Solid state quantum optics 1: Quantum optics with super conducting wires

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What is quantum optics?

Name: quantum effects of light

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Experiment with trapped ions, ultra cold atoms, etc only use classical light

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In practice: quantum effects of atoms and light

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(name is actually bad)

Quantization of light

Take complete set of normalized modes (e.g. plane waves) fulfilling

$$\nabla \times \nabla \times \vec{u}_n = \frac{\omega_n^2}{c^2}\vec{u}_n \qquad \int d^3r \; \vec{u}_n^\dagger \vec{u}_m = \delta_{n,m}$$

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Expand fields:

$$\vec{E} = \sum_n \frac{\sqrt{\omega_n}}{2\sqrt{\epsilon_0}}[(q_n + ip_n)\vec{u}_n(\vec{r}) + H.C.]$$

$$\vec{B} = \sum_n \frac{1}{2\sqrt{\epsilon_0 \omega_n}}[(-iq_n + p_n)\nabla \times \vec{u}_n(\vec{r}) + H.C.]$$

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Maxwell's equations fulfilled if $\dot{q}_n = \frac{\partial H}{\partial p_n} \qquad \dot{p}_n = -\frac{\partial H}{\partial q_n}$

with H being the classical energy $H = \int d^3r \left(\frac{\epsilon_0}{2}\vec{E}^2 + \frac{\vec{B}^2}{2\mu_0}\right)$

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q_n and p_n are canonically conjugated variables=> $[\hat{q}_n, \hat{p}_m] = i\hbar\delta_{n,m}$

Quantization of light

Introduce creation and annihilation operators .....

$$\hat{\vec{E}} = \sum_n \sqrt{\frac{\hbar\omega_n}{2\epsilon_0}} [\hat{a}_n \vec{u}_n(\vec{r}) + H.C.]$$

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Also works with spatially varying $\epsilon(r)$

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Use the D field and change mode functions:

$$\nabla \times \nabla \times \left(\frac{\epsilon_0 \vec{u}_n}{\epsilon(\vec{r})}\right) = \frac{\omega_n^2}{c^2}\vec{u}_n \qquad \int d^3r \ \frac{\epsilon_0}{\epsilon(\vec{r})}\vec{u}_n^\dagger \vec{u}_m = \delta_{n,m}$$

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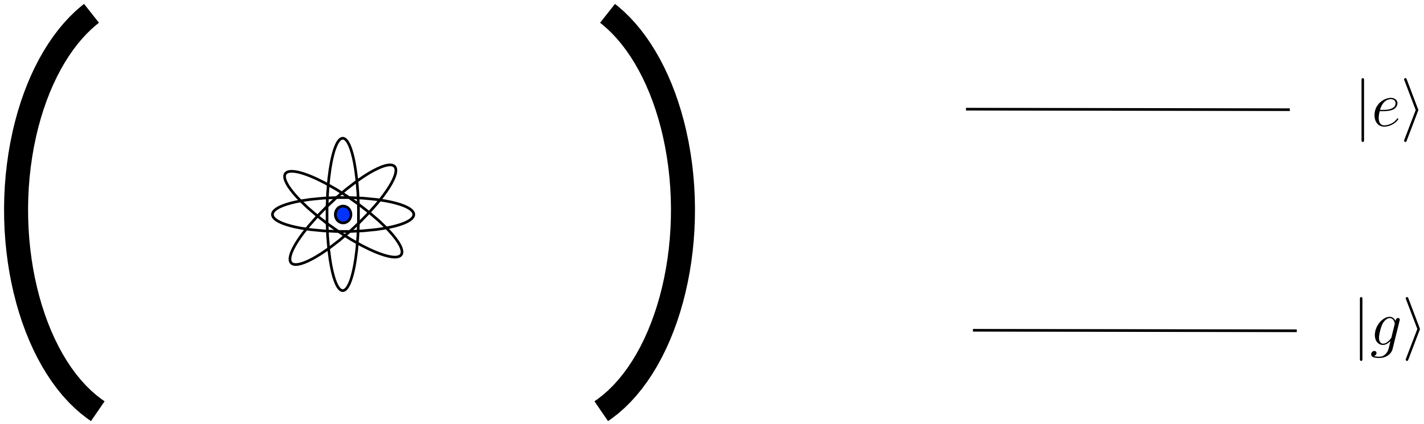
Can be done for any geometry with dielectrics, metals*, etc.

*Metals: real mess unless ideal (just boundary condition)

Jaynes Cummings Model

Two level atom in a cavity



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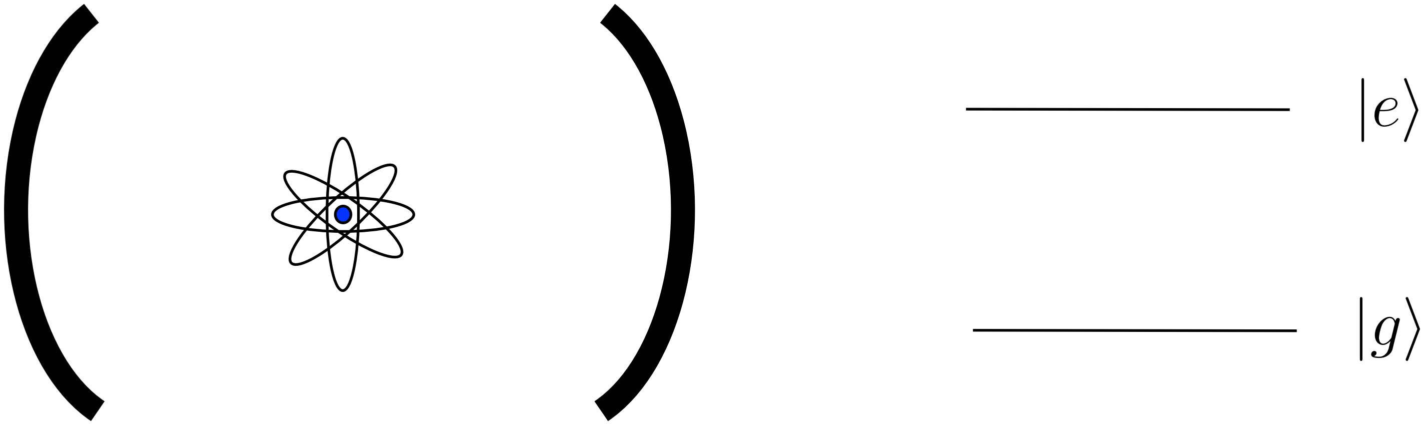

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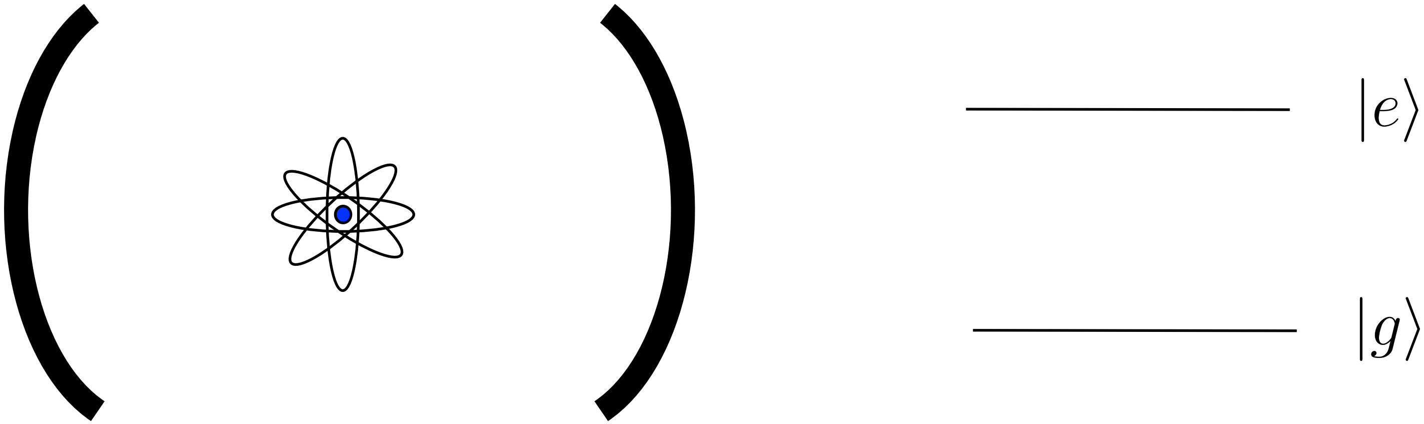

Only a single mode resonant

$$\hat{\vec{E}} = \sqrt{\frac{\hbar\omega_0}{2\epsilon_0}}[\hat{a}\vec{u}(\vec{r}) + H.C.]$$

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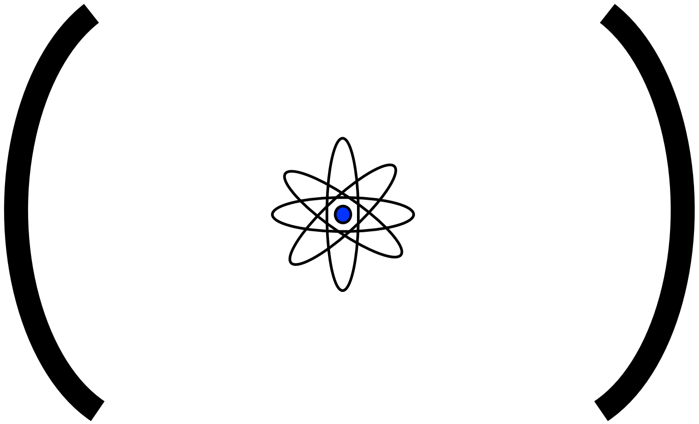

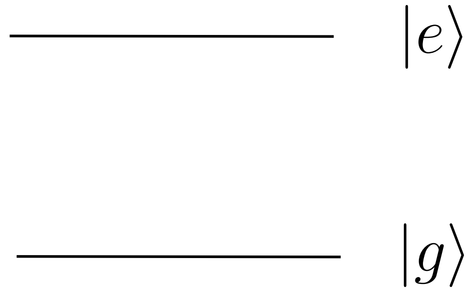

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Dipole and rotating wave approximation $H = g(\hat{a}^\dagger\sigma_- + \sigma_+\hat{a})$

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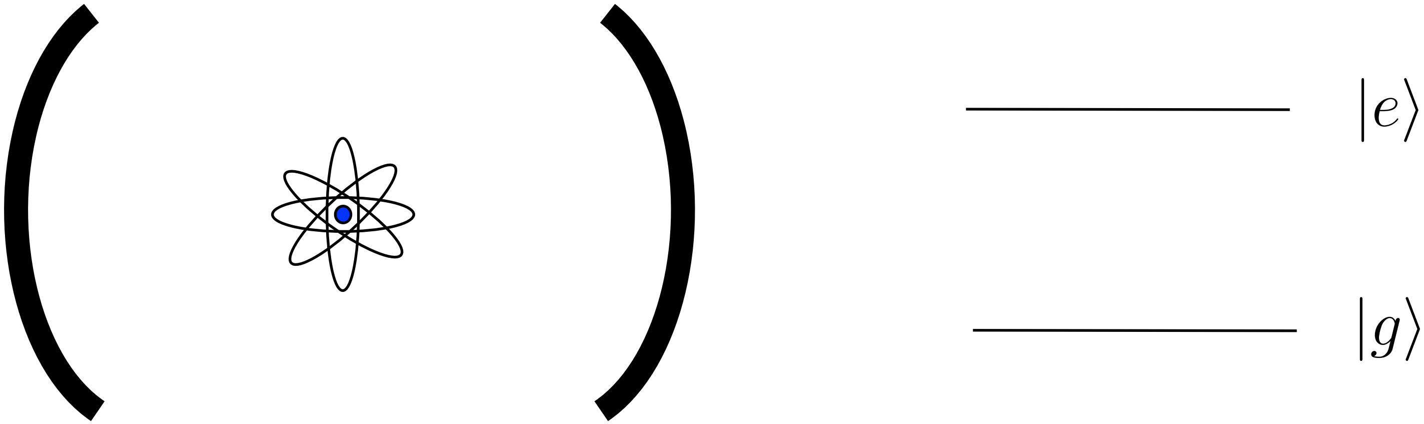

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Dipole and rotating wave approximation $\quad H = g(\hat{a}^\dagger\sigma_- + \sigma_+\hat{a})$

Coupling constant $\quad g = \sqrt{\frac{\hbar\omega}{2\epsilon_0}}\vec{u}^\dagger\vec{d}_0 \propto \frac{1}{\sqrt{V}}$

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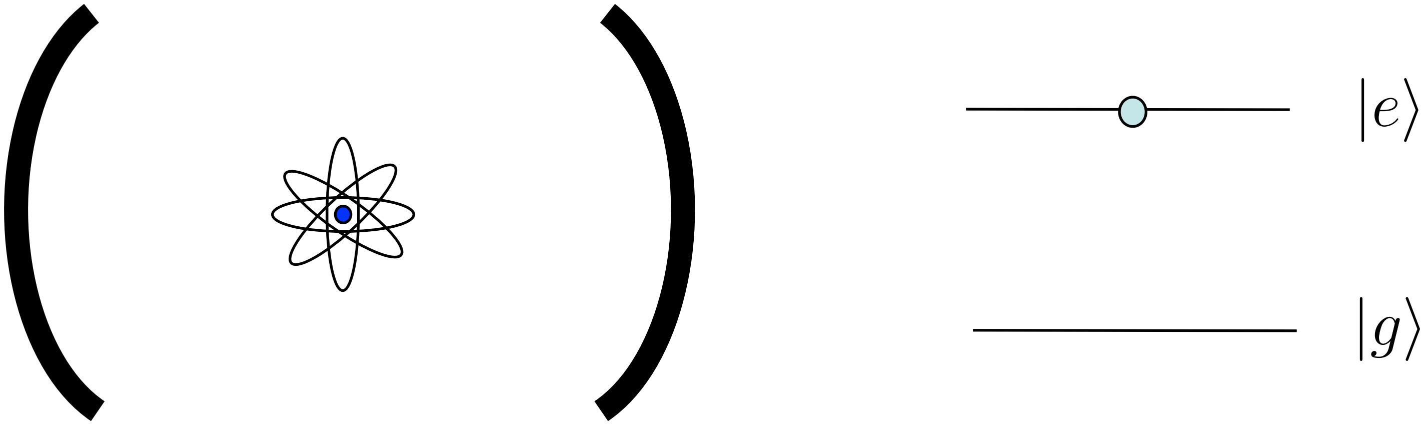

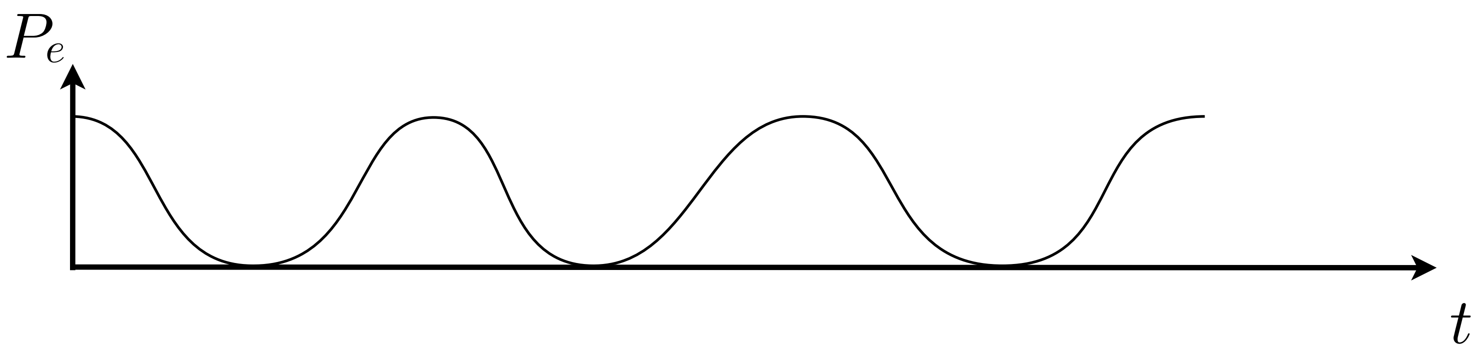

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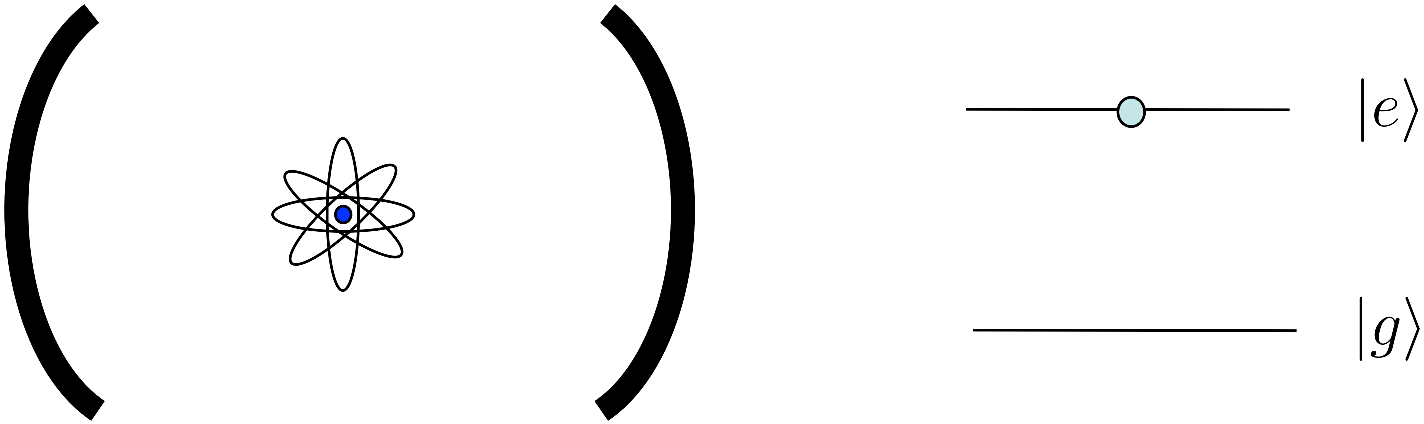

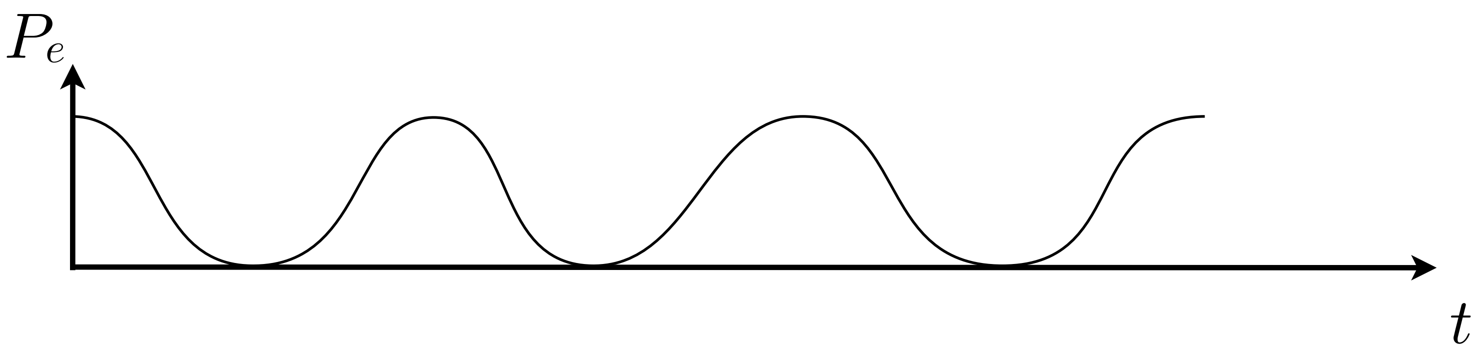

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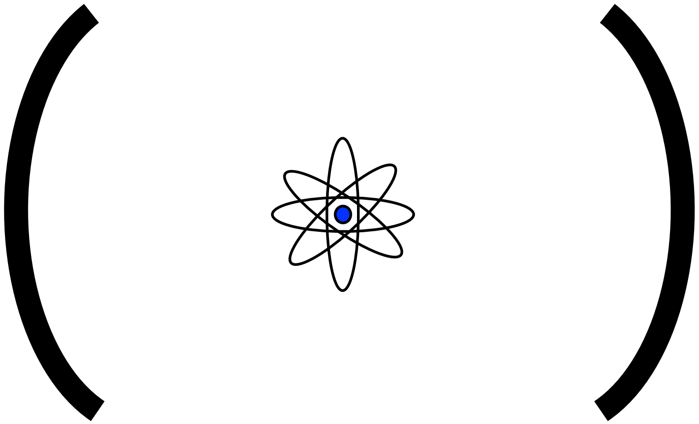

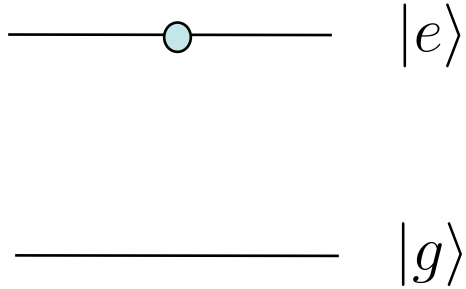

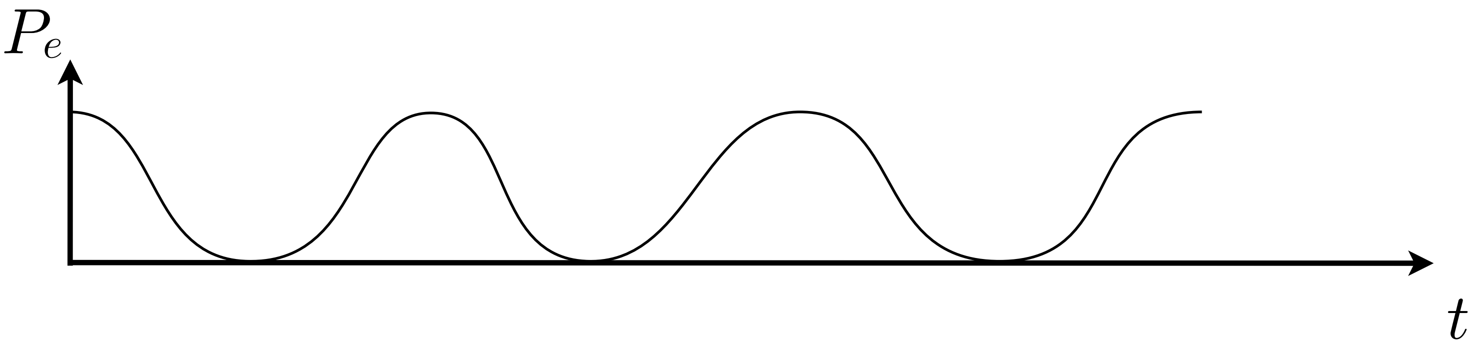

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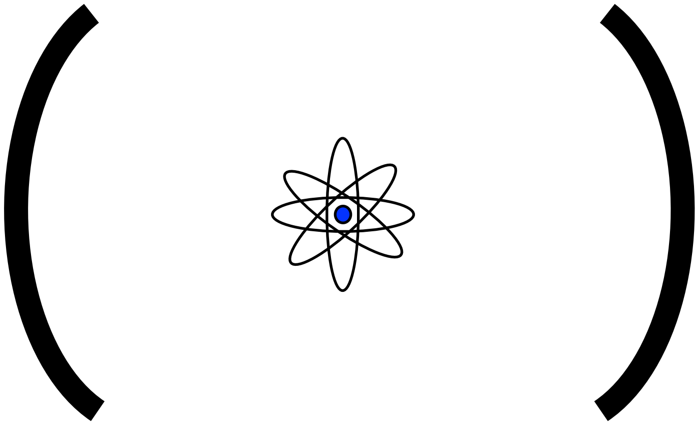

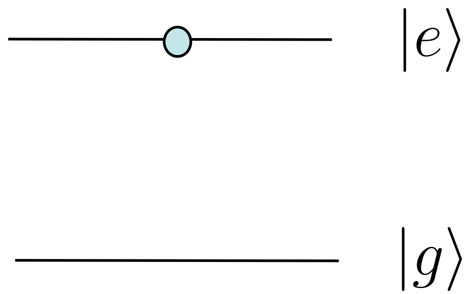

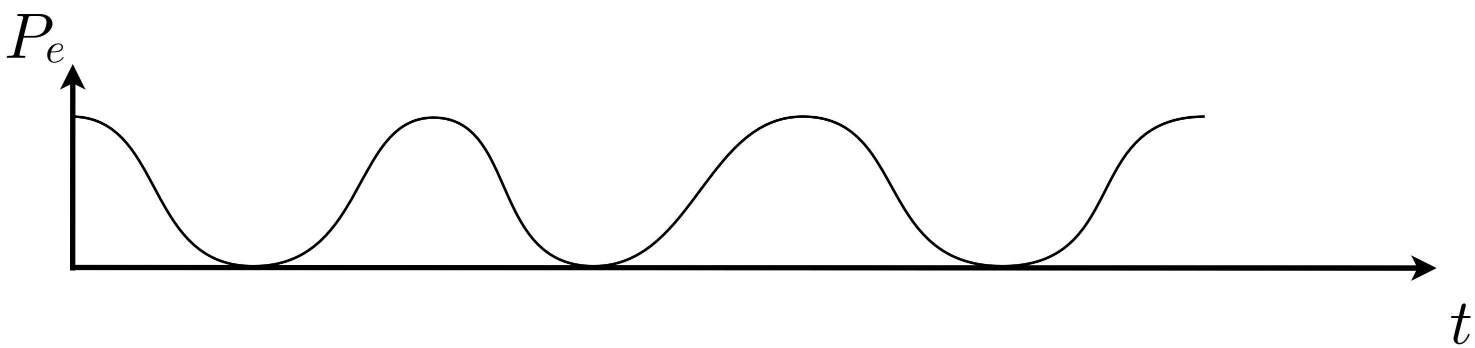

Problem: losses

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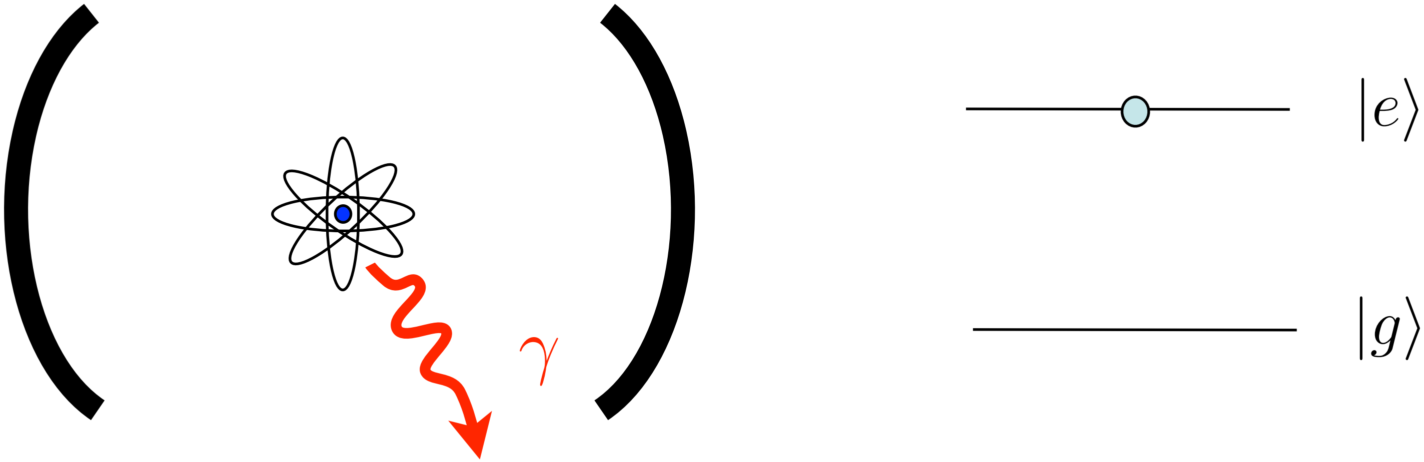

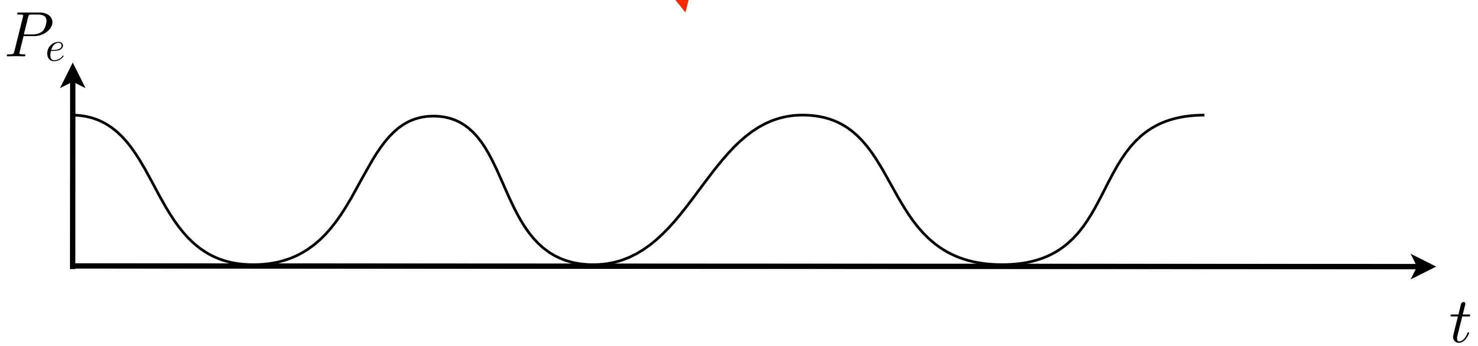

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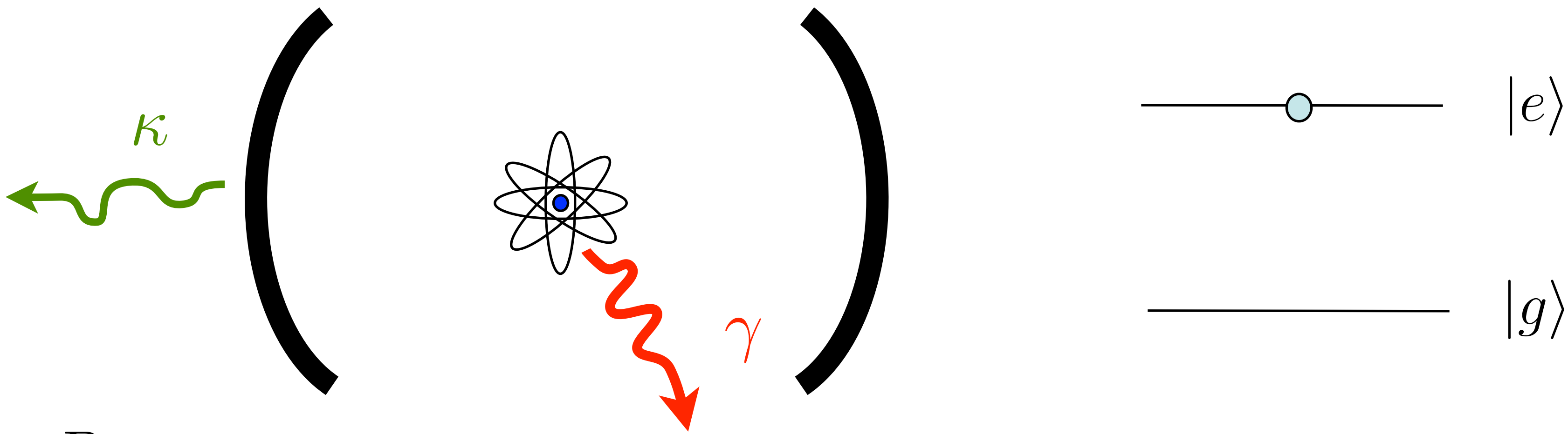

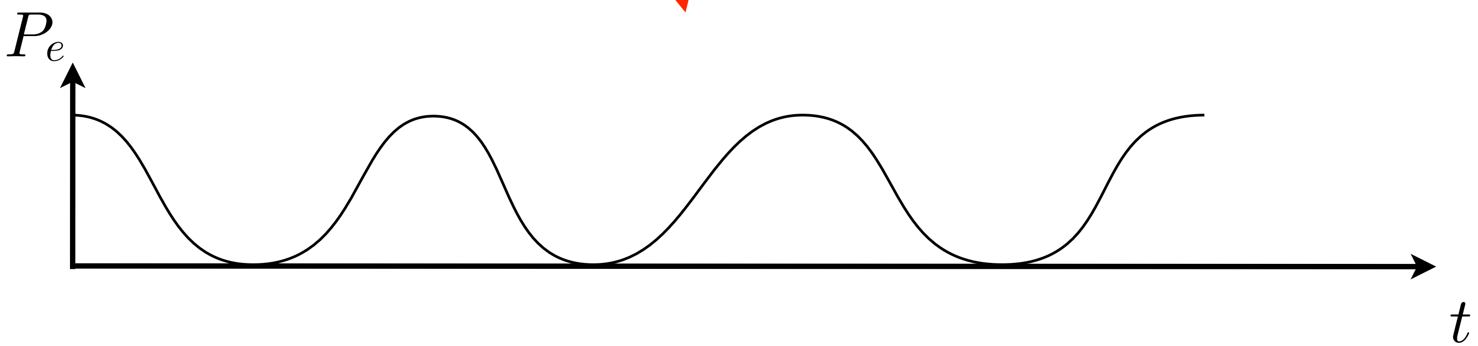

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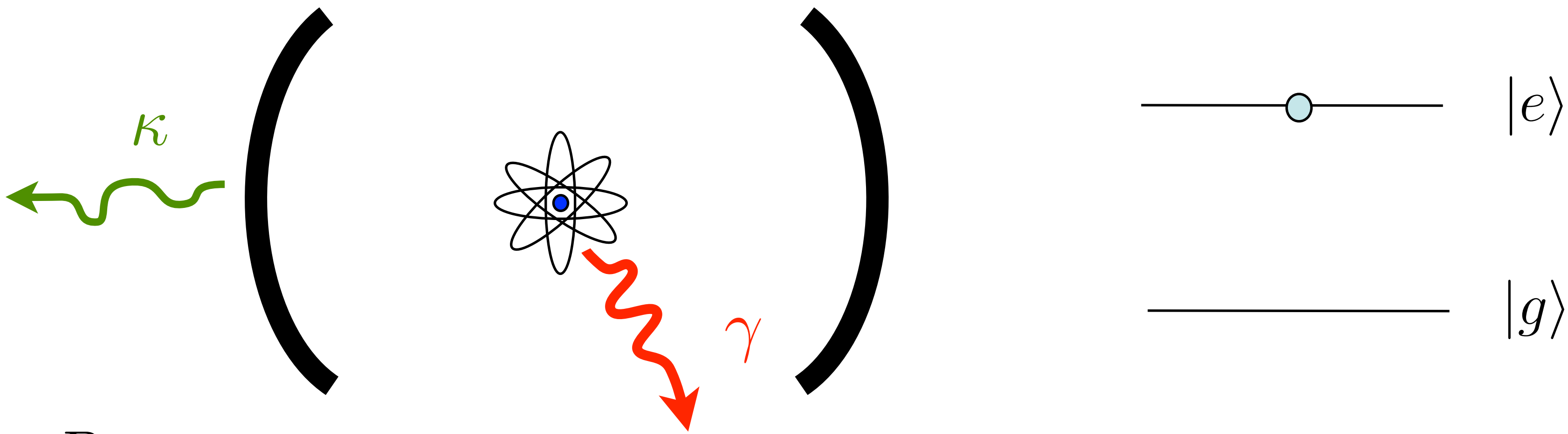

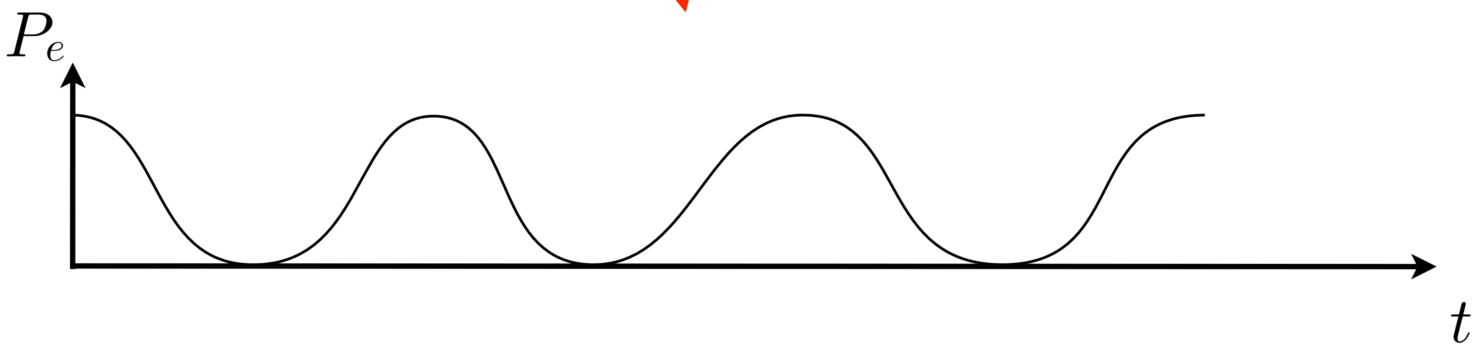

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Seeing this requires $\kappa,\gamma \ll g$

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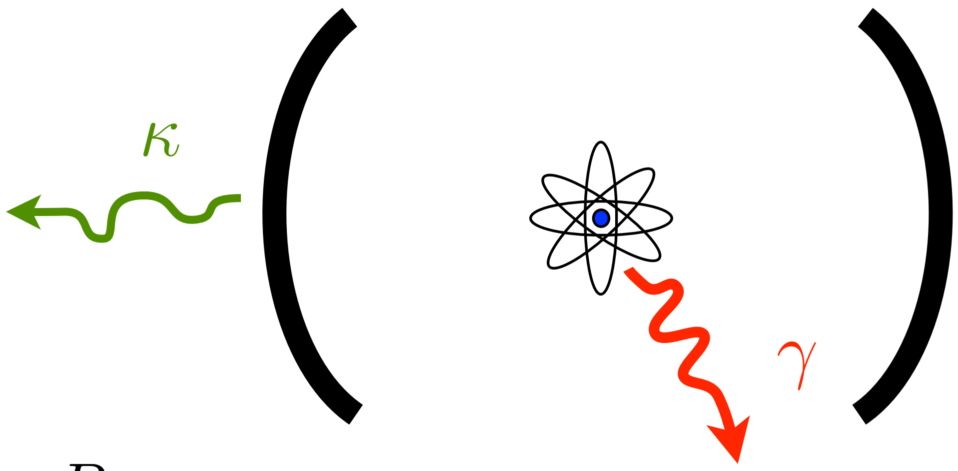

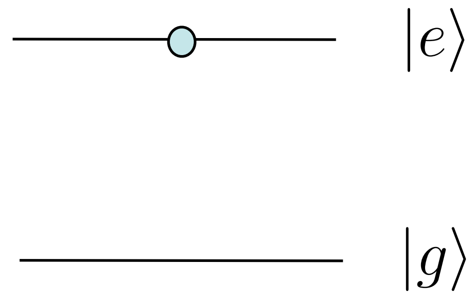

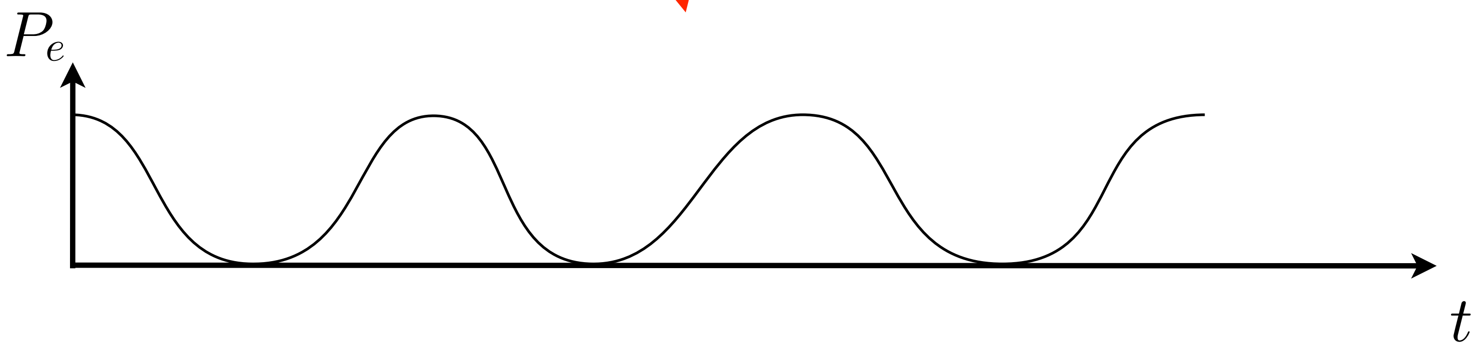

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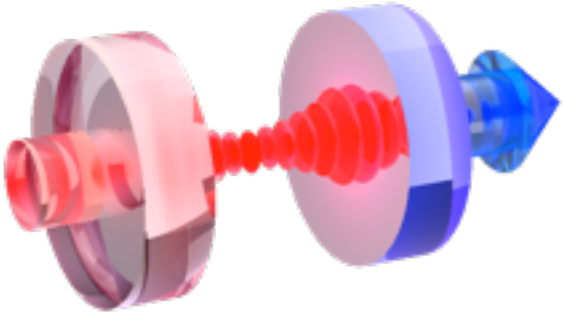
Small cavities / low loss

Experiments

Optical:



Microwave and Rydberg atoms

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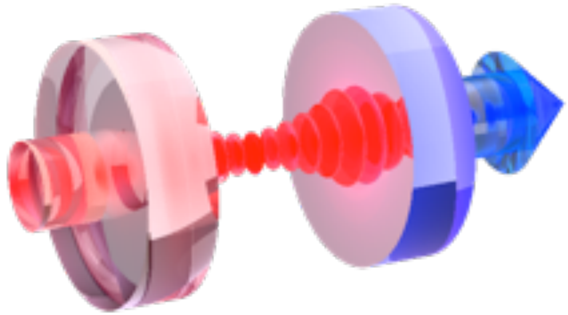

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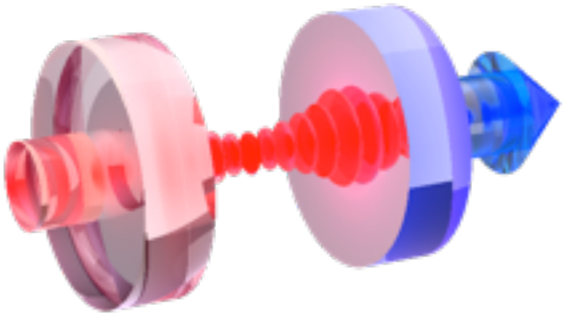

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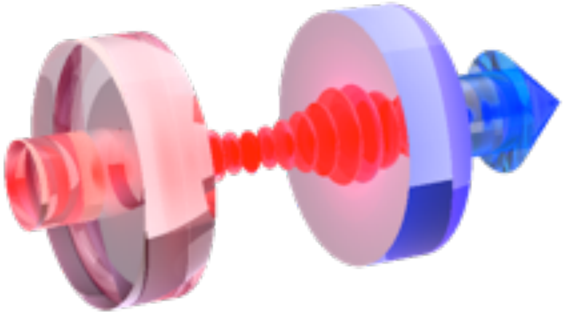

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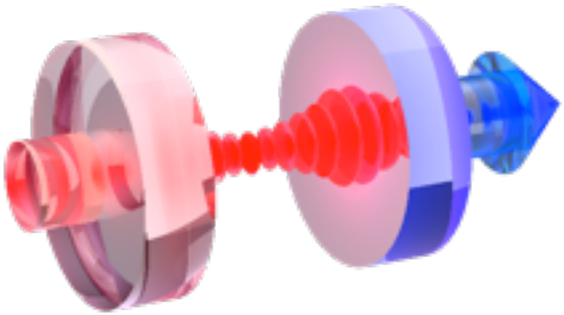

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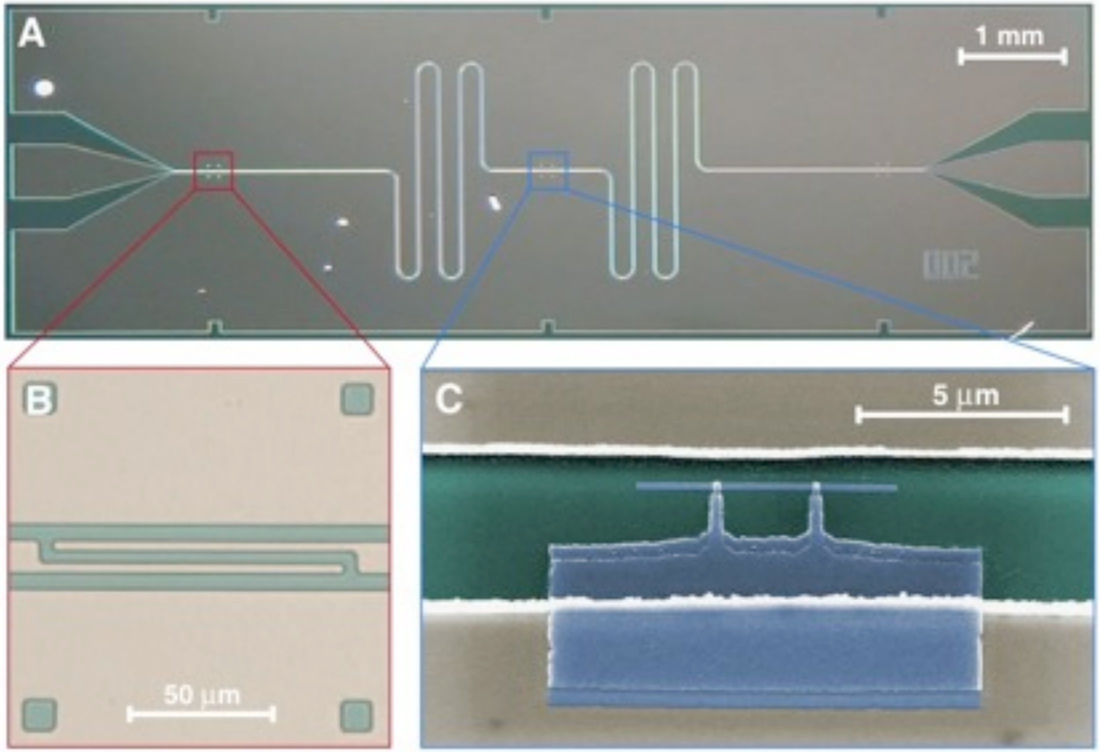
Problem transit time: t~a few times $1/g$

A solid state realization*



Schoelkopf, Yale and Wallraff, ETH

g~(2π) 200 MHz κ~ (2π) 2.4 MHz γ~(2π)300 kHz

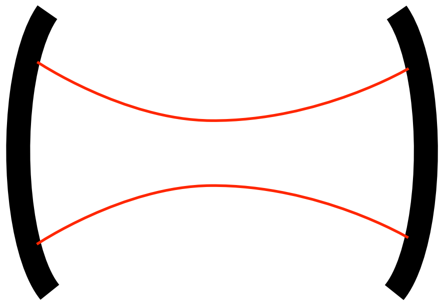
Trapping is easy and the atom stay there

* Also quantum dot work: Imamoglu, Yamamoto, Lodahl etc...

