas exist for the NFs of multiple-mode devices, for which

$$a_j(z) = \Sigma_k[\mu_{jk}(z)a_k(0) + \nu_{jk}(z)a_k^+(0)].$$

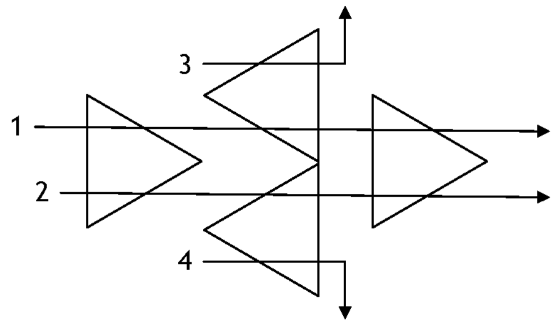
- The signal NF is $[(2H_0 - 1)T + (2G_2 - 1)(1 - T)]/H_0T$, where

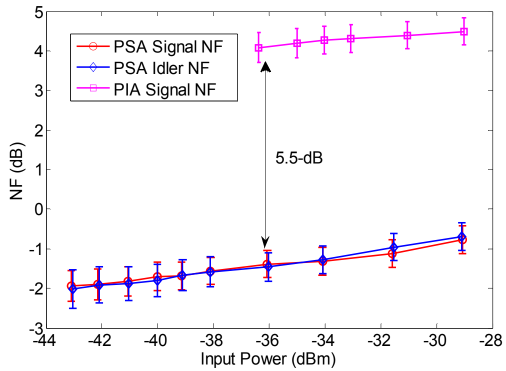
$$H_0 = G_2G_1 + (G_2 - 1)(G_1 - 1) + 2[G_2(G_2 - 1)G_1(G_1 - 1)]^{1/2}$$

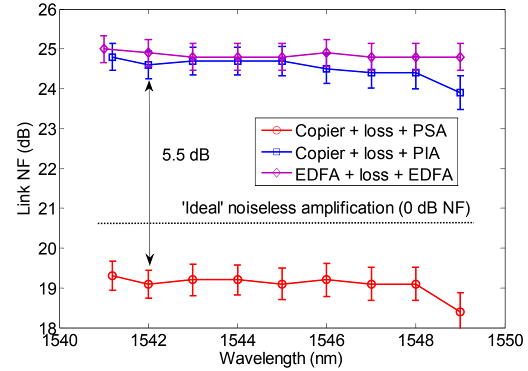
is composite gain, G_1 and G_2 are individual gains and T is transmission (loss).
- Recall that the NF of a balanced PI link is 2L – 1, where L = 1/T.
- In the high-loss regime, the NF of a CP link ≈ L/2 (6-dB reduction)! ☺

[C. McKinstrie and Z. Tong, Opt. Express **13**, 4986 (2005); **18**, 14820 (2010); **18**, 19792 (2010).]

Fiber link with two-mode PS amplification







- A parametric amplifier with one input is PI, and has a NF of 3 dB (excess noise from idler vacuum fluctuations), but an amplifier with two inputs is PS and has a NF of -3 dB (signals add coherently, but fluctuations add incoherently): 6-dB reduction!
- The first amplifier augments the signal (1) and generates an idler (2). Both modes are attenuated by a transmission fiber (3, 4 are loss modes). The second amplifier is PS.
- For a balanced PI link the NF ≈ 2L, where L = 1/T is the loss.
- For the PS link the NF ≈ L/2: 6-dB reduction!
- In recent experiments, 5.5-dB NF reductions were observed!
- PI copier plus PS amplifier operates in a format-independent manner!

[Z. Tong, Opt. Express **18**, 15426 (2010) and Nat. Photon. **5**, 430 (2011).]

Entropy and information

- Discrete symbols (variables) $x_i \in X$: $H(X) = -\Sigma_i \, p_i \ln p_i$, p_i is probability of x_i.
- H is the average of the individual uncertainties ($-\ln p_i$).
- High *a priori* uncertainty $\leftrightarrow$ the high potential information content.
- If each $p_i = 1/w$ (word length), then $H(X) = \ln w$.