Limitations on coupling

Coupling depend on mode volume $g = \sqrt{\frac{\hbar\omega}{2\epsilon_0}} \vec{u}^\dagger \vec{d}_0 \propto \frac{1}{\sqrt{V}}$

Small volume => strong coupling



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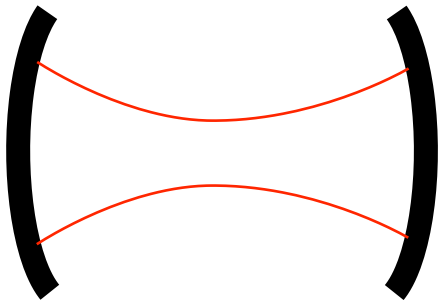

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Counter example:

ν = 50 Hz => λ=6000 km= ~ radius of Earth

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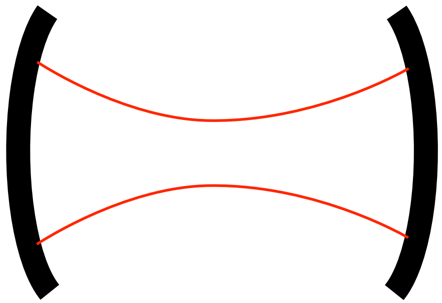

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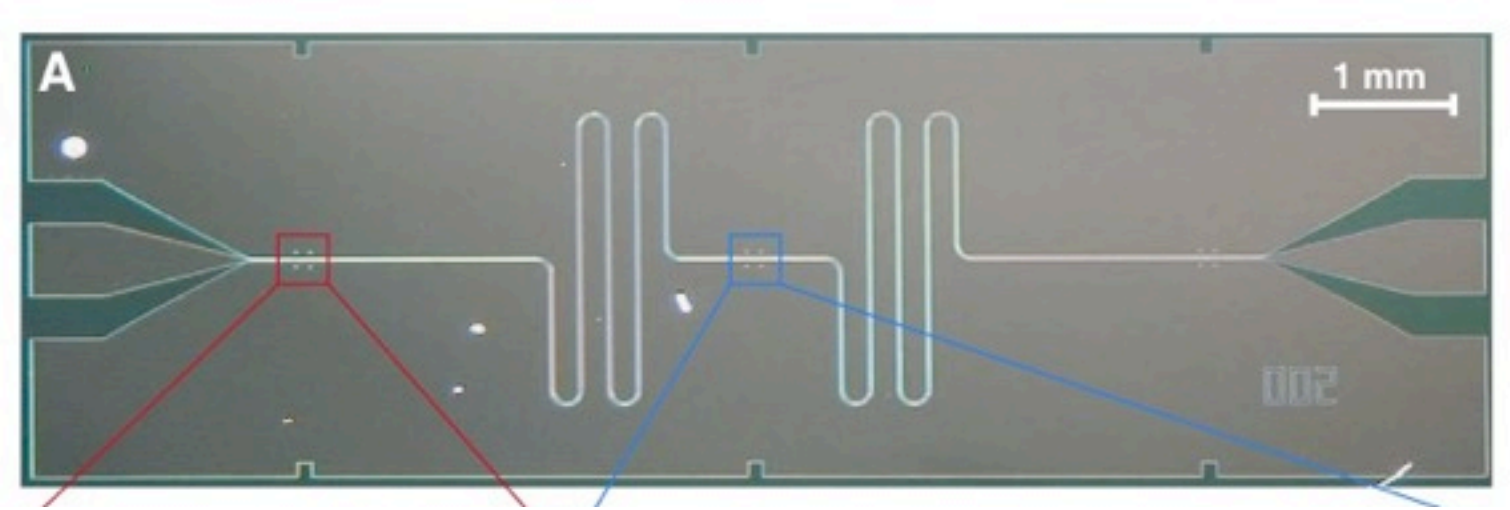
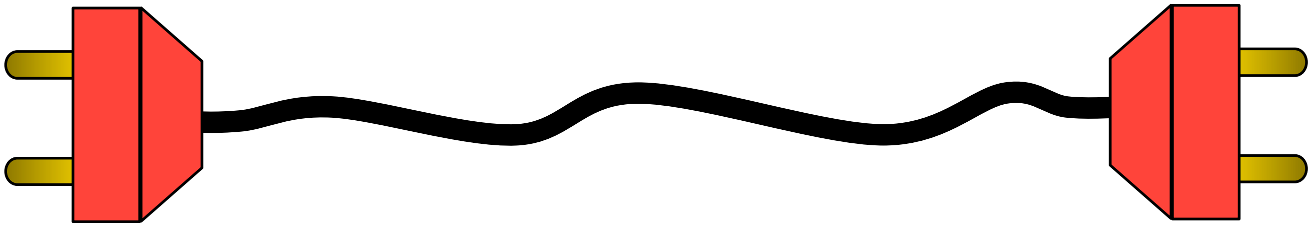
Counter example:

Conductors change things

ν = 50 Hz => λ=6000 km= ~ radius of Earth

Wires



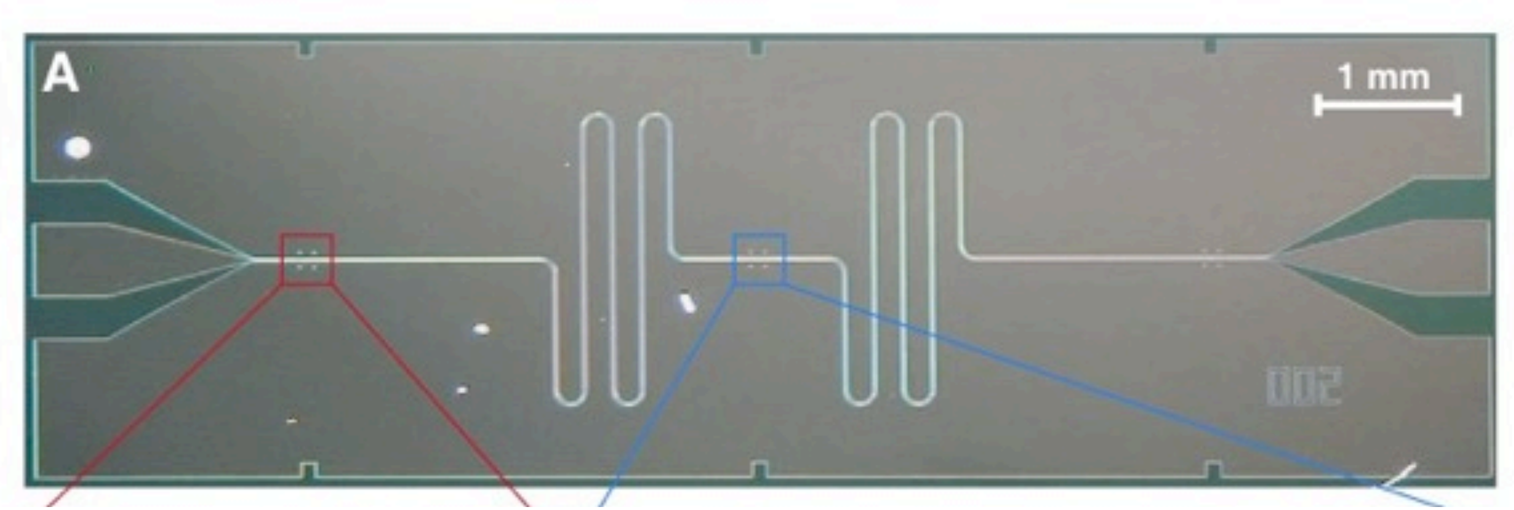


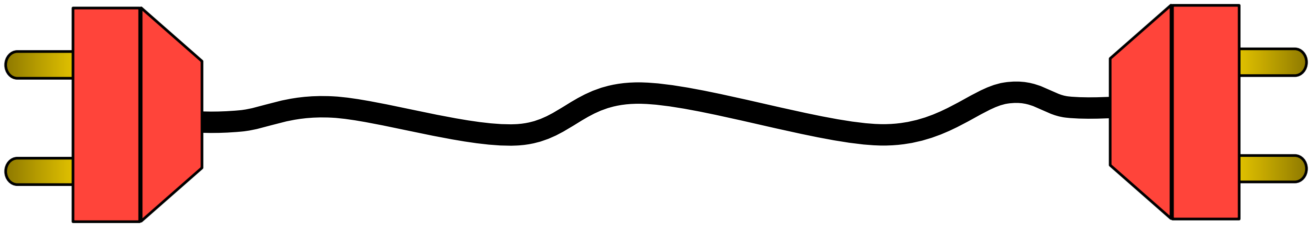
Describing the wire:

1) Field

2) Charges

Wires



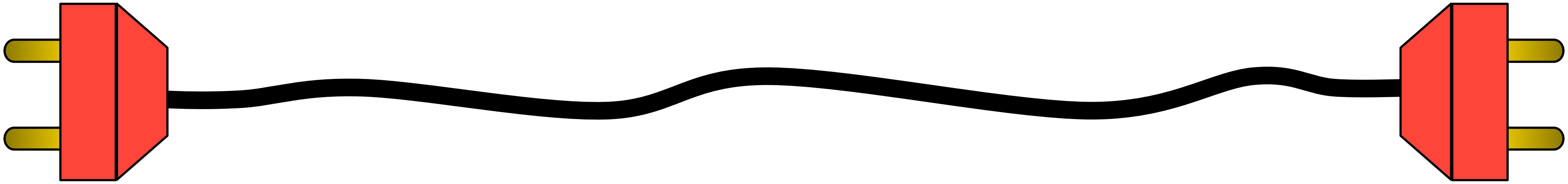


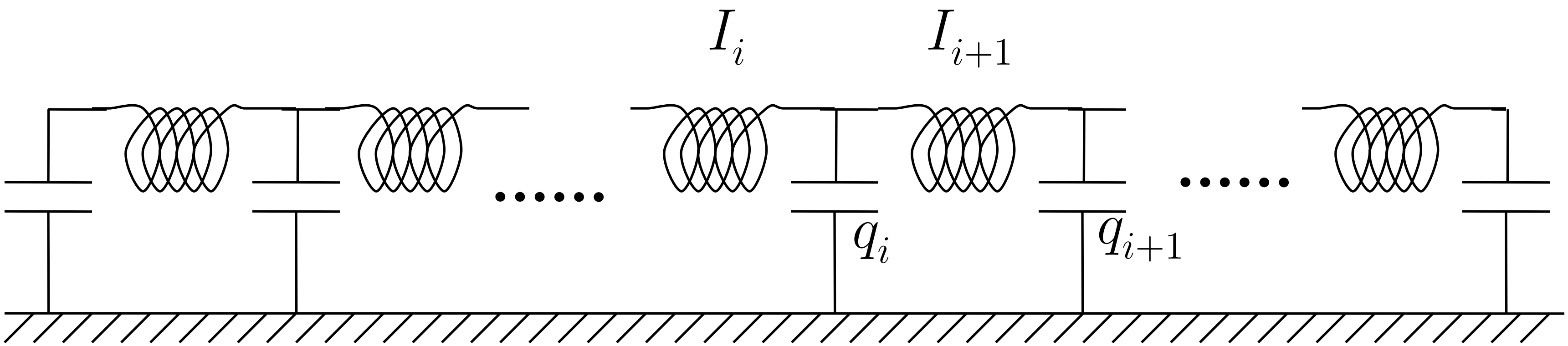
Describing the wire:

1) Field

2) Charges => The approach I will take
(will be useful for me later)

Describing the wire

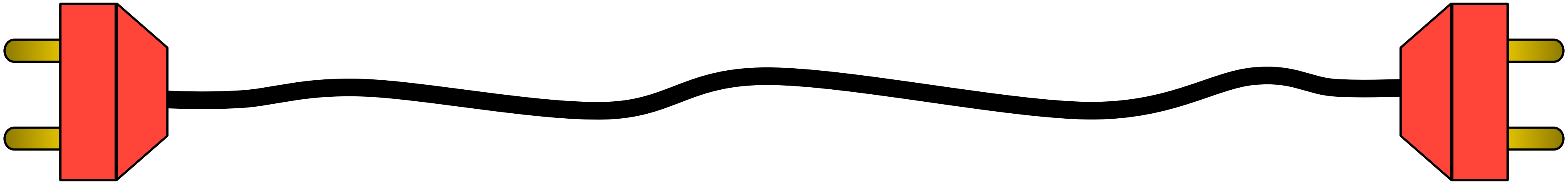


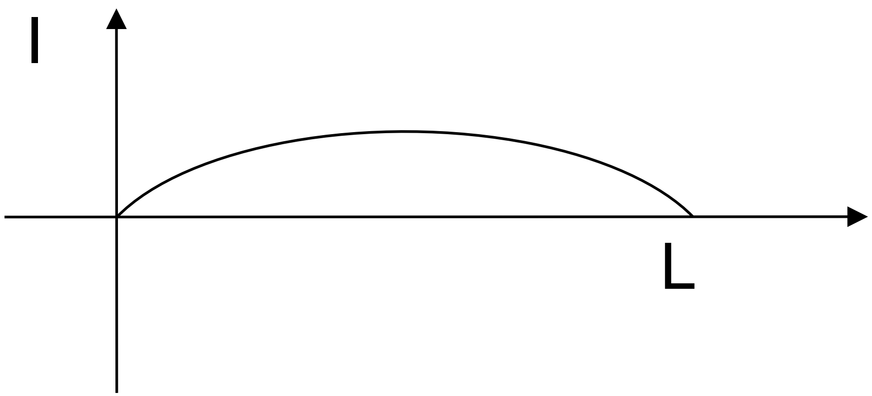


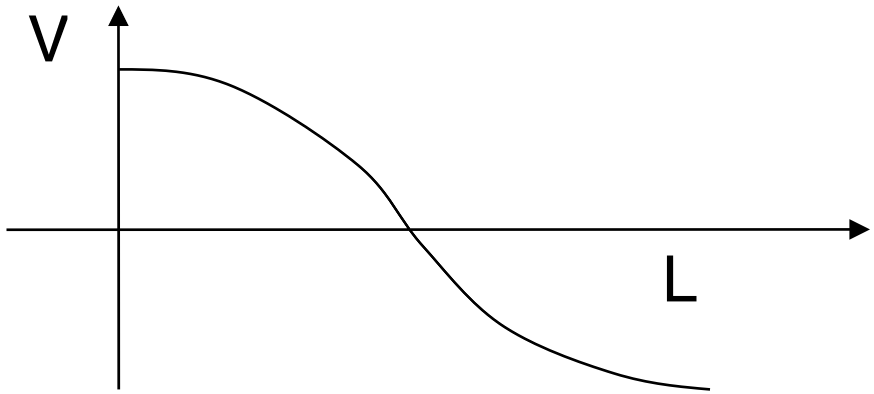
Equations of motion: $$\frac{d\lambda}{dt} = -\frac{dI}{dx} \qquad l\frac{dI}{dt} = -c\frac{d\lambda}{dx}$$

Wave equation: $$\frac{d^2I}{dt^2} = v^2\frac{d^2I}{dx^2} \qquad v = \frac{1}{\sqrt{lc}}$$

Standing waves



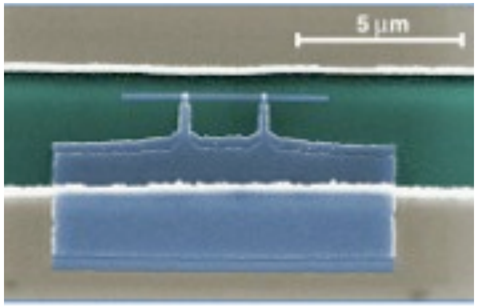




Discrete modes: $\omega = n\frac{\pi v}{L}$ L ~ mm or cm ω ~ GHz

Tune into resonance with transition in atoms => interact

Atom = Cooper pair box



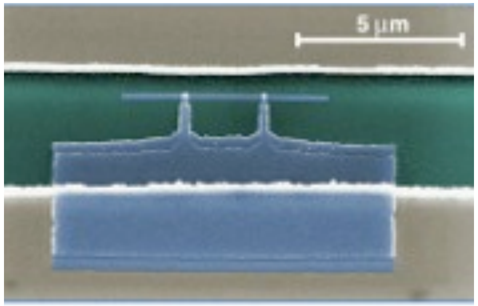
One Cooper pair can jump on and off

$|0\rangle$: no pair on box

$|1\rangle$: one pair on box

$$H = \frac{1}{2}(E_{\rm el}\sigma_x + E_J\sigma_z)$$

Atom = Cooper pair box



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$| 1 \rangle$: one pair on box

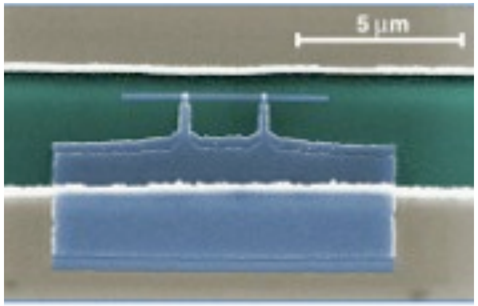
$$H = \frac{1}{2}(E_{\rm el}\sigma_x + E_J\sigma_z)$$

Electrostatic energy

$$\sigma_x = |0\rangle\langle 0| - |1\rangle\langle 1|$$

Can be controlled by gate voltage => Tune to zero

Atom = Cooper pair box



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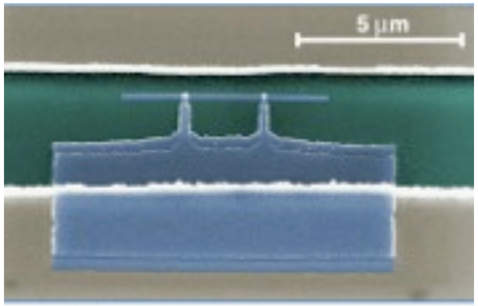

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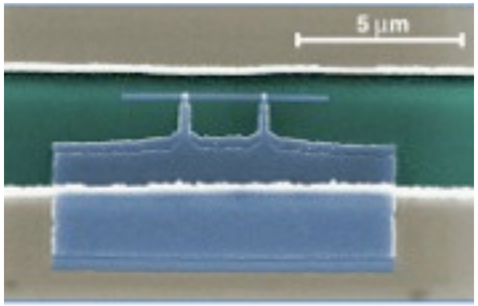

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Dressed states:

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

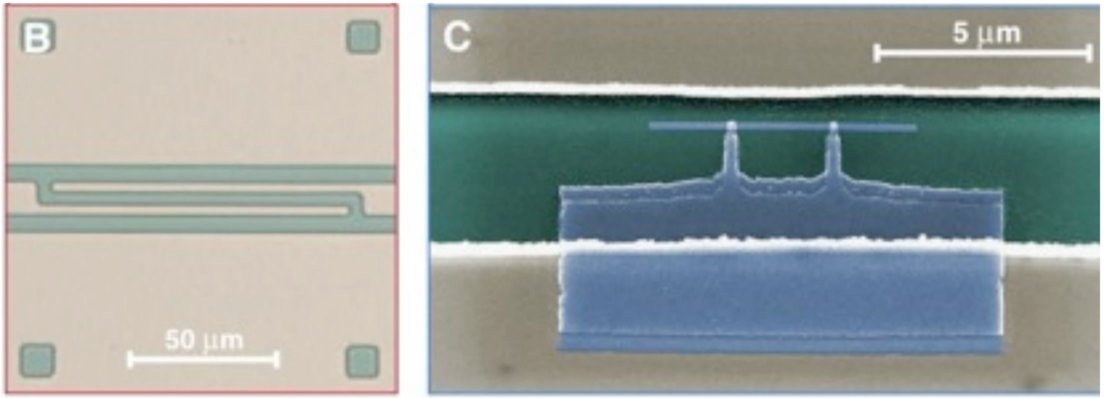
Two level system with dipole allowed transition

Describing the interaction

Rigorous description:

1. Write down Hamiltonian (the energy) - Harmonic oscillator
2. Expand on eigenmodes (standing waves)
3. Identify canonical variables (x_n ~ V_n , p_n ~ I_n)
4. Quantize by imposing [x_n, p_m]= $i\delta_{mn}$
5. Raising and lowering operators a_n ~ x_n+ip_m
6. Electric field: $\hat{\vec{E}}(\vec{r}) = \sum_n \vec{f}_n(\vec{r})(\hat{a}_n + \hat{a}_n^\dagger)$
7. Interaction $H = -\hat{\vec{d}} \cdot \hat{\vec{E}} = -(\sigma_+ + \sigma_-)\vec{D} \cdot \hat{\vec{E}}$
8. Rotating wave approximation $H = g(\sigma_+ \hat{a} + \sigma_- \hat{a}^\dagger)$

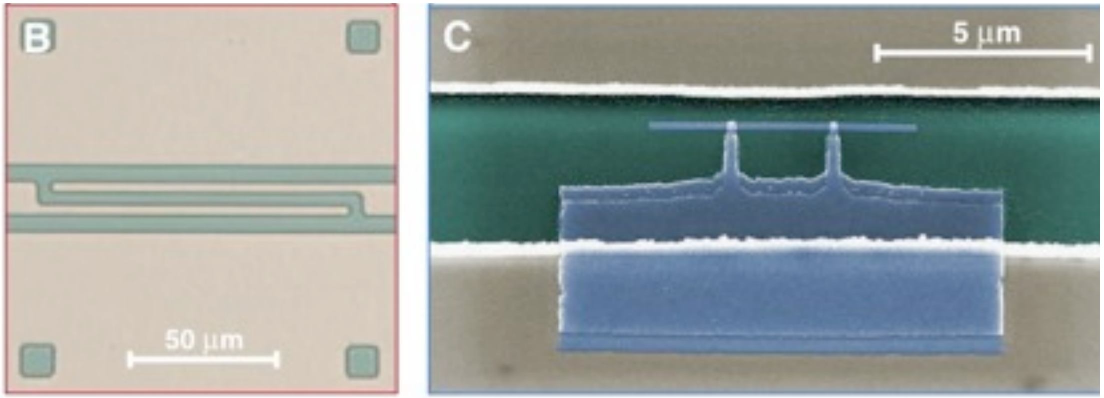
The coupling



h

Charging energy $\frac{Q^2}{C} \sim \hbar\omega$ $C \sim \epsilon_0 L$

The coupling



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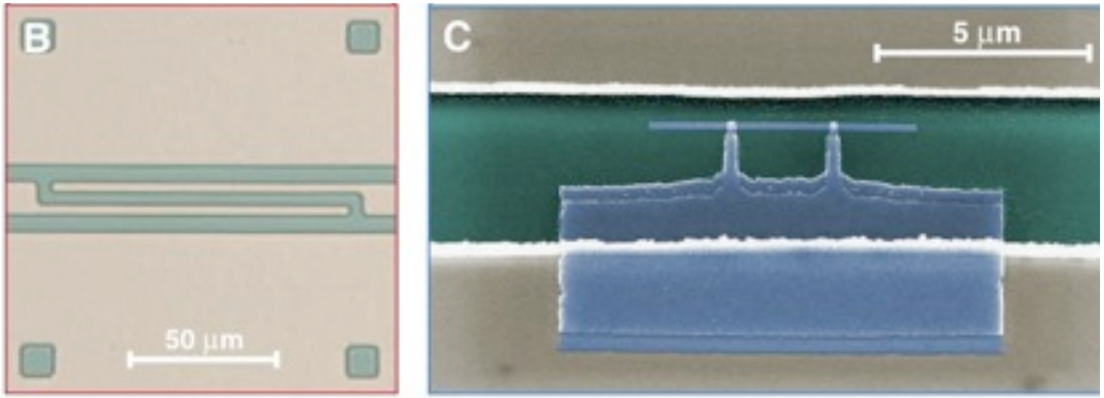
Field: $E_0 \sim \frac{Q}{\epsilon_0 L h} \sim \sqrt{\frac{\hbar\omega}{\epsilon_0 L}} \frac{1}{h}$

The coupling



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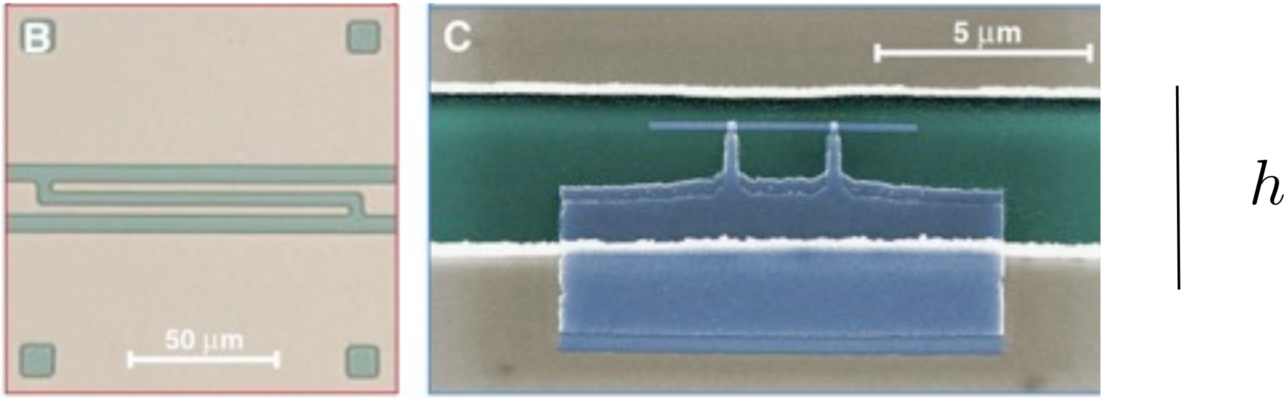
Same conclusion from field $H \sim \int d^3r(\epsilon_0 E^2 + B^2/\mu_0) \sim \hbar\omega$

The coupling



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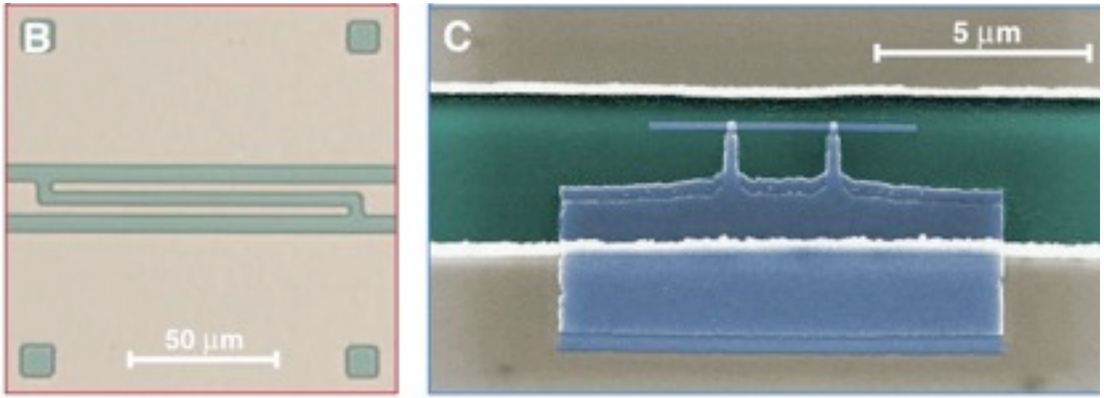

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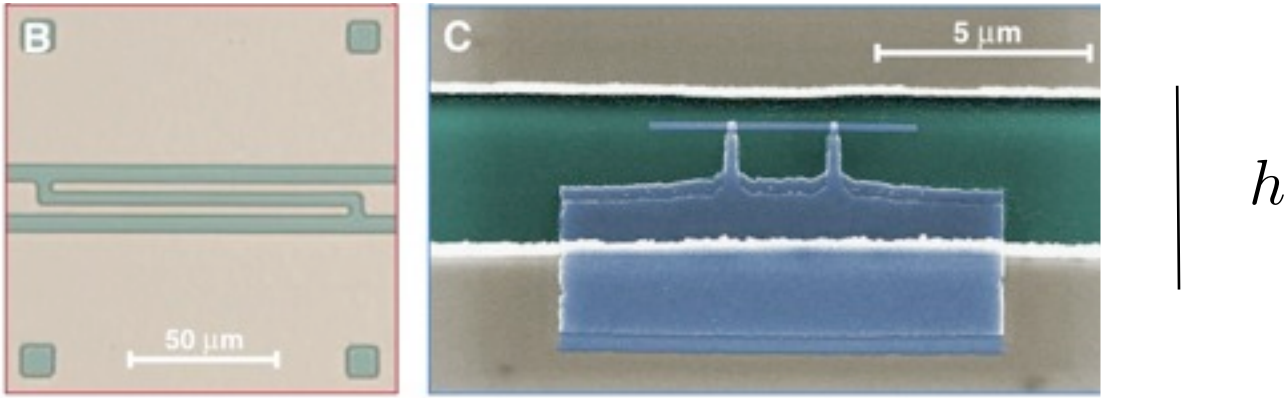
Strong confinement => strong coupling

The coupling



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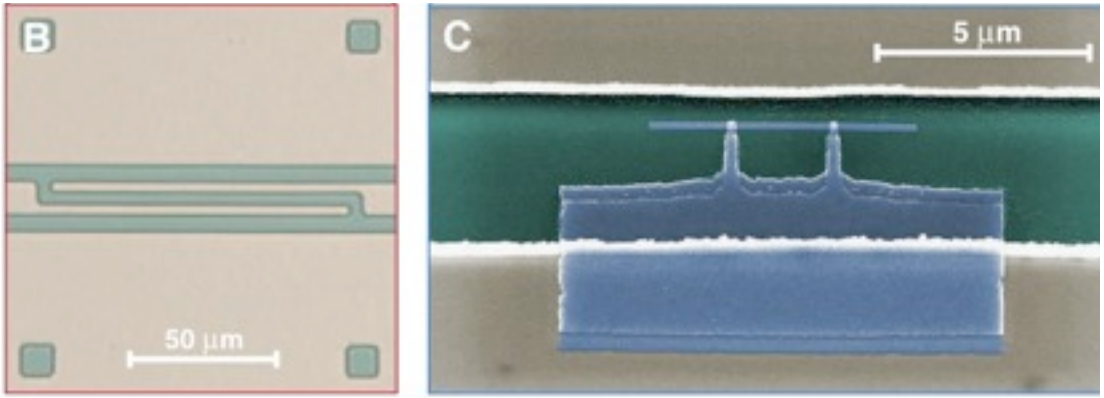
Strong confinement => strong coupling h ~ 5 µm

The coupling



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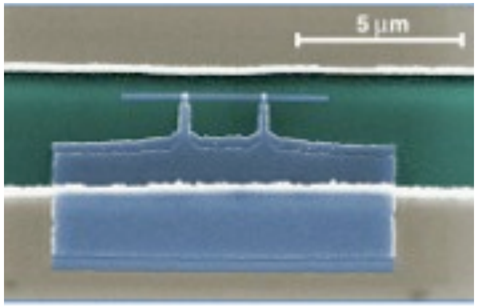
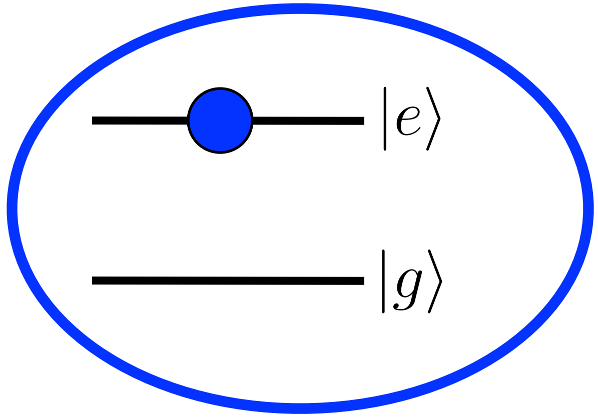
Strong confinement => strong coupling h ~ 5 µm ≪λ~ 10 cm

Resonant interaction



Coupling is strong enough make coherent coupling

Resonant interaction



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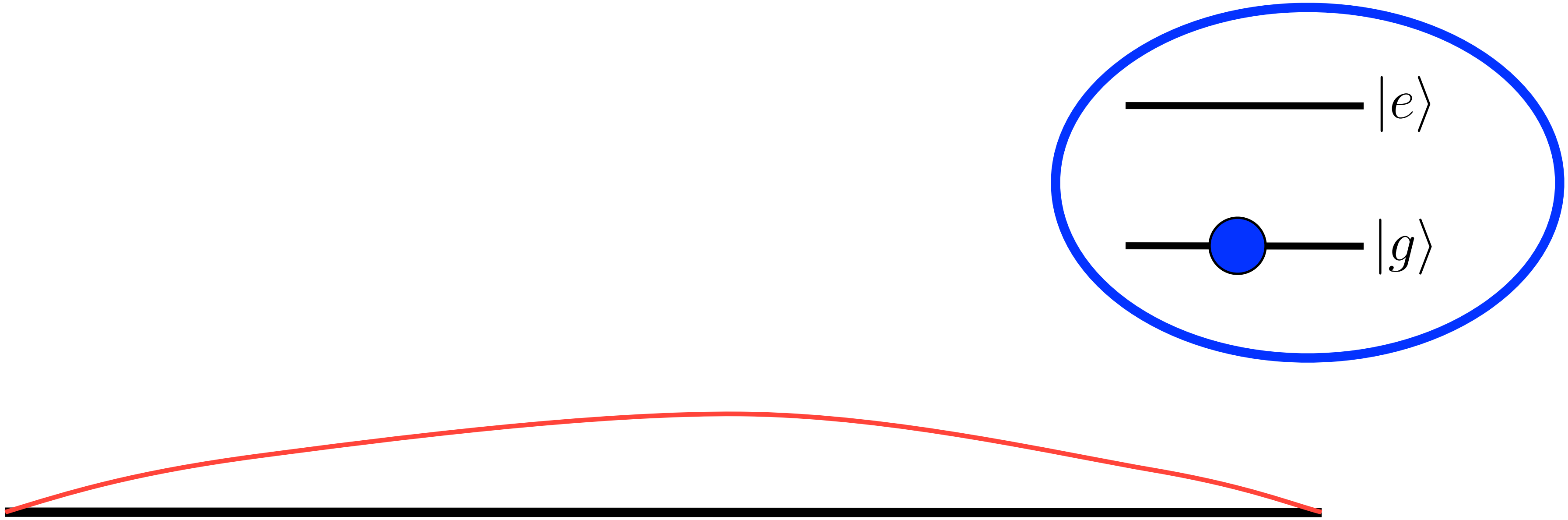

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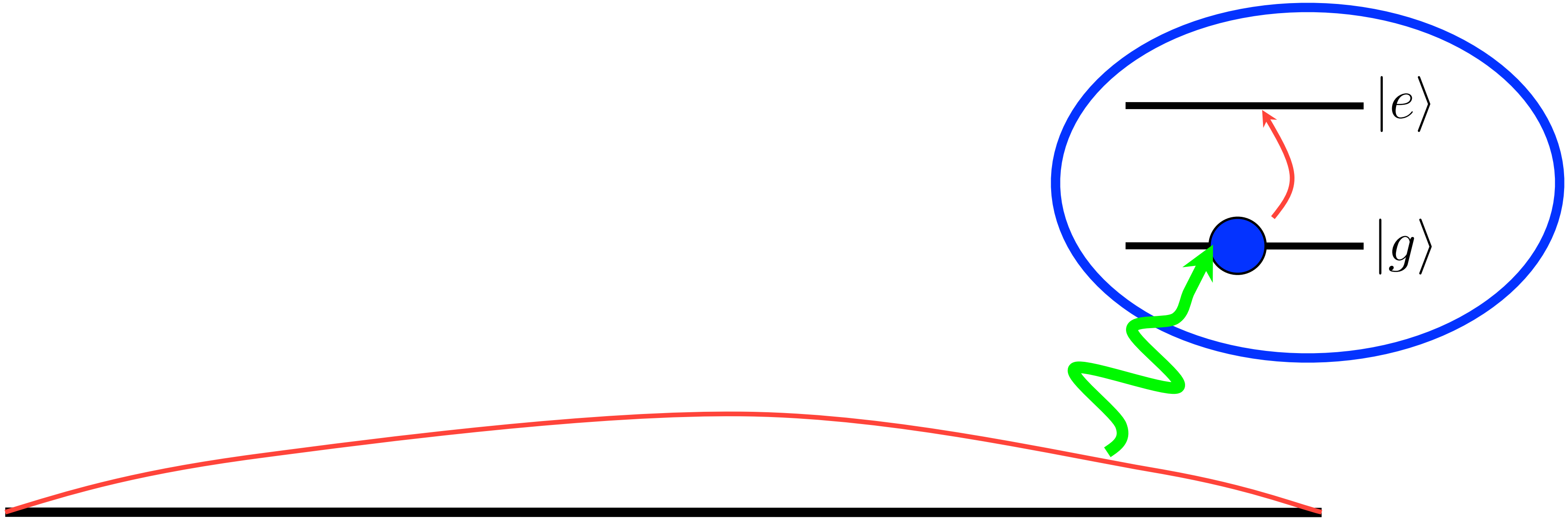

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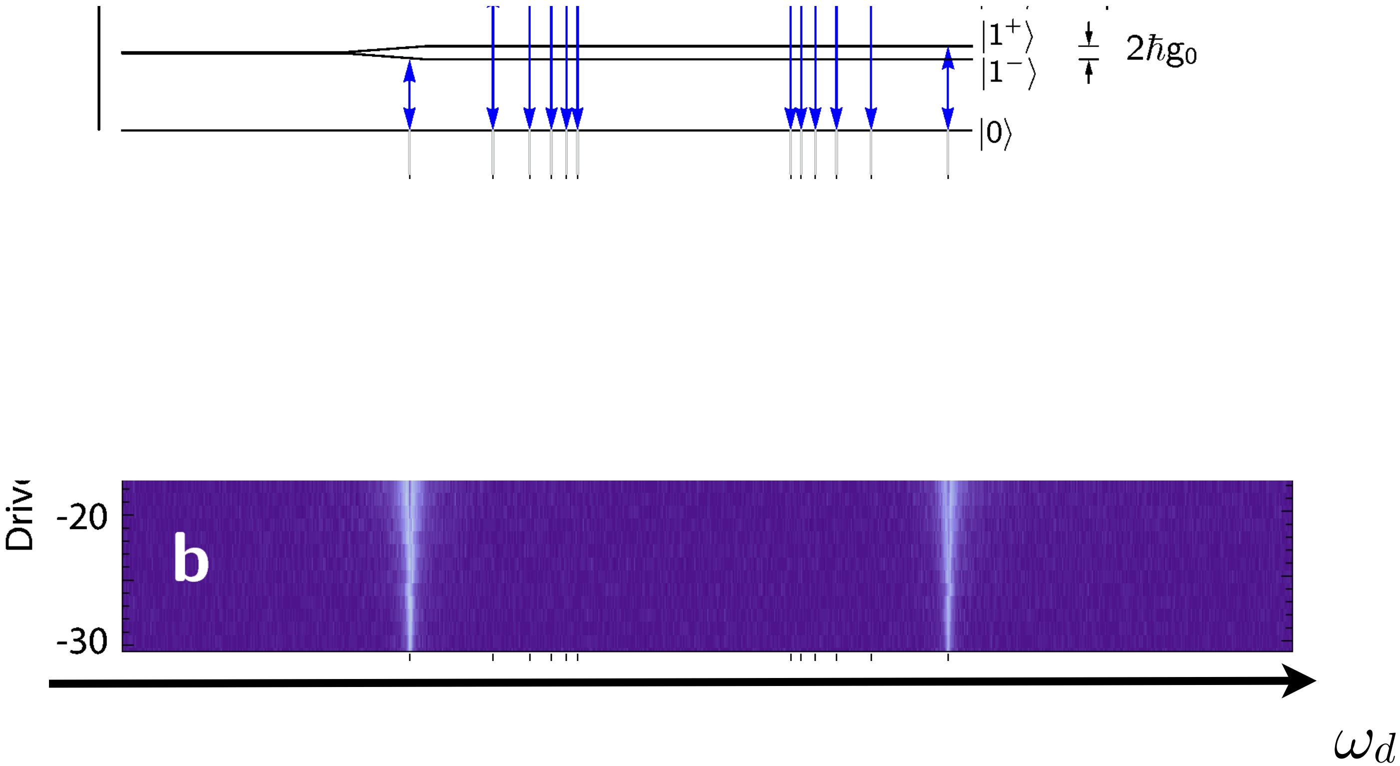
Jaynes Cummings Spectroscopy



L. S. Bishop, R. J. Schoelkopf *et al*, Nat. Phys. **5**, 105 (2009)

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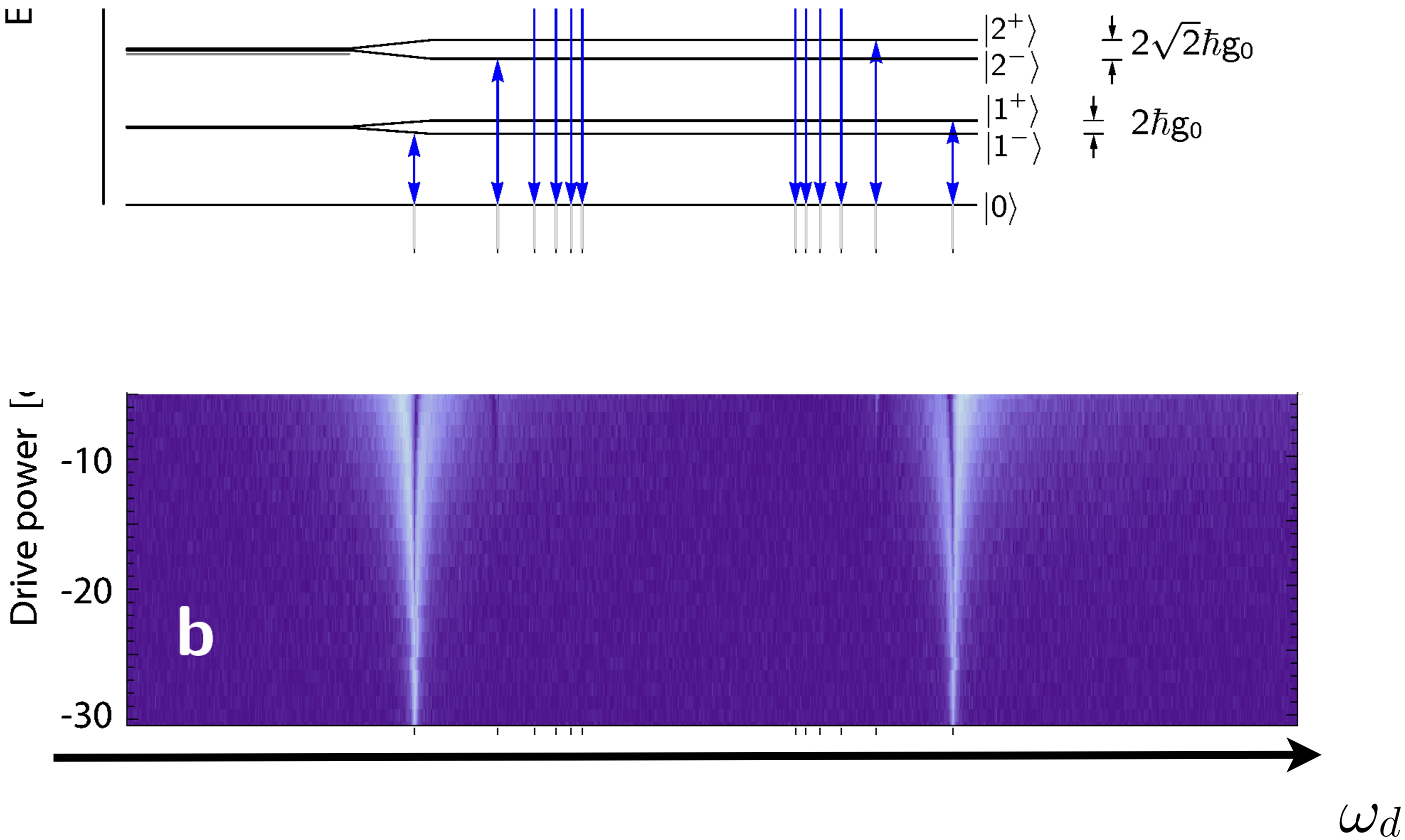

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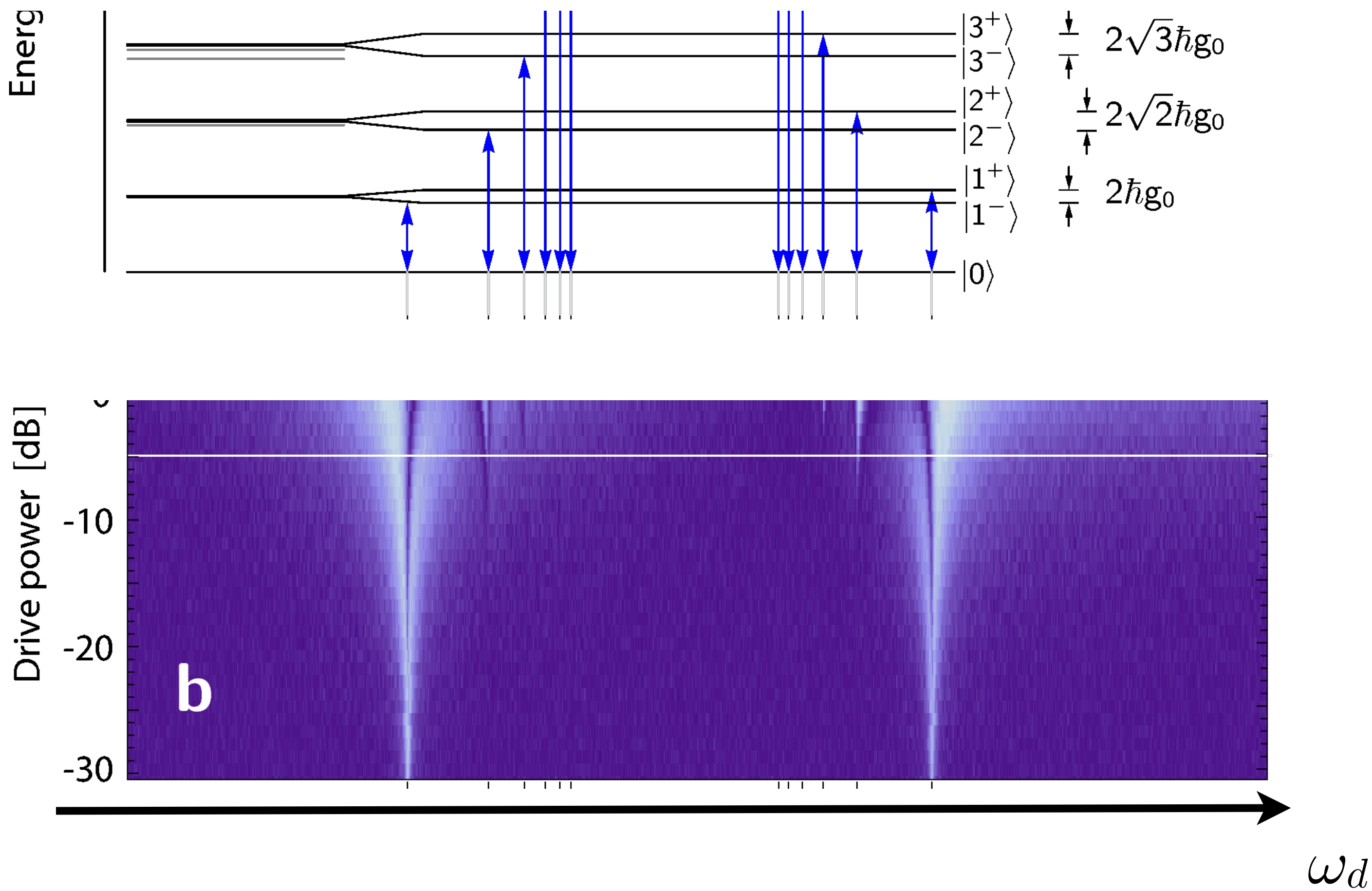

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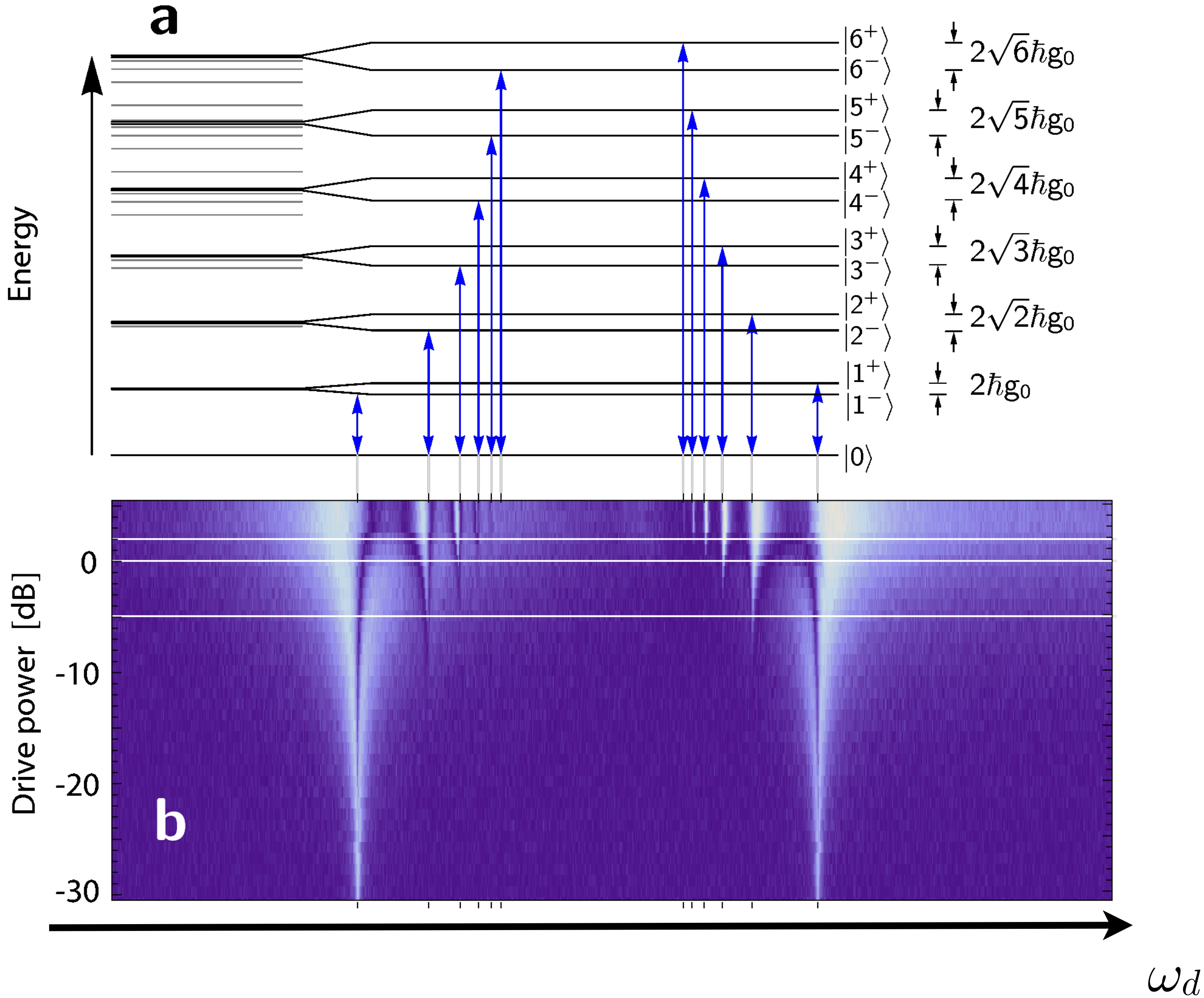

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Conclusion (1)

Conductors enables strong confinement of electric fields

Enables a strong coherent interaction

The best realization of the model system of quantum optics - the Jaynes Cummings model

Application: Quantum Information

Applications to quantum information

1. Build quantum computer

2. Hybrid devices

Applications to quantum information

1. Build quantum computer

    Very promising approach

2. Hybrid devices

Applications to quantum information

1. Build quantum computer

   Very promising approach

   Currently super conductors is the only real competitor to trapped ions

2. Hybrid devices

How to build a quantum computer

1. Qubits

   Low decoherence => weak interactions

2. Control

3. Read Out

4. Gates

   Some interaction between qubits => strong interactions

5. Coupling to light

   Useful for quantum communication

How to build a quantum computer

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   Low decoherence => weak interactions

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   Contradiction

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   Useful for quantum communication

Hybrid quantum computers

Atomic like

Atoms, ions, ...

- Long coherence times (ms-s)
- Coupling to light
- Identical

- Scaling hard
- Trapping hard

Solid state

Quantum dots, super conductors,...

- Short coherence time (ns-μs)
- Coupling to light harder
- Different

- Microfabrication
- It is solid

Hybrid quantum computers

Atomic like

Atoms, ions, ...

- Coupling to light
- Identical

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Solid state

Quantum dots,

- Coupling to light harder
- Different

- Microfabrication
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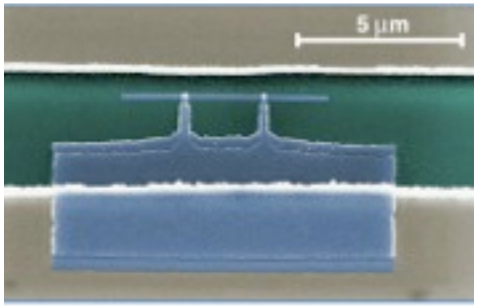
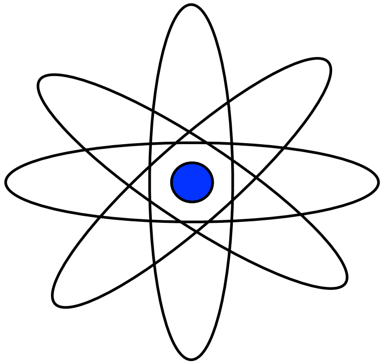
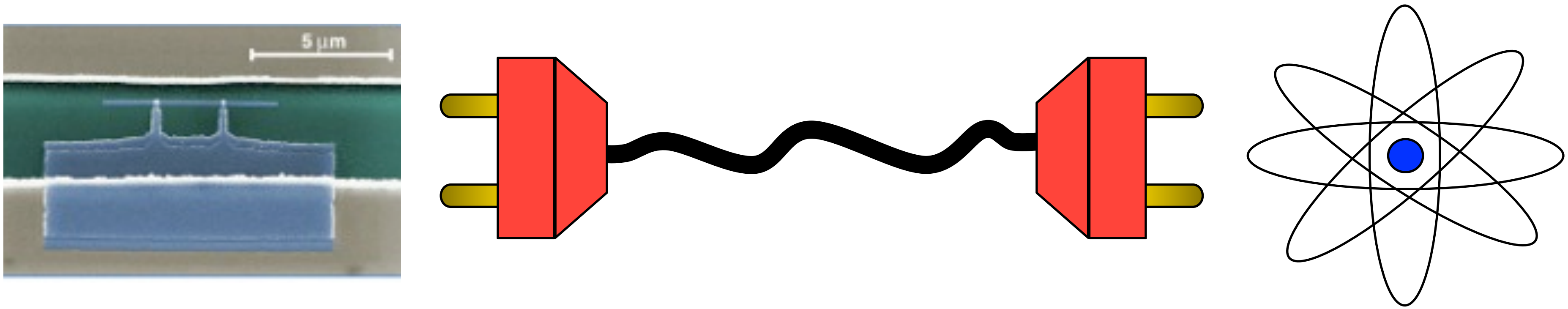
Can we get the best of both worlds?