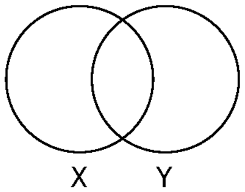
- Continuous variable $x \in X$: $H(X) = \int p(x) \ln[p(x)] \, dx$.
- For a Gaussian distribution with variance $\langle x^2 \rangle$, $H(X) = \ln(2\pi e \langle x^2 \rangle)^{1/2}$.
- If $\langle x^2 \rangle$ is specified (average signal power), the associated Gaussian distribution has maximal entropy.

- Base-2 logarithms $\rightarrow$ bits, base-e $\rightarrow$ nats.

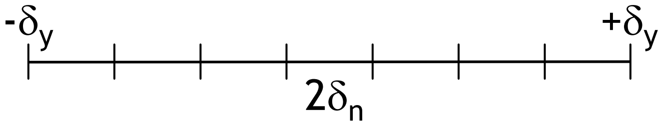
[C. Shannon, BSTJ **28**, 379 and 623 (1948), Proc. IRE **37**, 10 (1949); T. Cover, EoIT (Wiley, 2006); E. Desurvire, CaQIT (Cambridge, 2009).]

Mutual information

- Real signals are always corrupted by noise (QM).
- Model noise addition as an IO process: ideal signal $x \to$ real signal y.
- The mutual entropy (shared information) $H(X{:}Y) = H(Y) - H(Y|X)$ is the extra entropy associated with the signal.
- For additive noise, $y = x + n$ and $H(Y|X) = H(N)$.



- N is Gaussian (QM), Gaussian X (and Y) maximizes the information.
- Shannon: $H(X{:}Y) = \ln[2\pi e\langle y^2\rangle]^{1/2} - \ln[2\pi e\langle n^2\rangle]^{1/2} = \ln(1 + \langle x^2\rangle/\langle n^2\rangle)^{1/2}$.
- $(1 + \langle x^2\rangle/\langle n^2\rangle)^{1/2} = \delta_y/\delta_n$ = number of distinguishable signals (word length).



- H depends logarithmically on the SNR.

[C. Shannon, BSTJ **28**, 623 (1948) and Proc. IRE **37**, 10 (1949).]

Multiple input modes

- Let $X = [x_i]^t$ be a signal vector, $N = [n_i]^t$ be a noise vector and $Y = X + N$.
- The distributions are specified by the covariance matrices $K_x = [\langle x_i x_j \rangle]$, $K_n = [\langle n_i n_j \rangle]$, and $K_y = K_x + K_n$.
- Then $H(X{:}Y) = \ln(\Delta_y/\Delta_n)^{1/2}$, where $\Delta_j = \det(K_j)$. (Hint: Trivial if K_j diagonal.)
- Example: Two input modes with independent noise.
- $H(X{:}Y) = \ln[(1 + \sigma_{11}/\sigma_n)(1 + \sigma_{22}/\sigma_n) - (\sigma_{12}/\sigma_n)^2]^{1/2}$, where the strength parameters $\sigma_{ij} = \langle x_i x_j \rangle$ and $\sigma_n = \langle n_i^2 \rangle$.
- For specified σ_{11} and σ_{22}, $H(X{:}Y)$ decreases as σ_{12} increases.
- To maximize the mutual information, use independent signals, in which case $H_t = H_1 + H_2$. In general, $H_t = \Sigma_i H_i$, which depends linearly on the DoFs.
- The information capacity (b/s) $C = FH(X{:}Y)$, F = positive signal bandwidth, and the spectral efficiency (b/s-Hz) $S = C/B$, B = total positive bandwidth.

[C. Shannon, BSTJ **28**, 623 (1948) and Proc. IRE **37**, 10 (1949); C. McKinstrie, J. Sel. Top. Quantum Electron. **18**, 794 (2012).]

Spectral efficiencies of multiple-stage links

- In erbium-doped and Raman fiber amplifiers, each light mode interacts with a material mode (1-light-mode devices).
- In parametric amplifiers (PAs), each signal mode interacts with an idler. However, in phase-insensitive (PI) operation, the output idlers are discarded (effectively 1-light-mode devices).
- $S_q = \ln(1 + \sigma_x/\sigma_n)^{1/2}$, where σ_x and σ_n are the signal and noise strengths.
- For a balanced link, the argument of S is decreased by the NF of the link.
- Standard PI link: NF = 1 + 2s(L – 1), both quadratures are transmitted (×2).
- 1-mode PS link: NF = 1 + s(L – 1), only the in-phase quadrature is transmitted (×1): No net improvement (linear beats logarithmic)!
- Copier plus 2-mode PS link: NF ≈ sL/2, both quadratures are transmitted: Potential improvement of 2 b/s-Hz. Requires dark idler bandwidth!