“The standard solution”





“The standard solution”



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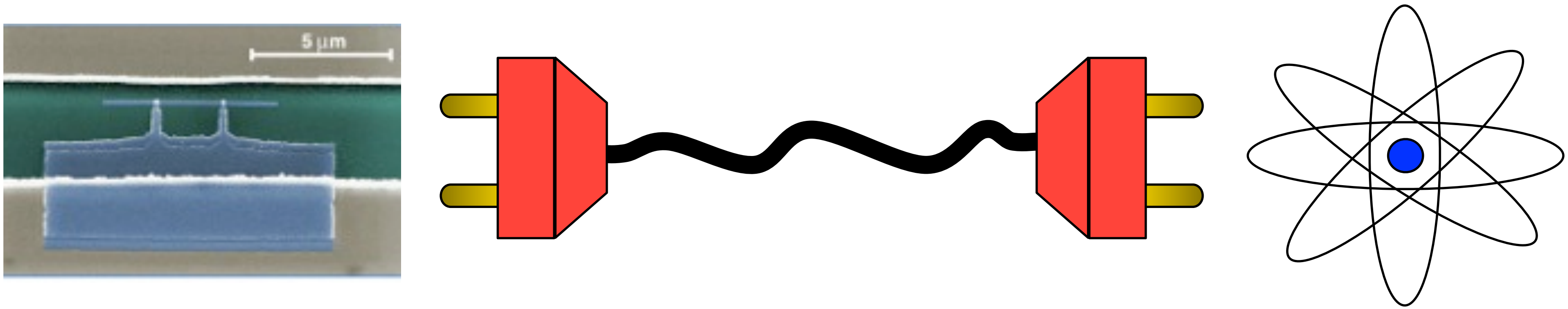
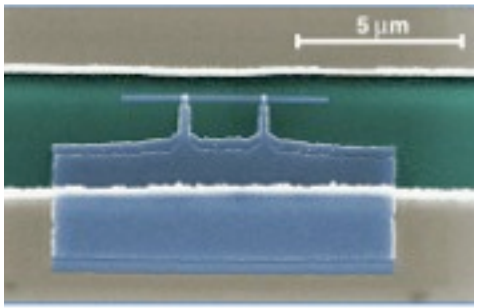
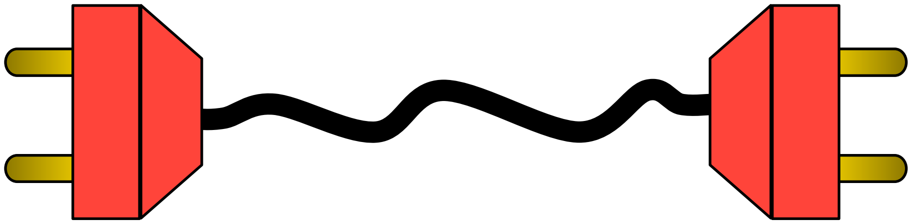
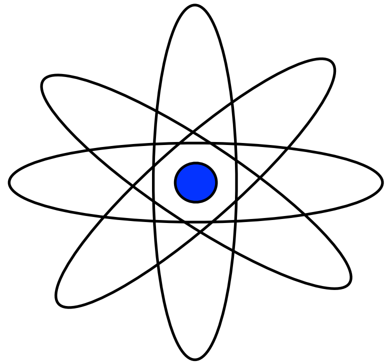


>20 Theory articles

“The standard solution”





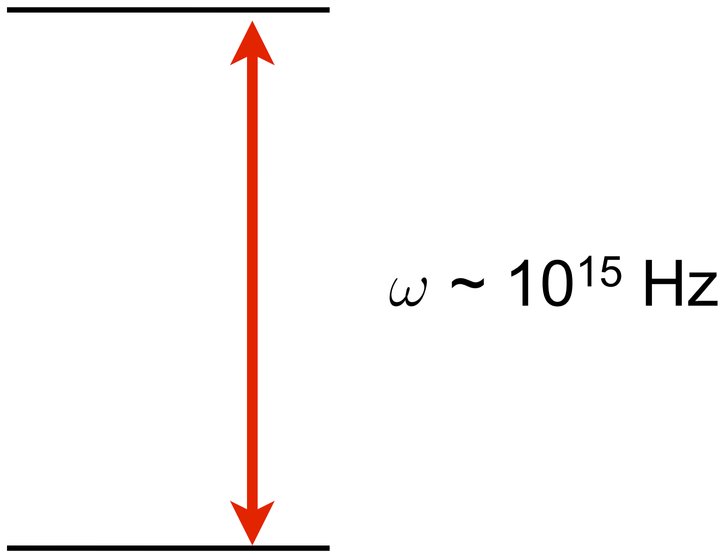


>20 Theory articles

3 Experimental

Atomic levels

Striplines: ω ~ GHz



Atomic levels

Striplines: ω ~ GHz

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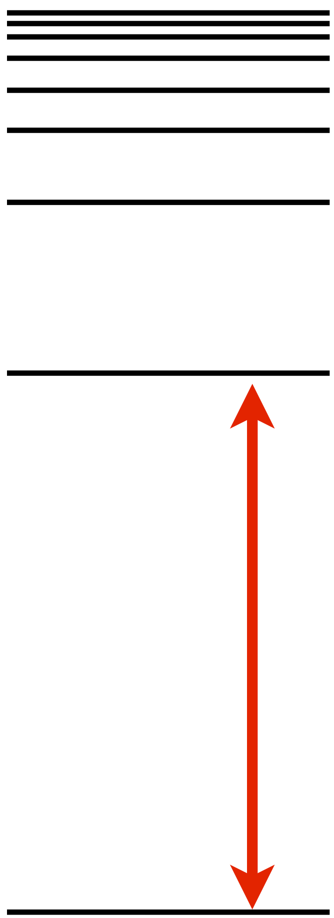

ω ~ 10^{15} Hz

Atomic levels

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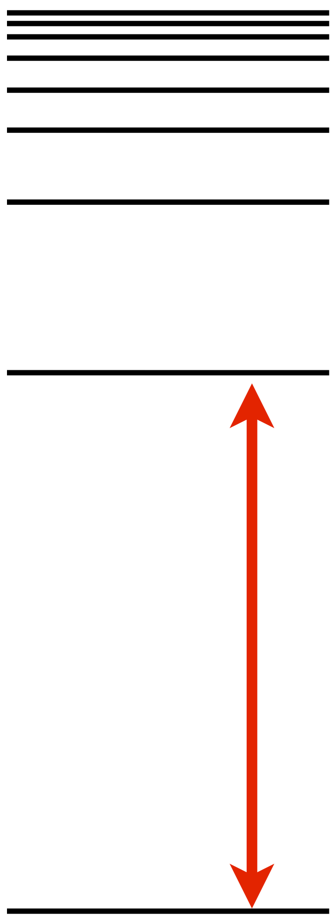

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$$E_n \propto \frac{1}{n^2}$$

$$\Delta E_n \propto \frac{1}{n^3}$$

n ~ 50-100 (Rydberg level)

=> ω ~ GHz

Atomic levels

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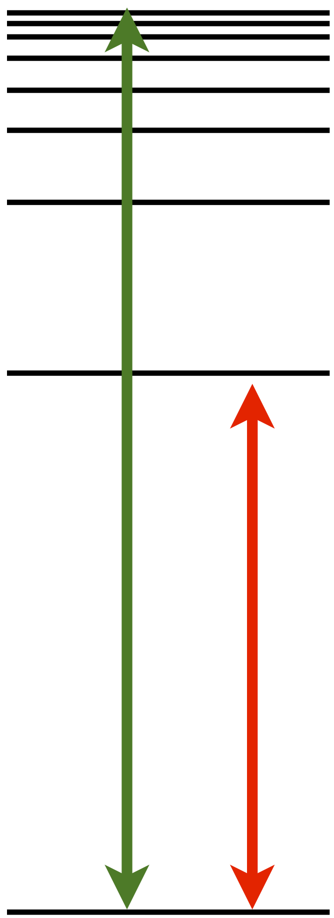

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n ~ 50-100 (Rydberg level)

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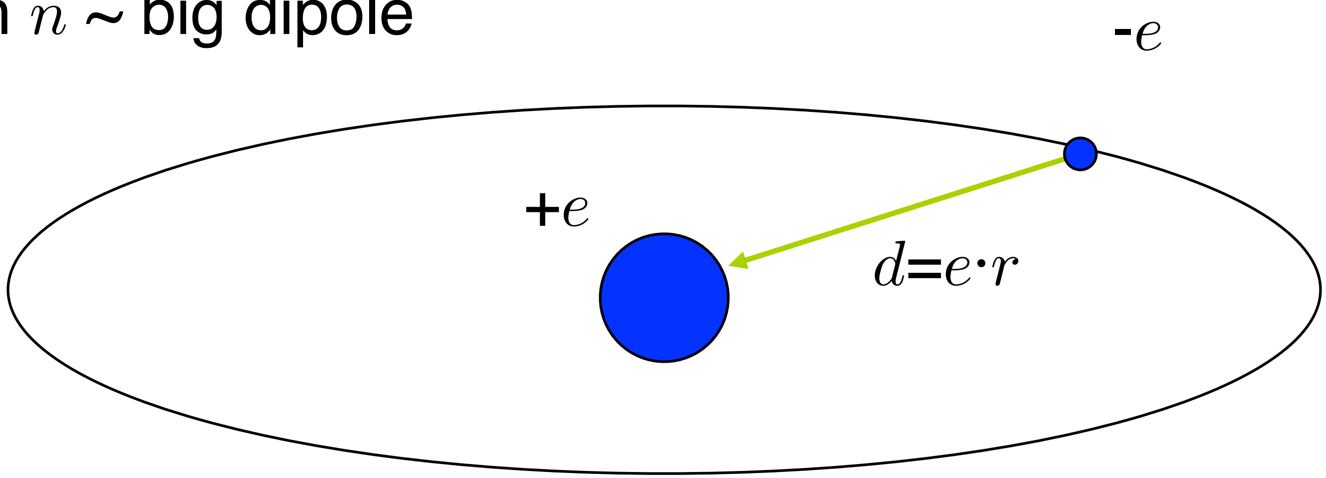
Can be excited by laser field

Rydberg atoms

High n ~ big dipole



r= n^2 a_0 $\Rightarrow$ d large

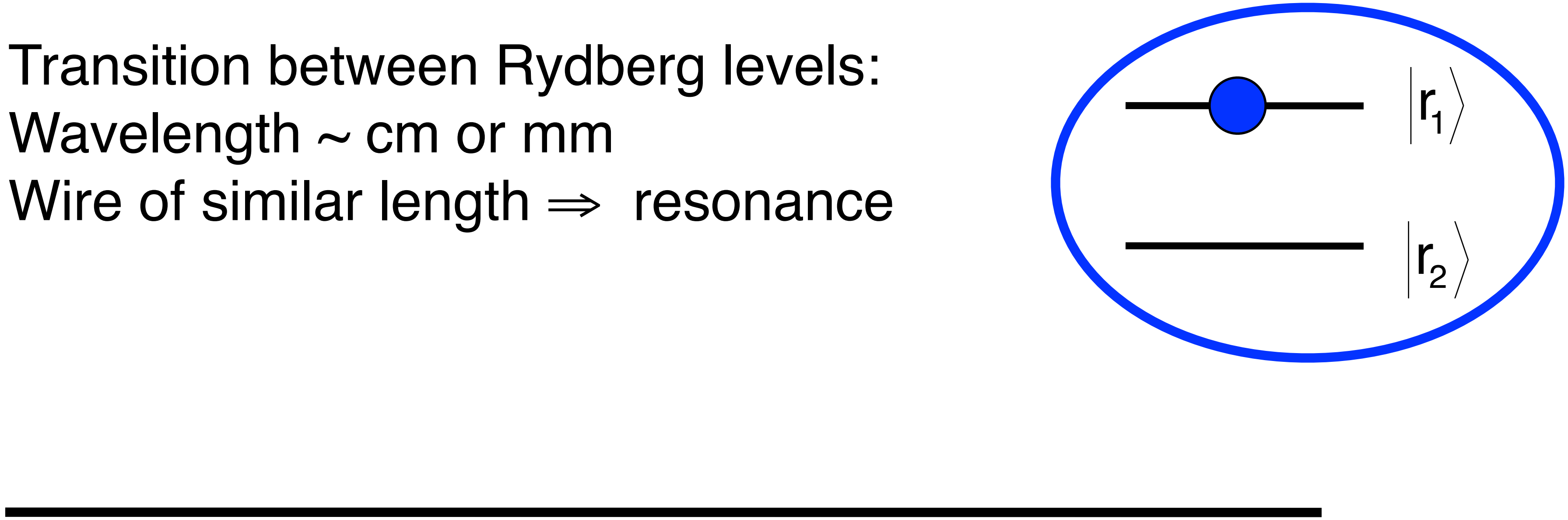
Large r $\Rightarrow$ small acceleration

$\Rightarrow$ weak radiation / long life times

Atom close to conductor: dipole interact with field from wire

Resonant interaction

Transition between Rydberg levels:
Wavelength ~ cm or mm
Wire of similar length ⇒ resonance



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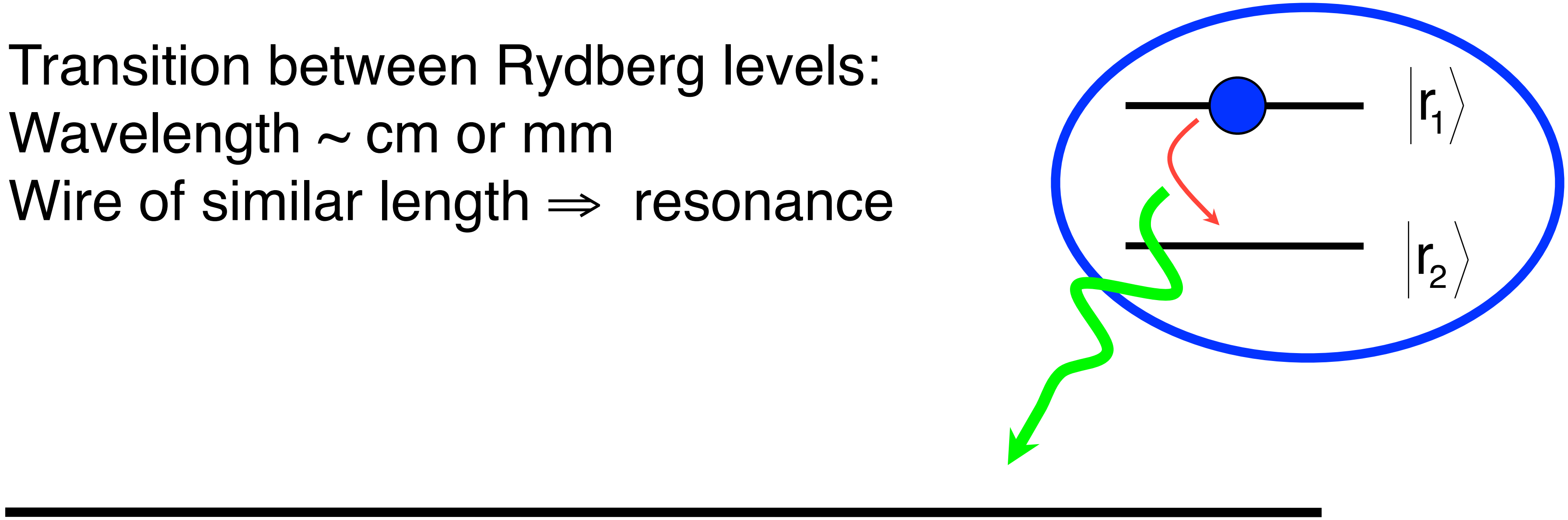

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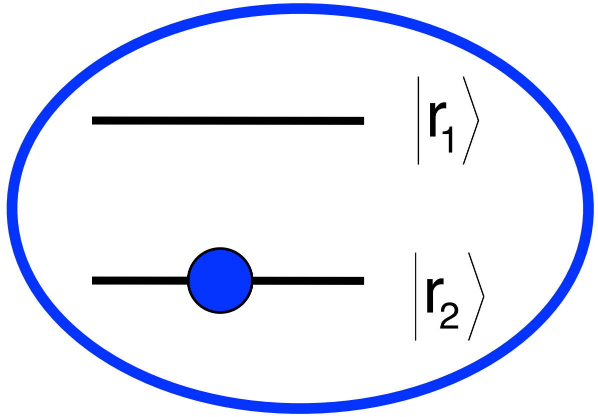

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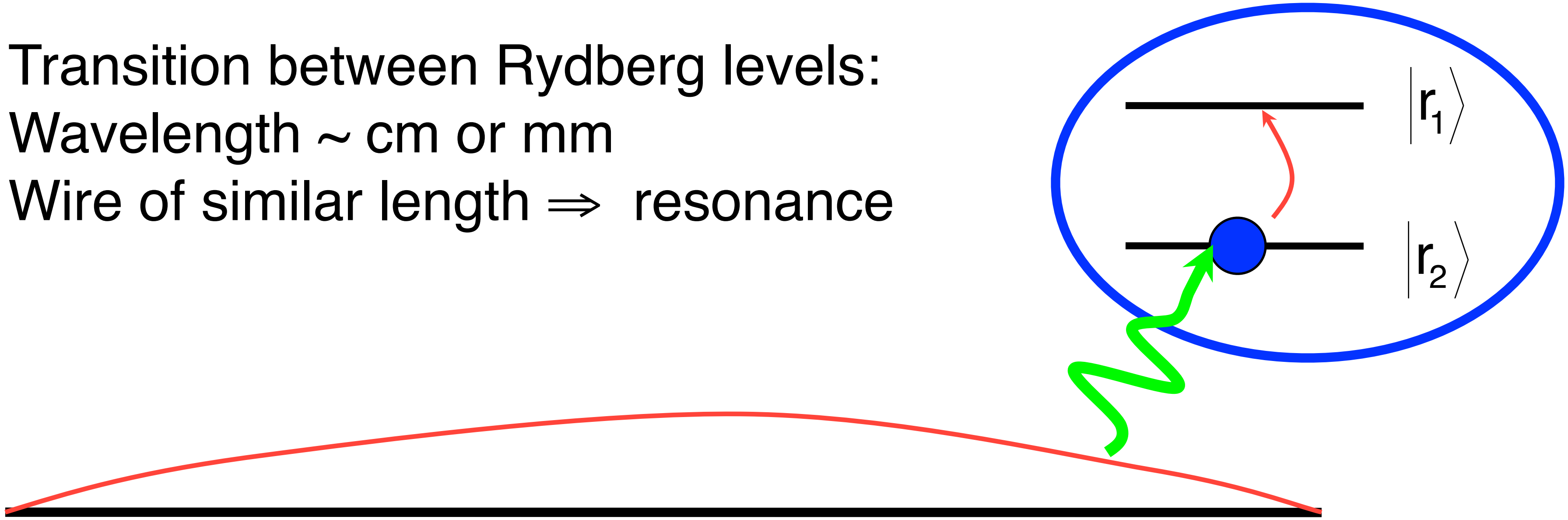

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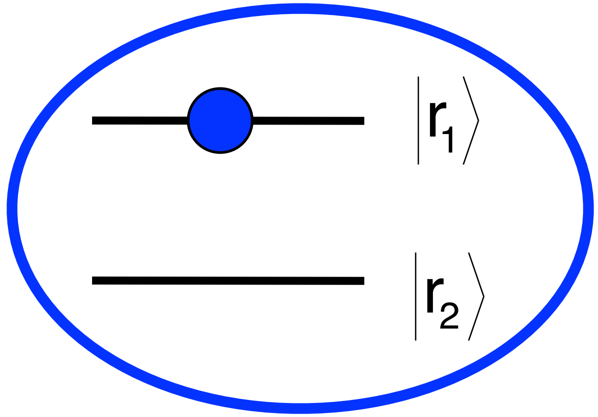
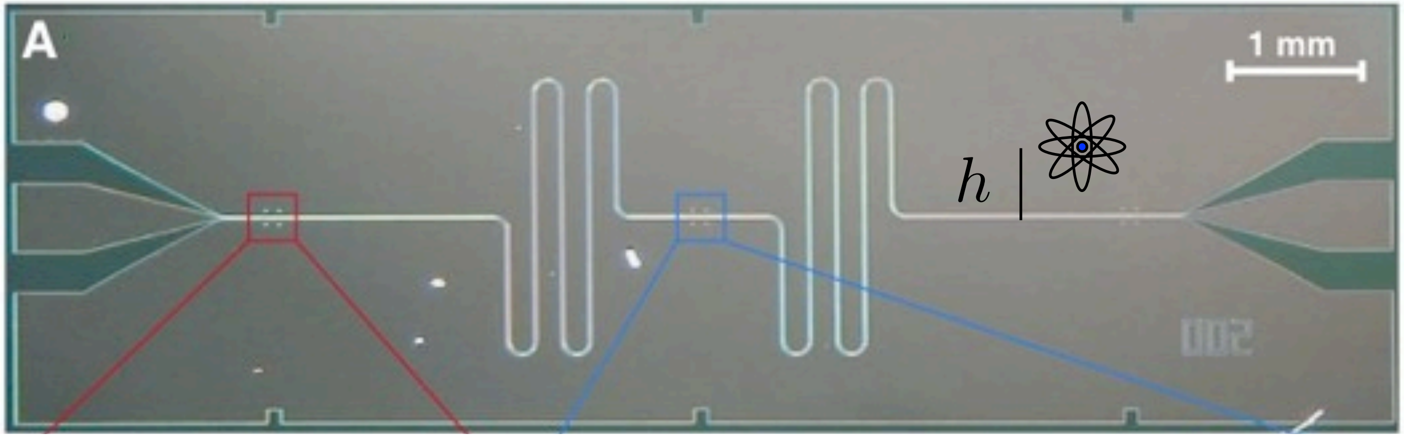


Describing the interaction



Field energy: $H \sim \int d^3r(E^2 + B^2) \sim \hbar\omega$ => $E_0 \sim \sqrt{\hbar\omega/V}$

Coupling: $H = g(\sigma_+ \hat{a} + \sigma_- \hat{a}^\dagger)$

$$g = \vec{E}_0 \cdot \vec{D} = D\sqrt{\hbar\omega/V}$$

A. S. Sørensen, van der Wal, Childress, and Lukin, Phys. Rev. Lett. **92**, 063601 (2004).

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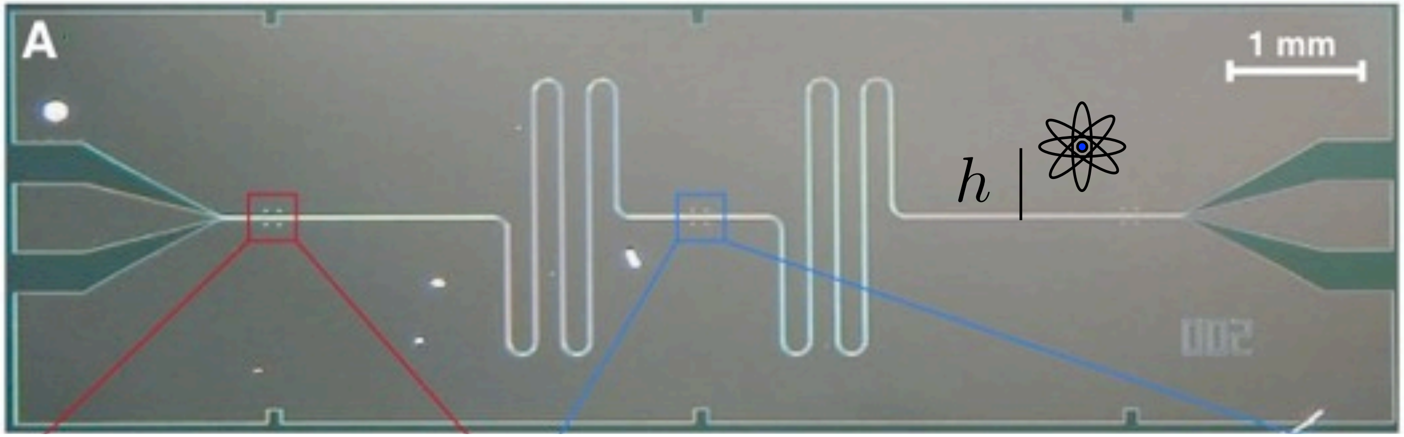

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n=50 $\Rightarrow$ L~ 3 mm, ω=2π 50 GHz, h=10 μm => g= 2π 3 MHz

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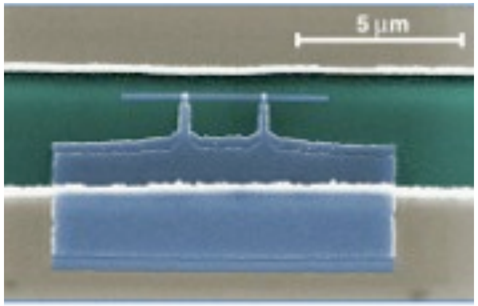
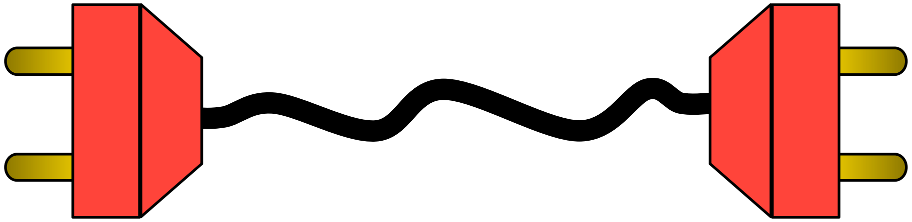
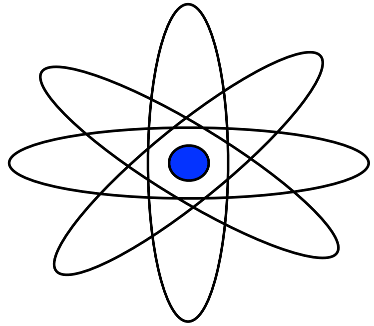
Combining atoms and solid







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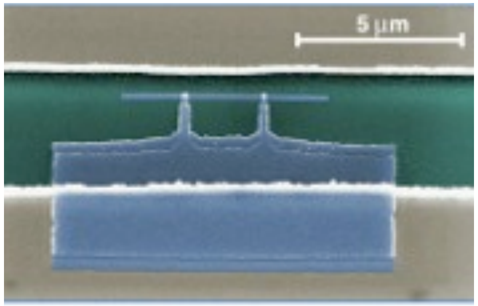
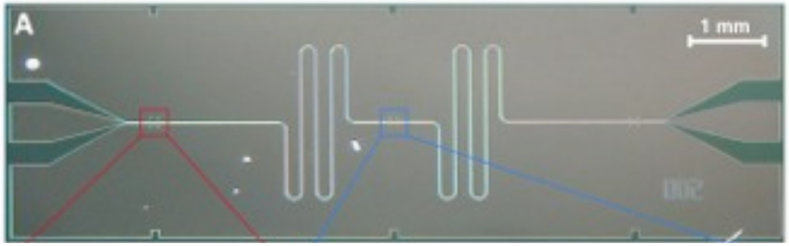
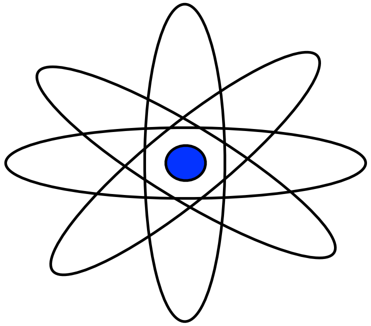
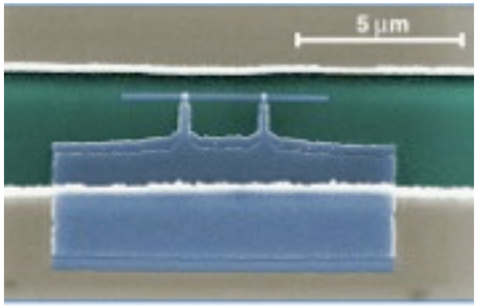


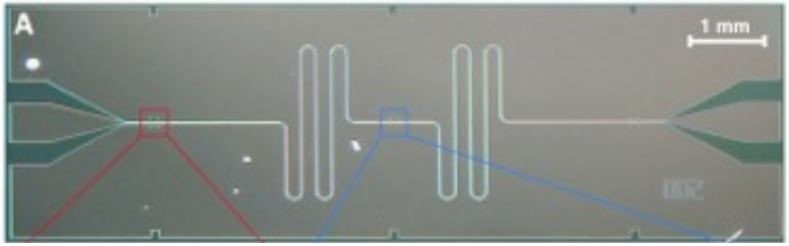


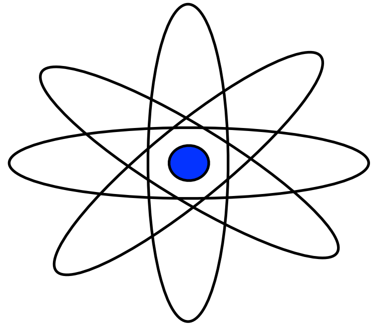


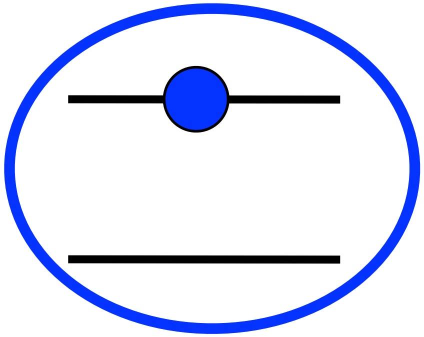
Strip lines can have strong coupling to
1) Solid state qubits
2) Single atoms

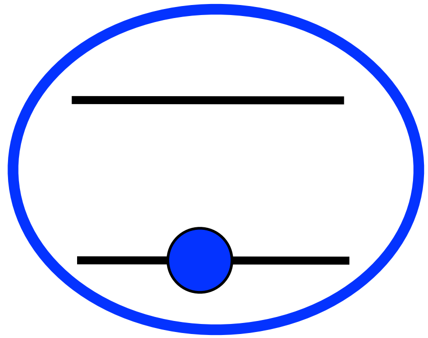
Combining atoms and solid





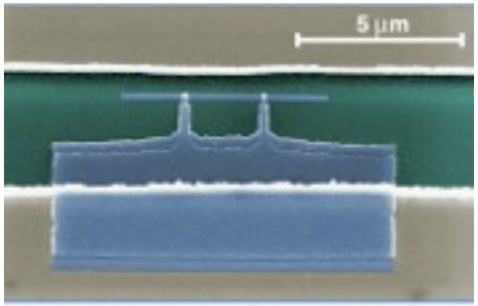


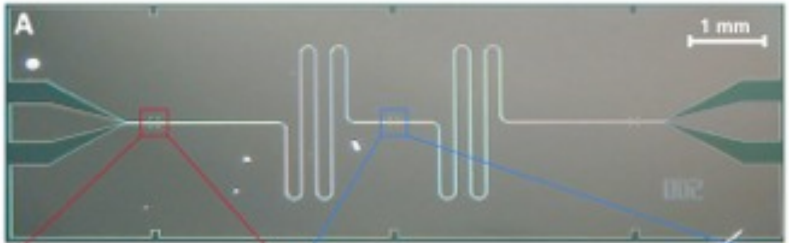


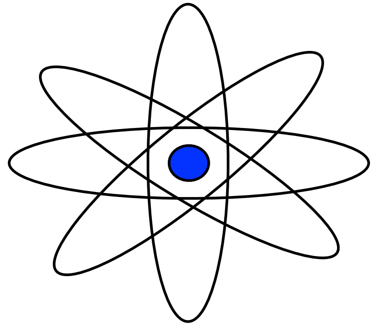


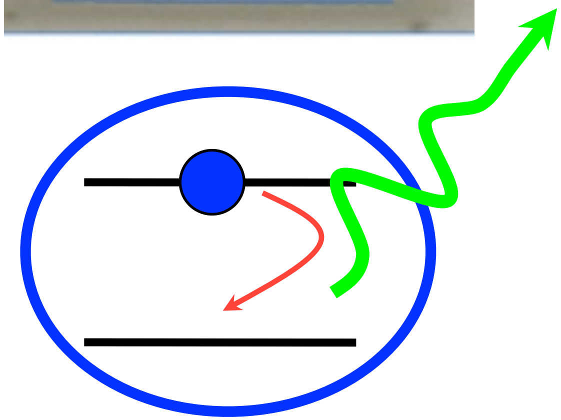
Strip lines can have strong coupling to
1) Solid state qubits
2) Single atoms

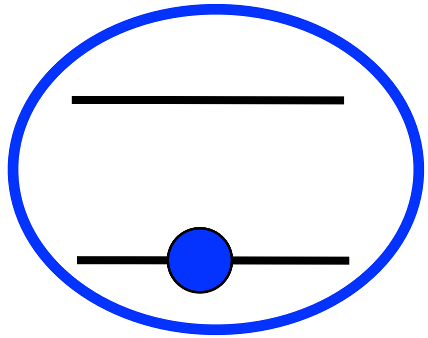
Combining atoms and solid











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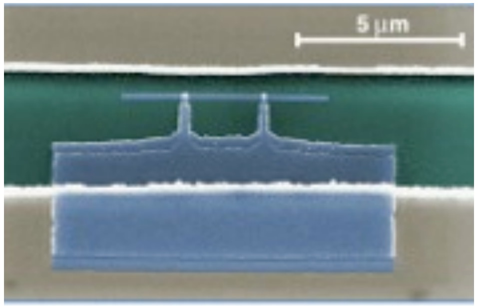

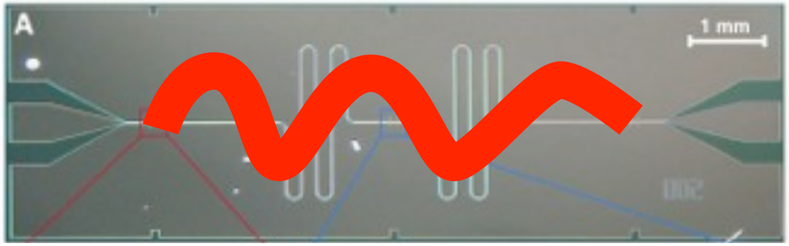

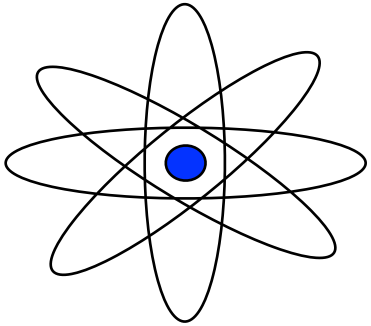

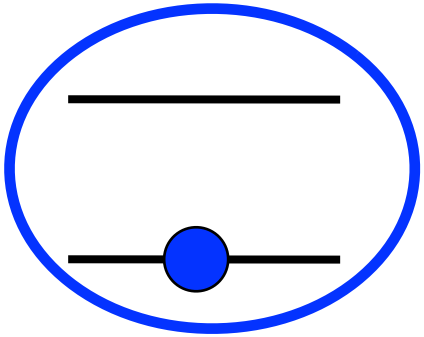

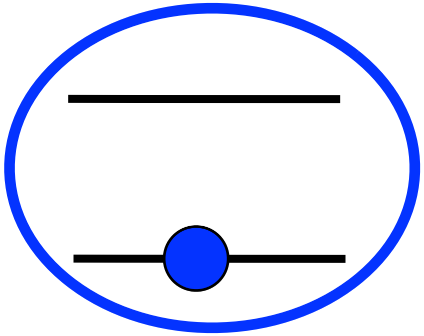

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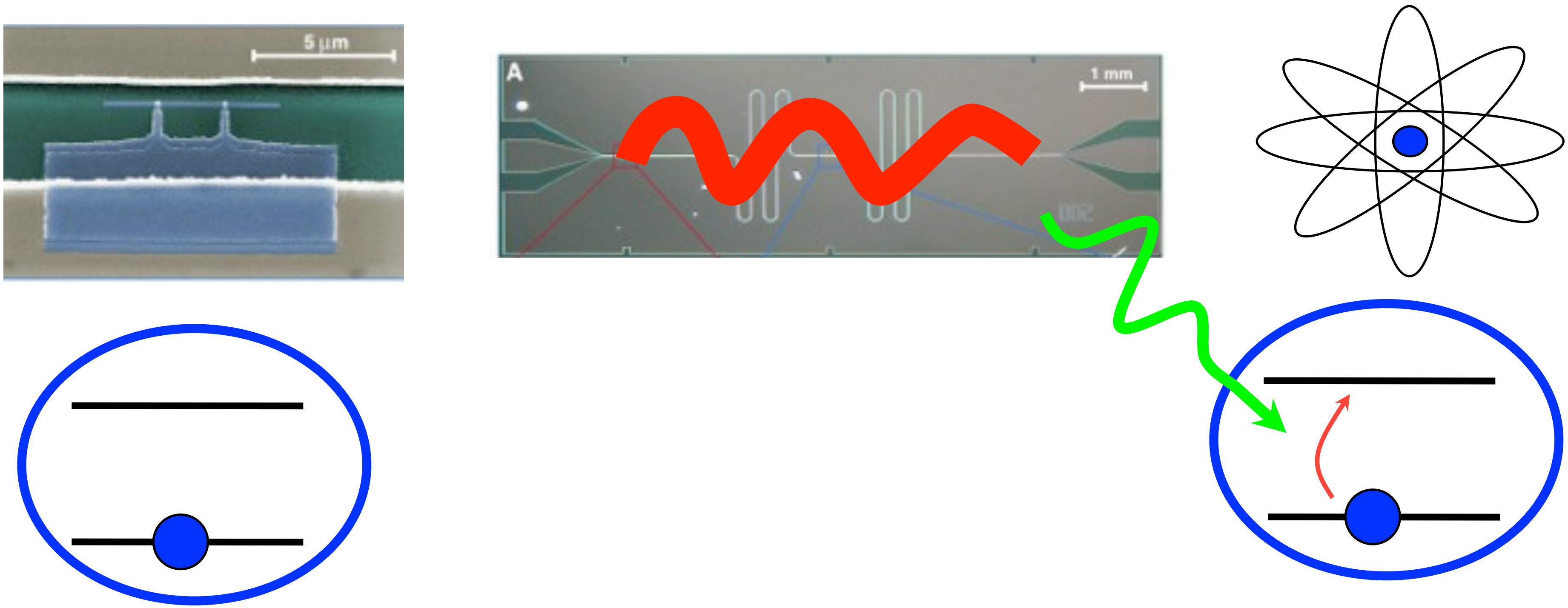

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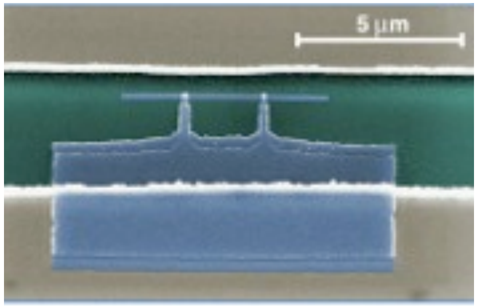

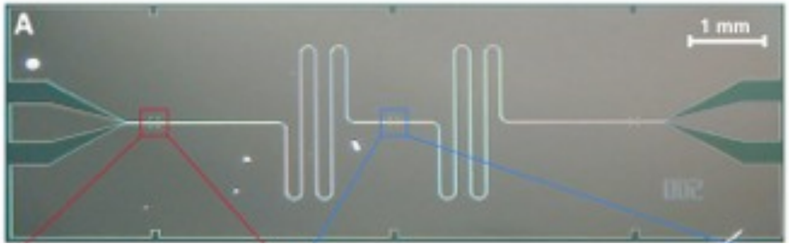

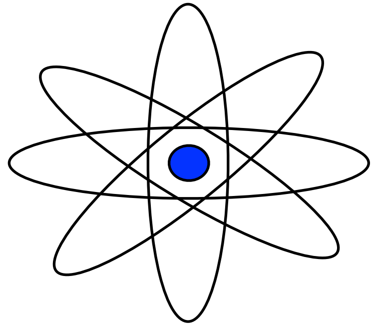

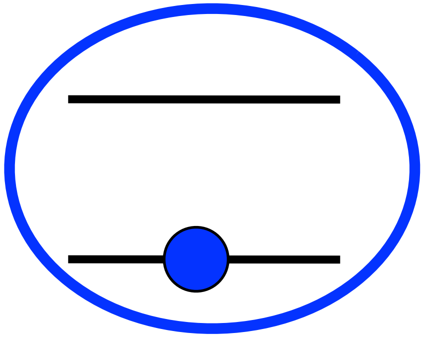

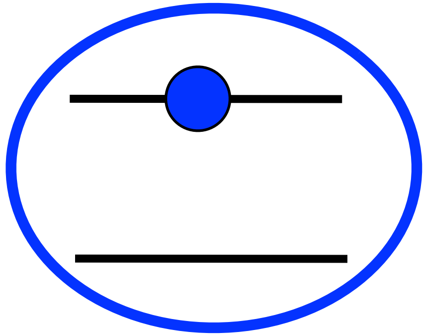

Strip lines can have strong coupling to
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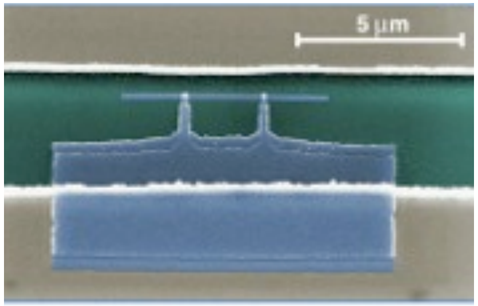
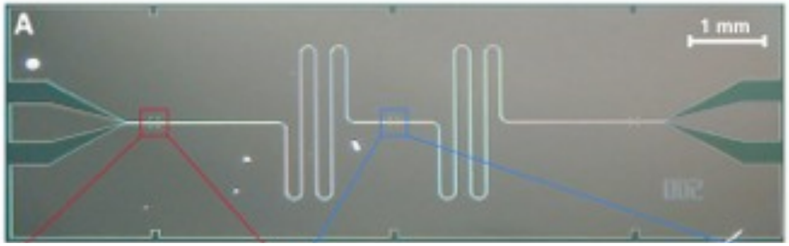
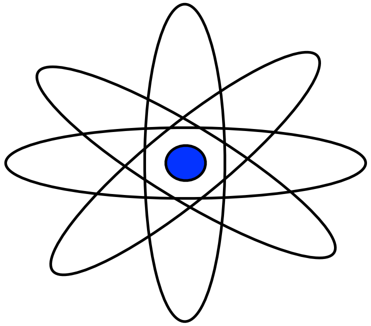
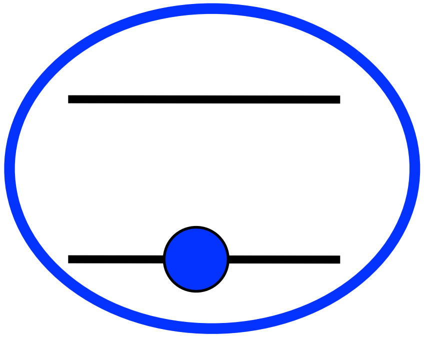
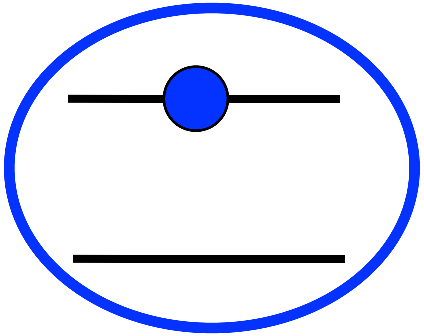
Combining atoms and solid











Strip lines can have strong coupling to
1) Solid state qubits ✓ Works
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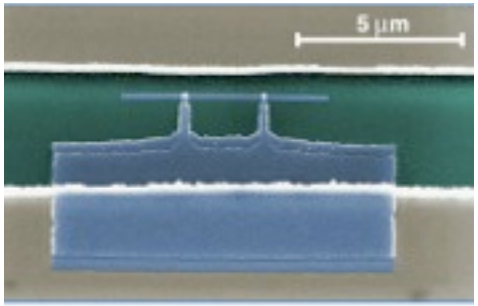
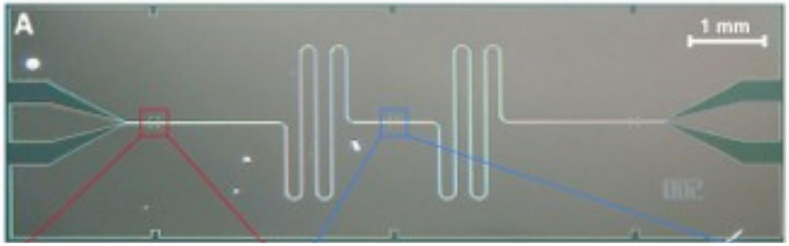
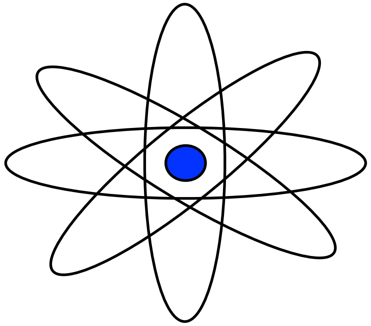
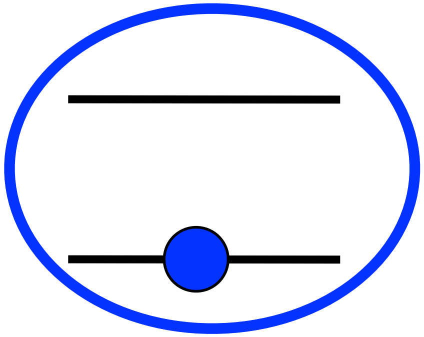
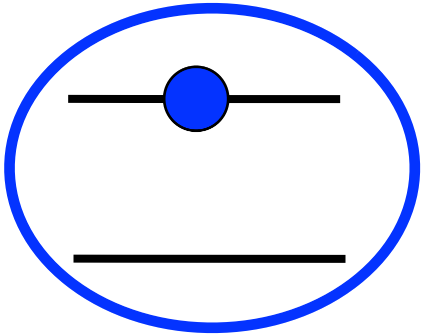
Combining atoms and solid











Strip lines can have strong coupling to
1) Solid state qubits ✓ Works
2) Single atoms In theory

Conclusion (2)

Hybrid systems may combine the best of two worlds
Atoms: long coherence times, identical, connect to light
Solid state: scalability, micro fabrication, no trapping

Stripline cavities is a promising solution
Can be coupled to solid state qubits (works!)
Can be coupled to atoms (theory)

This is all very easy and we will have a working quantum computer soon

“Nothing has been swept under the carpet”

“Nothing has been swept under the carpet”

Former Danish prime minister Poul Schlüter in parliament April 25th 1989

For discussion

What has been swept under the carpet?

Which things could potentially go wrong if one tries to build this device?

Potential problems

- Still need to trap and cool atoms, surface forces on atoms are big near surface
- Light and superconductors bad combination. One optical photon ~1 eV= many broken Cooper pairs
- No windows in setup at 100 mK
- Atoms have low decoherence when in vacuum. Decohere due to interaction with solid when close to surface

Solutions

A continuous transition

Quantum optics

Atoms

Ions

Solid state

Super Conductors

Electrically defined quantum dots

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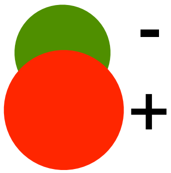
Polar Molecules



Heteronuclear molecule = small electric dipole
Rotation frequency ~ GHz

No need for laser to excite to high lying level

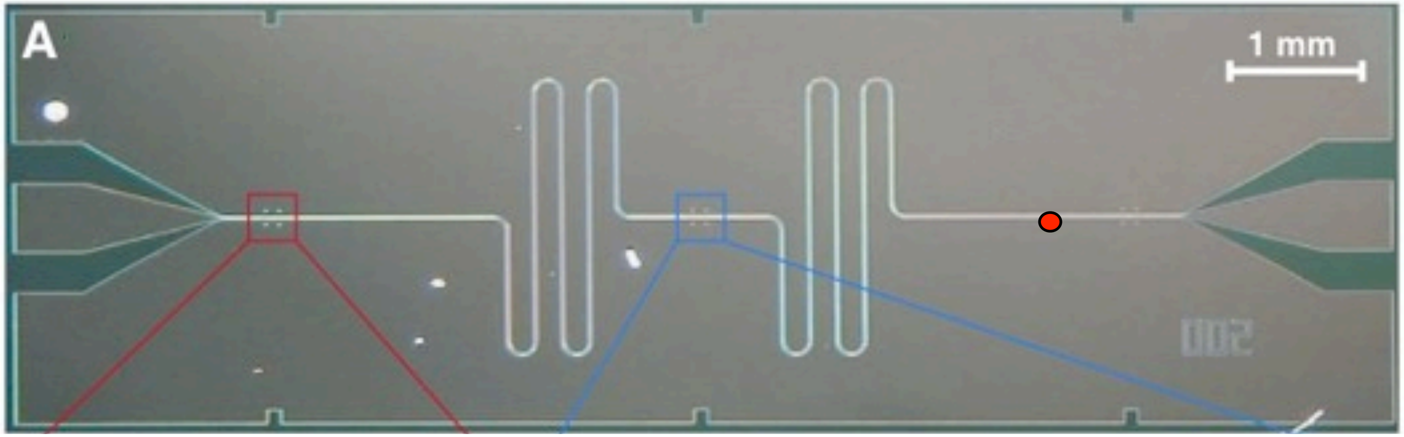
Problem: dipole moment much smaller

g~ (2π) 10-100 kHz

Cavity decay: similar or bigger.

Rabl, DeMille, Doyle, Lukin, Schoelkopf, and Zoller, Phys. Rev. Lett. **97**, 033003 (2006)
André, DeMille, Doyle, Lukin, Maxwell, Rabl, Schoelkopf and Zoller, Nature Physics **2**, 636 (2006)

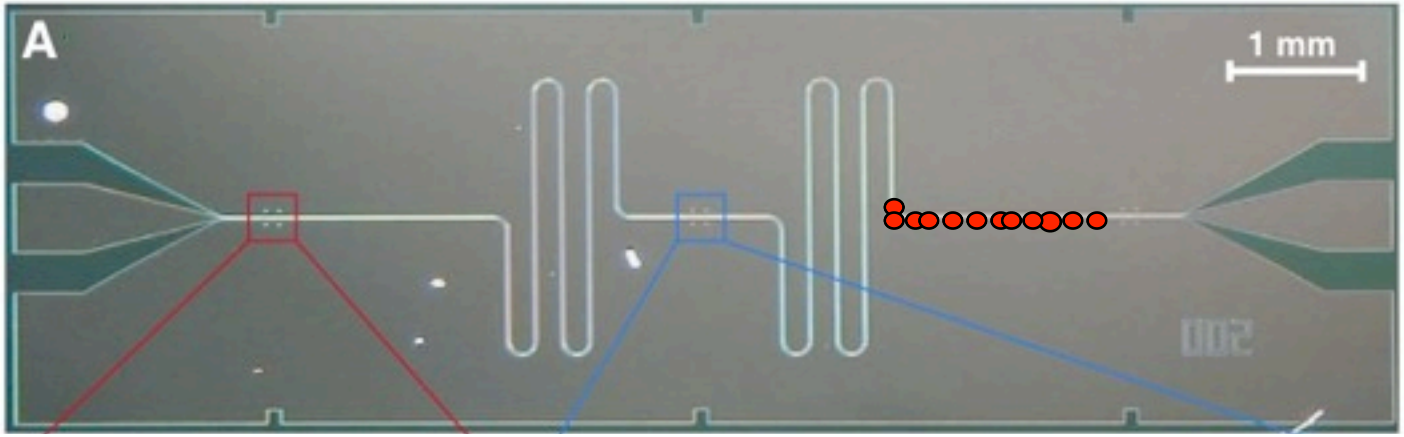
Collective enhancement



What happens if there are N molecules?

$$H = ga\sigma_+ + \text{H.C.}$$

Collective enhancement



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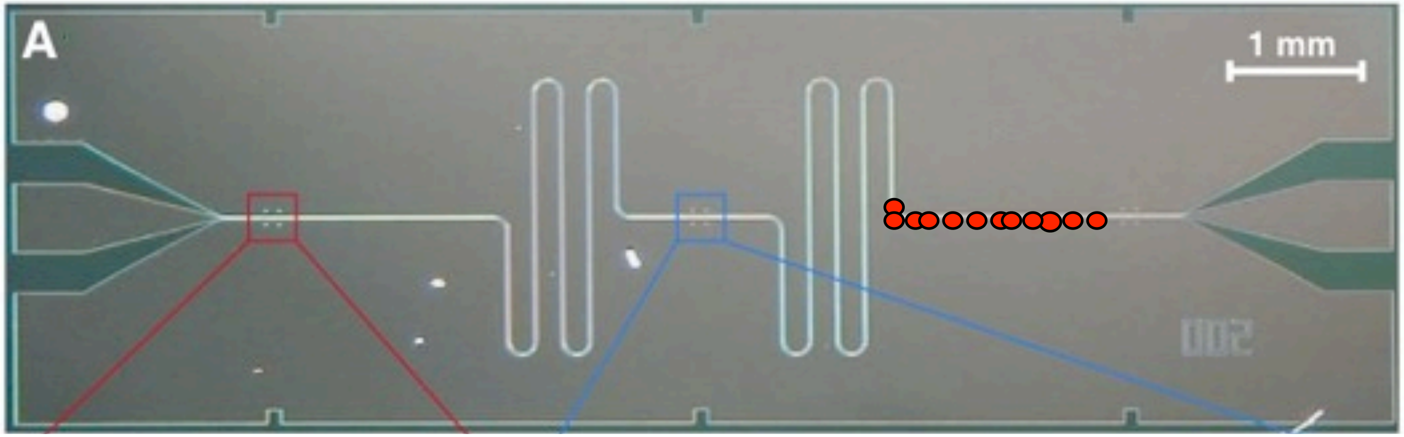

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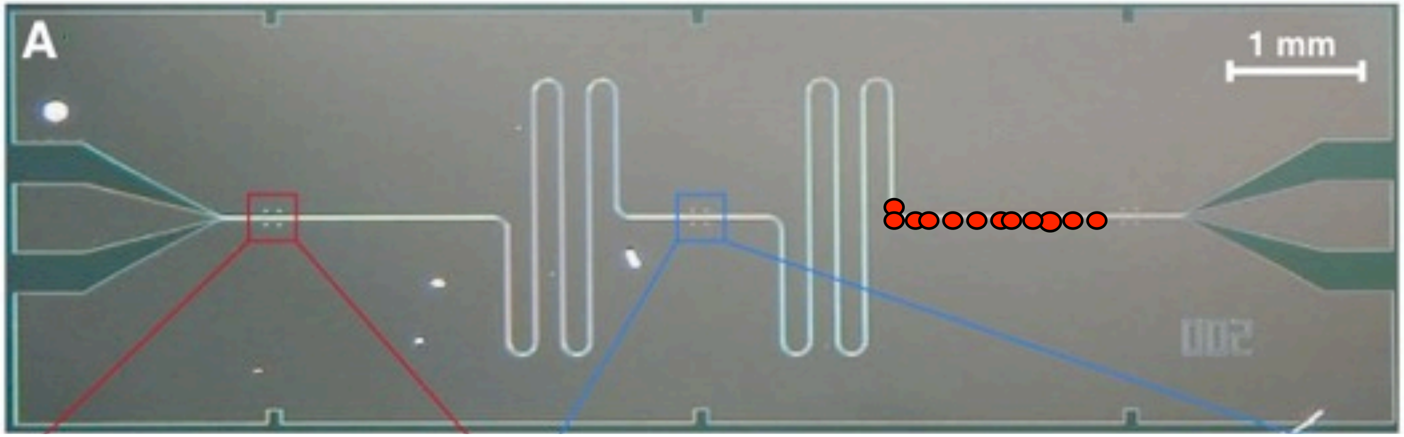

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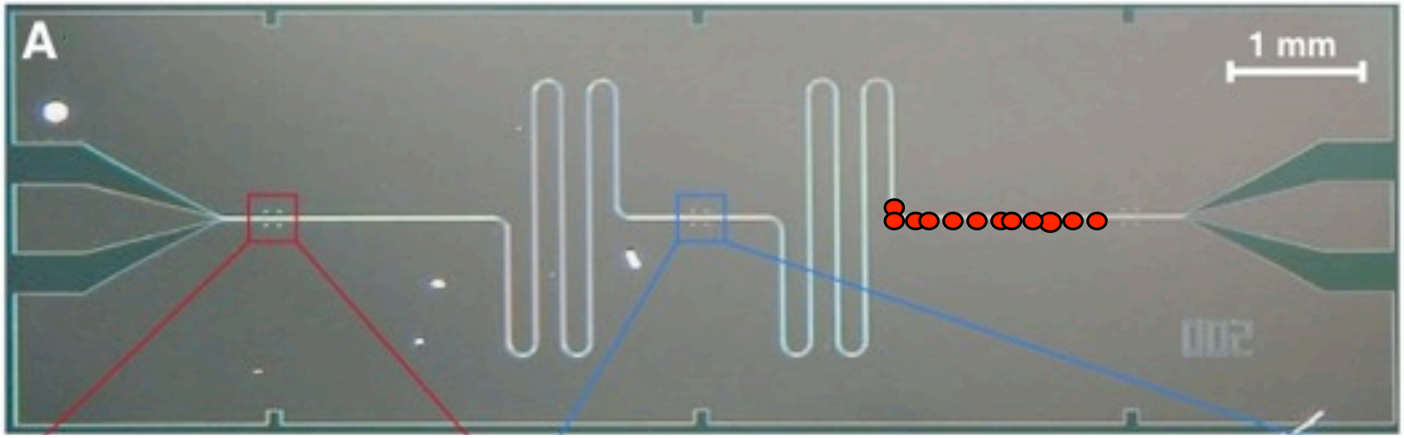
$$H = ga \sum_l \sigma_{+,l} + \text{H.C.}$$

$$|1\rangle_{\text{Cavity}} |00....0\rangle_{\text{mol}} \rightarrow \frac{1}{\sqrt{N}} |0\rangle_{\text{Cavity}} \sum_l |00...1_l...0\rangle_{\text{mol}}$$

Coupling enhanced by factor of $\sqrt{N}$

Collective enhancement



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‘Absorption’ not surprising:

N spins absorb N times better

Rabl, DeMille, Doyle, Lukin, Schoelkopf, and Zoller, Phys. Rev. Lett. **97**, 033003 (2006)

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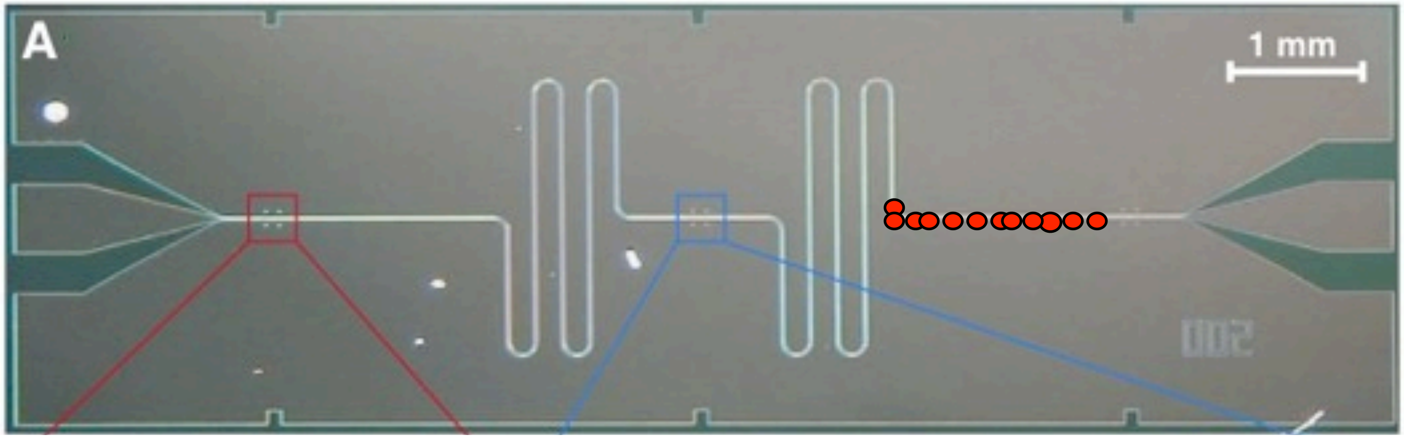

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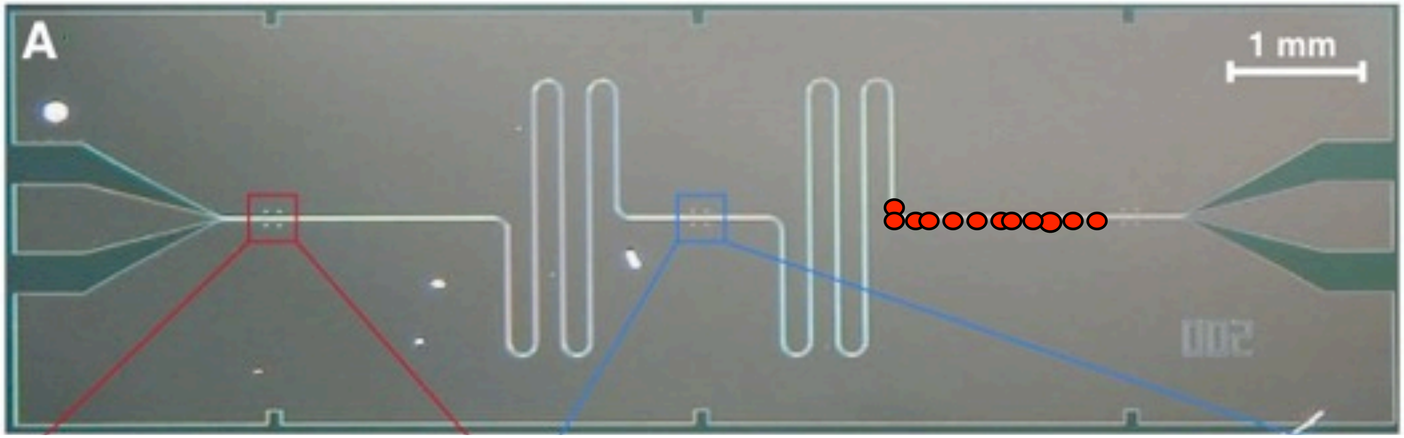

$$|1\rangle_{\rm Cav}|00....0\rangle_{\rm mol} \leftrightarrow \frac{1}{\sqrt{N}}|0\rangle_{\rm Cav}\sum_l |00...1_l...0\rangle_{\rm mol}$$

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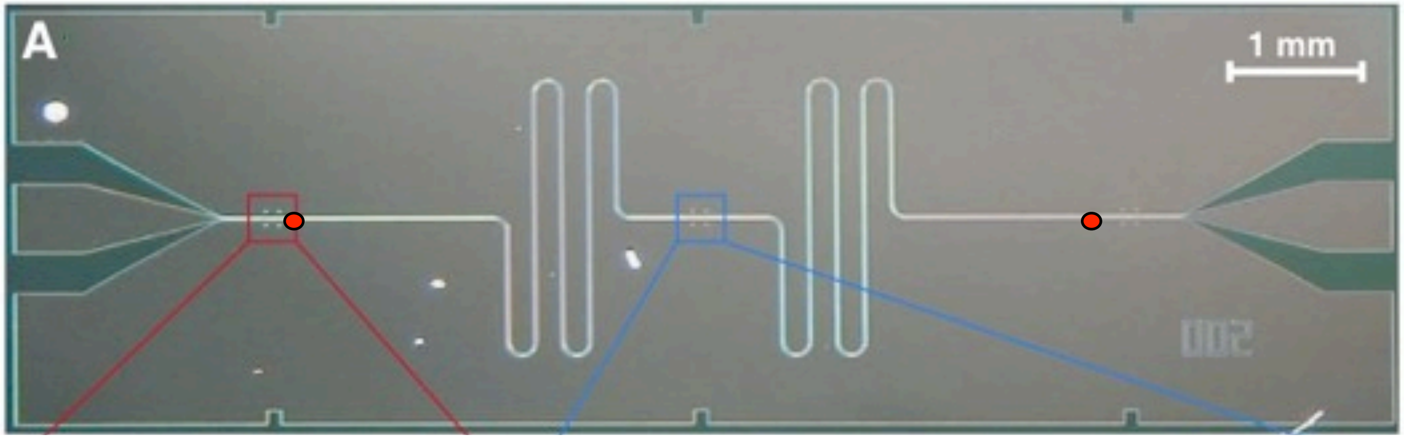
Coupling to collective excitation in ensemble possible

Rabl, DeMille, Doyle, Lukin, Schoelkopf, and Zoller, Phys. Rev. Lett. **97**, 033003 (2006)

Coupling two molecules



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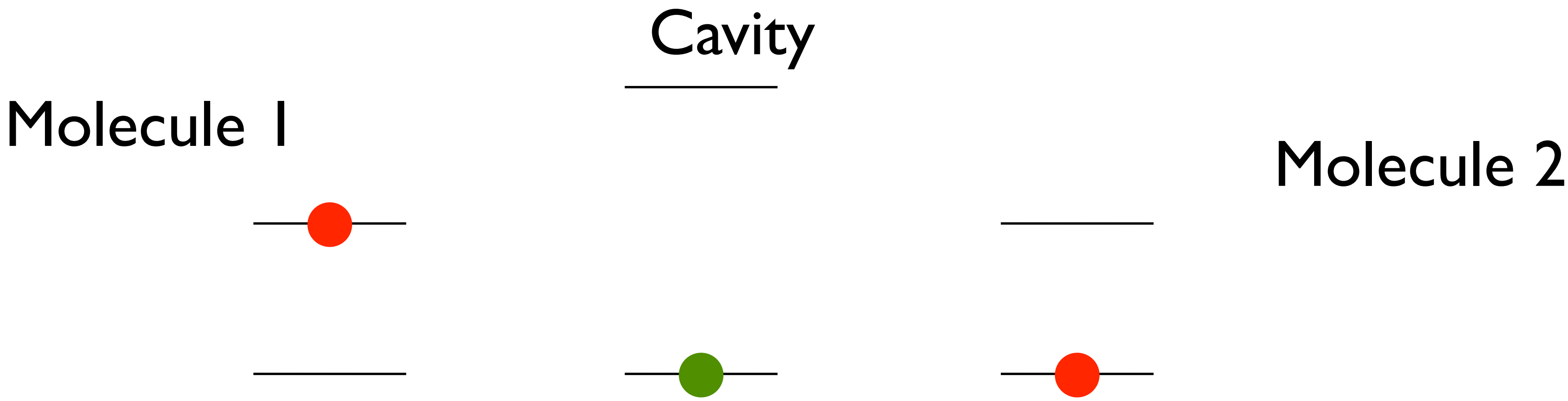

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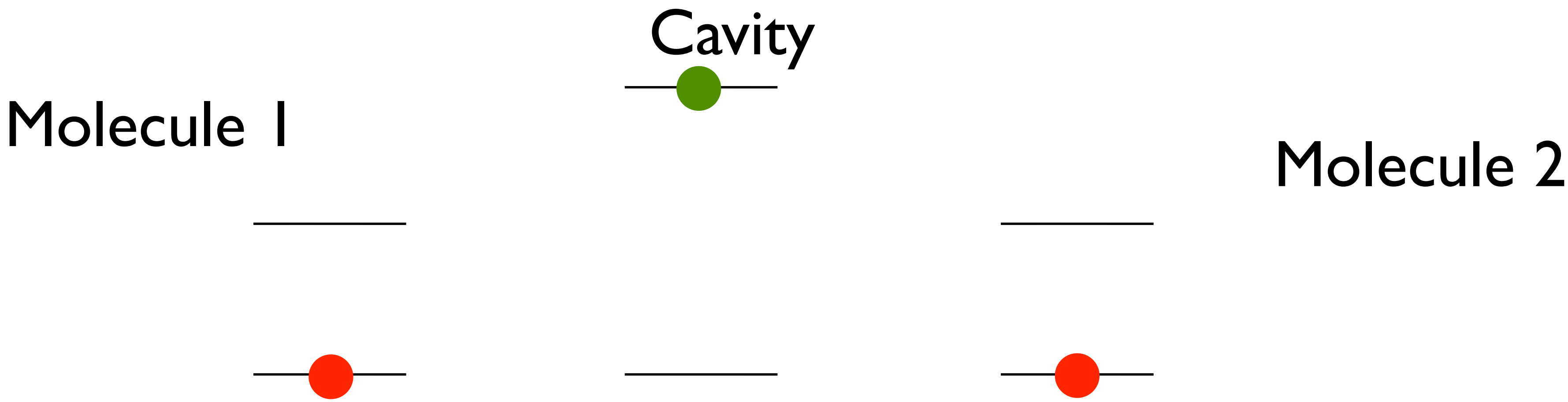

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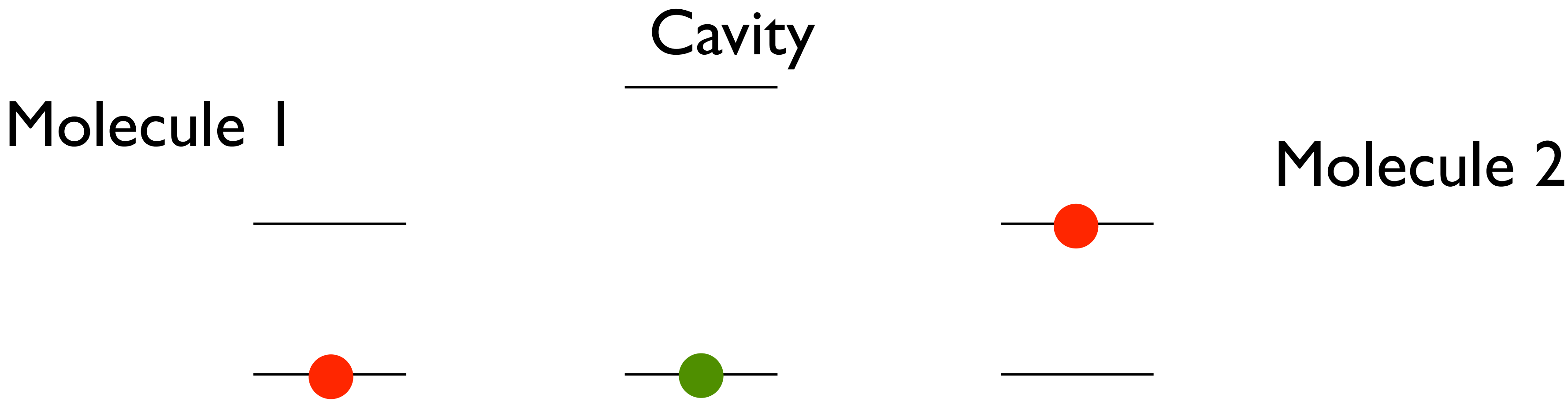

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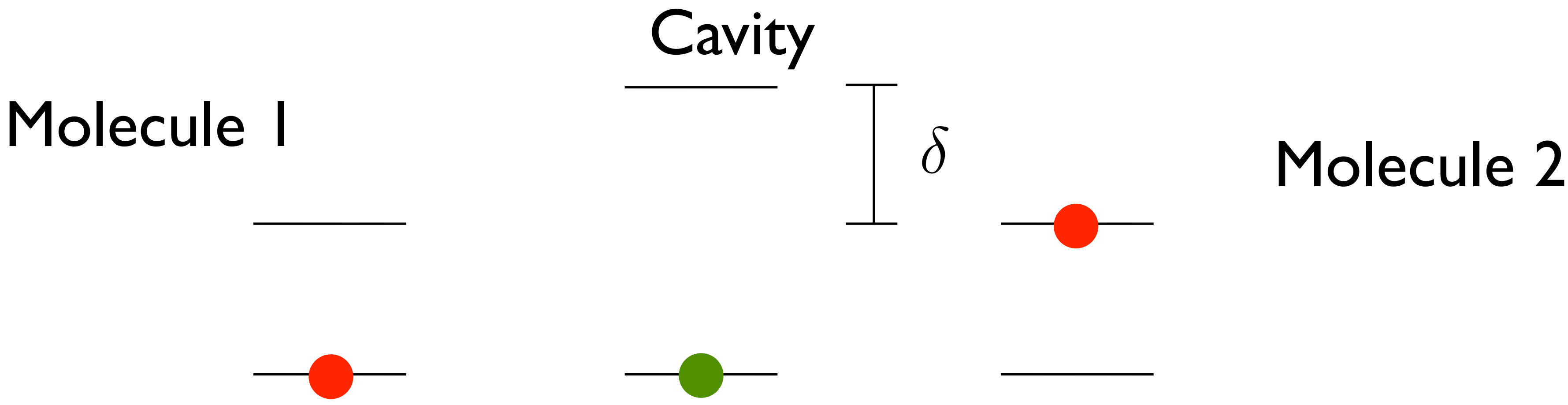

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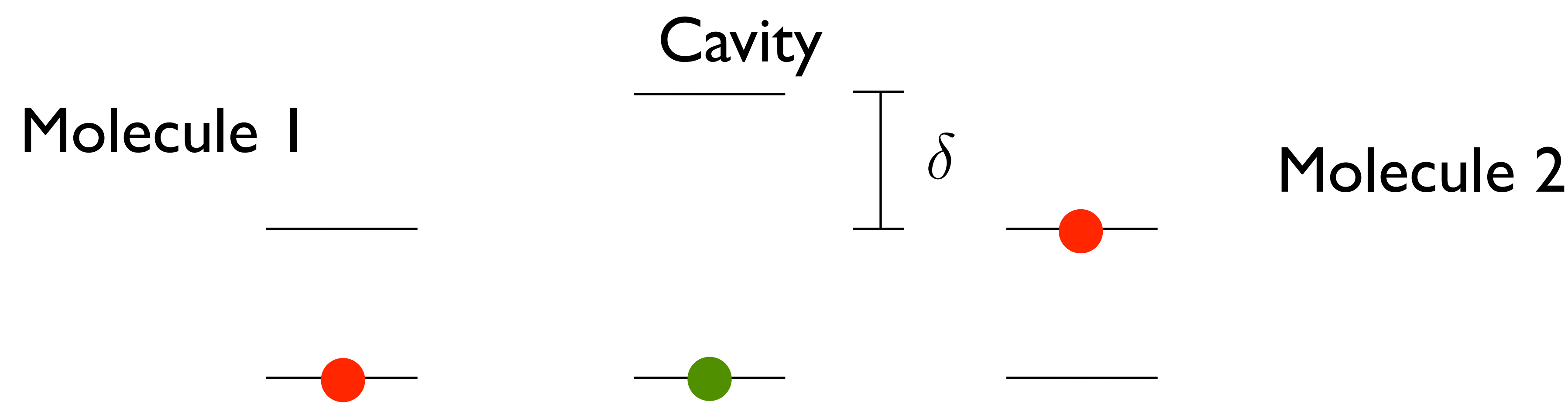

Coupling: $g_{\mathrm{eff}} = \dfrac{g^2}{\delta}$

Decoherence: $\gamma_{\mathrm{eff}} = \dfrac{g^2}{\delta^2}\kappa$

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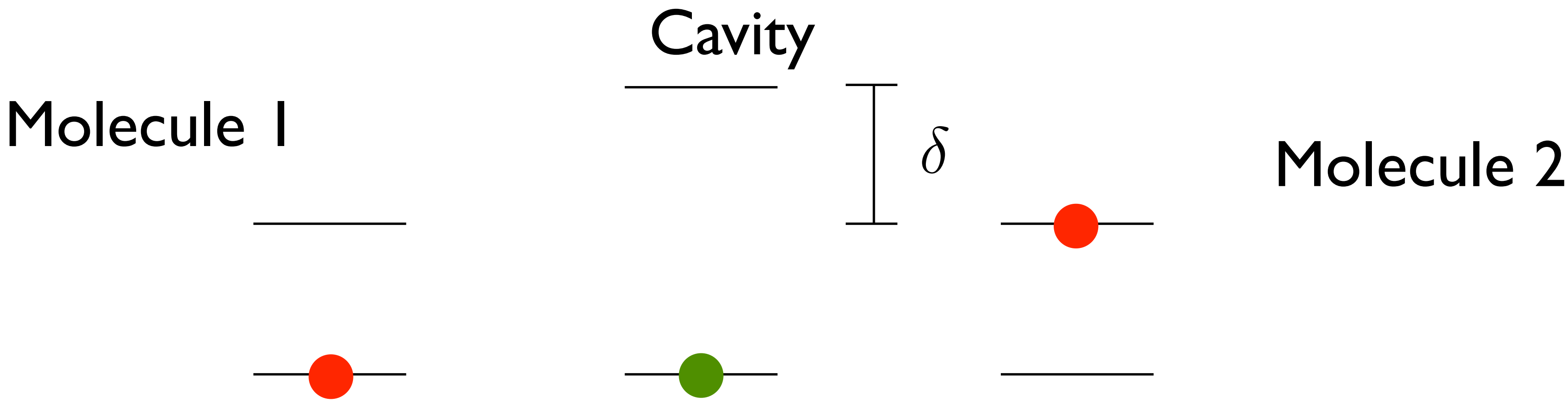

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Can connect two molecules through cavity

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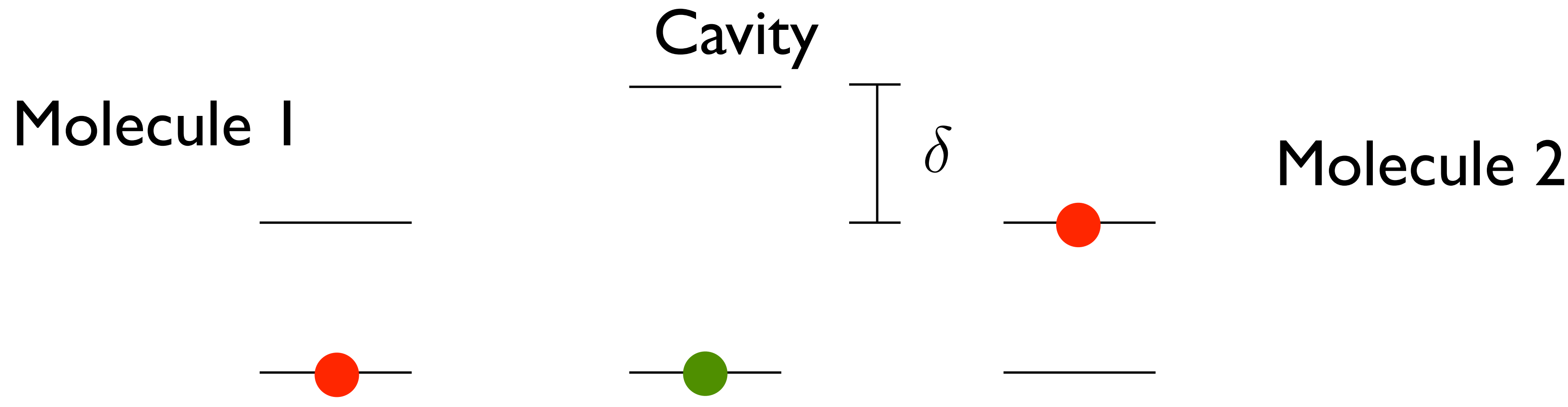

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Can connect two molecules through cavity

Cooling and trapping still challenging

André, DeMille, Doyle, Lukin, Maxwell, Rabl, Schoelkopf and Zoller, Nature Physics **2**, 636 (2006)

A continuous transition

Quantum optics

Solid state

Atoms

NV-centers

Super Conductors

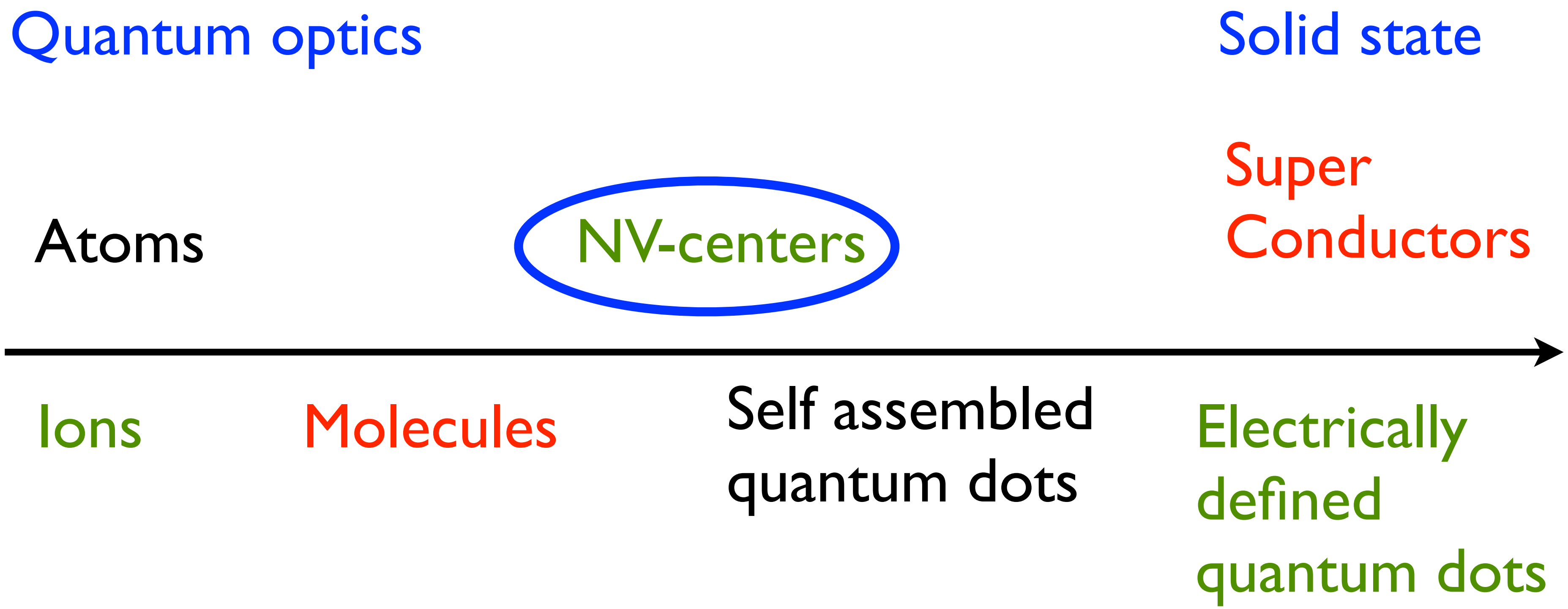
Ions

Molecules

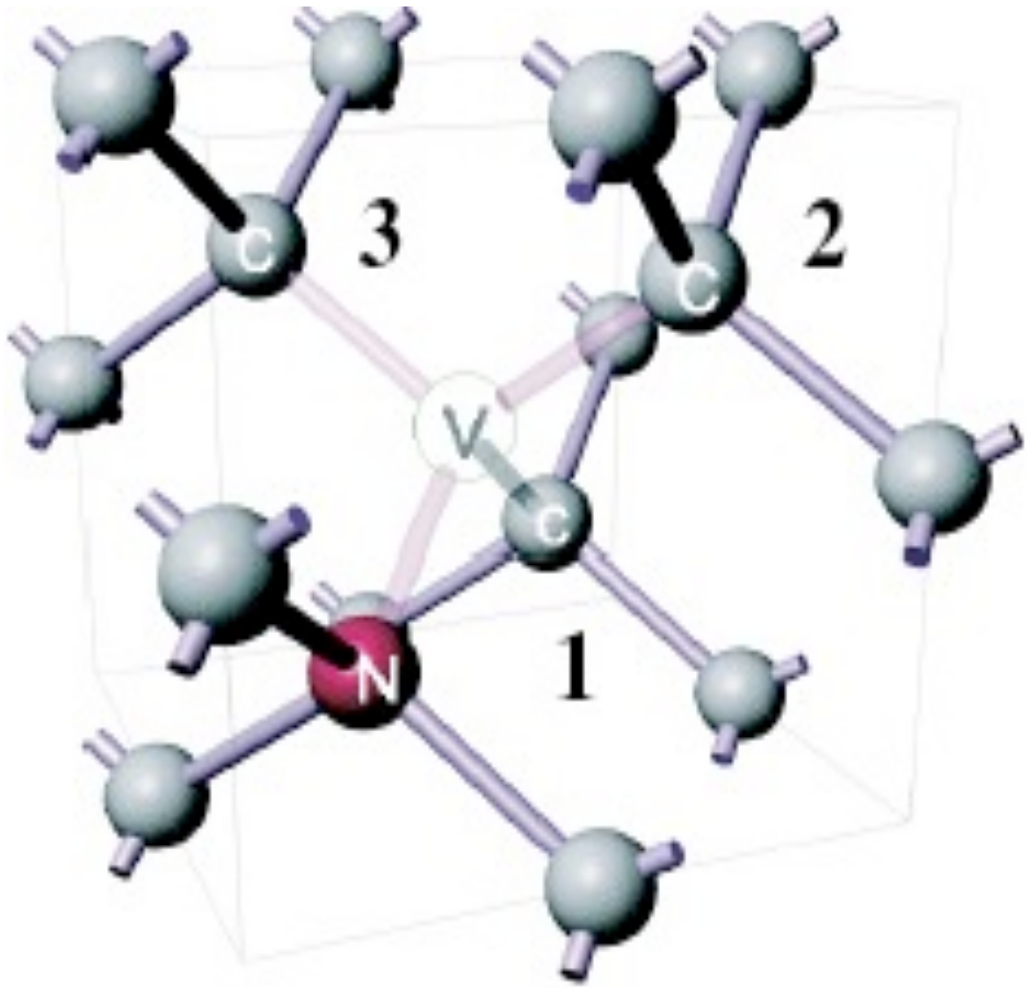
Self assembled quantum dots

Electrically defined quantum dots

A continuous transition



NV centers in diamond



Jelezko, Wrachtrup *et al.*, Stuttgart

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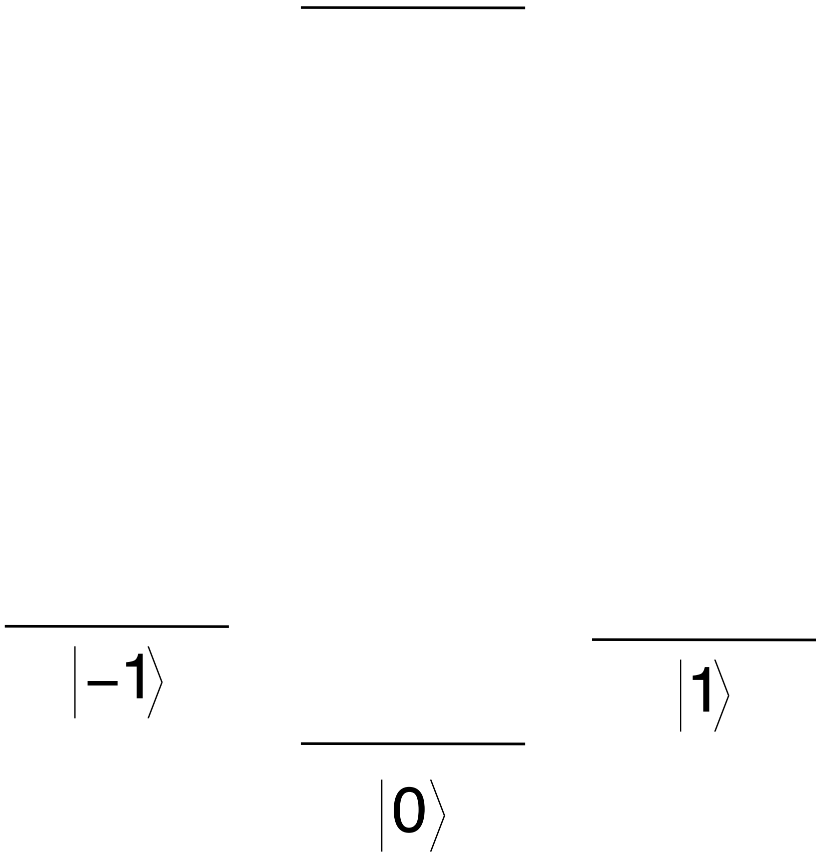

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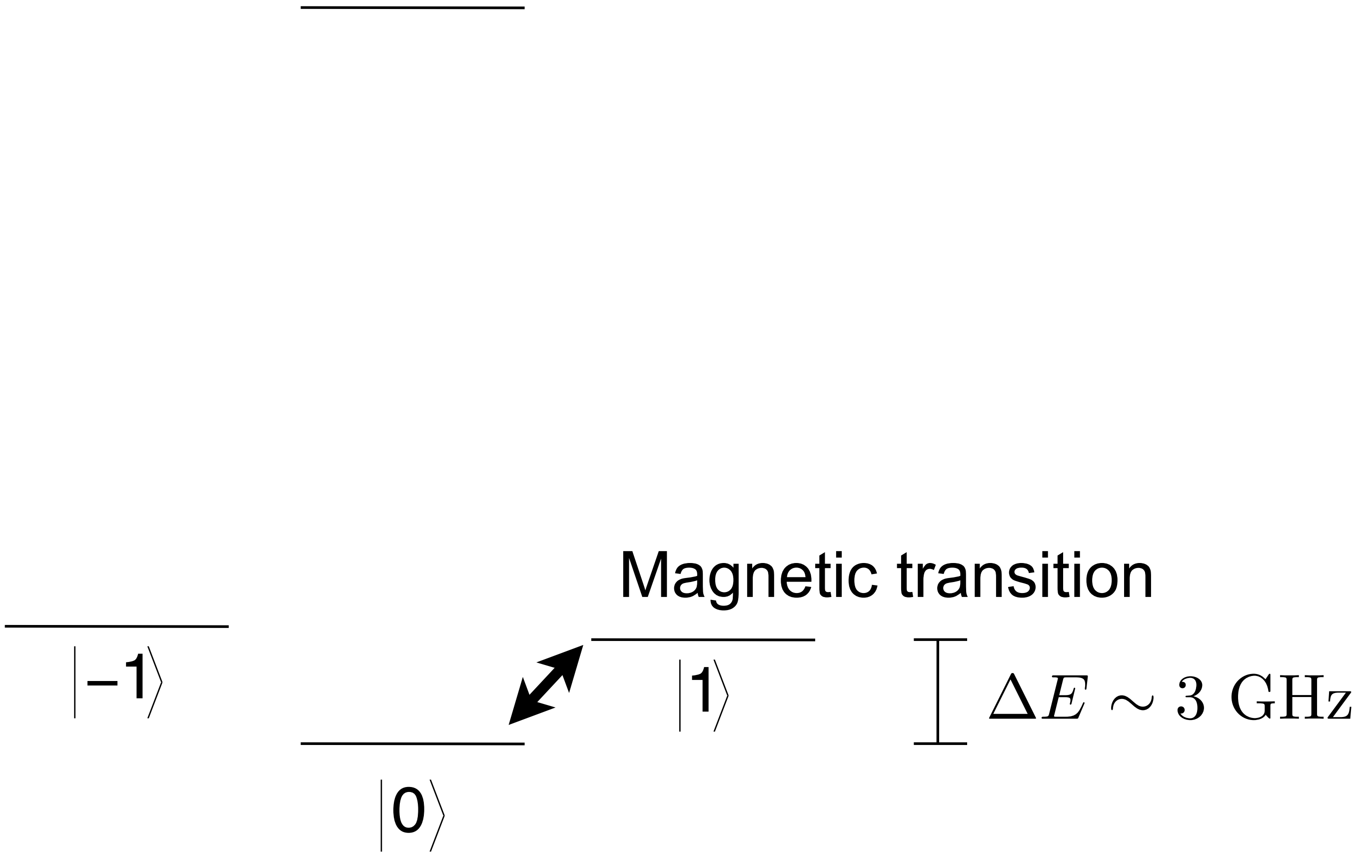

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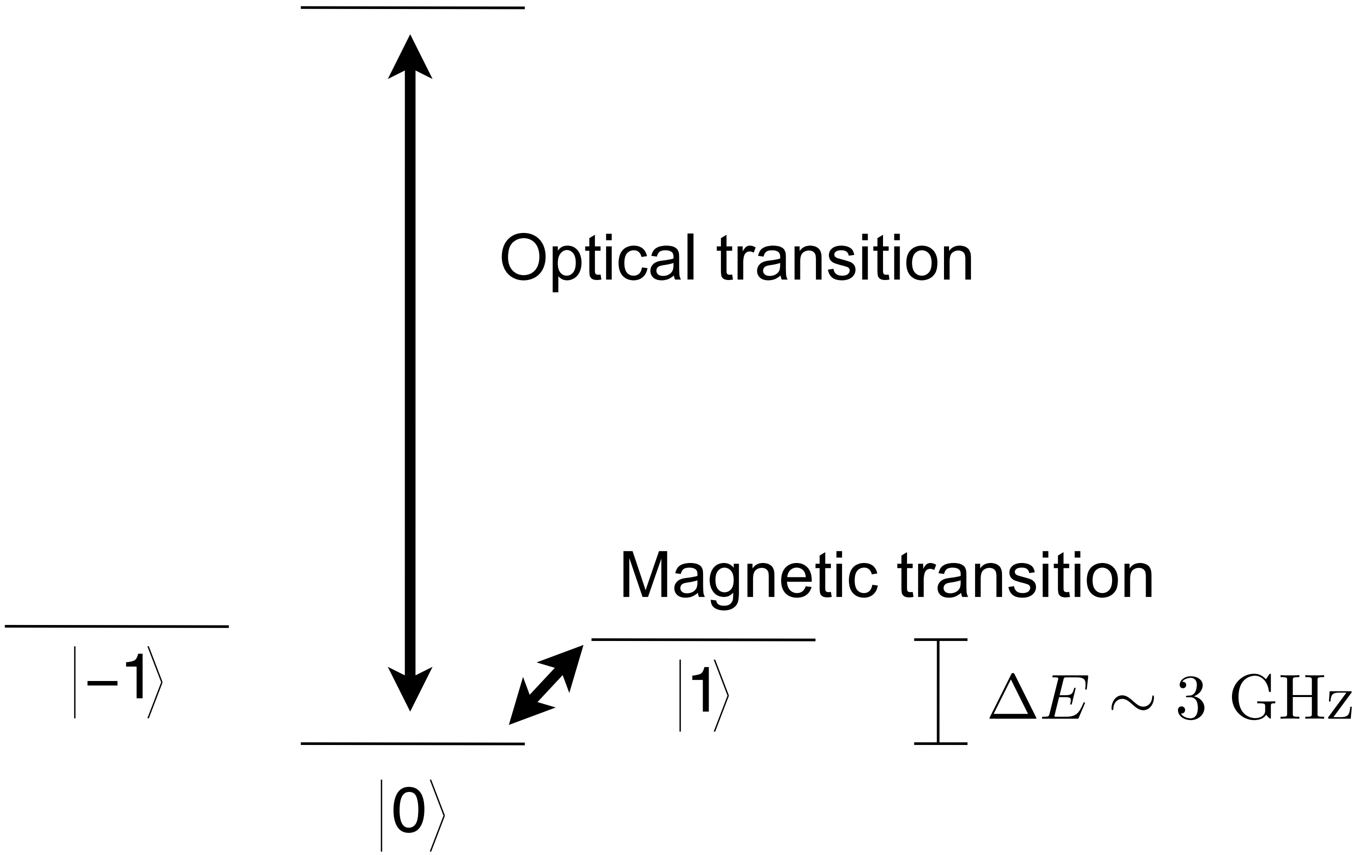
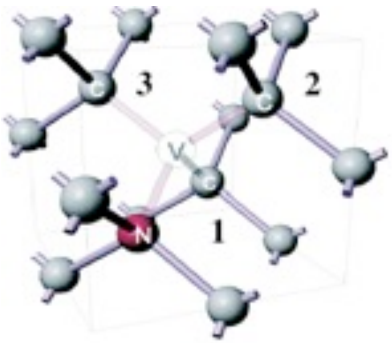
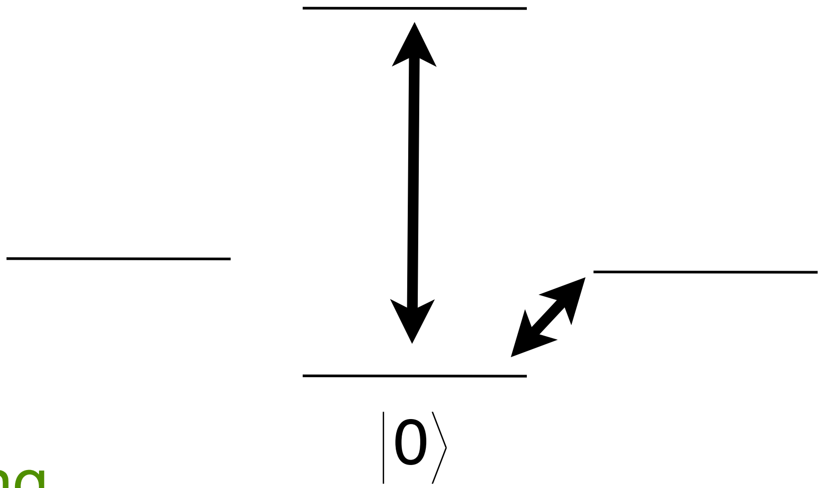


Jelezko, Wrachtrup *et al.*, Stuttgart

Properties of NV centers





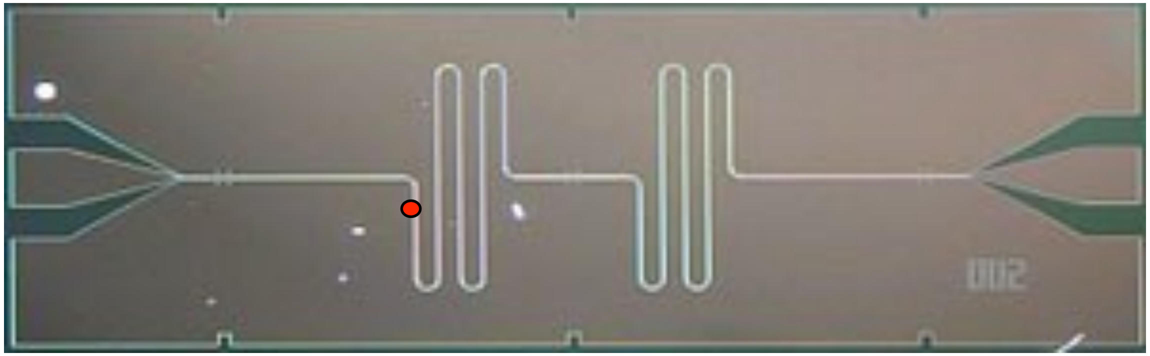
It is solid - no need for trapping

Ground state coherence is good
  Electron spin ~ ms
  Nuclear spin ~ s (even at room temperature)

Optical transitions are decent
  strong inhomogeneous broadening
  phonon emission
  selection rules not perfect

Coupling them is hard

Coupling to strip lines



Field energy: $H \sim \int d^3r(E^2 + B^2) \sim \hbar\omega$

Coupling: $g = B\mu_B = \mu_B\sqrt{\mu_0\hbar\omega/V}$ ~ 2π 10-100 Hz

Wesenberg, Ardavan, Briggs, Morton, Schoelkopf, Schuster, and Mølmer, PRL **103**, 070502 (2009)
Imamoglu, PRL **102**, 083602 (2009)
Verdú, Zoubi, Koller, Majer, Ritsch, and J. Schmiedmayer, PRL **103**, 043603 (2009)

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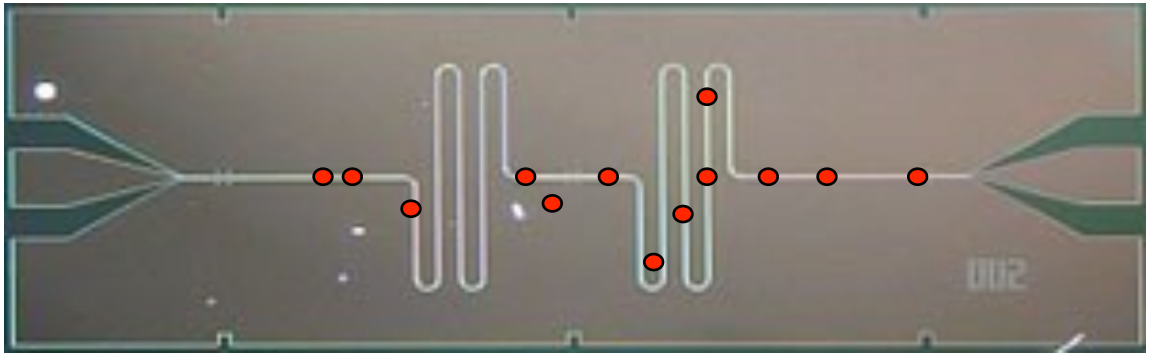

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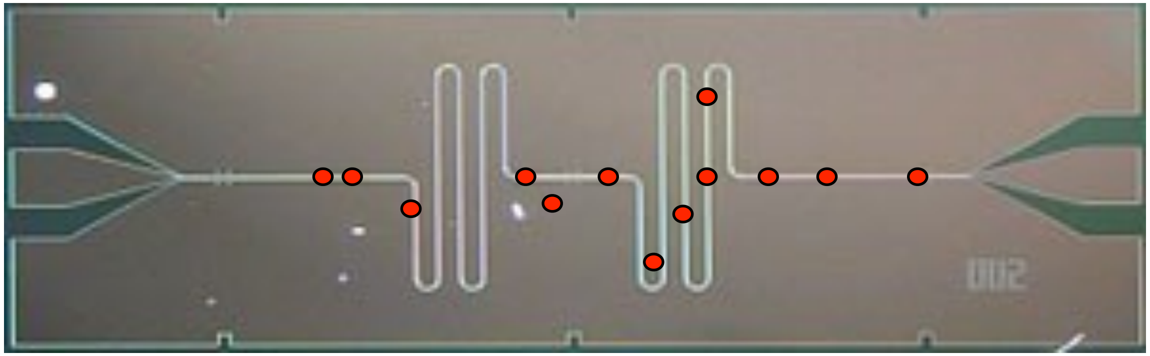
Coupling: $$g = B\mu_B = \mu_B\sqrt{\mu_0\hbar\omega/V}\cdot\sqrt{N}$$