[E. Desurvire, Opt. Lett. **25**, 701 (2000); C. McKinstrie, Opt. Express **13**, 4896 (2005); Z. Tong, Opt. Express **18**, 15426 (2010); C. McKinstrie, Opt. Express **19**, 11977 (2011).]

Quantum information

- The density matrix $\rho = |\psi\rangle\langle\psi|$ or $\Sigma_i\, p_i|\psi_i\rangle\langle\psi_i|$.
- For any operator o, $\langle\psi|o|\psi\rangle = \mathrm{Tr}(\rho o)$, because $\mathrm{Tr}(|p\rangle\langle q|) = \langle q|p\rangle$.
- Von Neumann entropy $S = -\mathrm{Tr}(\rho\ln\rho)$, $\ln\rho$ is defined by its power series.
- Example: If $|\psi\rangle = \Sigma_n\, c_n|n\rangle$, then $S = -\Sigma_n\, |c_n|^2 \ln(|c_n|^2)$, like the entropy H.
- Gordon-Kholevo information $I = S[\rho(X)] - \Sigma_x\, p_x S[\rho(x)]$, where $\rho(X) = \Sigma_x\, p_x\rho(x)$, and $x \in X$ is a signal.
- Like the mutual information $H(Y) - H(Y|X) = H(Y) - \Sigma_x\, p_x H(y|x)$.

[C. Bennett, Trans. Inform. Theory **44**, 2724 (1998), M. Nielsen, QCaQI (Cambridge, 2000), E. Desurvire, CaQIT (2009).]

Summary 2

- Conventional systems use coherent-state (CS) signals, which are like classical signals with intrinsic amplitude fluctuations.
- The input (CS) and output (after transmission through a sequence of amplifiers and attenuators) fluctuations have Gaussian statistics.
- The SC model (half-photon per mode of Gaussian fluctuations) predicts the quadrature and number fluctuations (variances and correlations) accurately.
- Standard links use 2-mode PI amplifiers (linear devices). Simple formulas exist for their noise figures and information capacities.
- Systems with 1-mode PS amplifiers have lower noise figures and lower information capacities (only in-phase quadratures are transmitted)!
- Systems with 2-mode PS amplifiers have lower noise figures and higher information capacities (but requires idler bandwidth)!
- Fiber nonlinearities reduce the link capacities (by increasing fluctuations).

[C. McKinstrie, OE **19**, 11977 (2011); JSTQE **18** 794 and 958 (2012).]

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List of acronyms 1

- ASK = amplitude-shift keying
- BS = Bragg scattering
- CM = classical mechanics
- CME = coupled-mode equation
- CW = continuous wave
- CR = commutation relation
- DCF = dispersion-compensating fiber
- DSF = dispersion-shifter fiber
- DPSK = differential phase-shift keying
- EDFA = erbium-doped fiber amplifier
- FC = frequency conversion
- FM = frequency matching
- FS = four-sideband
- FWM = four-wave mixing
- HNF = highly-nonlinear fiber
- HP = horizontally polarized
- IO = input-output
- LCP = left-circularly polarized
- LO = local oscillator
- MSF = micro-structured fiber
- MRW = Manley-Rowe-Weiss
- NSE = nonlinear Schrodinger equation
- PA = parametric amplifier
- PC = phase conjugation
- PCF = photonic-crystal fiber
- PD = parametric device
- PM = phase modulation
- PO = parametric oscillator
- PR = polarization rotation
- PI = phase-insensitive

List of acronyms 2

- PJ = phase jitter
- PMD = polarization-mode dispersion
- PS = phase-sensitive
- QM = quantum mechanics
- RBF = randomly-birefringent fiber
- RCP = right-circularly polarized
- RFA = Raman fiber amplifier
- RSF = rapidly-spun fiber
- SBF = strongly-birefringent fiber
- SBS = stimulated Brillouin scattering
- SRS = stimulated Raman scattering
- SNR = signal-to-noise ratio
- THG = third-harmonic generation
- VP = vertically polarized
- WC = wavelength conversion
- WDM = wavelength-division multiplexing
- ZDF = zero-dispersion frequency