Wesenberg, Ardavan, Briggs, Morton, Schoelkopf, Schuster, and Mølmer, PRL **103**, 070502 (2009)
Imamoglu, PRL **102**, 083602 (2009)
Verdú, Zoubi, Koller, Majer, Ritsch, and J. Schmiedmayer, PRL **103**, 043603 (2009)

Coupling to strip lines



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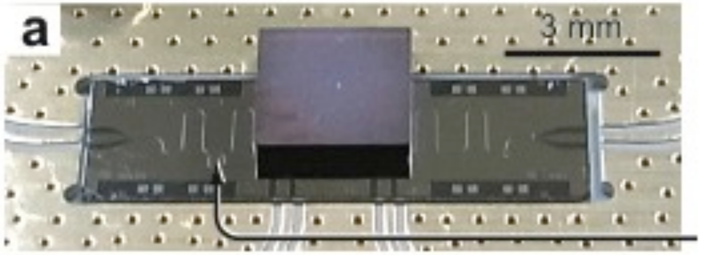
$$= \mu_B\sqrt{\mu_0\hbar\omega n}$$

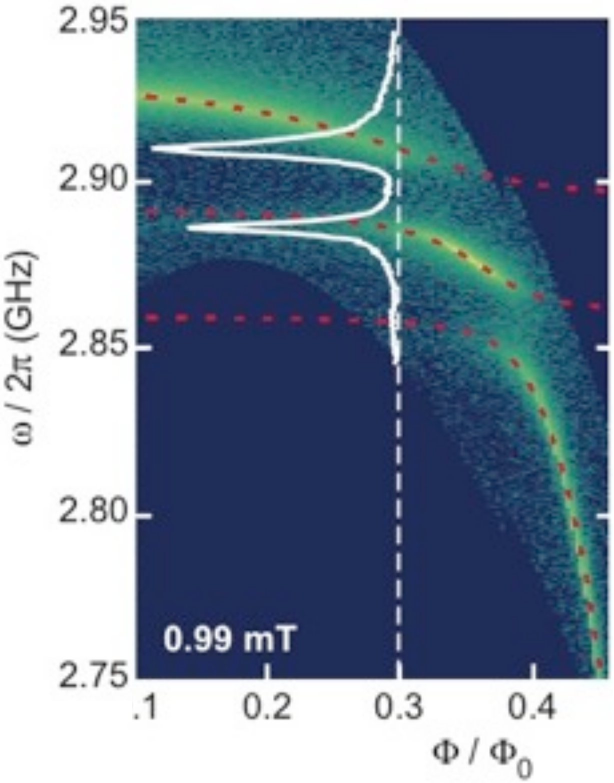
Wesenberg, Ardavan, Briggs, Morton, Schoelkopf, Schuster, and Mølmer, PRL **103**, 070502 (2009)
Imamoglu, PRL **102**, 083602 (2009)
Verdú, Zoubi, Koller, Majer, Ritsch, and J. Schmiedmayer, PRL **103**, 043603 (2009)

It works

Kubo, Ong, Bertet, Vion, Jacques, Zheng, Dréau, Roch, Auffeves, Jelezko, Wrachtrup, Barthe, Bergonzo, Esteve, Phys Rev Lett. **105**, 140502 (2010)



n~$10^{18}cm^{-3}$



See also Sears, *et al*, Phys. Rev. Lett. **105**, 140501 (2010)
and R. Amsüss *et al.*, Phys. Rev. Lett. **107**, 060502 (2011)

Conclusion (3)

Hybrid systems may combine the best of two worlds

Stripline cavities is a promising solution

Polar molecules:

Can be coupled to ensemble

Can coupled individual

Need trapping and cooling

Magnetic Coupling to ensembles of spins work

Magnetic coupling to individual spin too weak

This is all very easy and we will have a working quantum computer soon

Conclusion (3)

Hybrid systems may combine the best of two worlds

Stripline cavities is a promising solution

Polar molecules:

- Can be coupled to ensemble
- Can coupled individual
- Need trapping and cooling

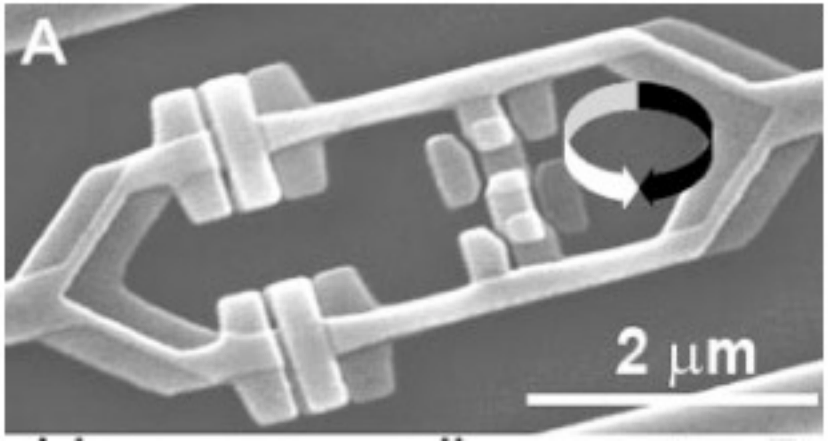
Magnetic Coupling to ensembles of spins work

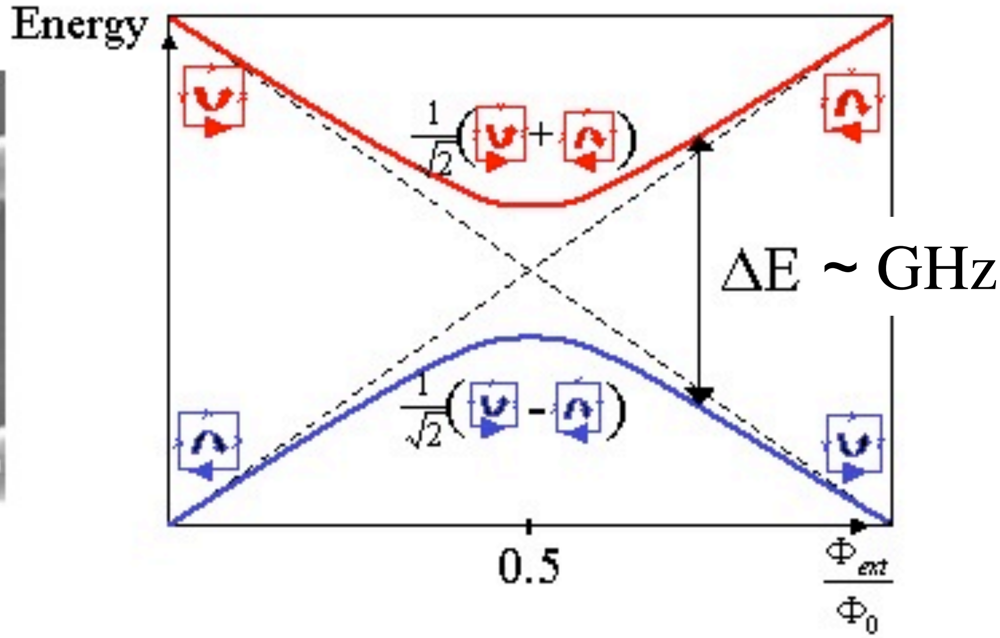
Magnetic coupling to individual spin too weak

We are making progress and maybe we will get somewhere

A different solution

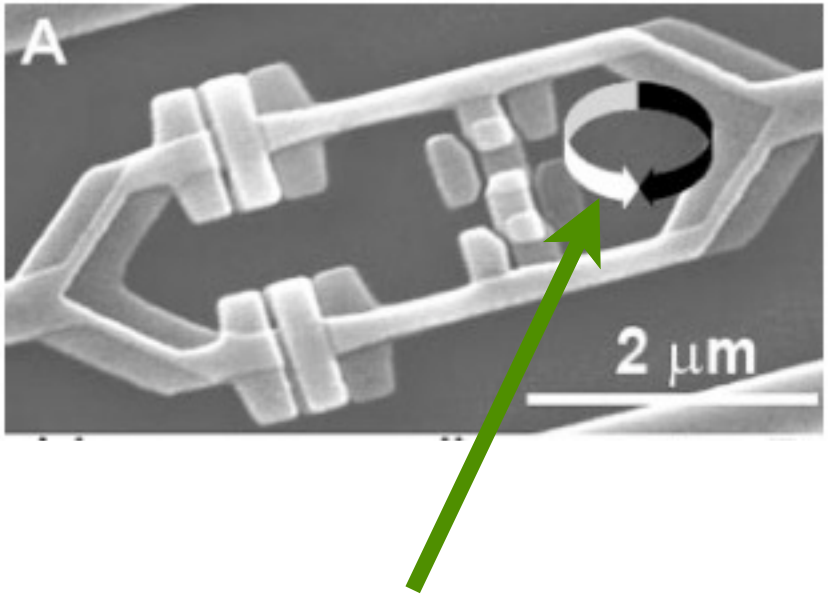
Flux qubits

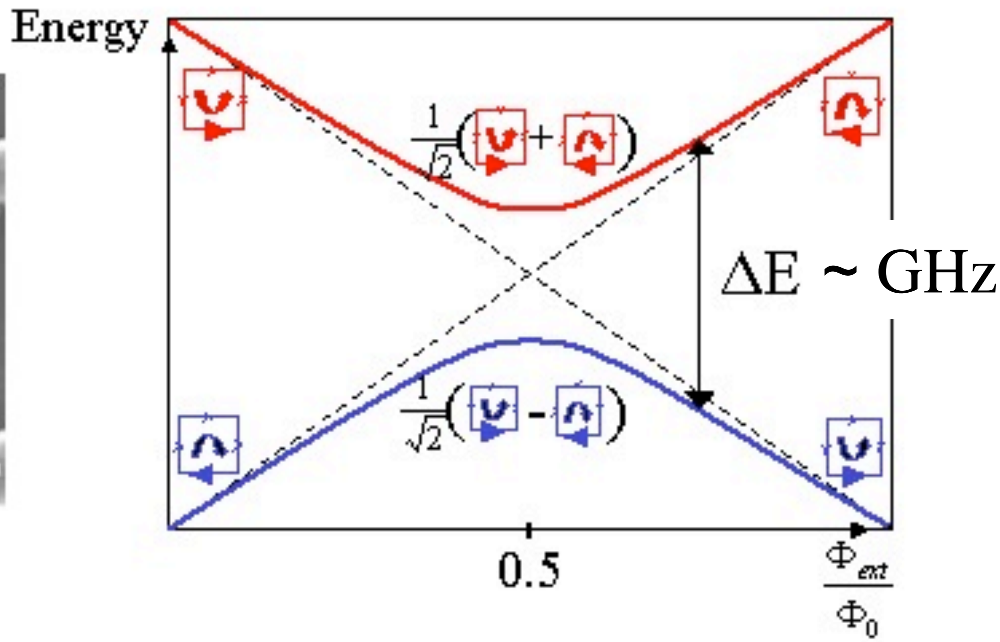




Mooij *et al.* Delft

Flux qubits



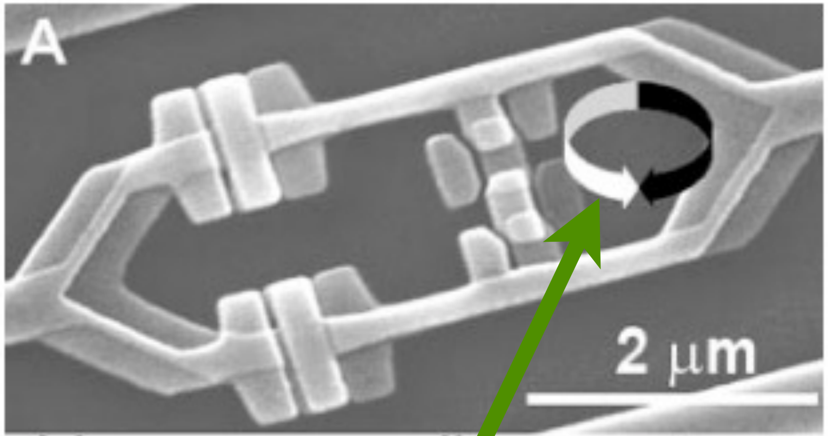


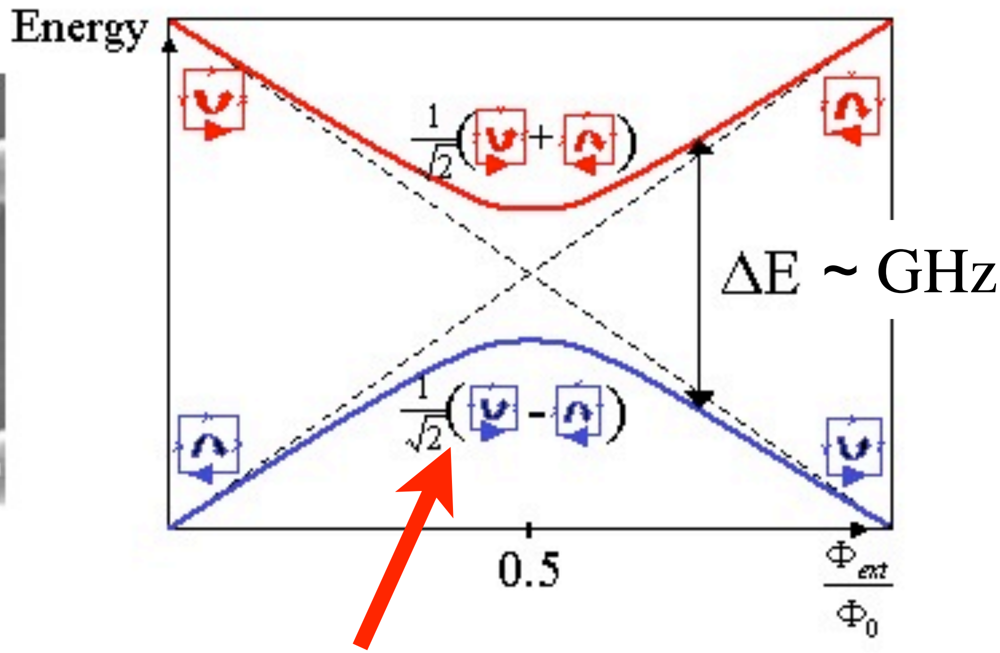
Current generate magnetic field

$$\hat{B} = B(\vec{r})(|\circlearrowleft\rangle\langle\circlearrowleft| - |\circlearrowright\rangle\langle\circlearrowright|)$$

Mooij *et al.* Delft

Flux qubits





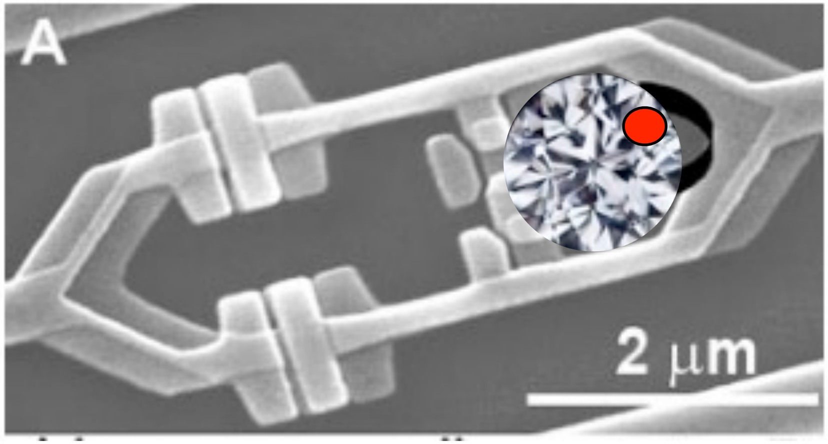
Current generate magnetic field

Degeneracy point:
Fluctuating magnetic field

$$\hat{B} = B(\vec{r})(|\circlearrowleft\rangle\langle\circlearrowleft| - |\circlearrowright\rangle\langle\circlearrowright|)$$

Mooij *et al.* Delft

Combining the two



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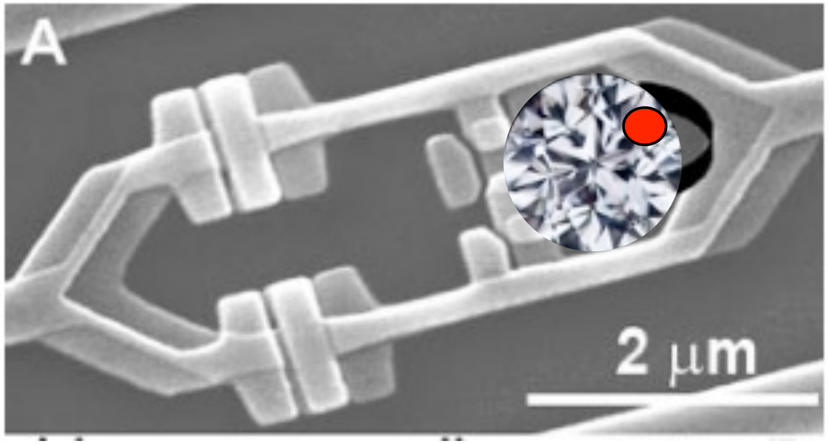
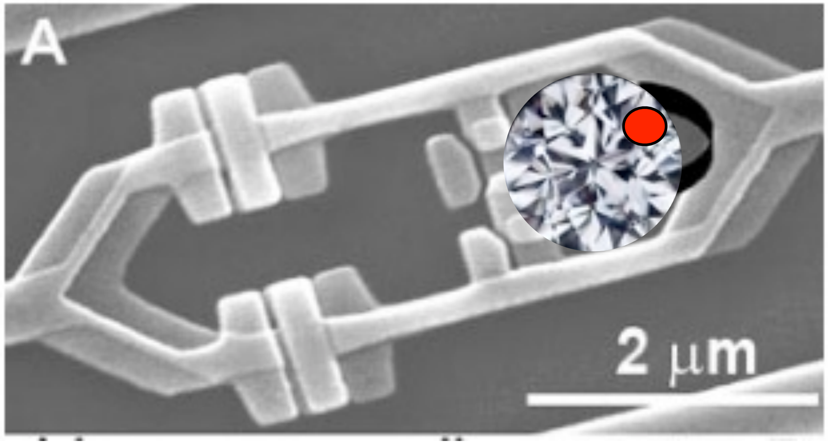


Flux and NV resonant => resonant transfer between flux qubit and NV

Combining the two



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$$H = g_e \mu_B \vec{S} \cdot \hat{B}$$

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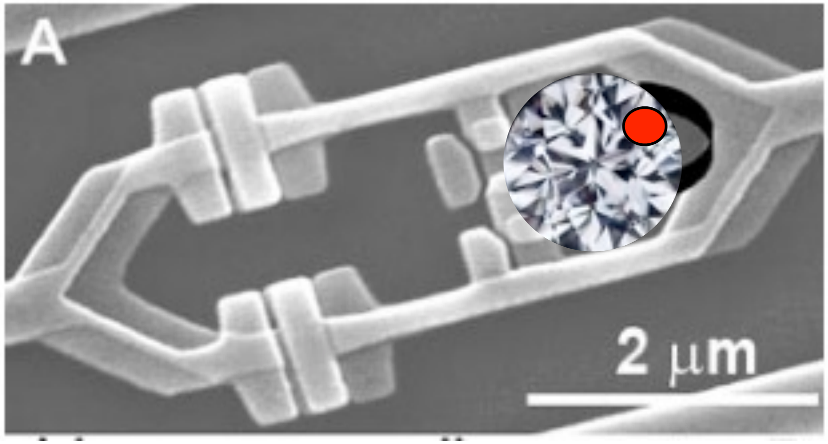


Flux and NV resonant => resonant transfer between flux qubit and NV

$$H = g_e \mu_B \vec{S} \cdot \hat{B}$$

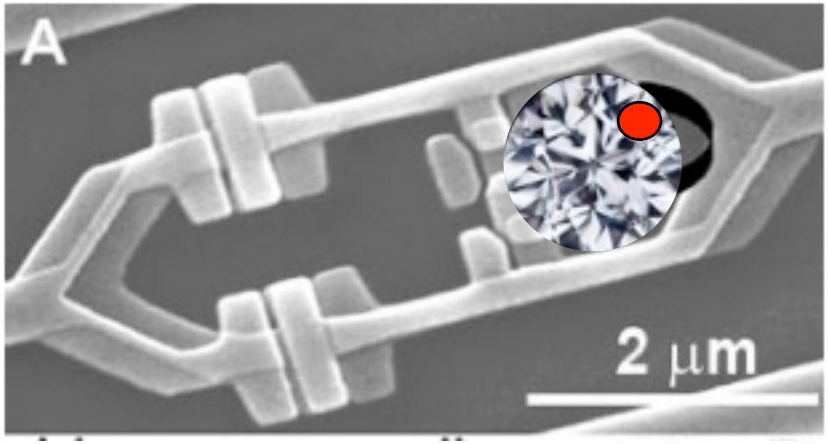
Rotating wave approximation

$$H = g(\sigma_- \tau_+ + \tau_- \sigma_+)$$

$$g \sim (2\pi) 15 \text{ kHz}$$

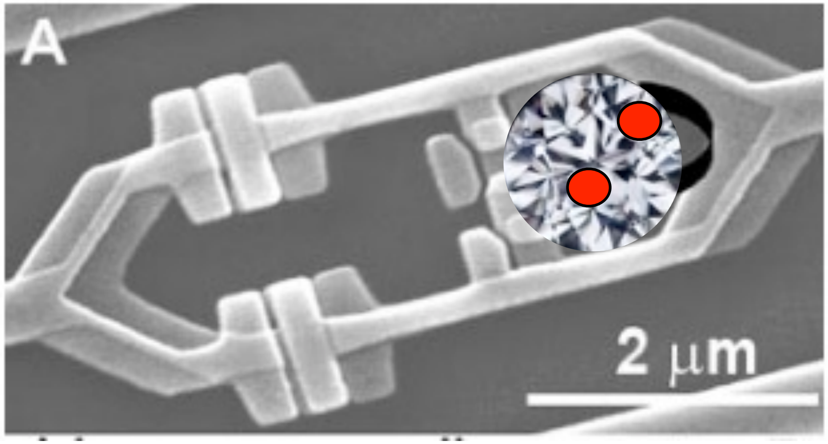
(For existing flux qubit)

Coupling two NVs



Problem: Flux qubit decohere, T_2 ,T_1~ μs

Coupling two NVs



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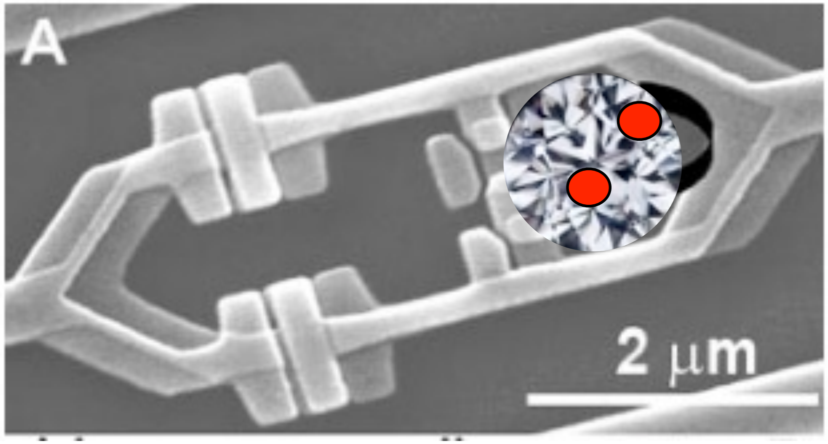
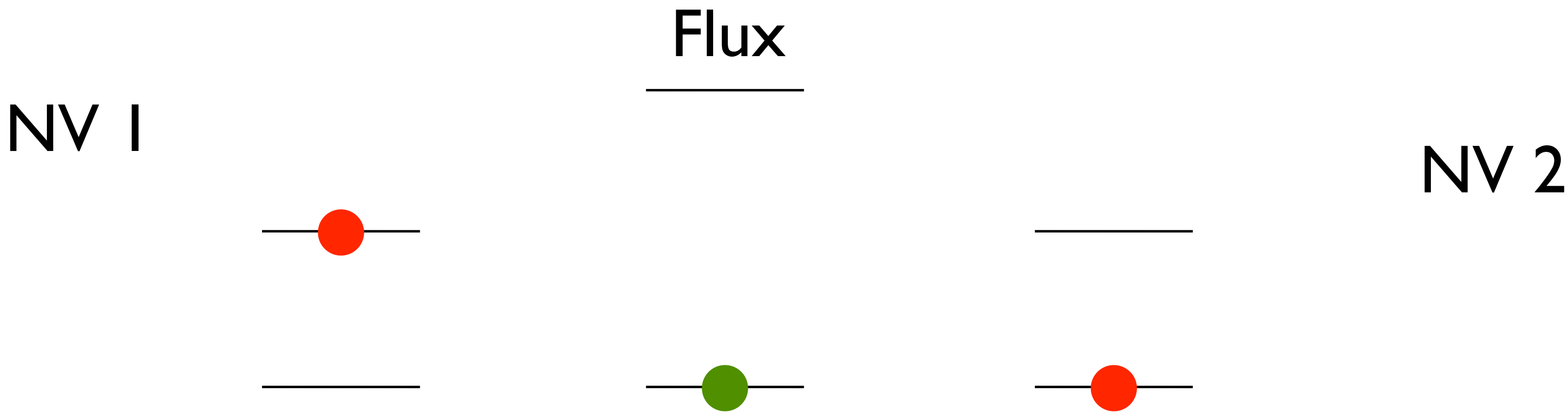


Problem: Flux qubit decohere, $T_2, T_1 \sim \mu s$

Can we couple two NVs through the Flux qubit?

Coupling two NVs



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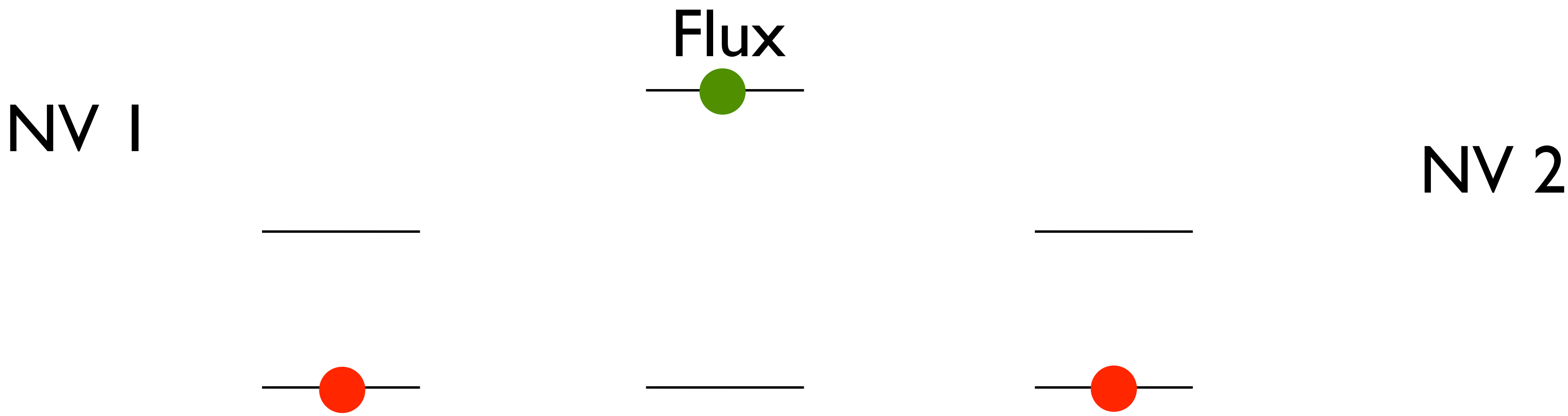

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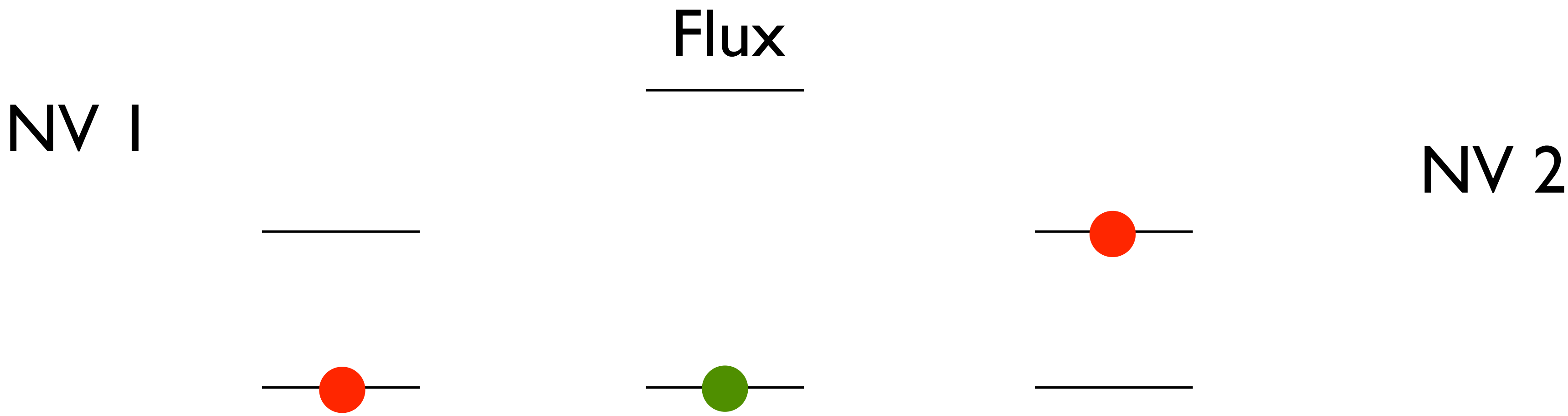

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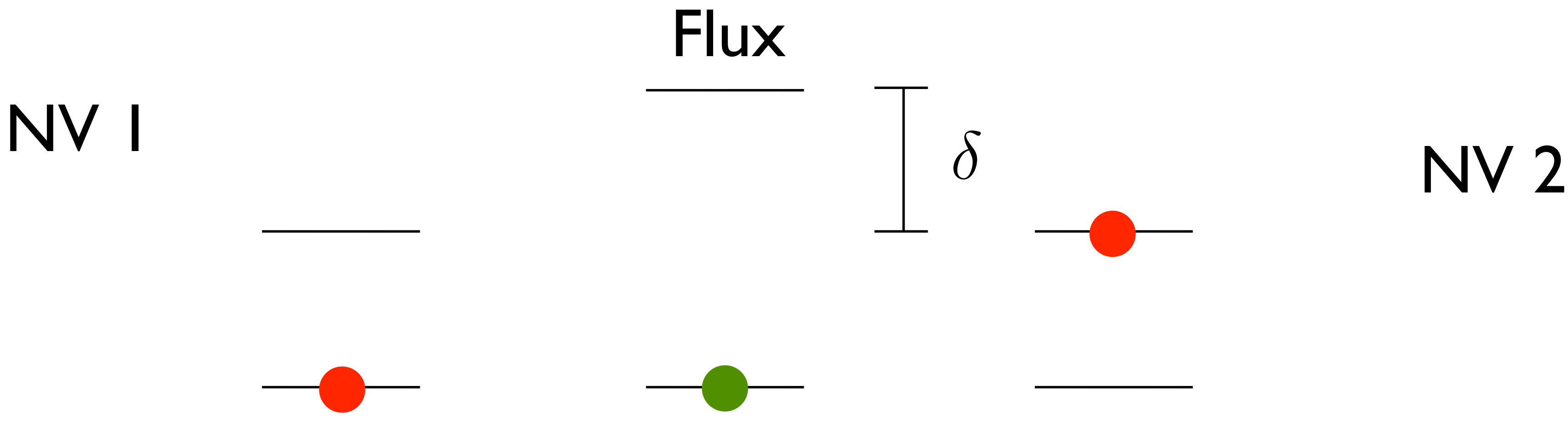

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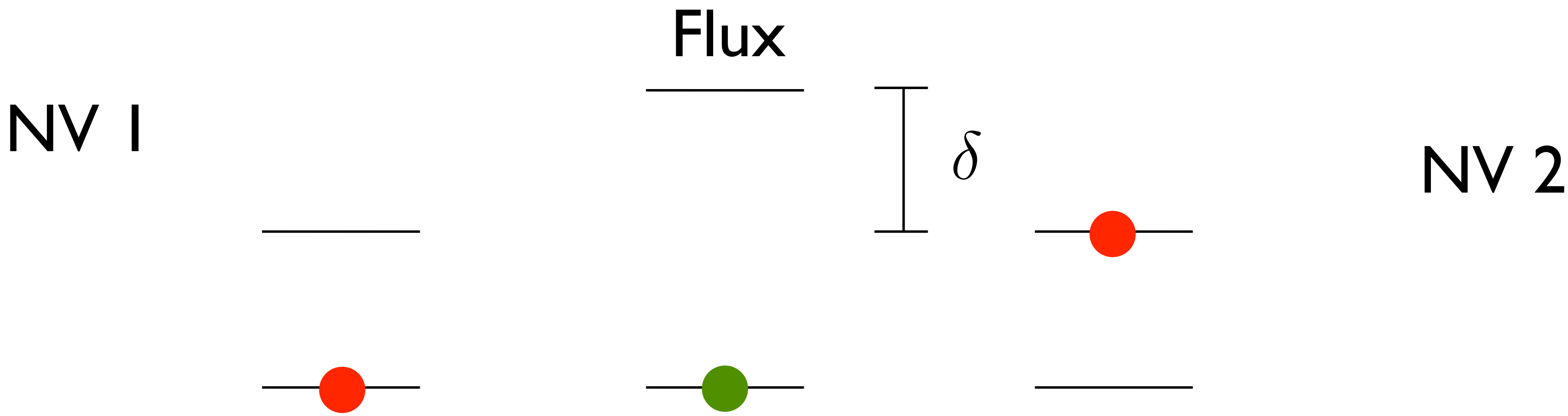
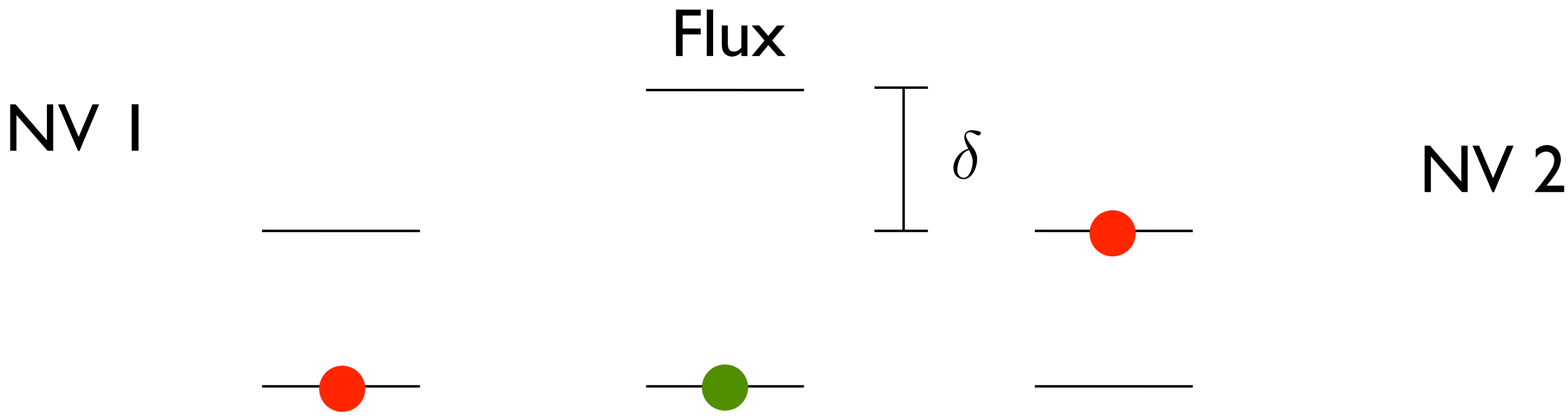


Coupling: $g_{\text{eff}} = \dfrac{g^2}{\delta}$

Coupling two NVs



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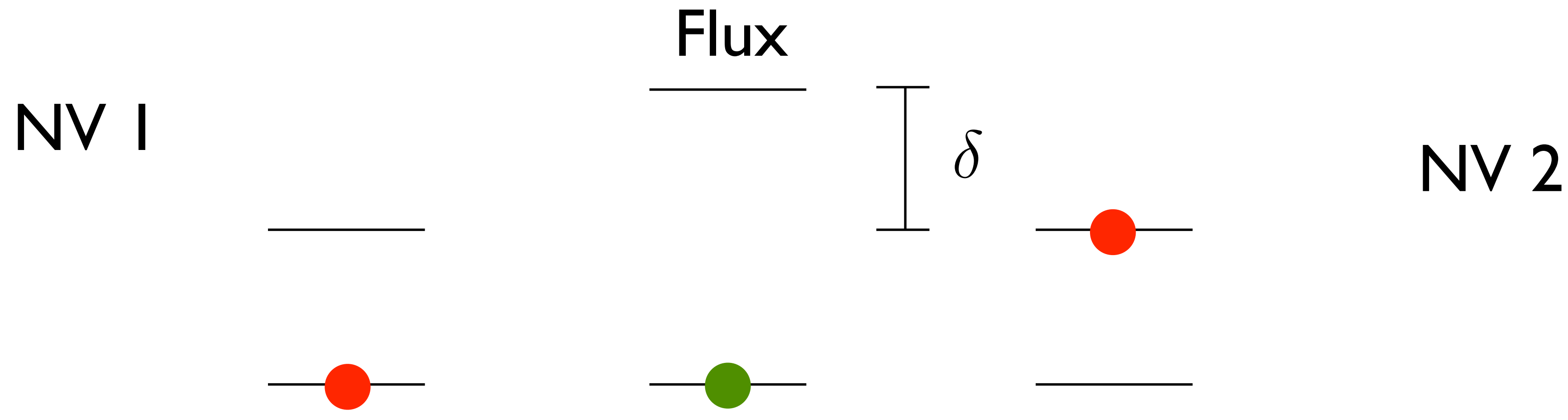

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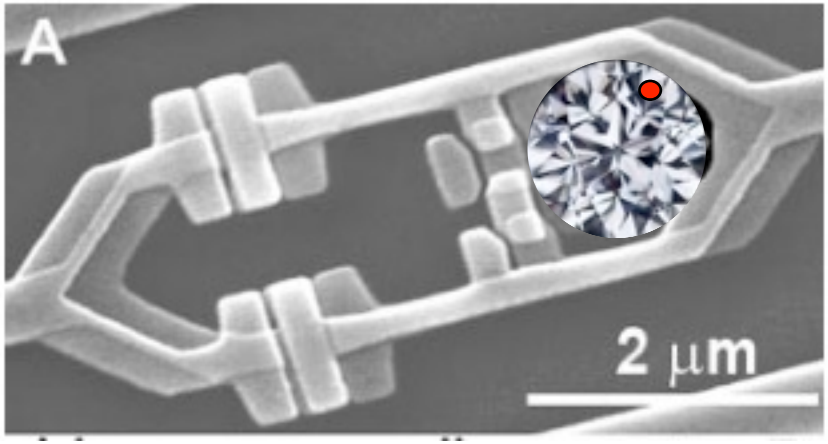
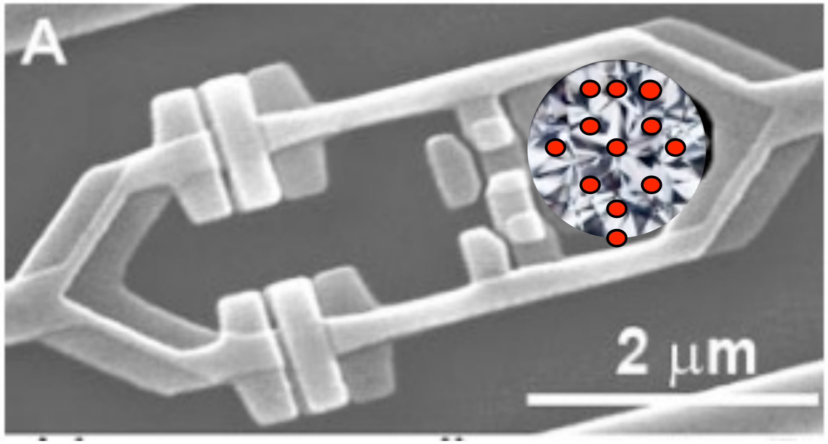
$\sqrt{\text{SWAP}}: \quad F \sim 99\%$

Collective enhancement



What happens if there are N centers?

Collective enhancement



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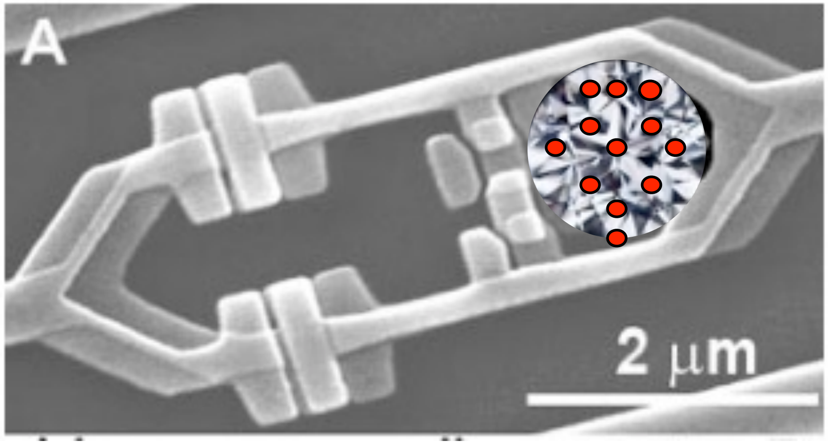

What happens if there are N centers?

$$H = g\tau_- \sum_l j_{+,l} + \mathrm{H.C.}$$

$$|1\rangle_{\mathrm{FQ}}|00....0\rangle_{\mathrm{NV}} \rightarrow \frac{1}{\sqrt{N}}|0\rangle_{\mathrm{FQ}} \sum_l |00...1_l...0\rangle_{\mathrm{NV}}$$

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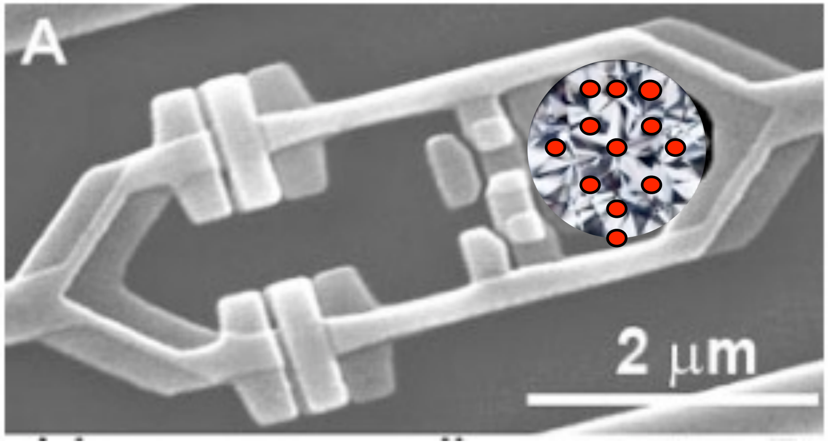

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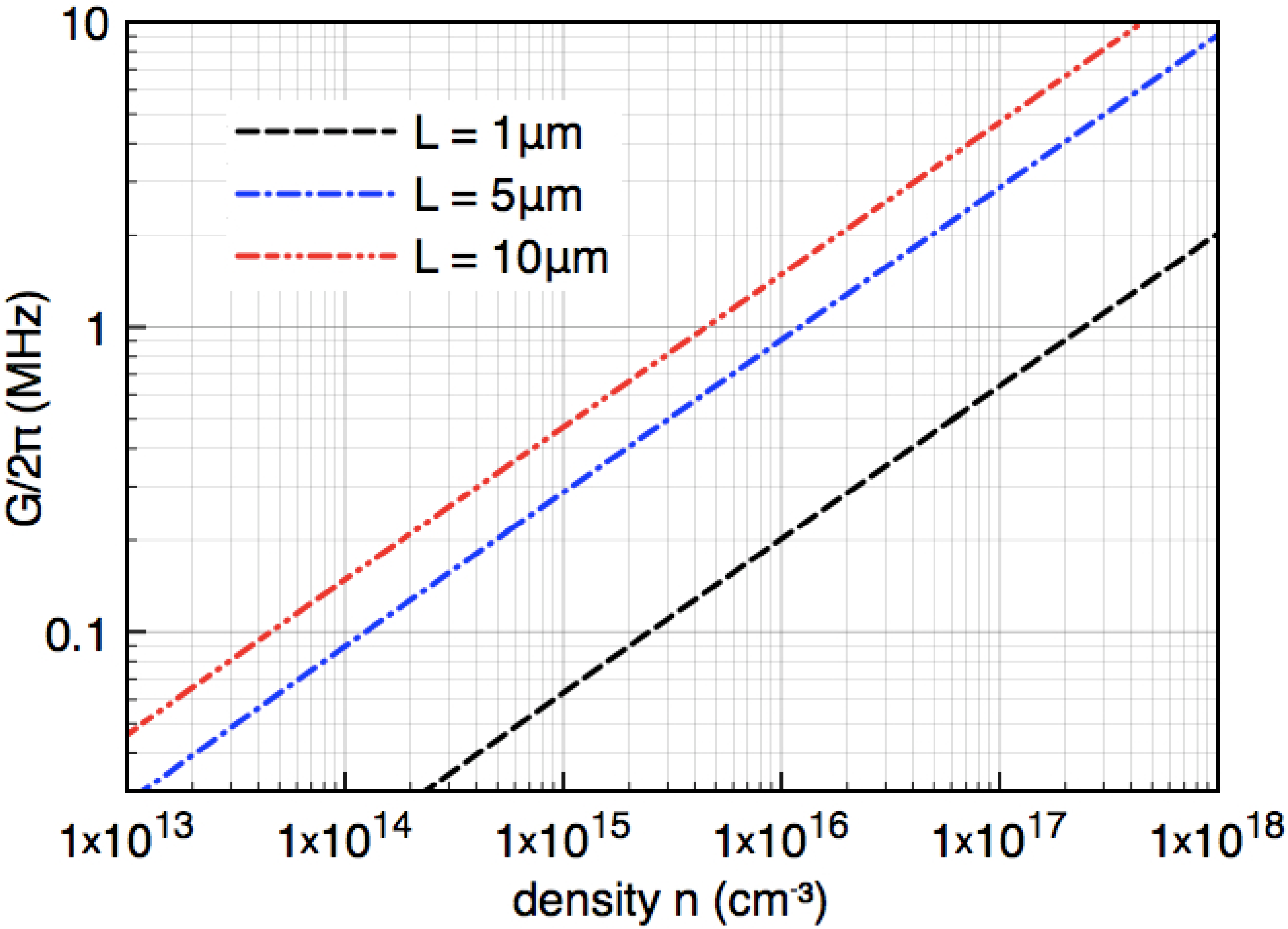
Coupling enhanced by factor of $\sqrt{N}$

Collective enhancement

$I_c = 0.5\ \mu\text{A}$



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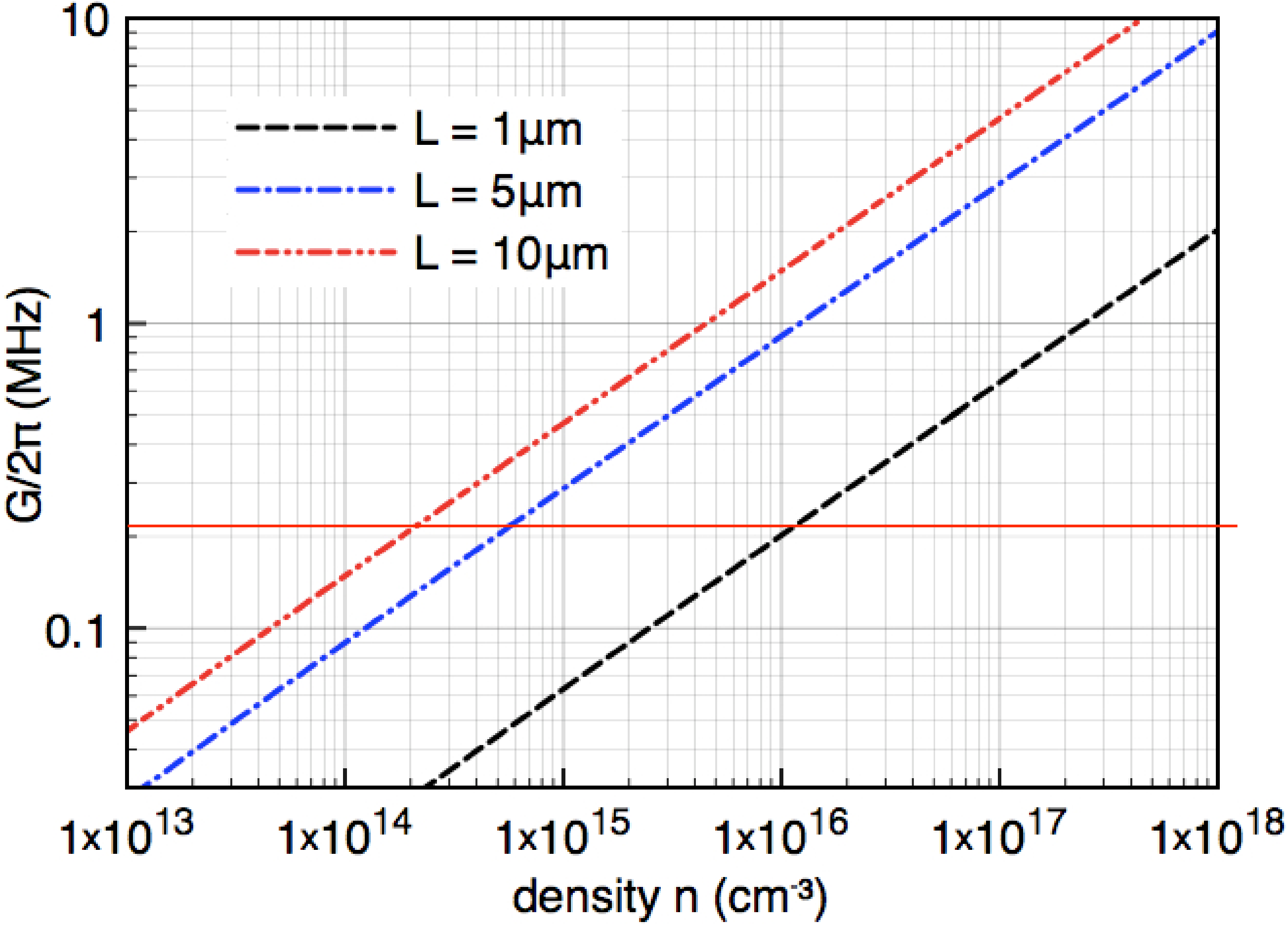

Flux qubit decoherence (Theory*)

*Makhlin, Schön, Shnirman, Rev. Mod. Phys. **73**, 400 (2001)

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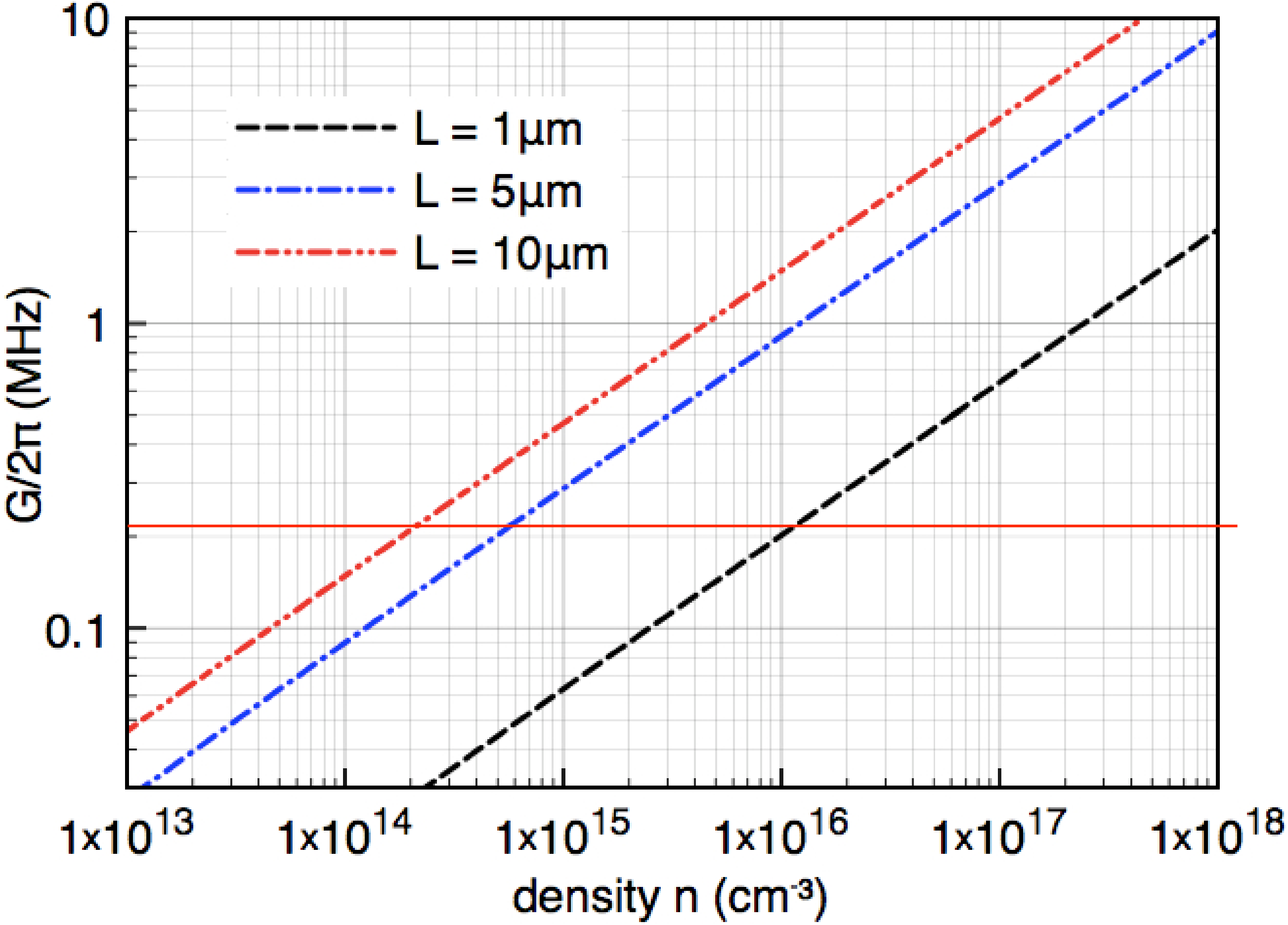


Flux qubit decoherence (Theory*)

Coupling may exceed decoherence at realistic densities

*Makhlin, Schön, Shnirman, Rev. Mod. Phys. **73**, 400 (2001)

Decoherence

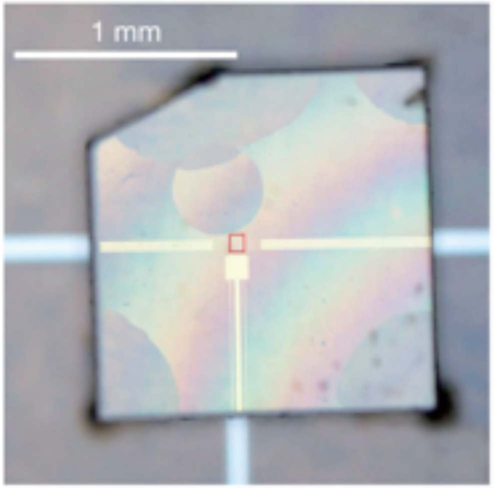
^{13}C: up to 200 MHz =>Bad. Use isotopically pure ^{12}C

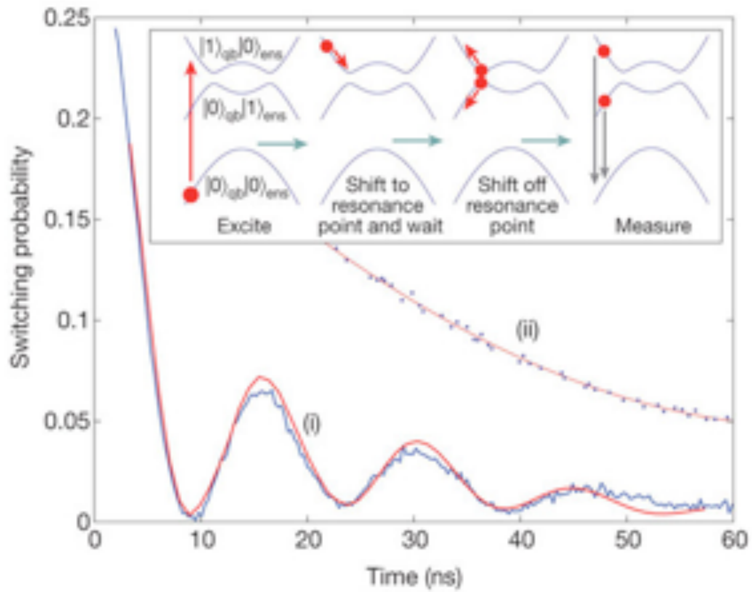
Four different orientations of centers=>
Separate spectrally by magnetic field

^{14}N: 2 MHz. Polarize or use only resonant

Dipole interactions with paramagnetic impurities
Less than flux-qubit if $n \lesssim 10^{18}\ \mathrm{cm}^3$

This also works!

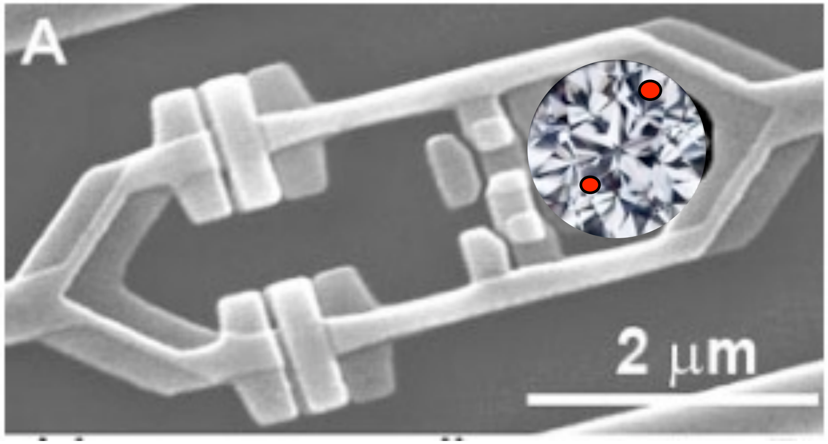




Quantum state transferred from flux qubit to NV centers and back

Pictures from: X. Zhu et al., Nature 478, 221 (2011)

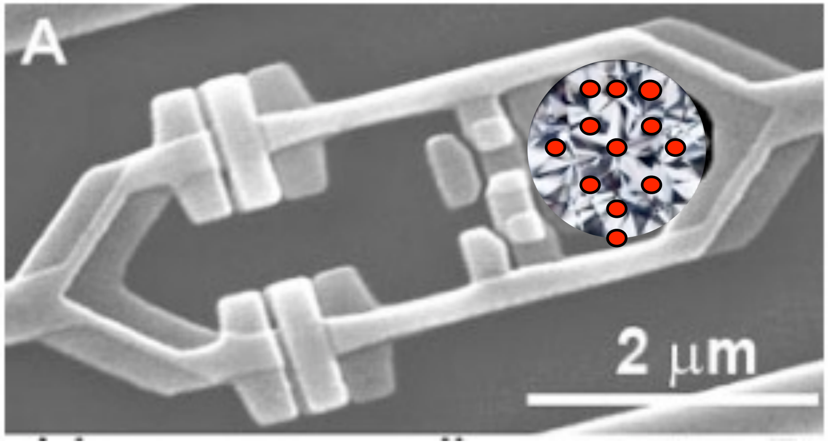
Possible architectures



1. Bits stored in individual NVs.
   Each NV connected to light.
   Two NVs connected through Flux qubits

2. Bits in super conducting circuit
   Ensemble of NVs used for long term storage
   NV ensemble used to connect to light
   (long coherence time may allow recooling of flux)

Possible architectures



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Comparison to strip lines

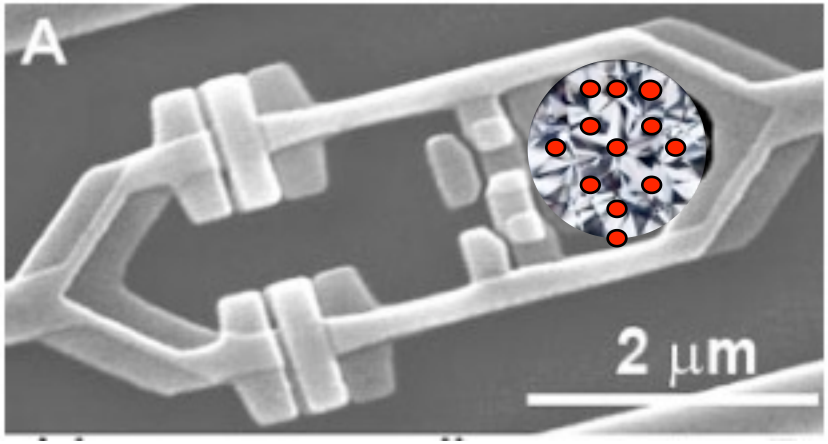
Striplines can have higher couplings to ensembles than existing flux qubits

But

- Single atom operation are within reach
- May change by careful design of flux qubits
- Smaller area => more compact, more homogeneous fields
- Fewer atoms => spin echo less demanding

Outlook

Hybrid systems may combine the best of two worlds



NV centers can have strong coupling to flux qubits

Possible applications:

Couple individual NV center, e.g, for quantum repeaters

Transfer flux qubit to ensemble of NV

Transfer to nuclear spin => very long coherence time (s)

Possible interface between superconductors and light?

D. Marcos, M. Wubs, J. M. Taylor, R. Aguado, M. D. Lukin, and A. S. Sørensen, Phys. Rev. Lett. **105**, 210501 (2010).

Collaborators

Harvard:
Caspar van der Wal (now Groningen)
Lillian Childress (now Yale)
Mikhail D. Lukin

Copenhagen:
Martijn Wubs (now DTU)

Madrid:
D. Marcos
R. Aguado

NIST, JQI
Jacob Taylor

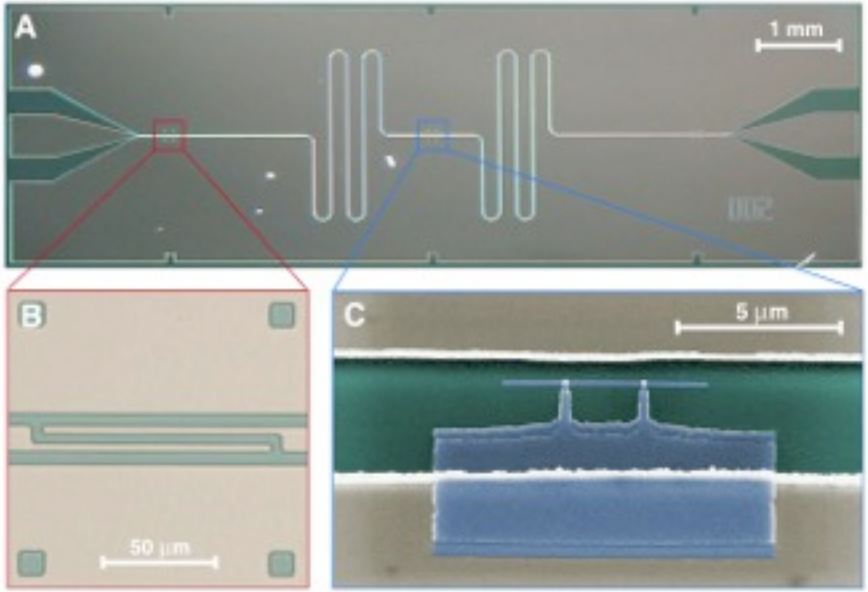
Solid state quantum optics 2: Extending to optical frequencies: Surface plasmons and single photon transistors

Anders S. Sørensen

Quantop, Danish Quantum Optics Center

Niels Bohr Institute, University of Copenhagen

Summary



The best realization of the model system of quantum optics - the Jaynes Cummings model

Conductors enables strong confinement of electric fields

Enables a strong coherent interaction

Extend to optical frequencies

Main motivation: quantum communication done with optical photons

Extend to optical frequencies

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Quantum cryptography works!

Extend to optical frequencies

Main motivation: quantum communication done with optical photons

Quantum cryptography works!   For short distances (< 100 km)

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Longer distances: requires that we store and process the information locally

Extend to optical frequencies

Main motivation: quantum communication done with optical photons

Quantum cryptography works!   For short distances (< 100 km)

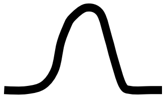
Longer distances: requires that we store and process the information locally

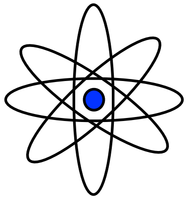
We need light matter-matter quantum interface at optical frequencies

Connecting atoms and light

Connecting atoms and light

Ideally:

Sandoghdar 



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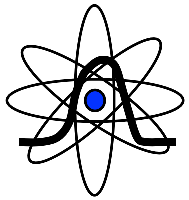

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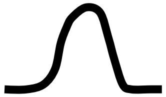
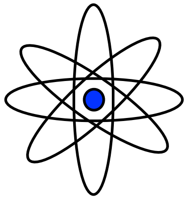
Sandoghdar 



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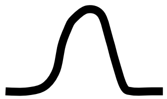
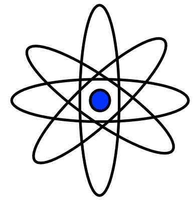
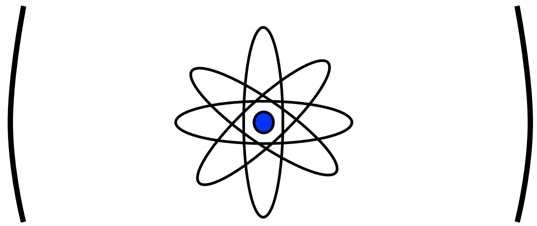
Sandoghdar  

Rempe 

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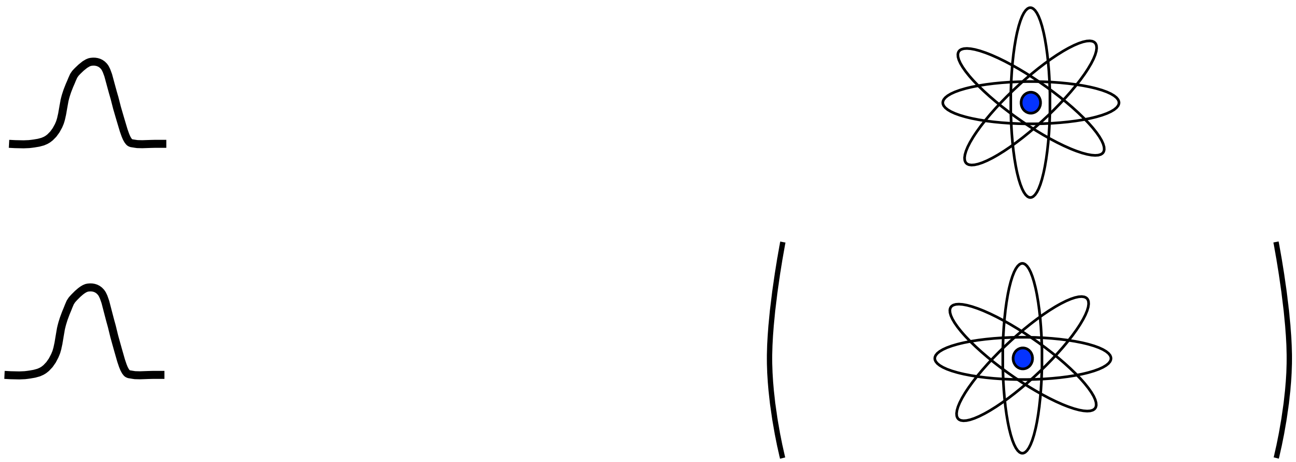

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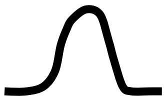

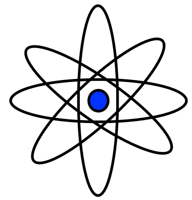

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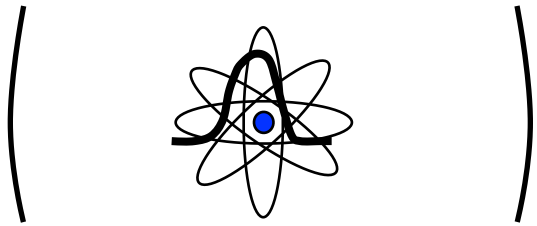
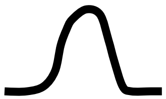
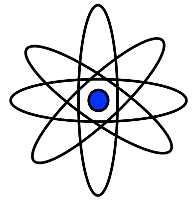


Connecting atoms and light

Ideally:

Hard

Sandoghdar 



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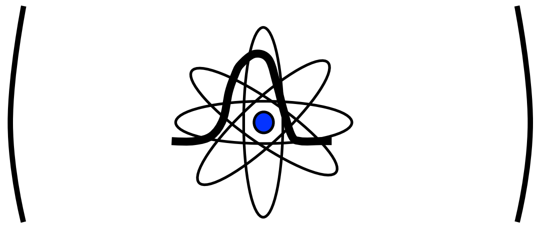

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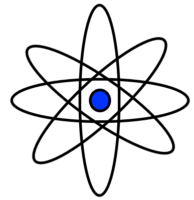
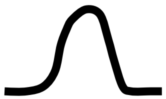
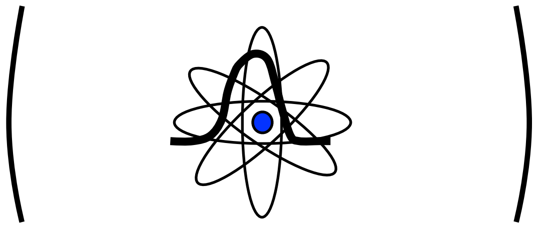
Ideally:

Hard



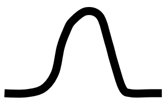
Sandoghdar

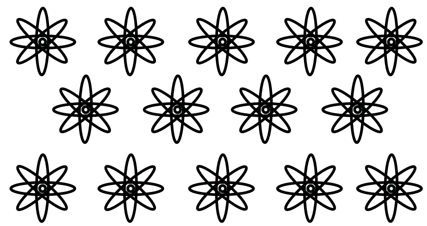




Rempe

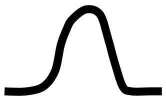
Easier

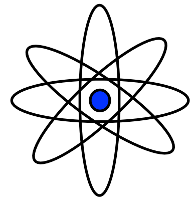




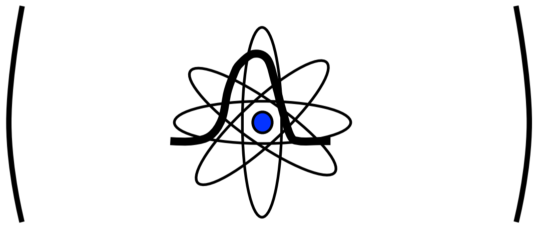
Connecting atoms and light

Ideally:

Sandoghdar 



Hard

Rempe 

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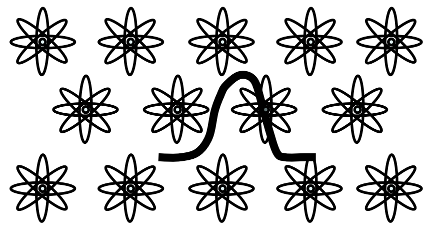

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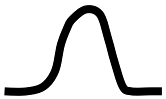

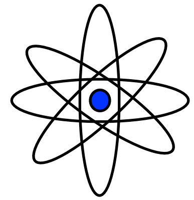

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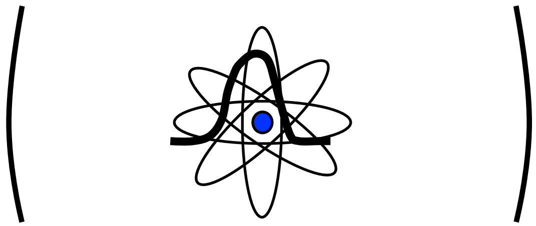

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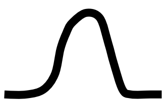

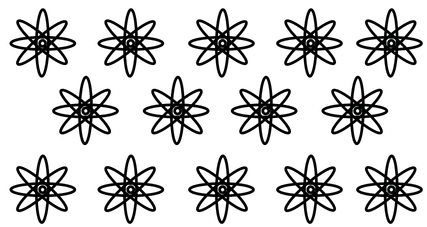

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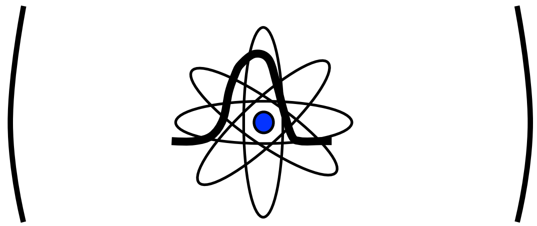


Easier

But coupling to single atom makes it easier to process the information

Surface plasmons

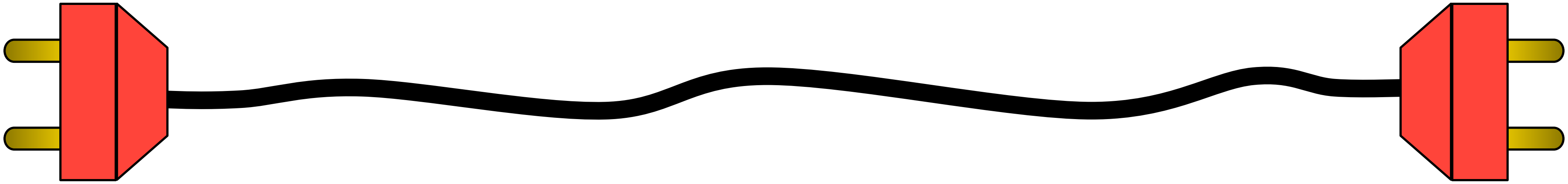
Metallic wire: current carried by charges

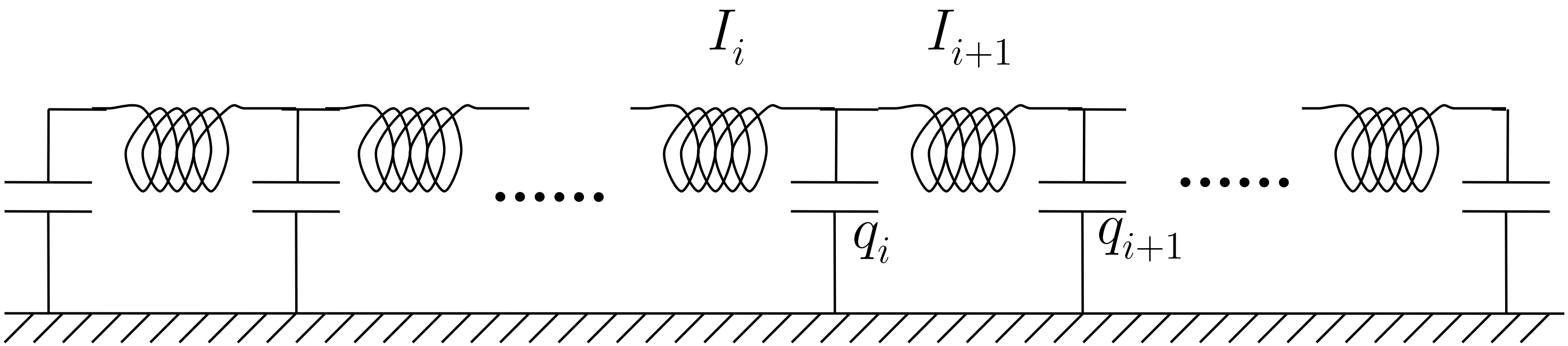
Alternating current of frequency ν:

R big: Signal runs on surface, $v \approx c$, $\Delta x \approx \lambda = c/\nu$

Perpendicular extension of field ~ $\Delta x \sim \lambda$

Describing the wire





Equations of motion: $$\frac{d\lambda}{dt} = -\frac{dI}{dx} \qquad l\frac{dI}{dt} = -c\frac{d\lambda}{dx}$$

Wave equation: $$\frac{d^2I}{dt^2} = v^2\frac{d^2I}{dx^2} \qquad v = \frac{1}{\sqrt{lc}}$$

Small wire

Current carried by electrons, electrons have mass:

Energy per unit length: $u = \frac{1}{2} l I^2 \quad + n\pi R^2 \, \frac{1}{2} m v^2$

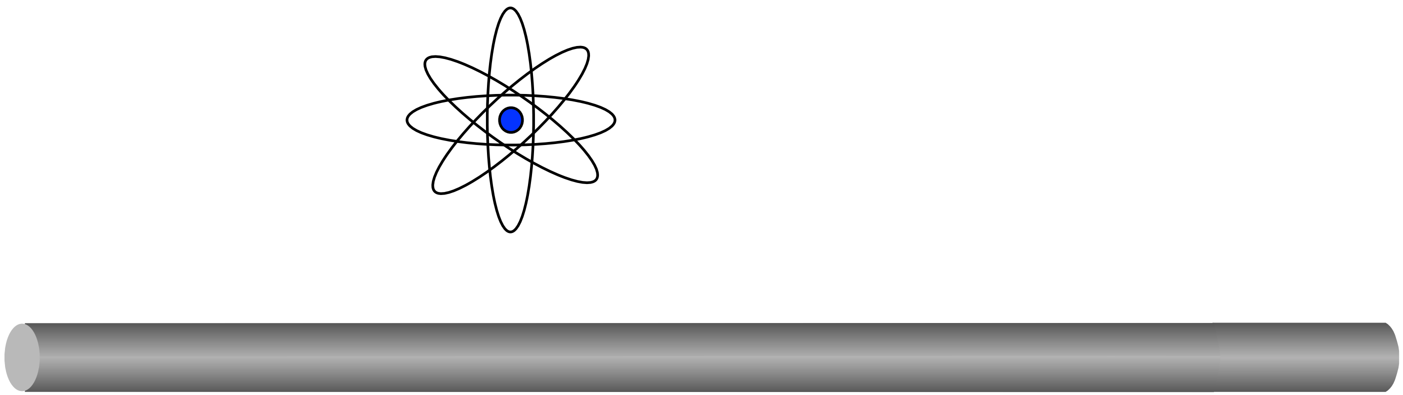
$= \frac{1}{2} l_{\text{eff}} I^2$ (using $I = n\pi R^2 q v$)

Effective inductance: $l_{\text{eff}} = l + \frac{m}{n\pi R^2 q^2} \propto \frac{1}{R^2}$ (small R)

Small R => strong confinement: $\Delta x \propto v \propto \frac{1}{\sqrt{l_{\text{eff}}}} \propto R$

Coupling atoms to wires

Atoms: any two level systems; real atom, quantum dot, color center, ...



Fermi's golden rule: $\gamma_{\text{plasmon}} \sim g^2 \rho \propto \frac{1}{R^3}$

$g \sim \sqrt{\frac{1}{V}} \sim \frac{1}{R}$ $\qquad \rho \propto \frac{1}{v_{\text{group}}} \propto \frac{1}{R}$

Make wire thin => mainly decay to plasmon modes

D.E. Chang, A.S. Sørensen, P.R. Hemmer, and M.D. Lukin, Phys. Rev. Lett **97**, 053002 (2006).

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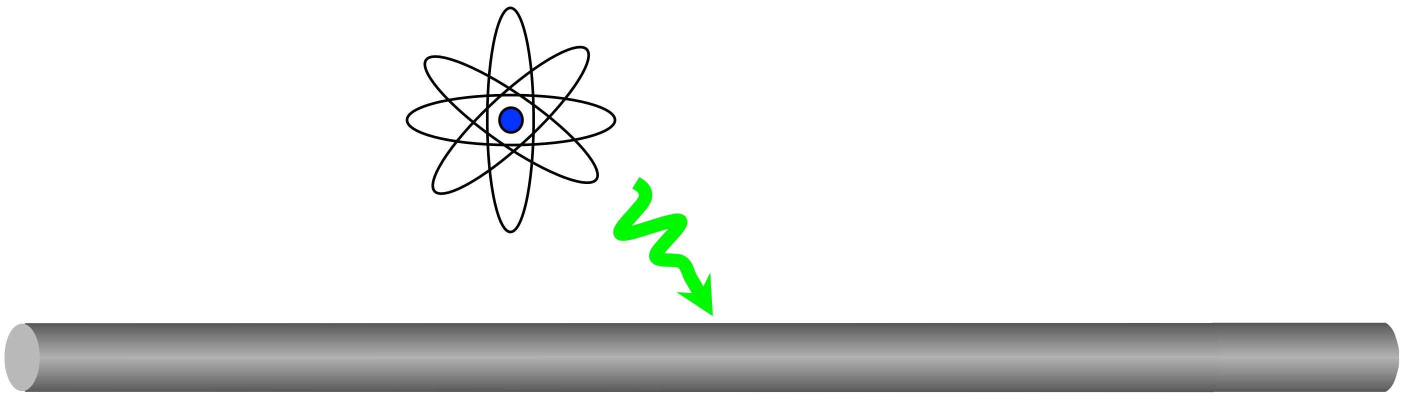

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The full theory

Description of metal: negative (and imaginary) ϵ

E.g. free electron model $\epsilon_0(\omega) = 1 - \dfrac{\omega_p^2}{\omega^2 + i\omega\gamma_p}$

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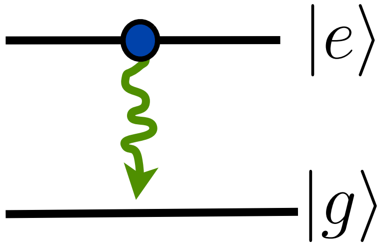
Ex. $H \sim \int d^3r \; \epsilon E^2$ don't work

Quantum vs. classical theory

Ex: Spontaneous emission



Dipole moment vanish $\langle \hat{\vec{d}} \rangle = 0$

No electric field $\vec{E}(\vec{r}) = G(\vec{r})\langle \hat{\vec{d}} \rangle = 0$

=> No radiation

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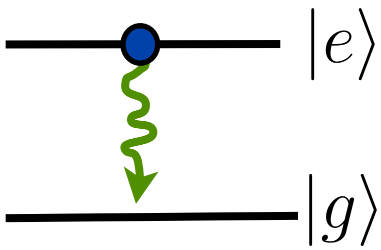

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=> No radiation

Quantize: $\hat{\vec{E}}(\vec{r}) = G(\vec{r}) d\sigma_-$

$$\hat{\vec{E}}^\dagger \hat{\vec{E}}(\vec{r}) = G(\vec{r})^2 d^2 \sigma_+ \sigma_- \sim |e\rangle\langle e|$$

Classical spontaneous emission

Harmonic oscillator with random phase

Dipole moment vanish $$\langle d \rangle \sim d_0 \langle \mathrm{e}^{\mathrm{i}\phi} \rangle = 0$$

Square of dipole does not $$\langle d^*(t+\tau) d(t) \rangle \sim d_0^2 \mathrm{e}^{\mathrm{i}\omega\tau} \neq 0$$

Radiation as before $$\left\langle \vec{E}^\dagger \vec{E} \right\rangle = G(\vec{r})^2 d_0^2$$

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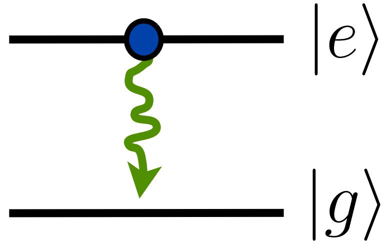
Rabi oscillation: phase lost during excitation

Classical spontaneous emission

Replace two level system by Harmonic oscillator

$$H = \sum_k g_k \sigma_- a_k^\dagger + H.C.$$



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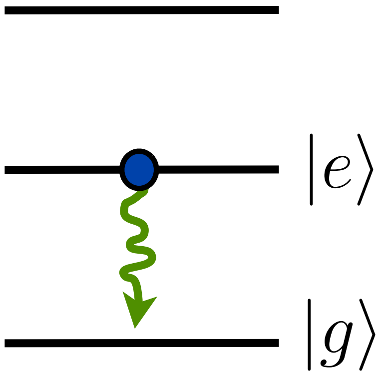

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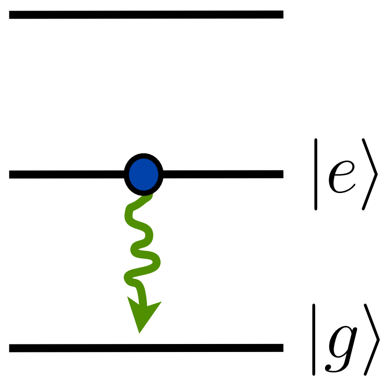
$$H = \sum_k g_k \sigma_- a_k^\dagger + H.C. \rightarrow \sum_k g_k b a_k^\dagger + H.C.$$



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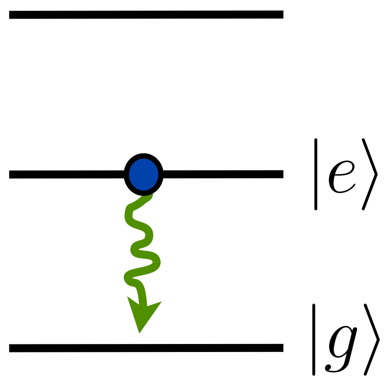

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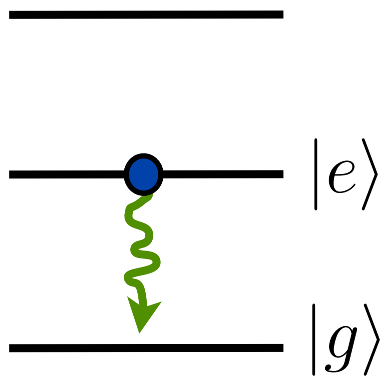

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Heisenberg equations of motion = Hamilton equations with hats

Classical spontaneous emission

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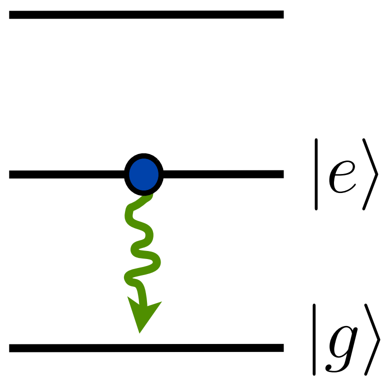

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$$\frac{\partial q}{\partial t} = p$$

$$\frac{\partial p}{\partial t} = -\omega^2 q$$

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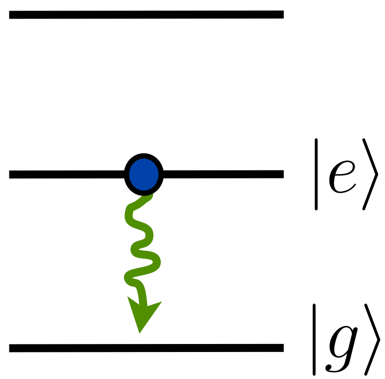

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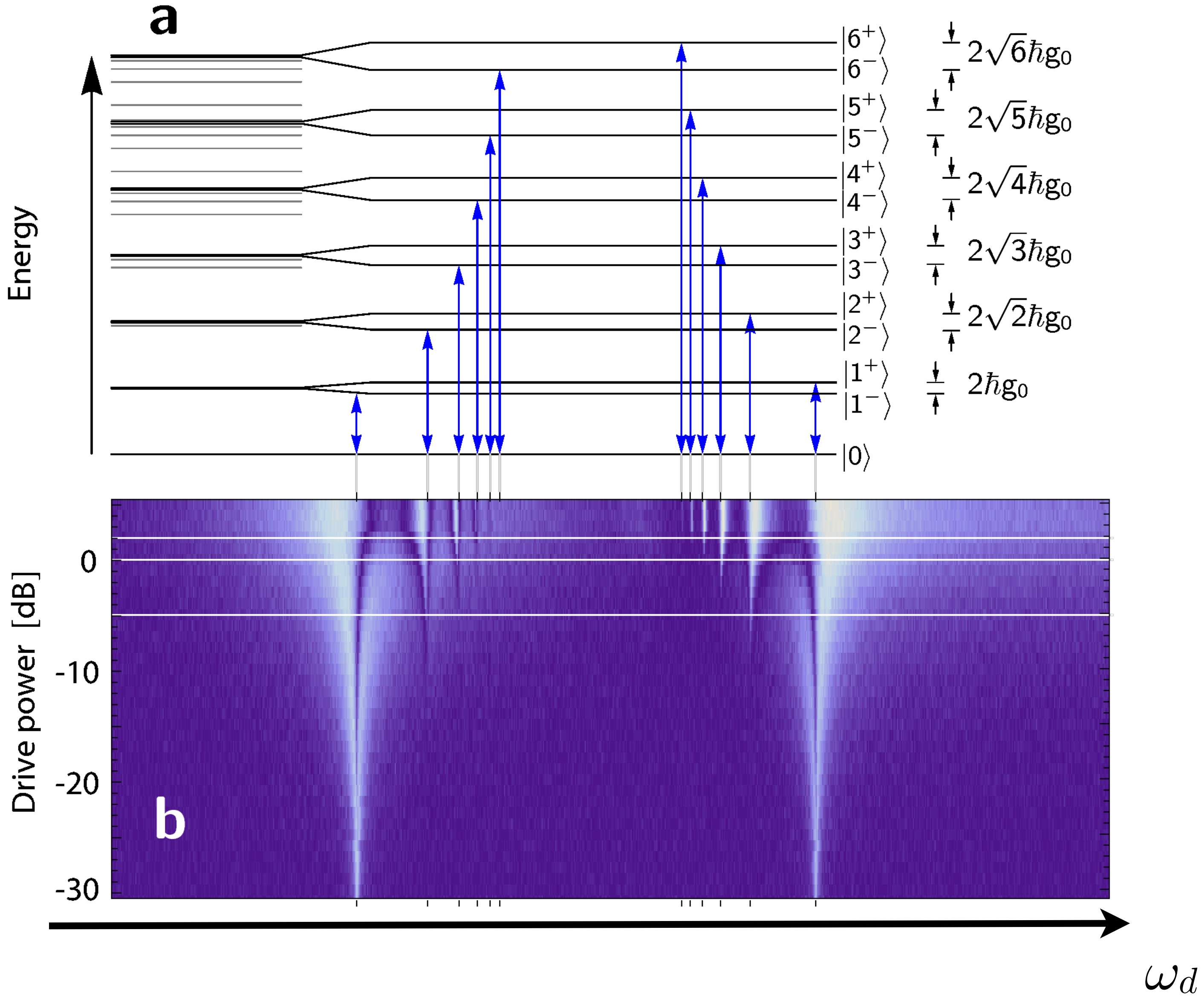
Coupled harmonic oscillators => classical and quantum the same

Heisenberg equations of motion = Hamilton equations with hats

$$\frac{\partial q}{\partial t} = p \qquad \frac{\partial \hat{q}}{\partial t} = \hat{p}$$

$$\frac{\partial p}{\partial t} = -\omega^2 q \qquad \frac{\partial \hat{p}}{\partial t} = -\omega^2 \hat{q}$$

Jaynes Cummings Spectroscopy



L. S. Bishop, R. J. Schoelkopf *et al*, Nat. Phys. **5**, 105 (2009)

Classical spontaneous emission

(Classical) Interaction Hamiltonian $H = d^- E^+ + E^- d^+$

Equations of motion

$$\frac{\partial d^+}{\partial t} \sim E^+(\vec{r} = 0)$$

$$\frac{\partial E^+(\vec{r})}{\partial t} \sim d^+ \delta(\vec{r})$$

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$$\frac{\partial E^+(\vec{r})}{\partial t} \sim d^+ \delta(\vec{r})$$ => $$E^+(\vec{r}) = E_0^+(\vec{r}) + G(\vec{r}, 0) d^+$$

Outcome:

$$\frac{\partial d^+}{\partial t} = (-\gamma + i\delta\omega) d^+$$

$$\gamma = \frac{2\omega_0^2 d_0^2}{\hbar \epsilon_0 c^2} \mathrm{Im}[G(\vec{r} = 0, \vec{r} = 0, \omega_A)]$$

The simpler calculation

Nothing quantum about spontaneous emission

The simpler calculation

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The correct quantum theory is the one that gives the same as the classical theory

The simpler calculation

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Use classical theory to calculate the Green's function

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Use classical theory to calculate the Green's function

Extract plasmon fraction

The simpler calculation

Nothing quantum about spontaneous emission

The correct quantum theory is the one that gives the same as the classical theory

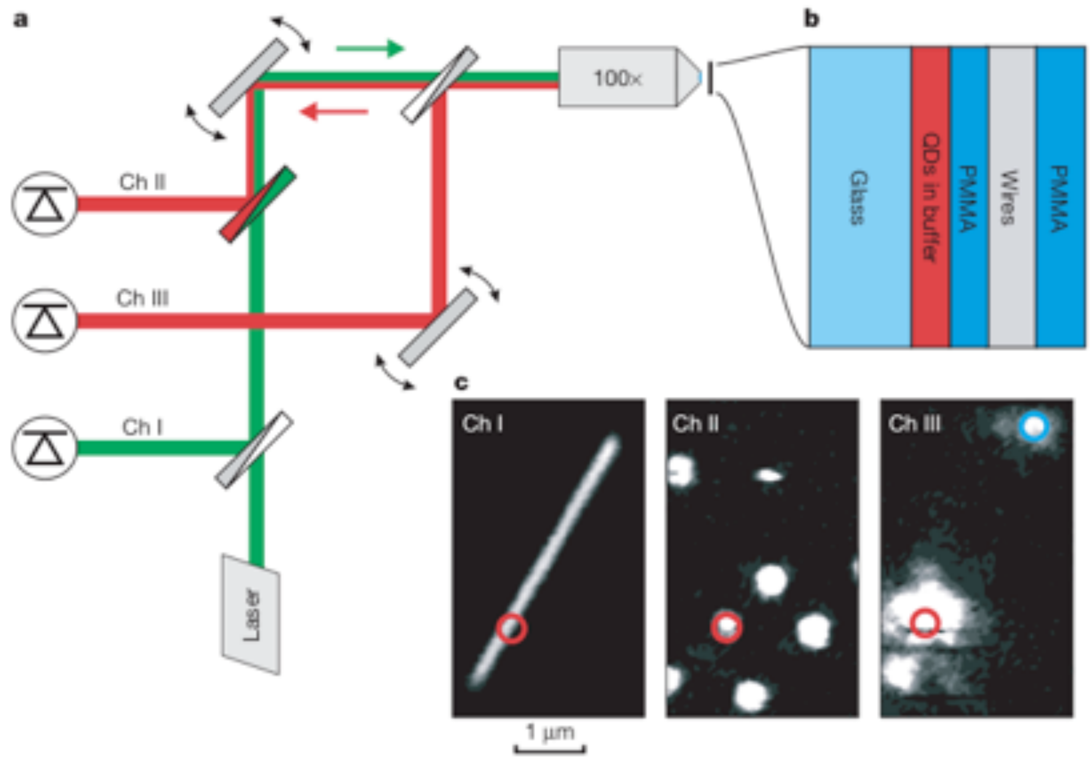
Use classical theory to calculate the Green's function

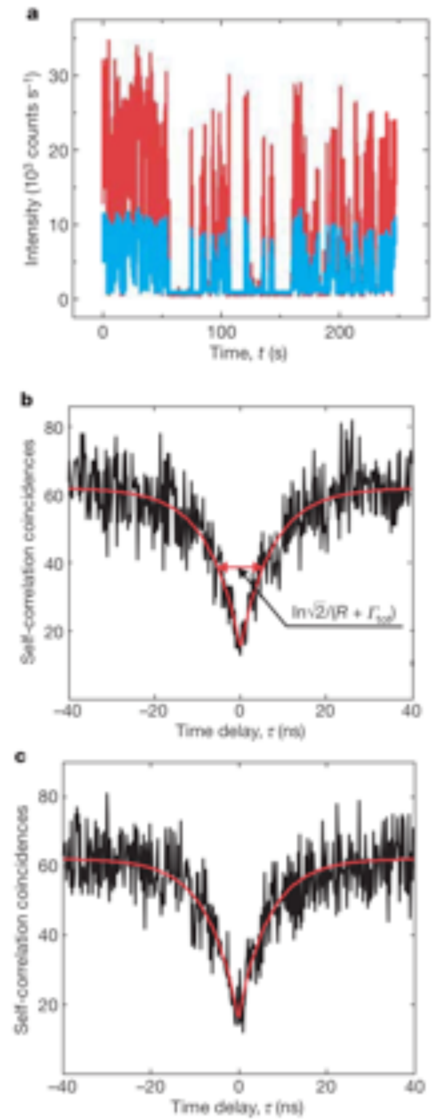
Extract plasmon fraction

... lots of math later, you recover simple physics

Narrow wire: $P_{\text{plasmon}} \sim 99.9\%$

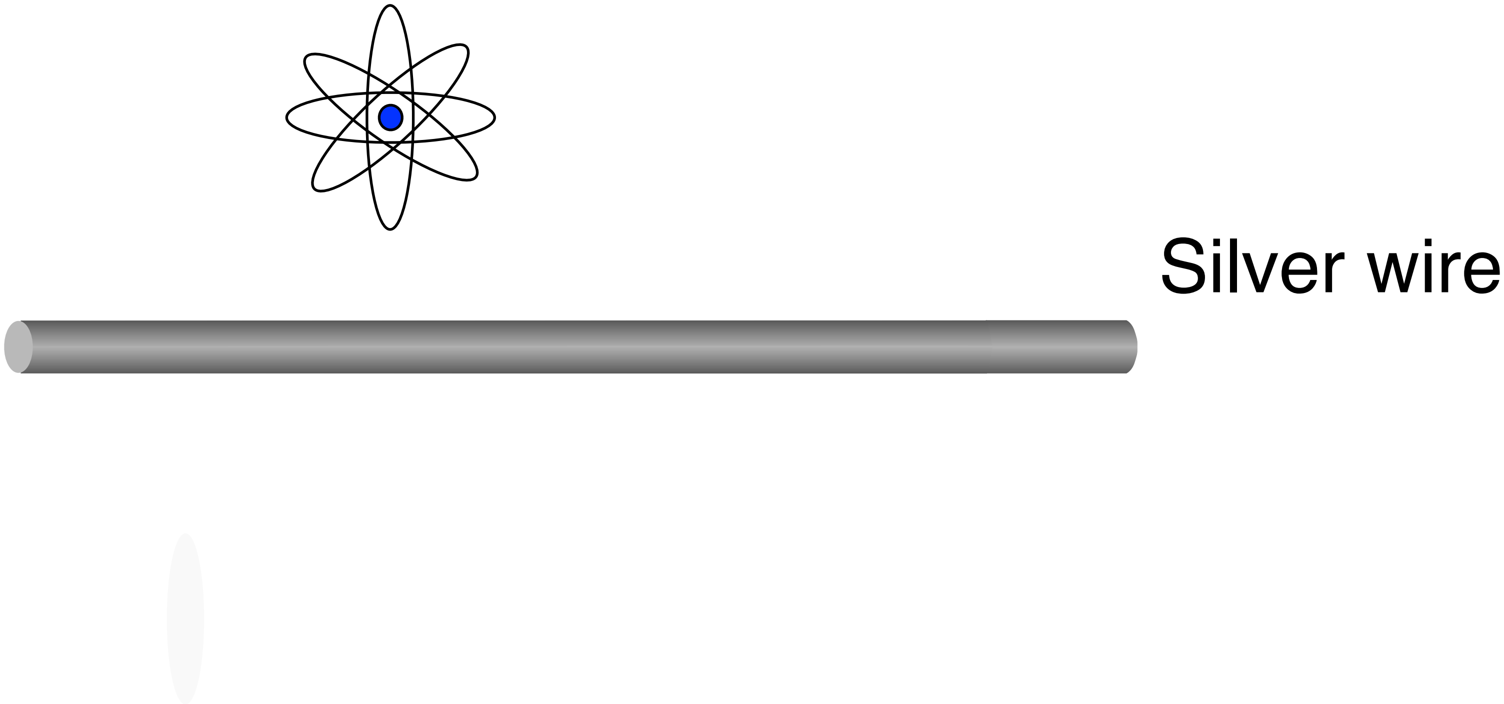
It works!





A.V. Akimov *et al.* (Harvard), Nature **450**, 402 (2007).
See also Y. Fedutik *et al.*, Phys. Rev. Lett. 99, 136802 (2007)
Ulrik Lund Andersen et al.

Single photon source



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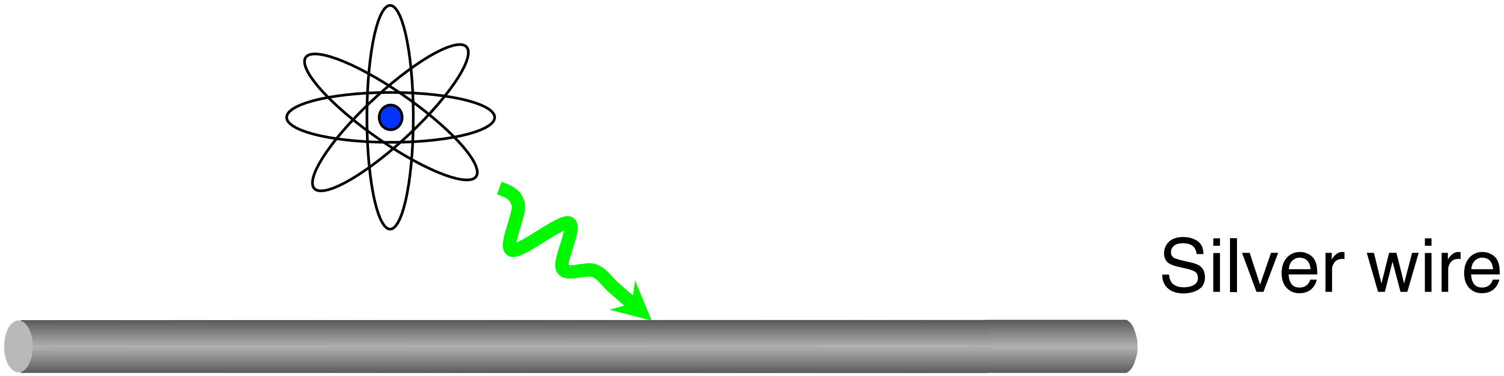

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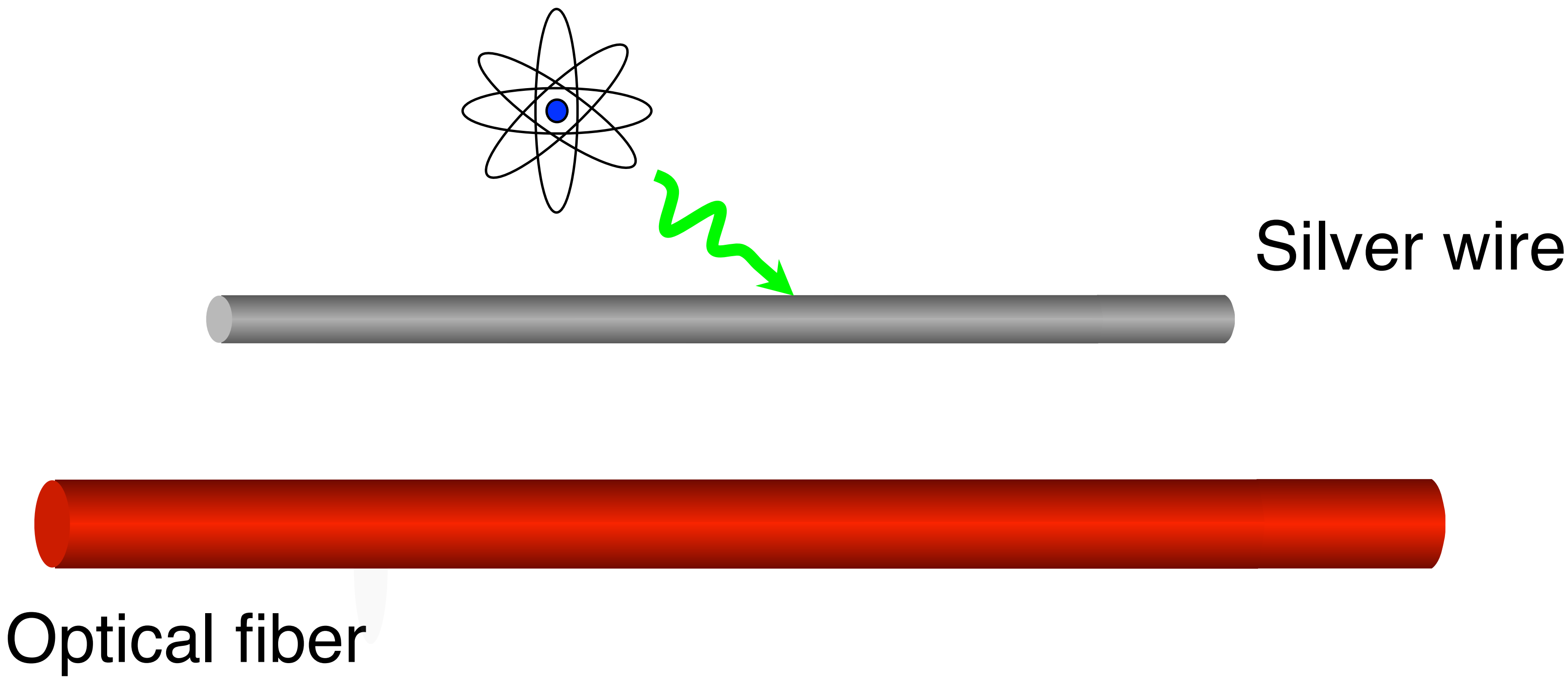

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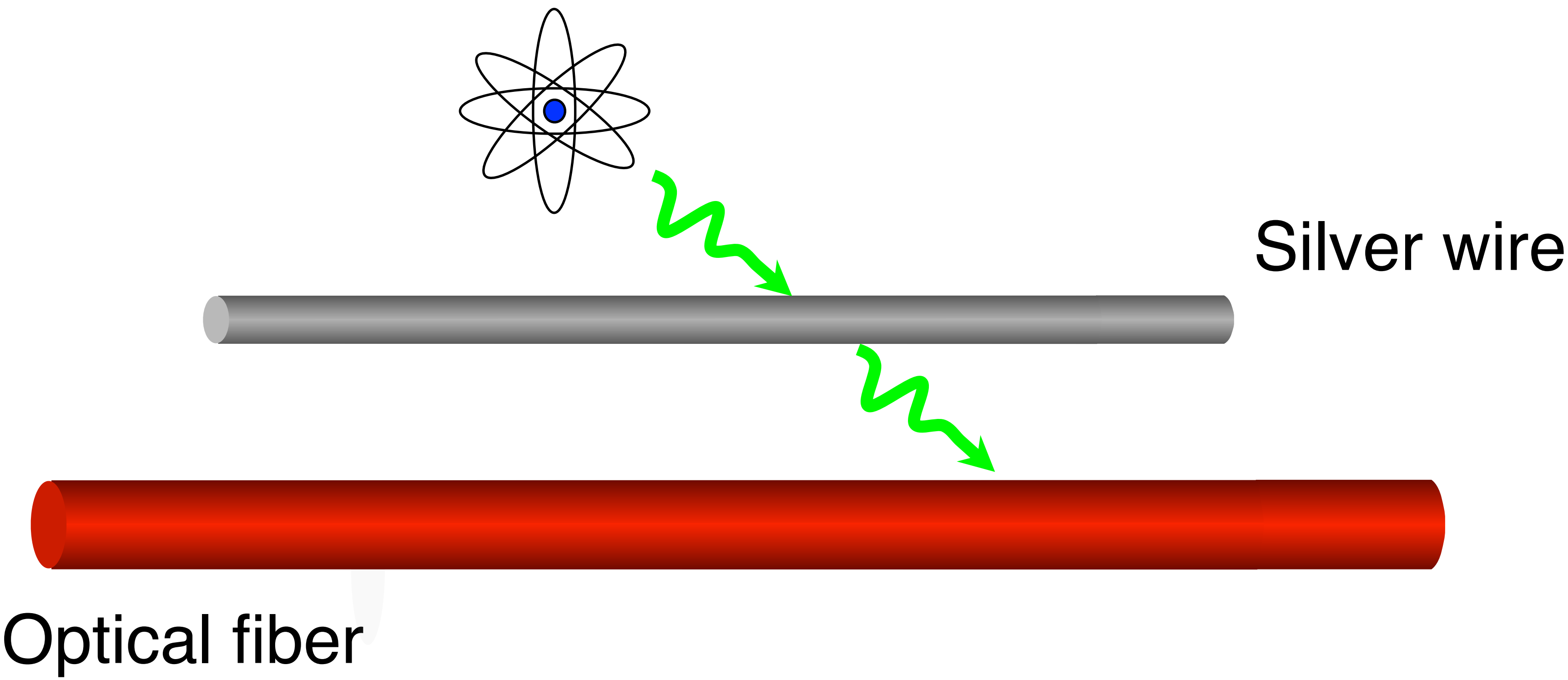

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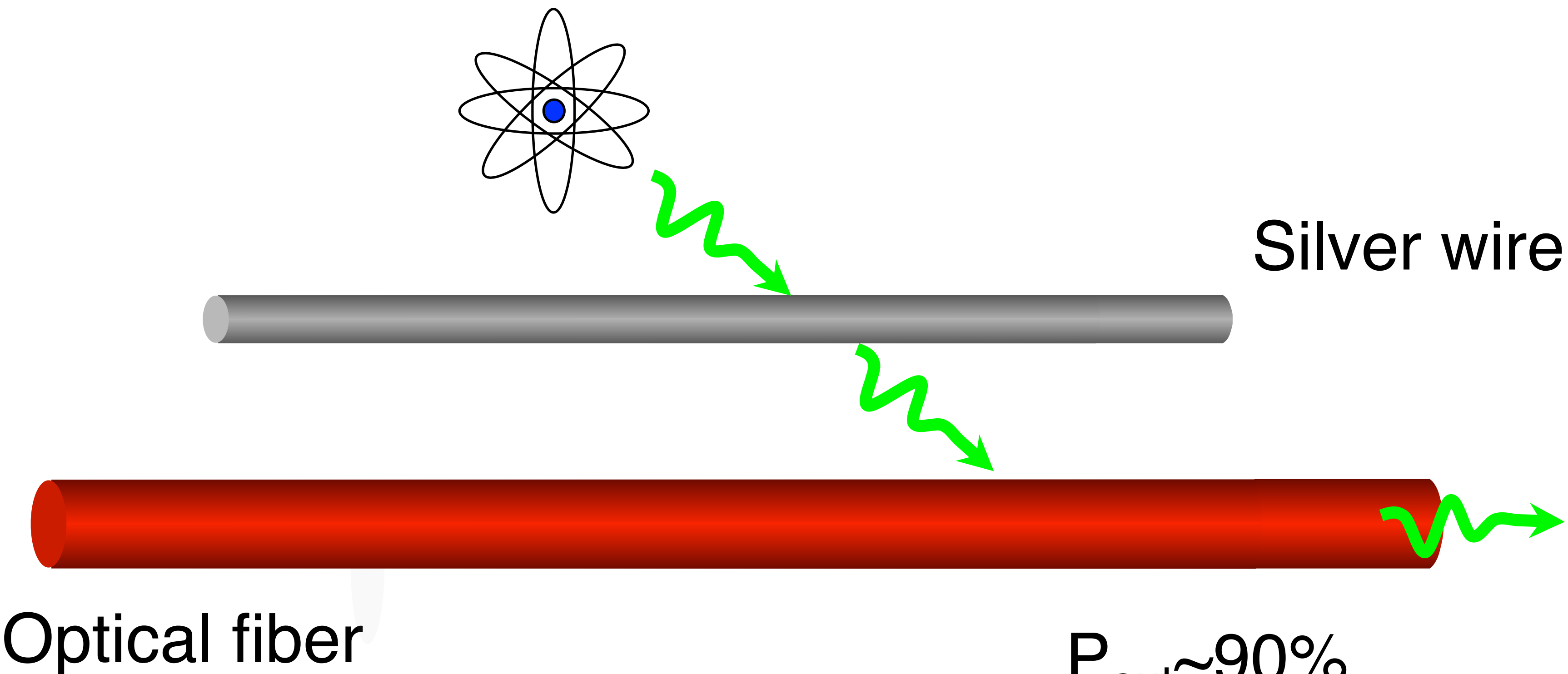
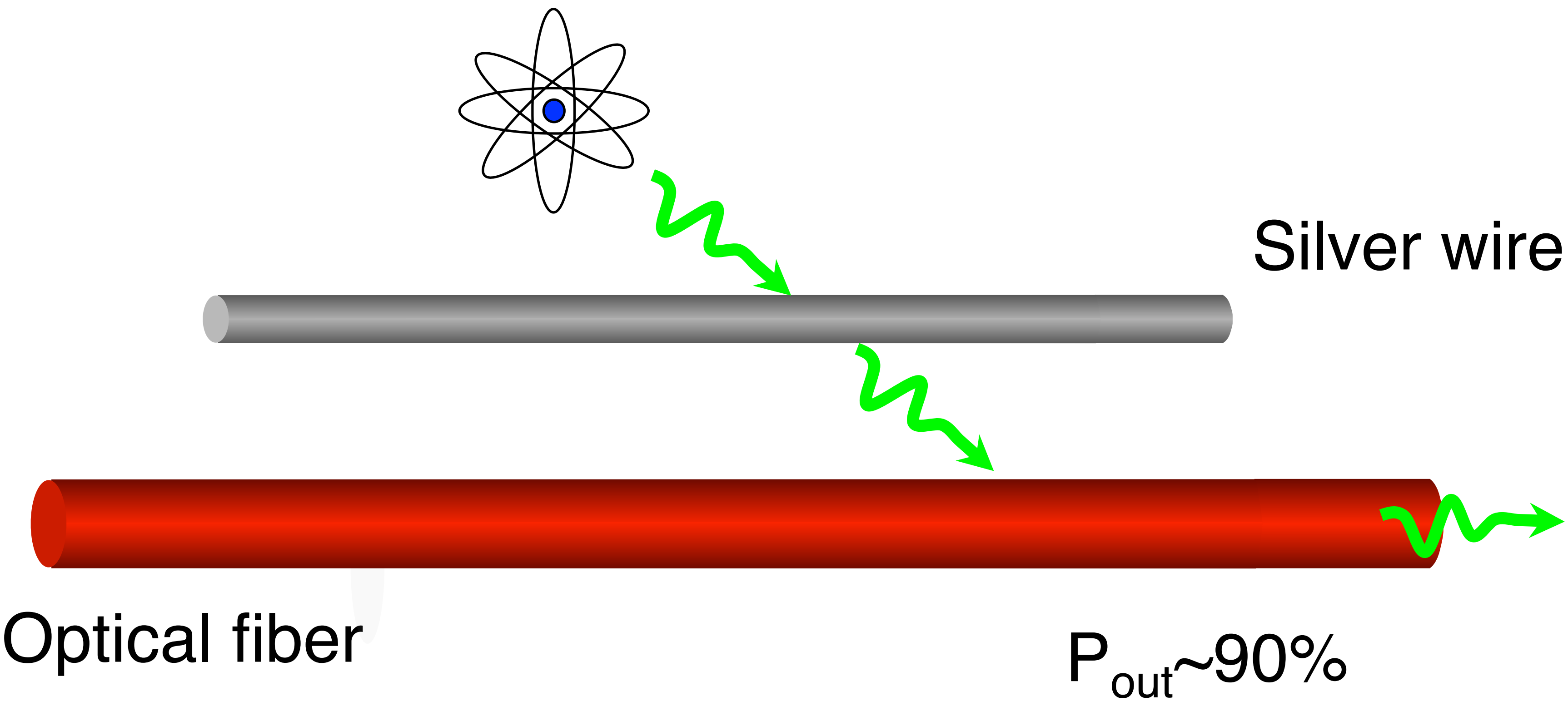


P_{out}~90%

(limited by Ohmic loss)

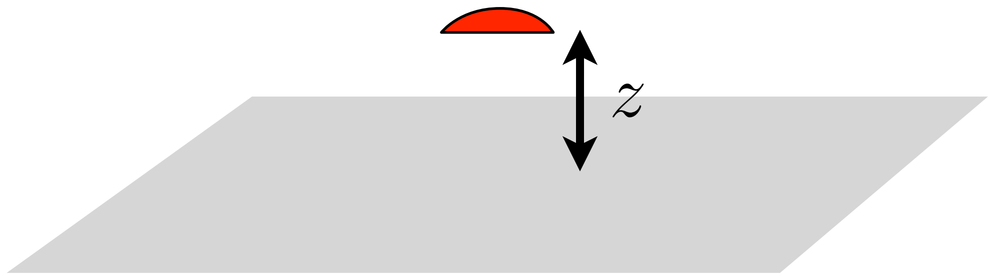
Single photon source



(limited by Ohmic loss)

Also useful for quantum repeater or scaling quantum computer

Experiments with Q. dots (P. Lodahl)

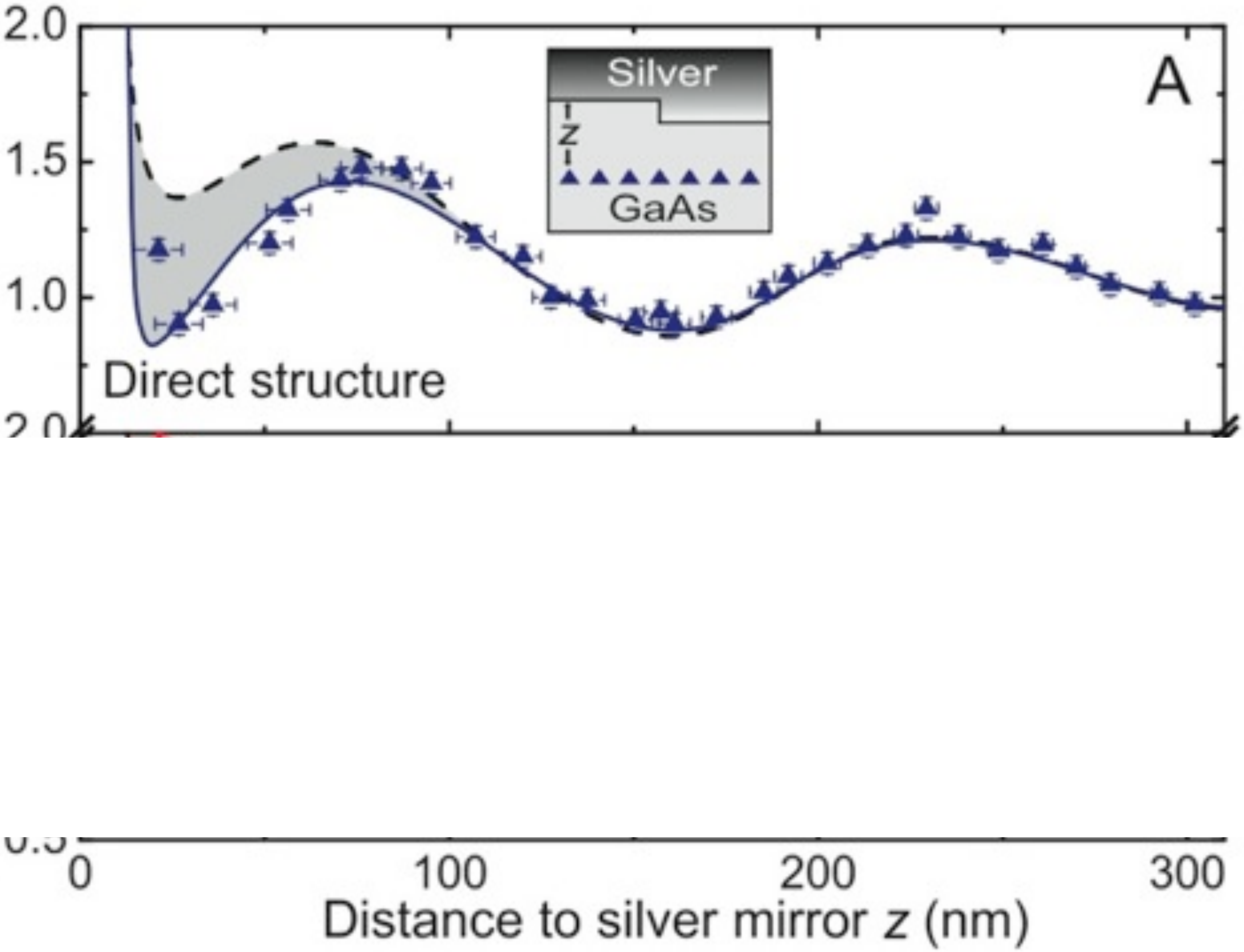


Decay has three contributions:

1. Radiation
2. Surface loss
3. Plasmons

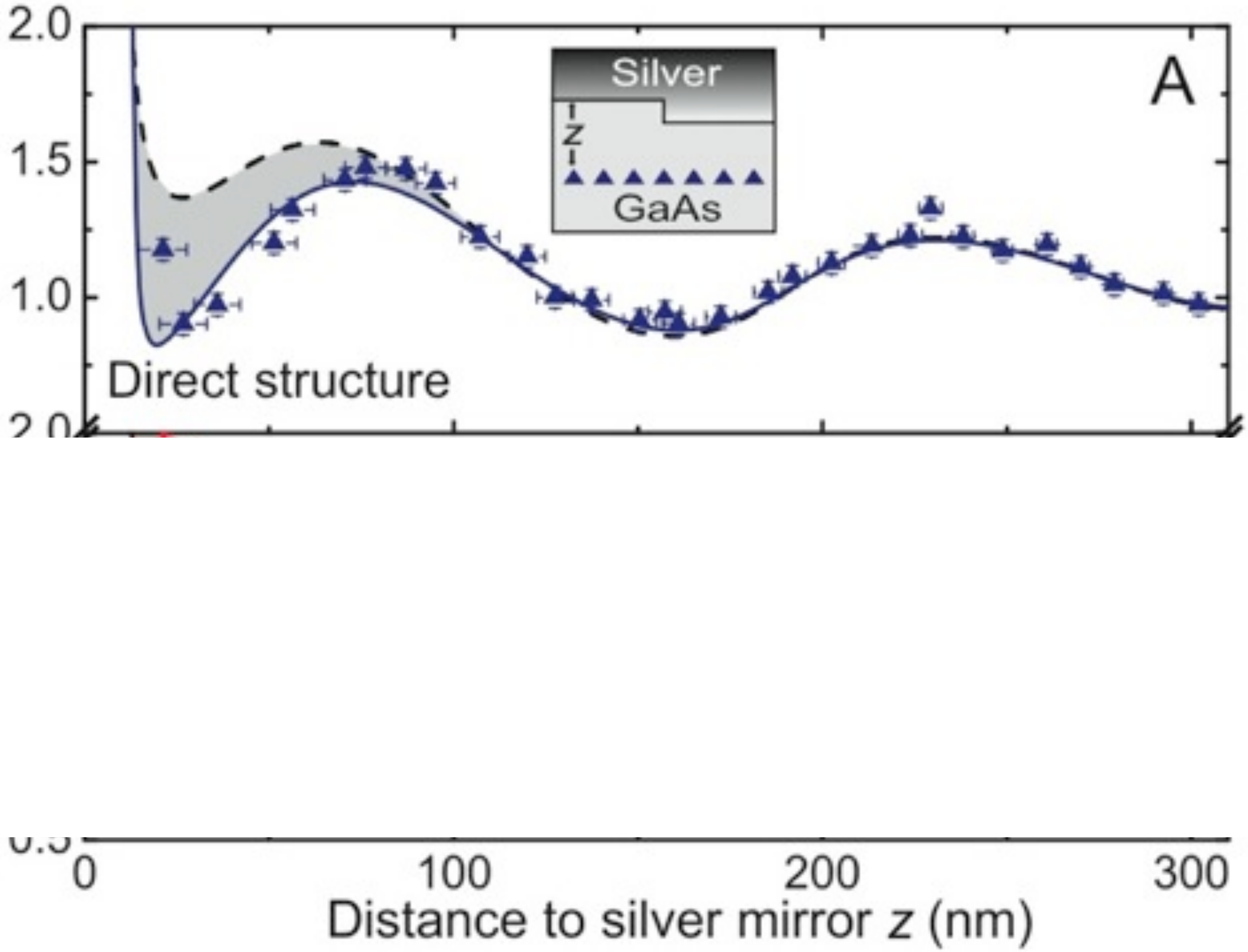
Map out each contribution by measuring $\gamma(z)$

Results



M. L. Andersen, S. Stobbe, A. S. Sørensen and P. Lodahl, Nat. Phys. **7**, 215 (2011).

Results



Explanation: quantum dots are not dots

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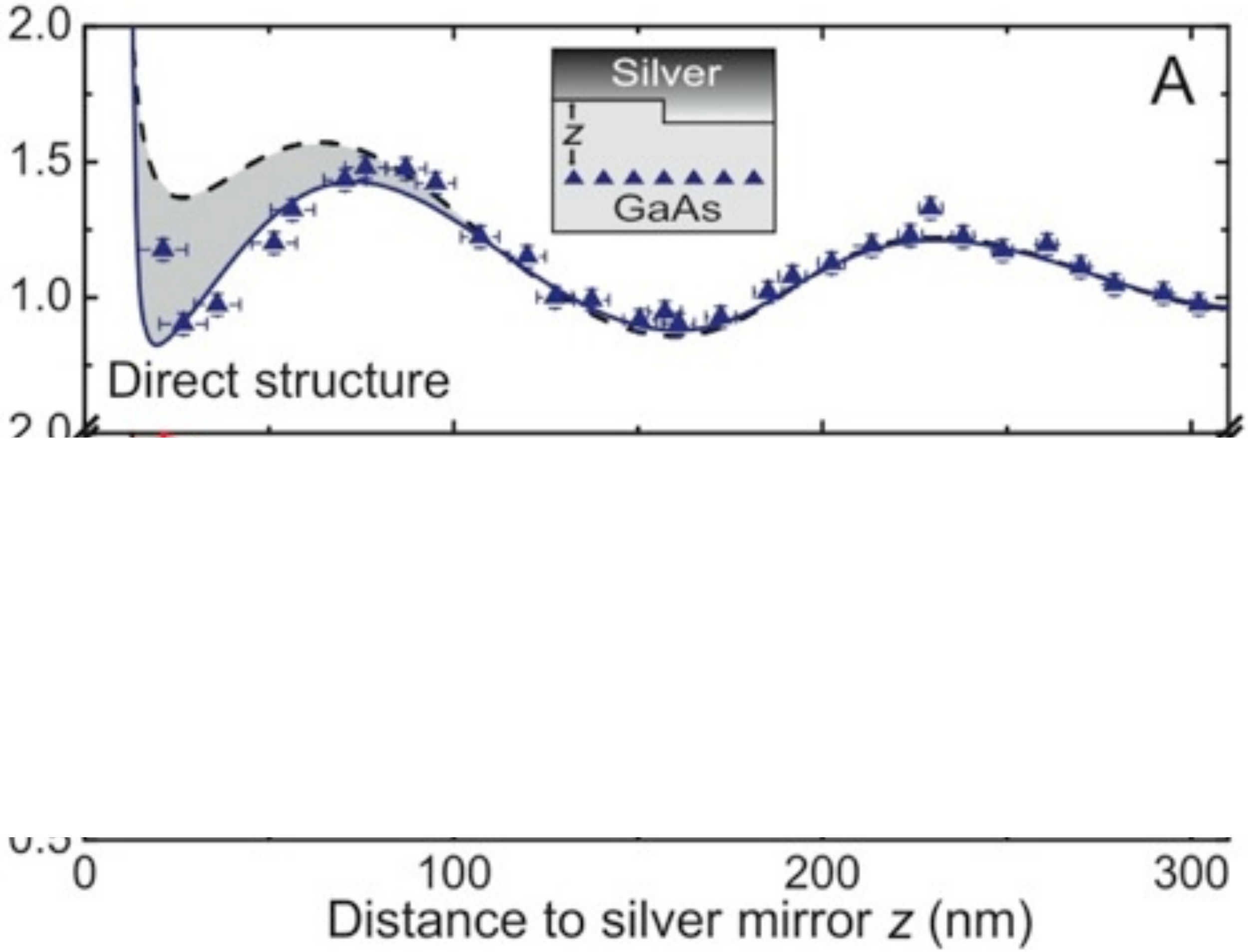

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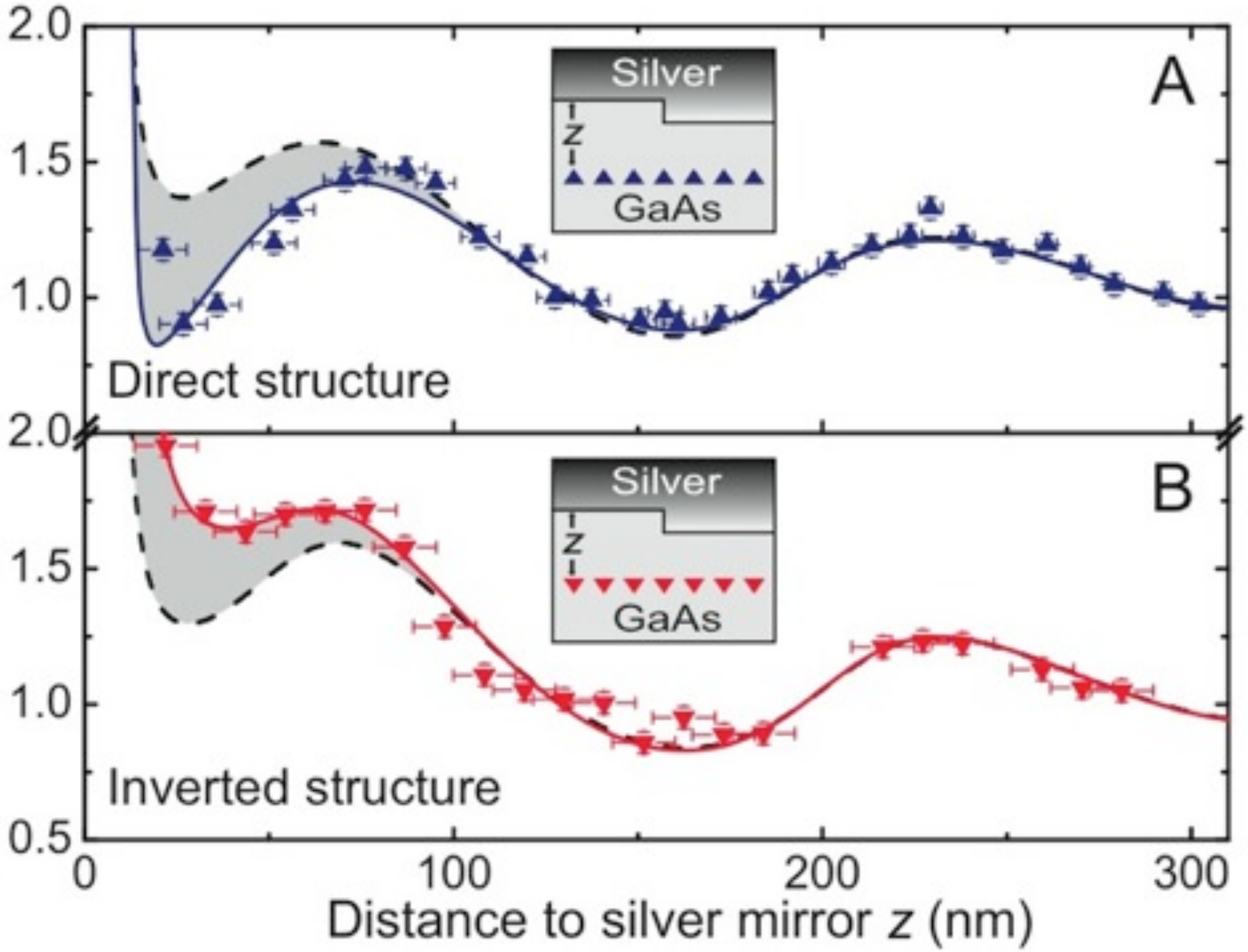

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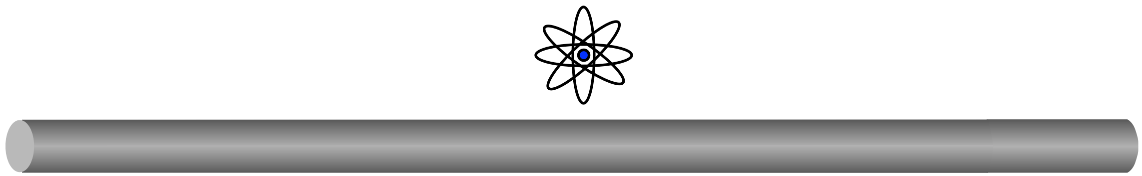
Conclusion (1)



- Strong confinement of fields near conductors permits strong coupling to atoms.

- Experiments have shown that the coupling works also optically

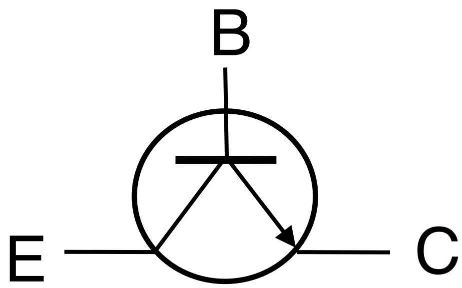
- Can be used to create single photon sources or connect quantum computers to light

- Remaining challenge: out coupling from plasmon to light

Non-linear optics

The Problem

Electronic transistor:



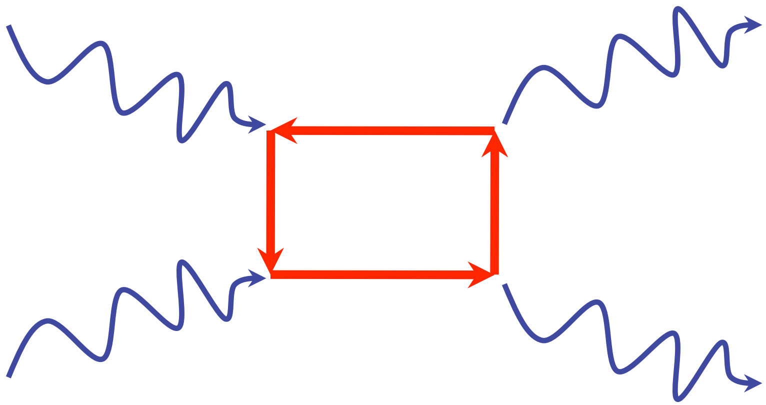
I_{CE} controlled by a small signal on B

Can we do the same with light?

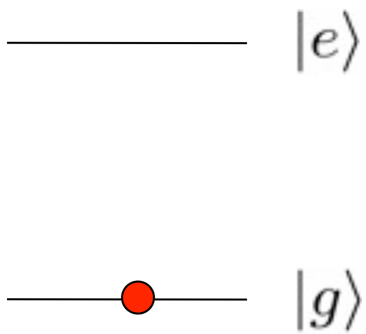
Problem: nonlinear process => requires interaction between photons

Non-linear optics

Feynman diagram:

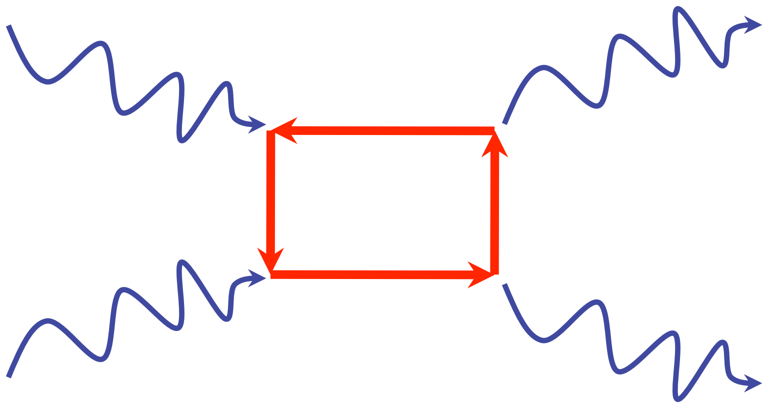


Atoms:

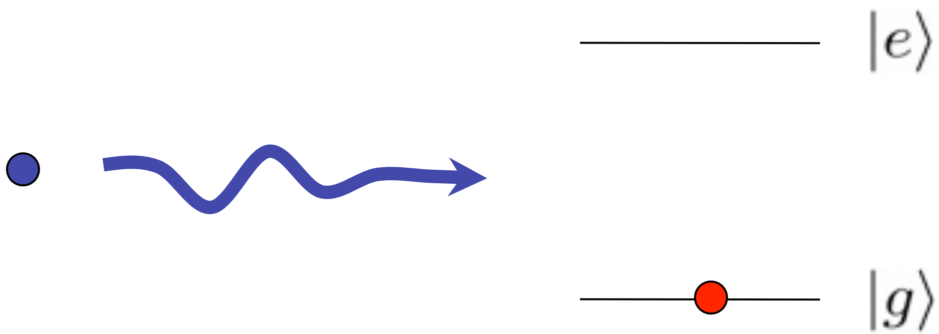


Non-linear optics

Feynman diagram:

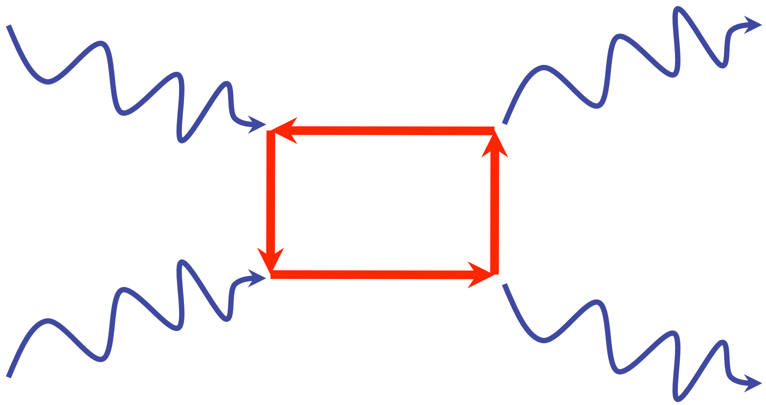


Atoms:

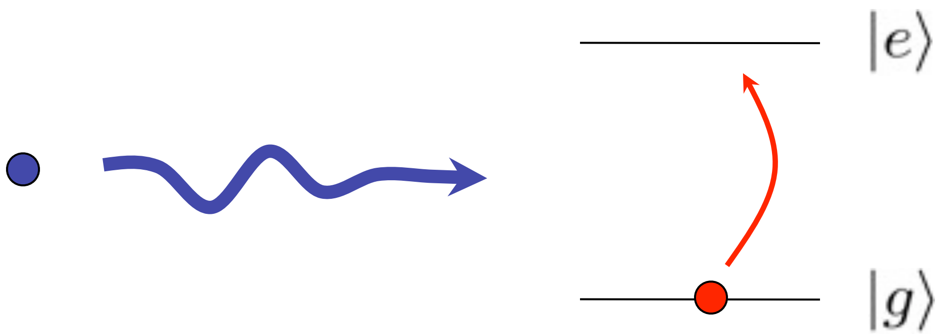


Non-linear optics

Feynman diagram:

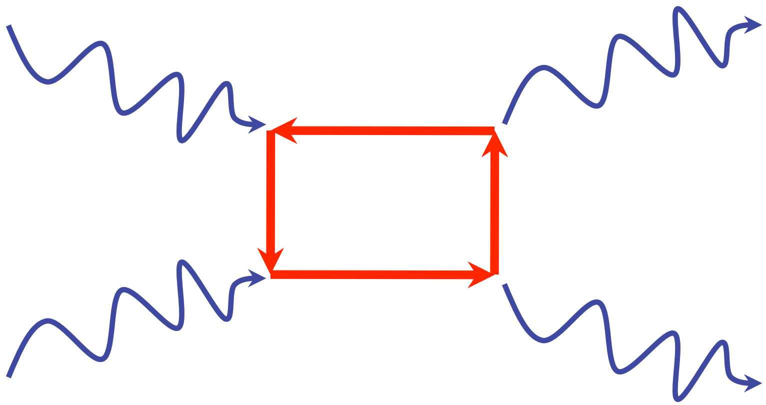


Atoms:

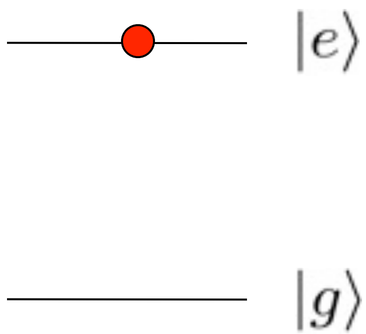


Non-linear optics

Feynman diagram:

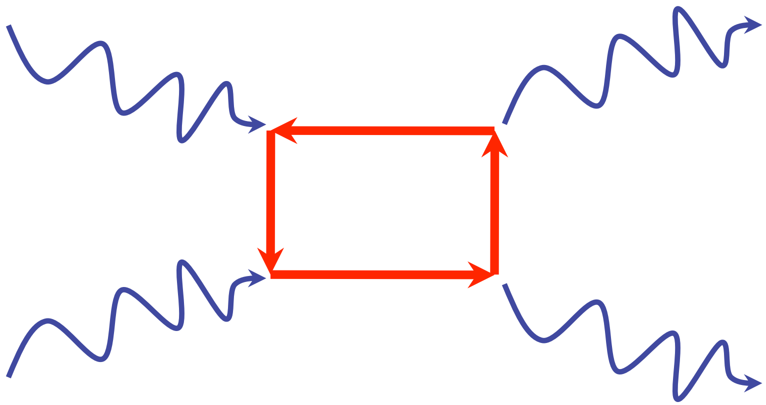


Atoms:

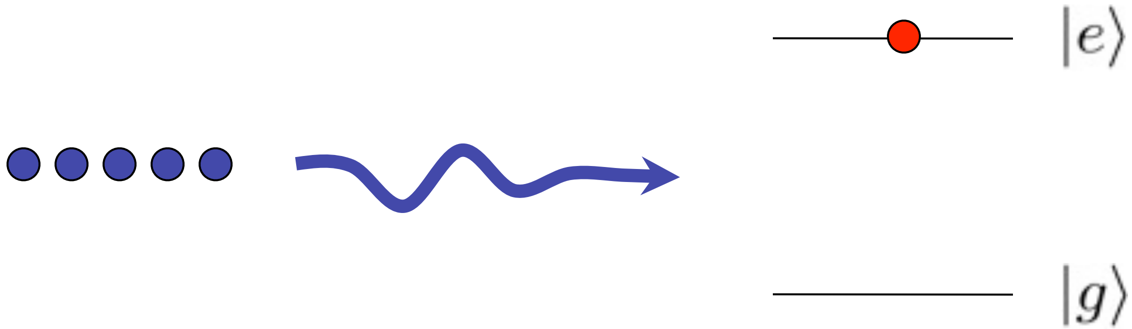


Non-linear optics

Feynman diagram:

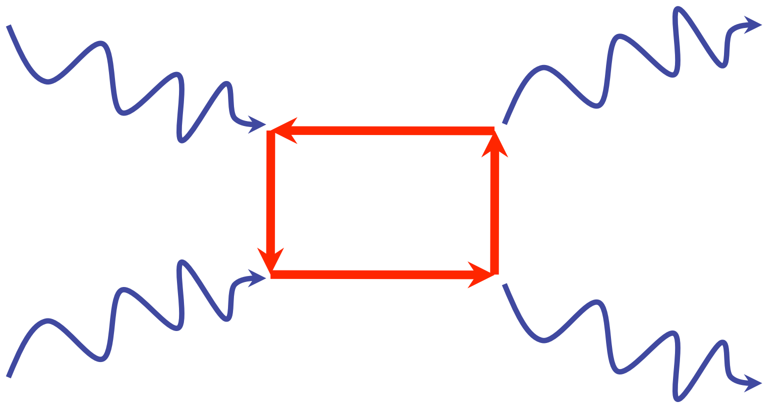


Atoms:

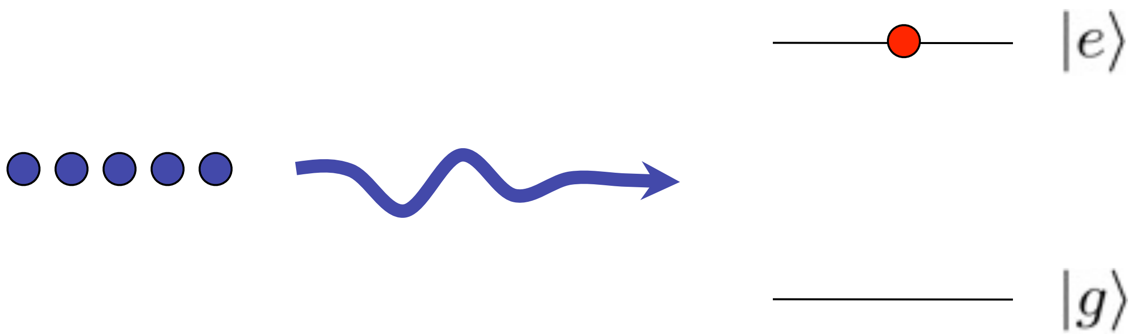


Non-linear optics

Feynman diagram:



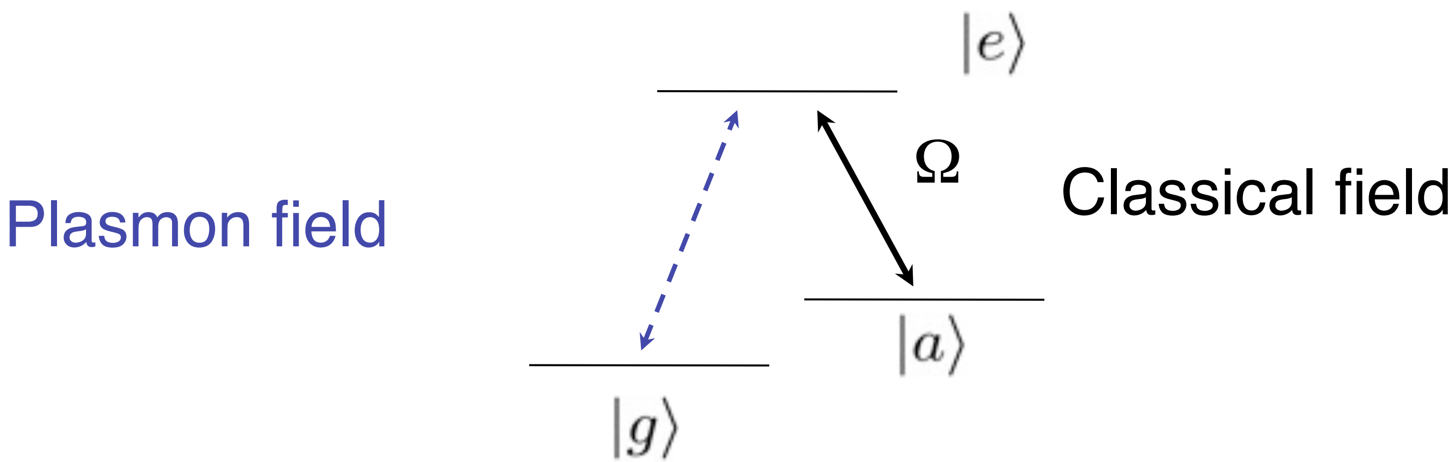
Atoms:



Strong non-linear interaction requires strong interaction between single photon and single atoms => surface plasmons

Λ-type atoms

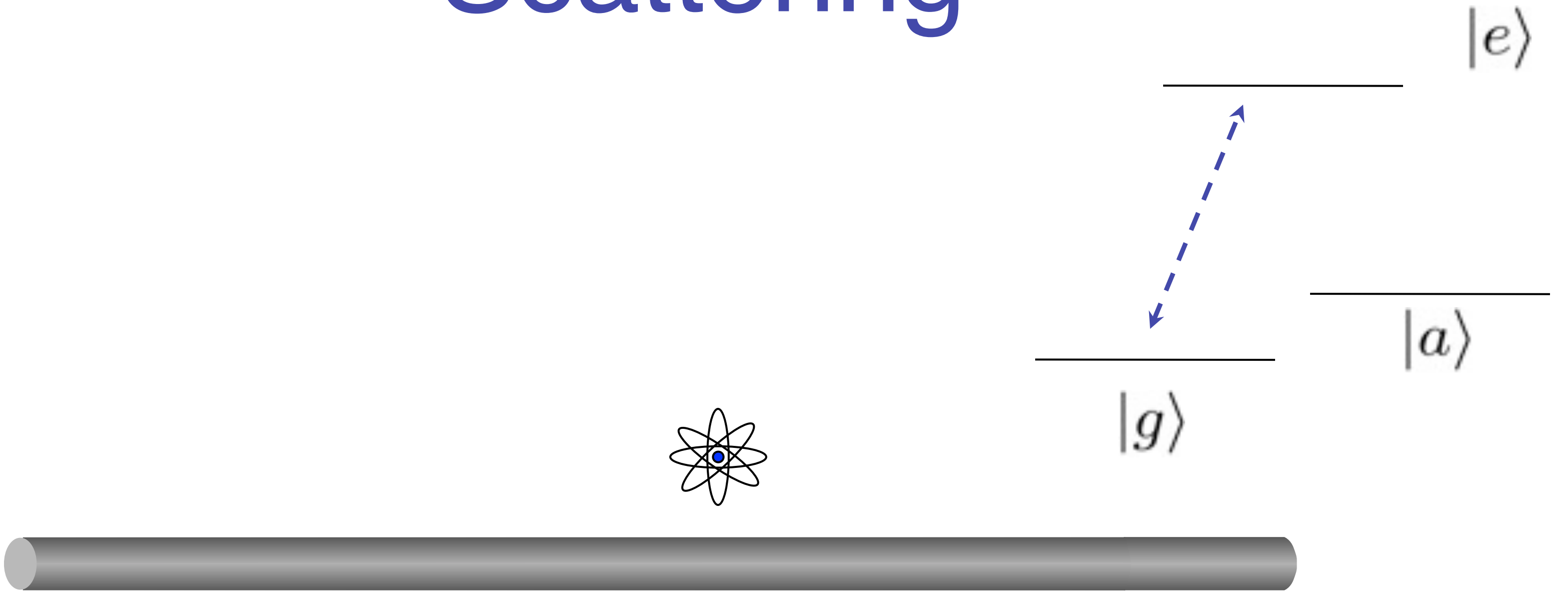
Assume that atoms have structure:



Not essential for non-linear effect, but:

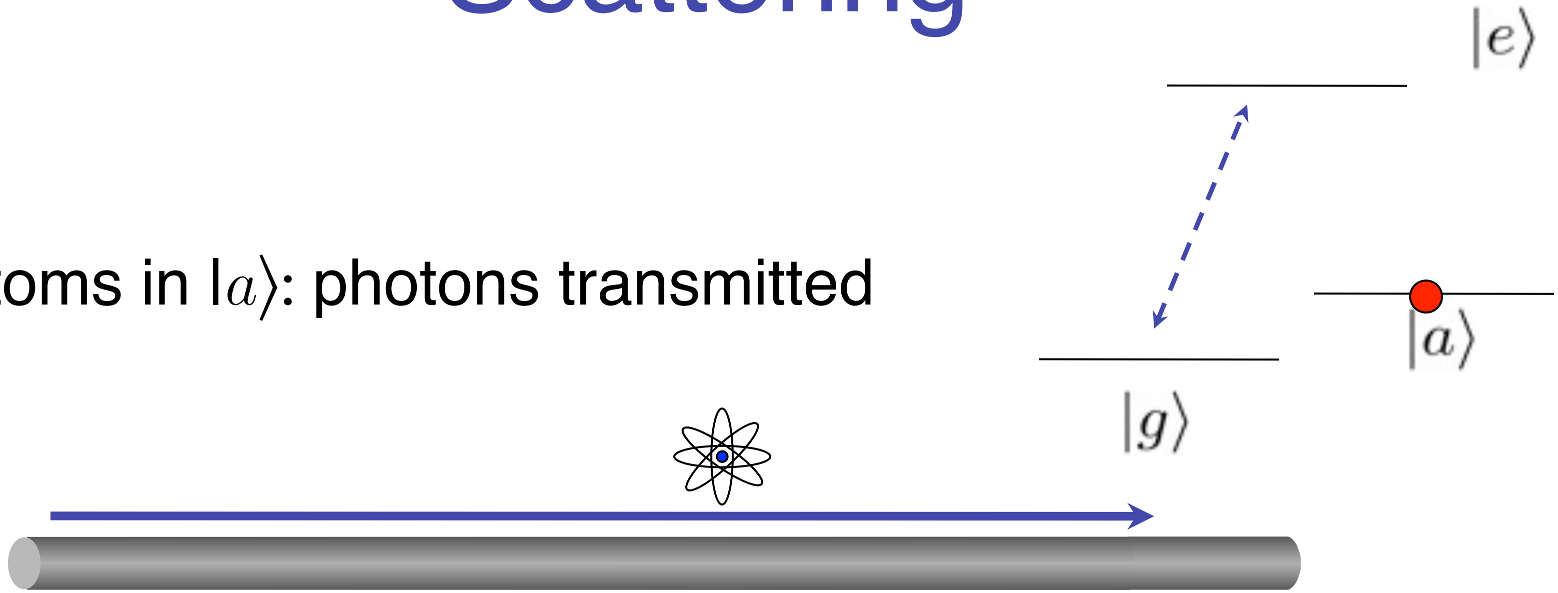
- equations are easy to solve
- can achieve almost ideal operation

Scattering



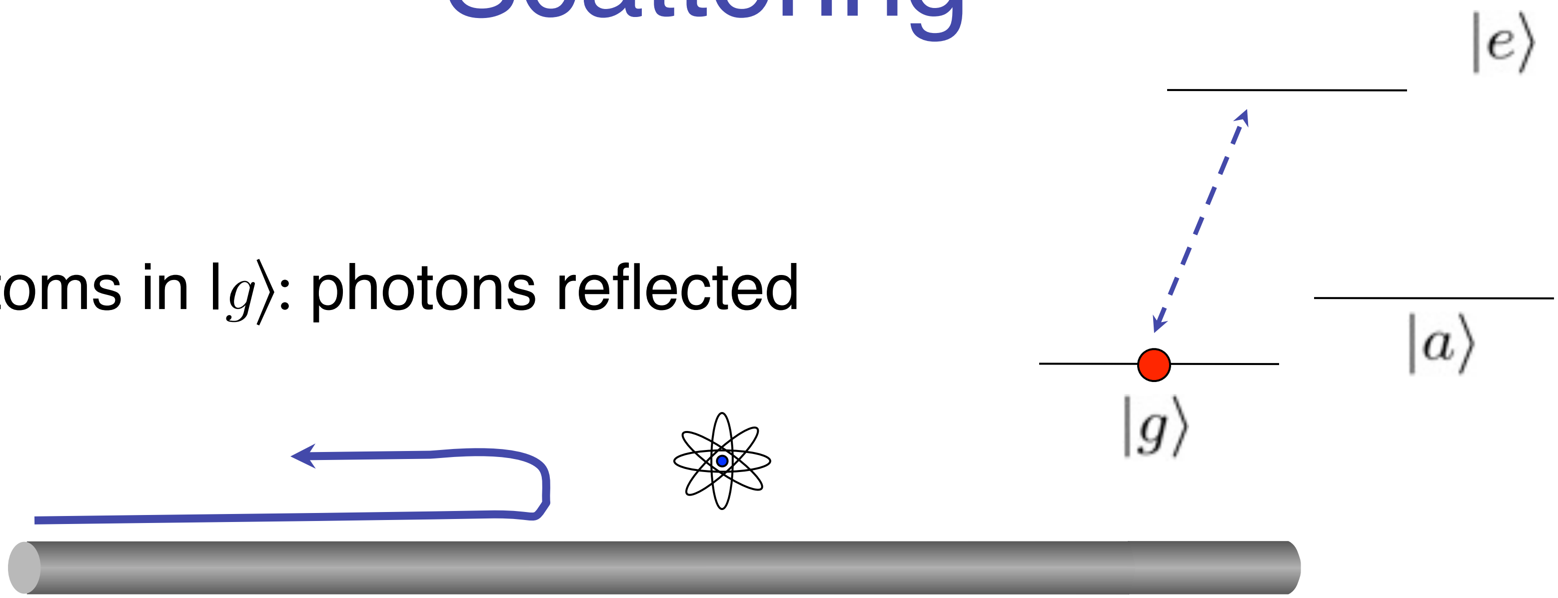
Scattering

Atoms in $|a\rangle$: photons transmitted

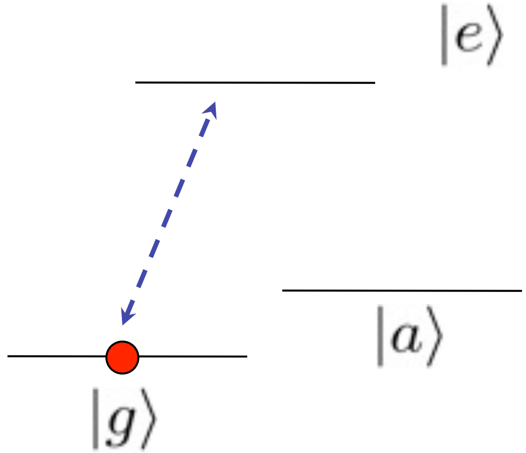


Scattering

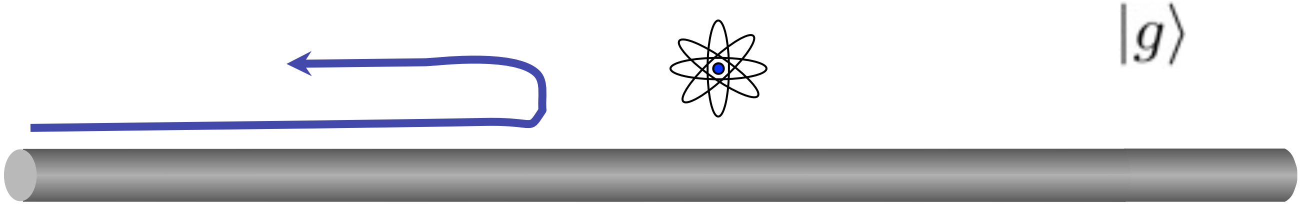
Atoms in $|g\rangle$: photons reflected



Scattering

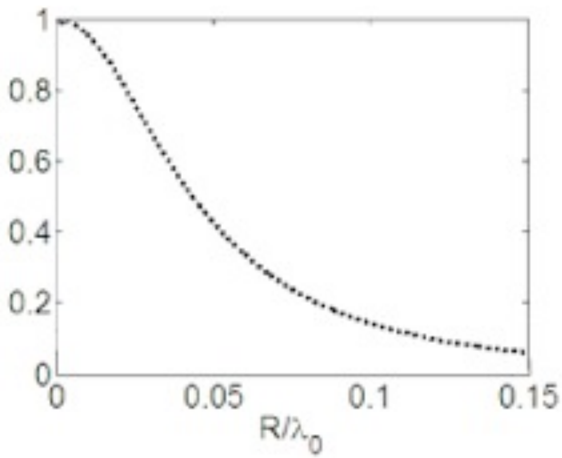


Atoms in $|g\rangle$: photons reflected



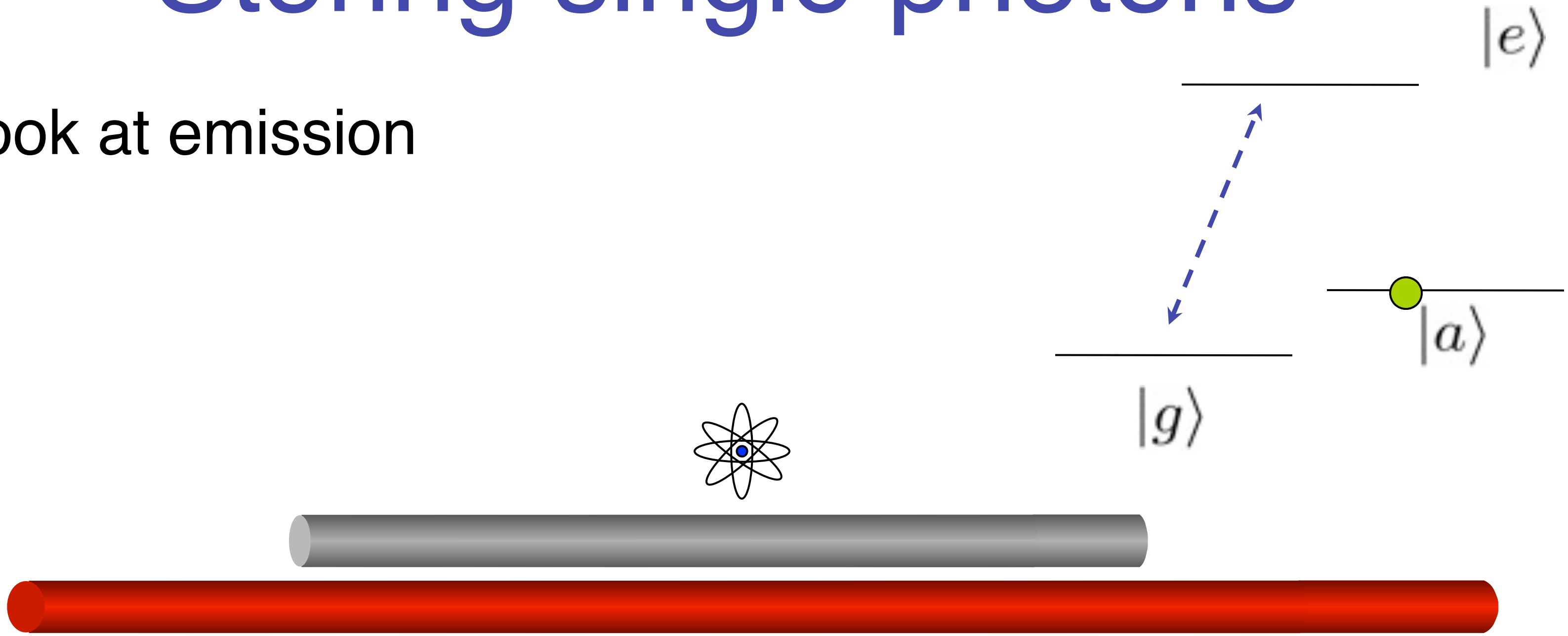
Reflection probability

(Silver, λ= 1μ m)



Storing single photons

1. Look at emission



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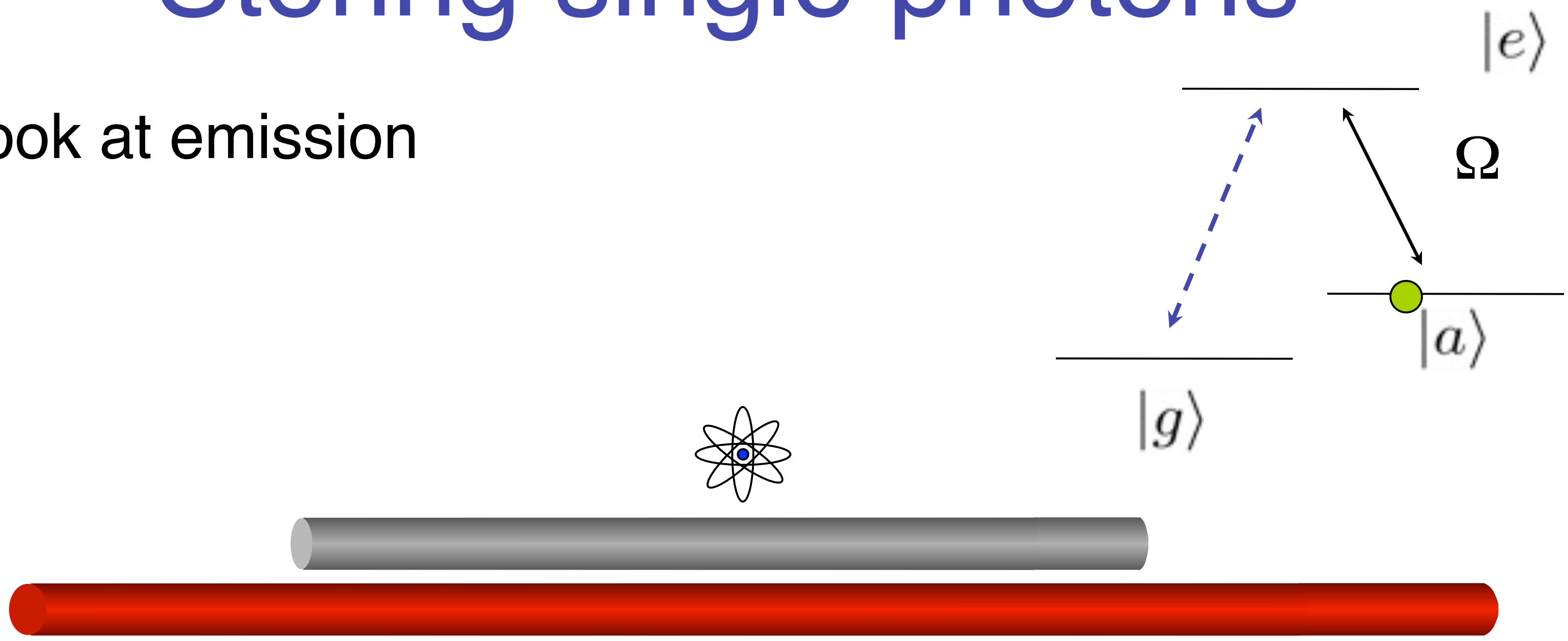

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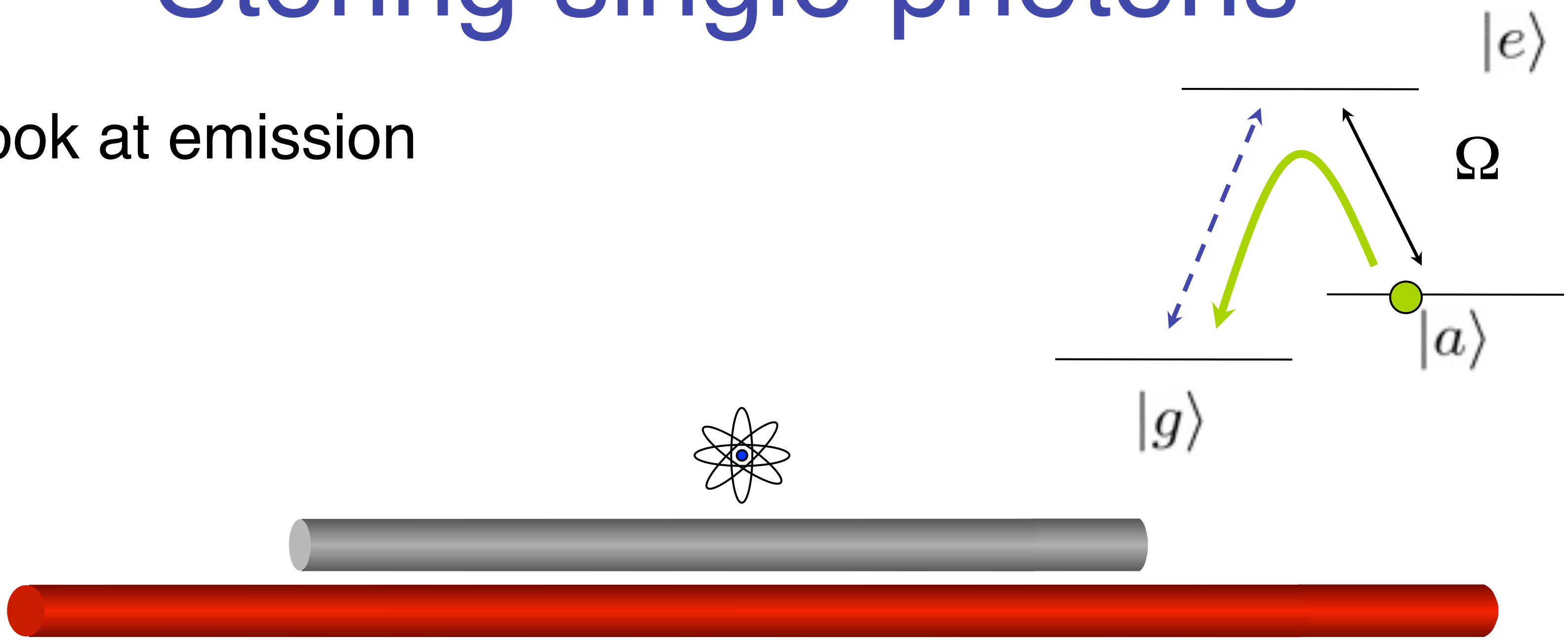

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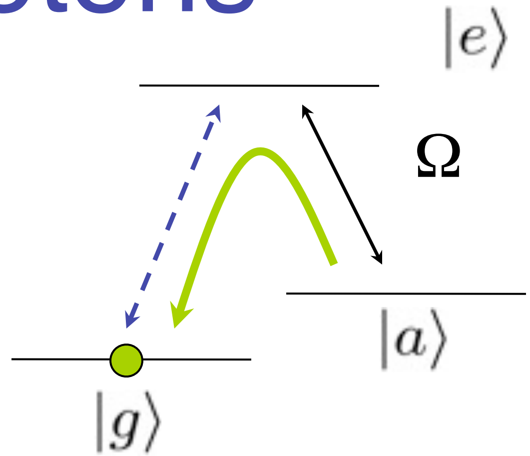

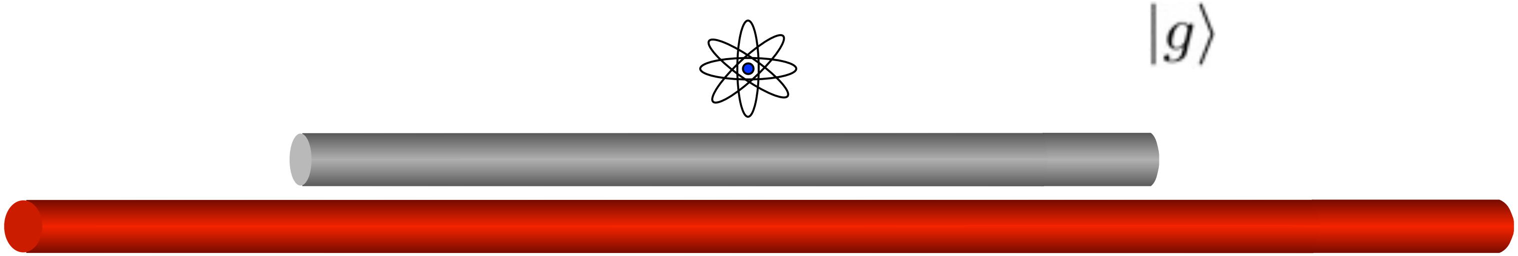

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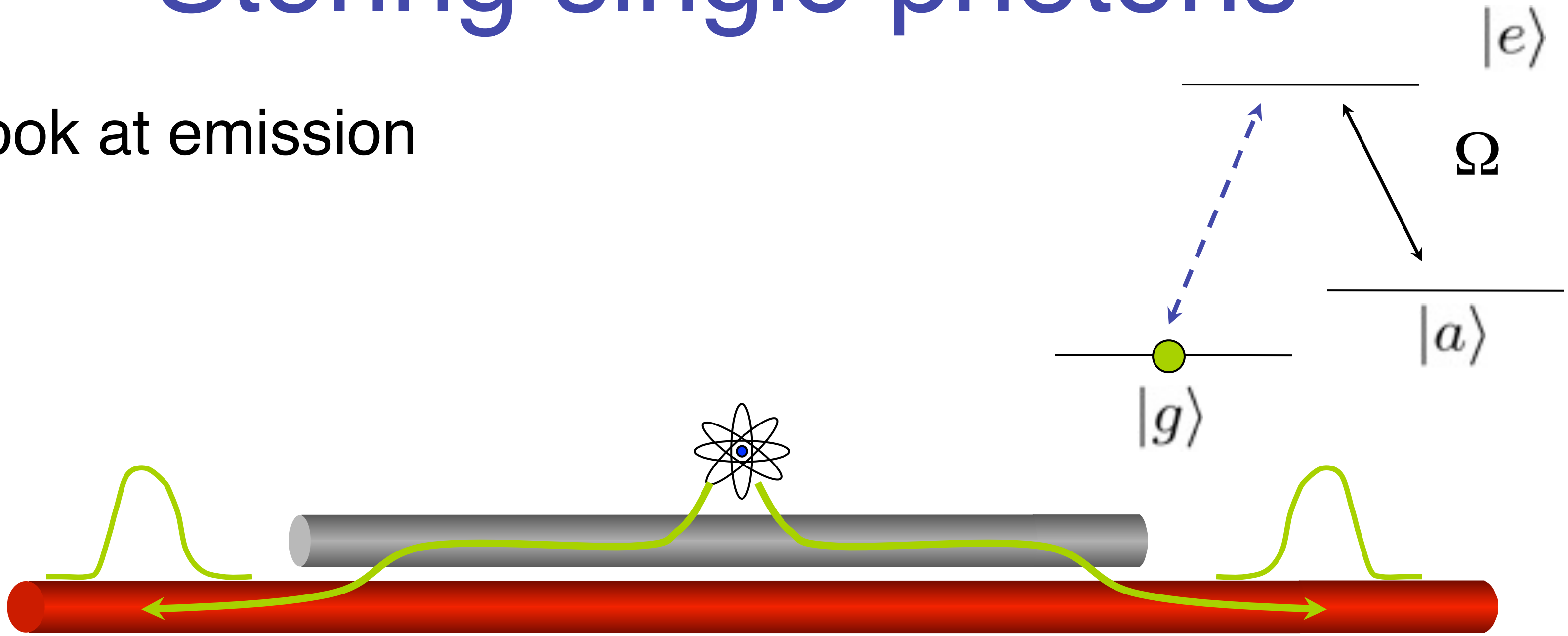

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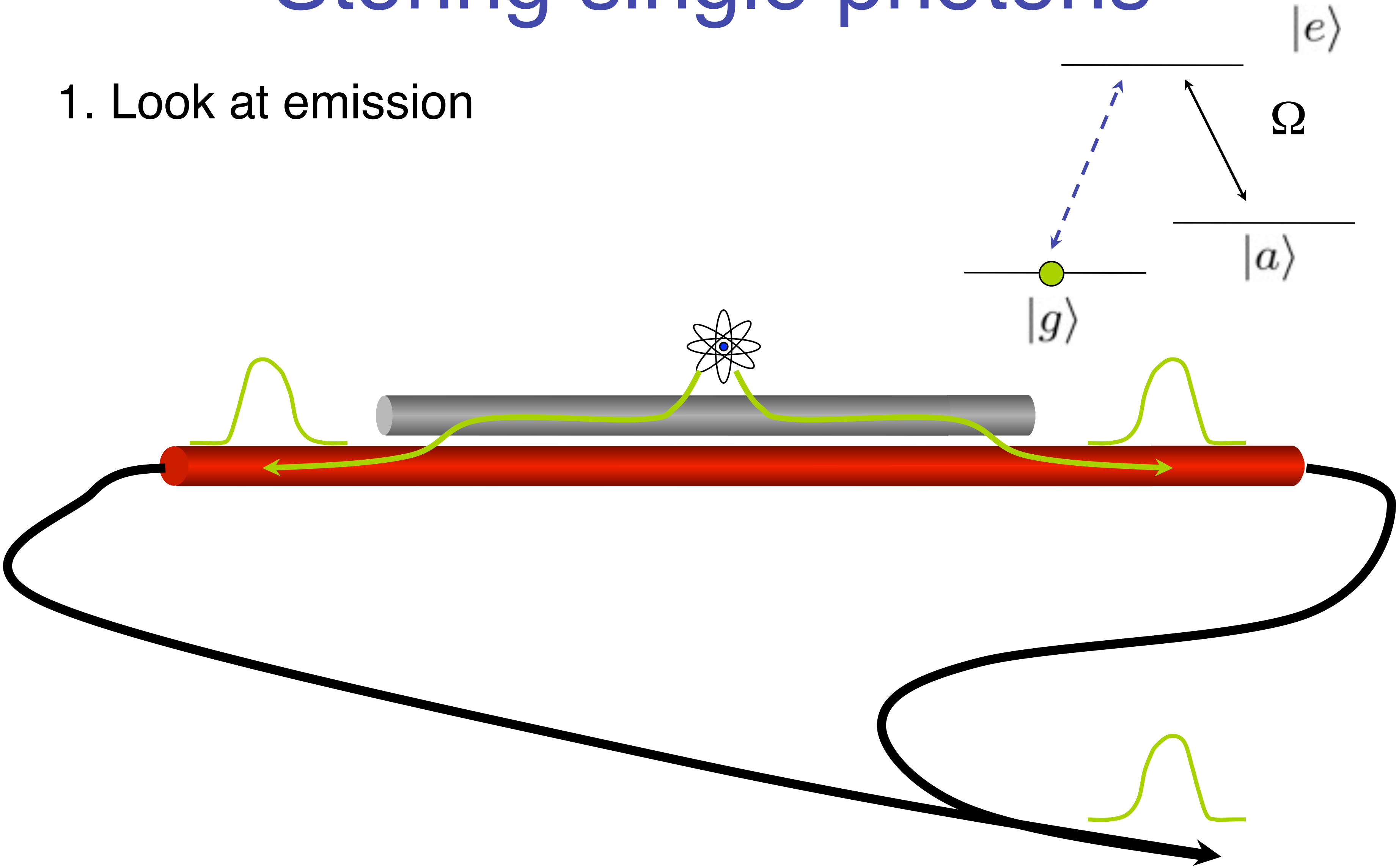
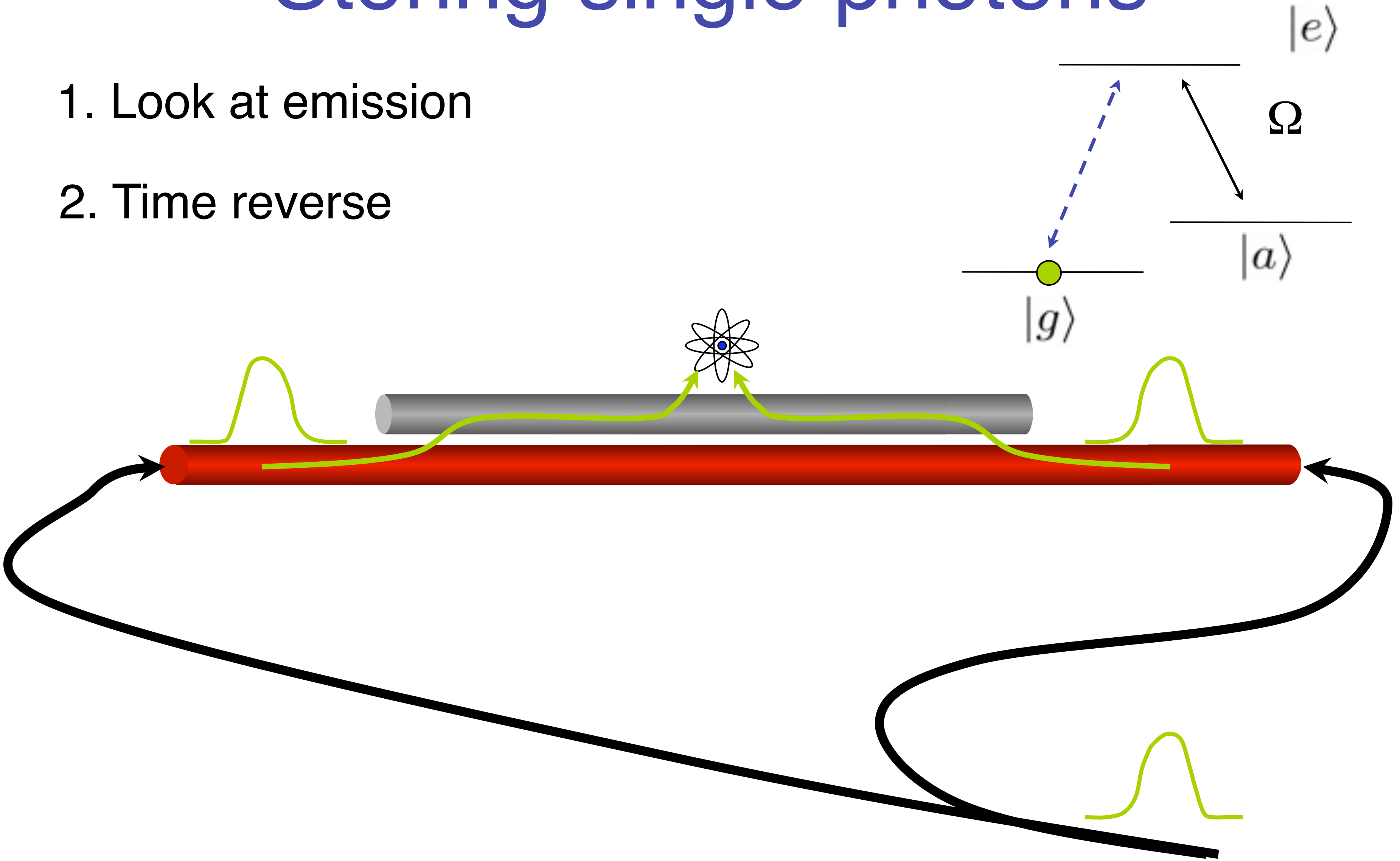


Storing single photons

1. Look at emission

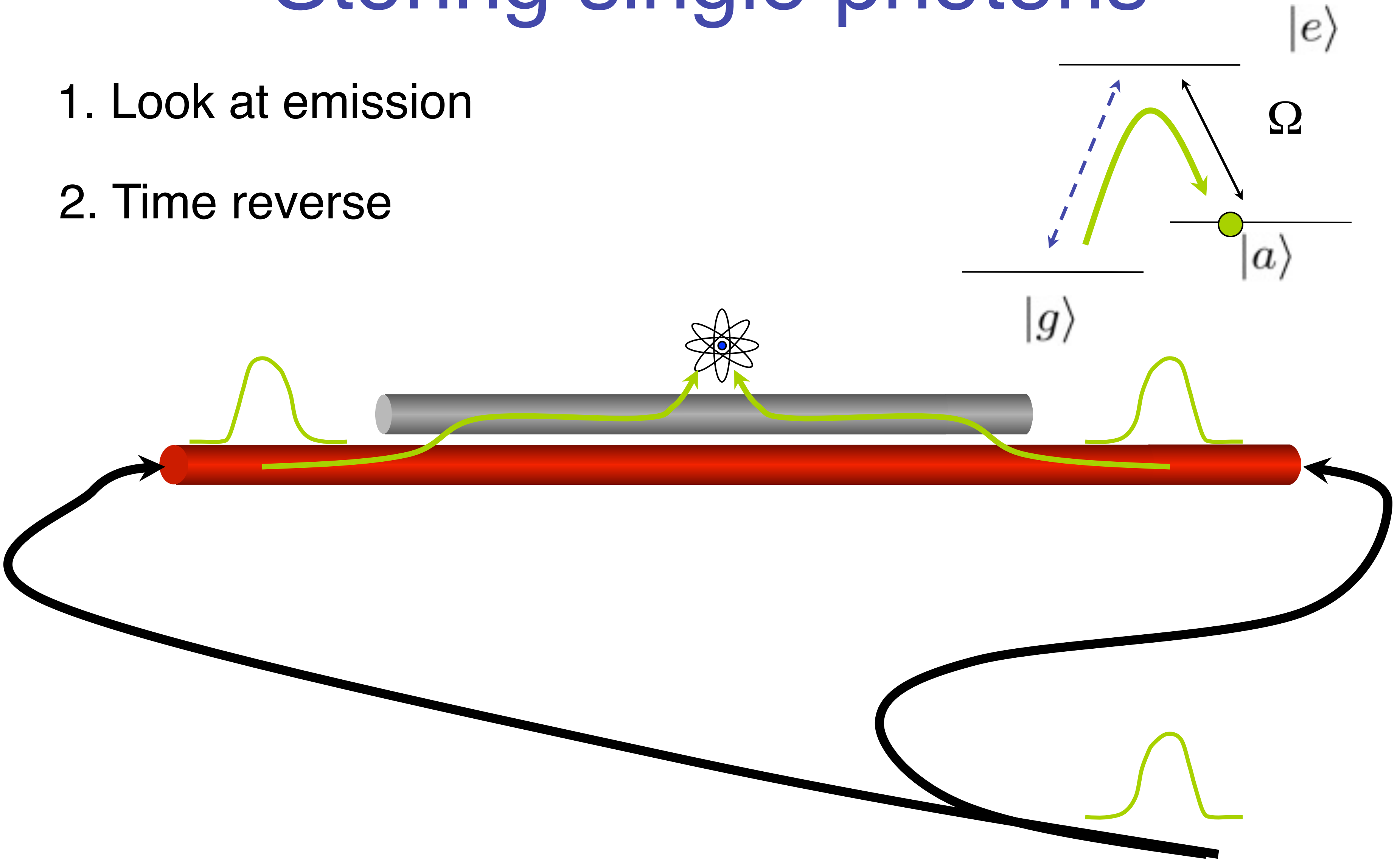
2. Time reverse



Storing single photons

1. Look at emission

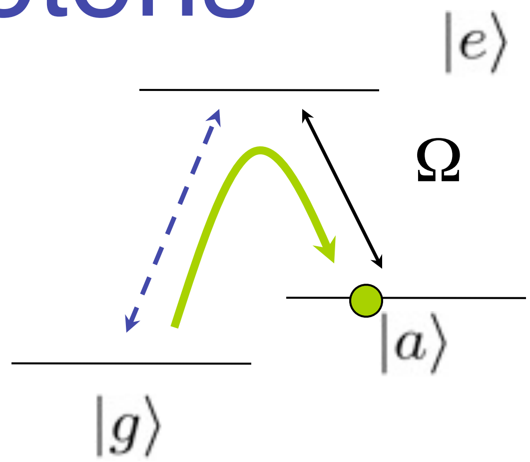
2. Time reverse

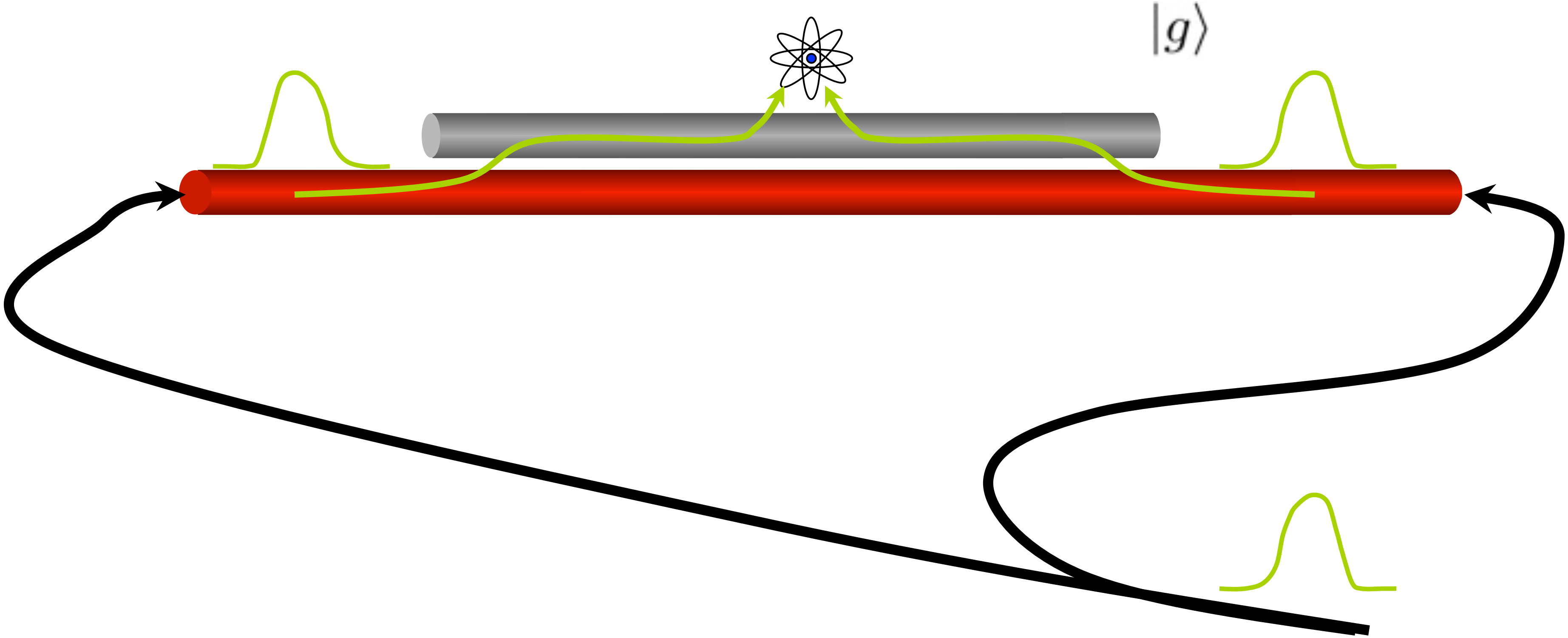


Storing single photons

1. Look at emission

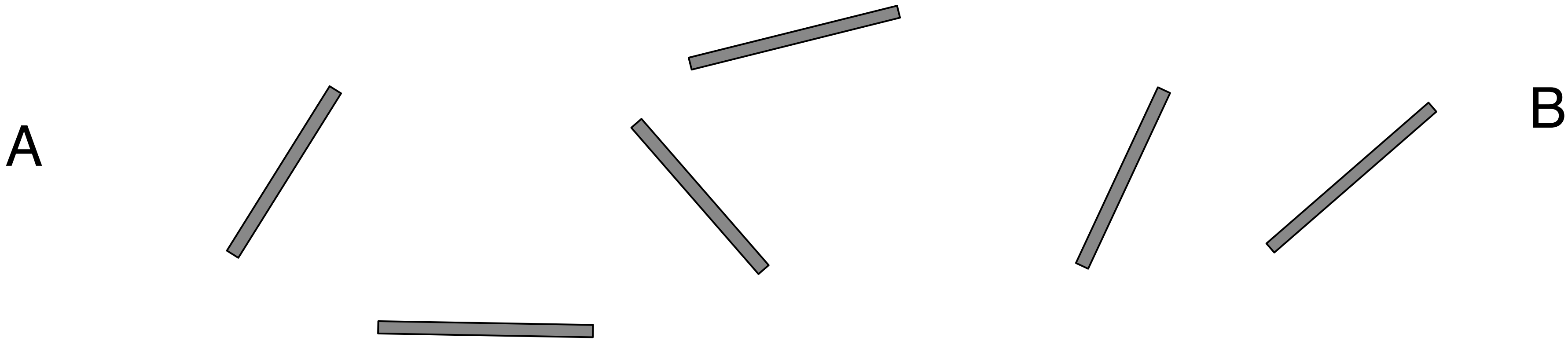
2. Time reverse $P_{\text{in}} = P_{\text{out}} \approx 90\ \%$





Time reversal

Equation of motion ~ beam splitter relation



Time reversal

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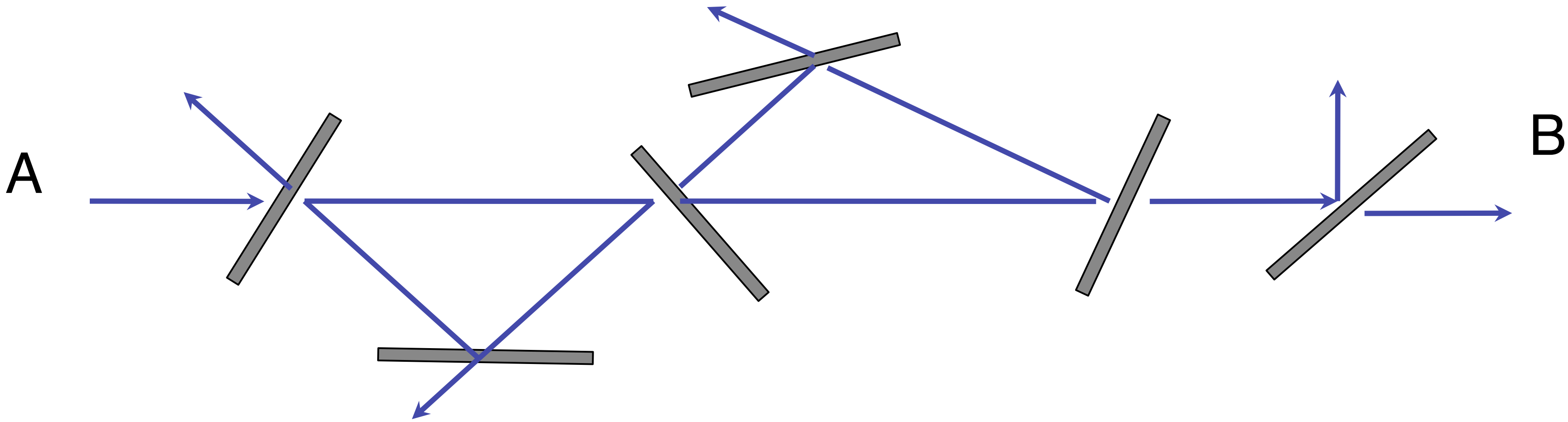

Time reversal

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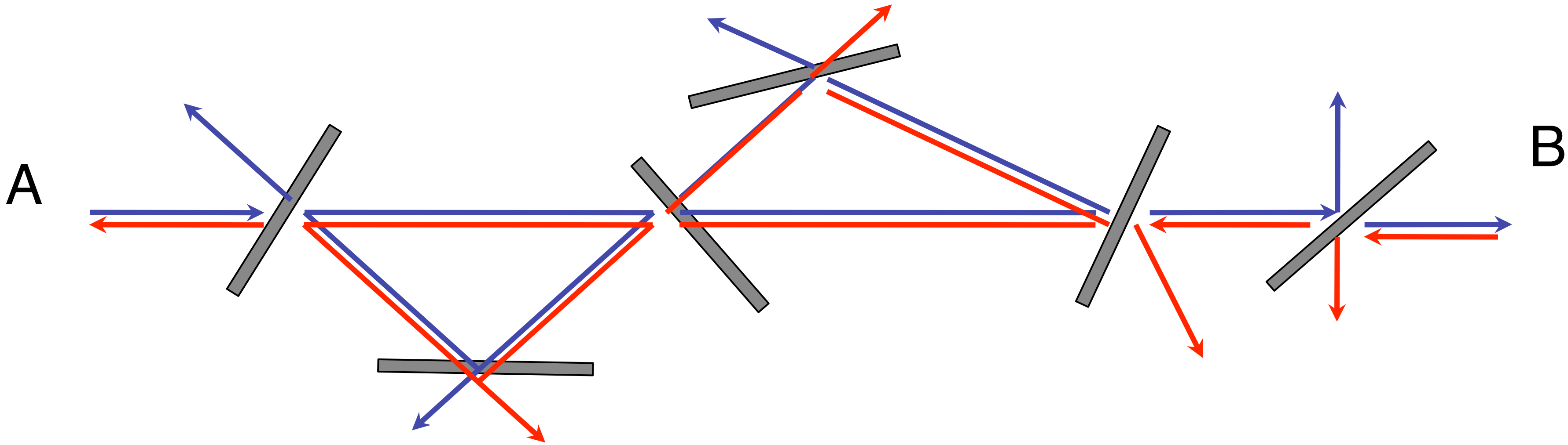

Time reversal

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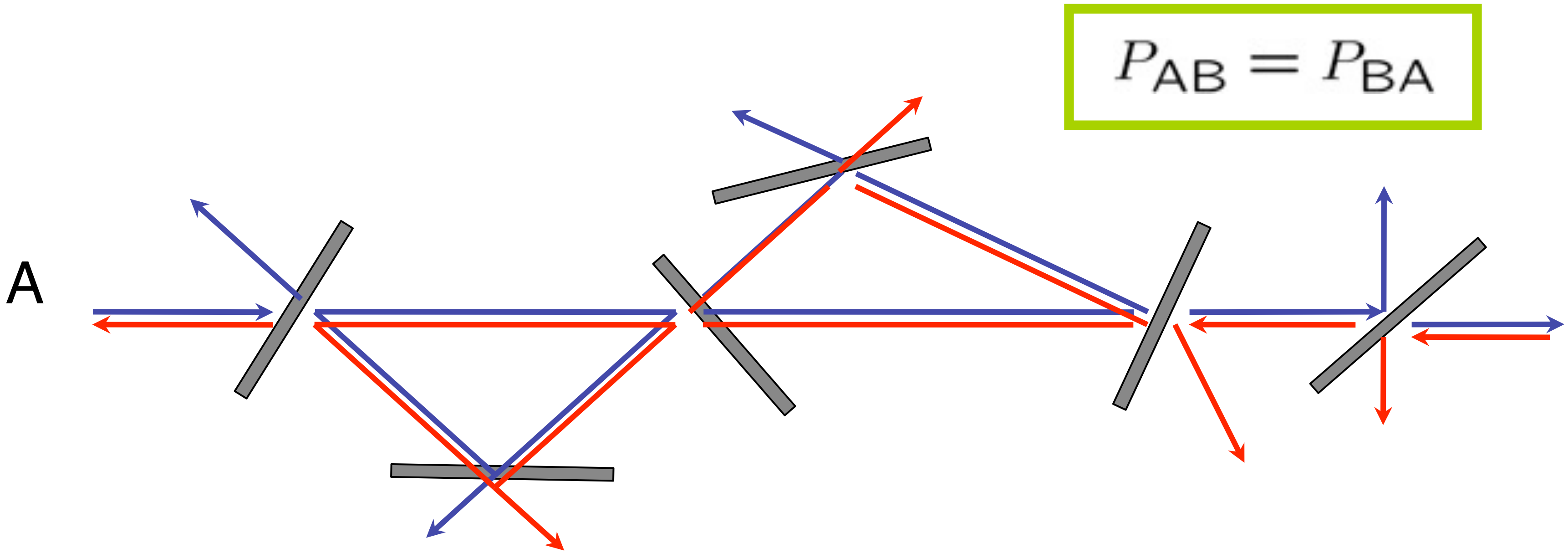
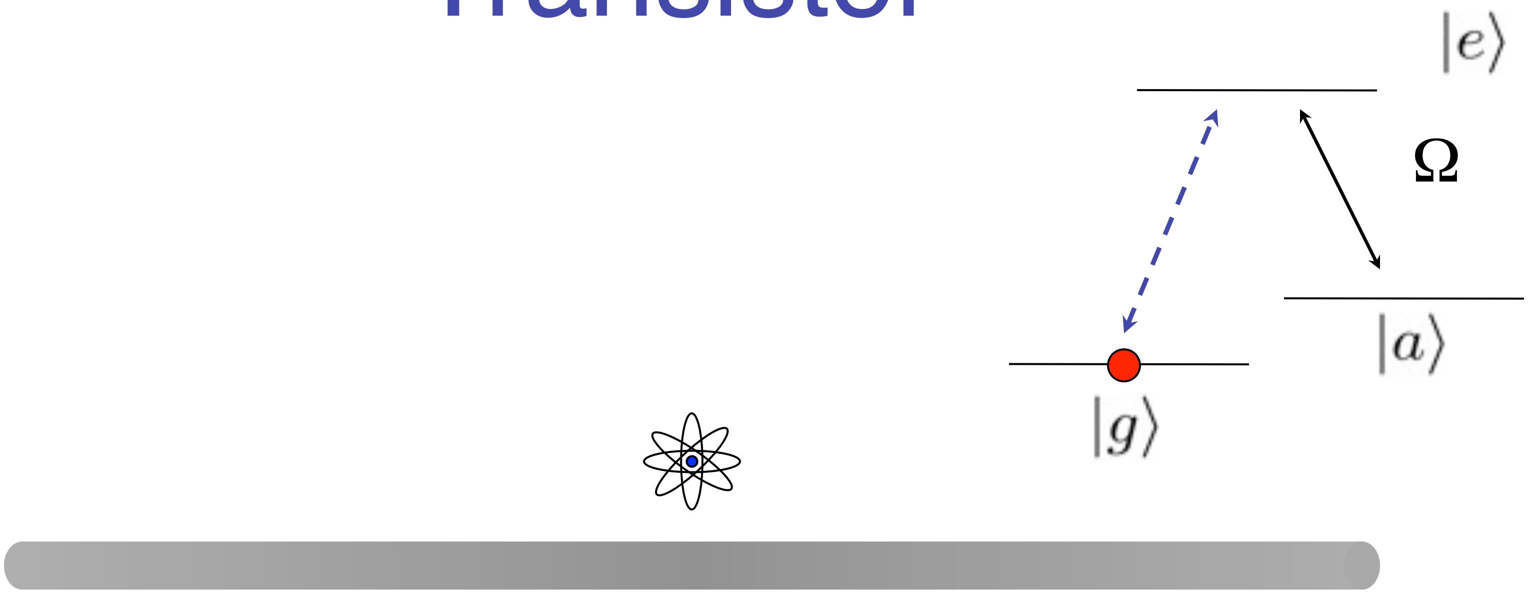


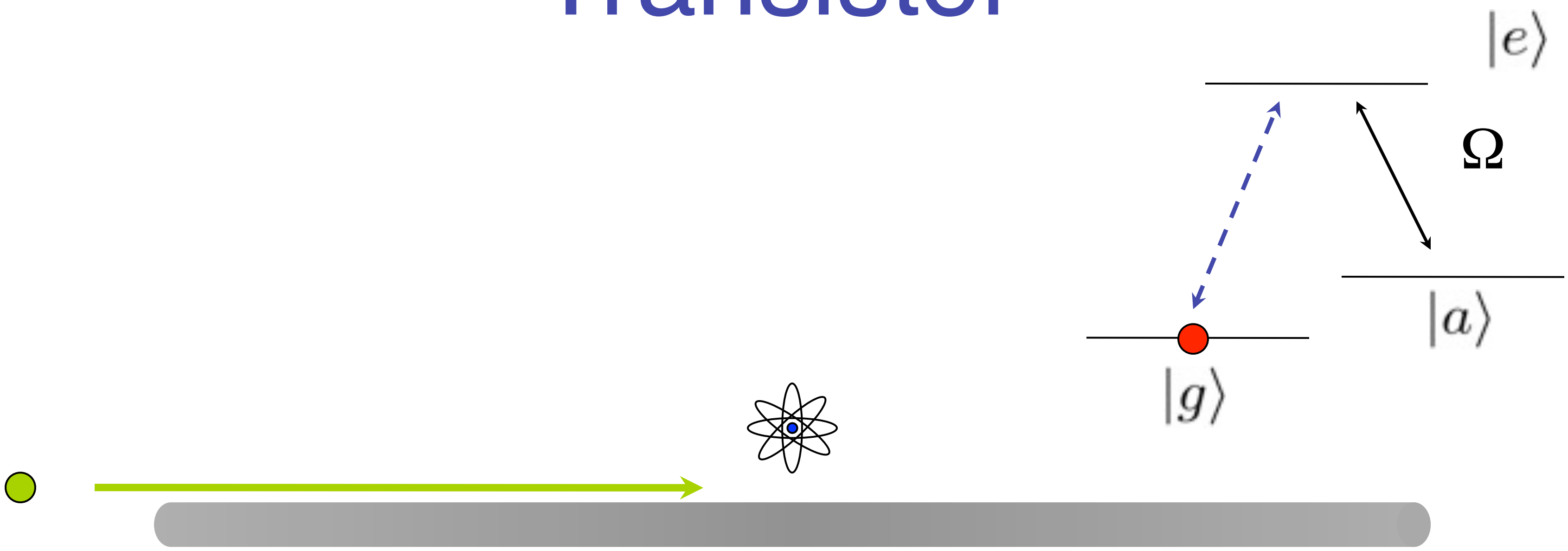
Transistor



1. Store "gate photon" using classical field (P≈90 %)
2. "Signal photons" transmitted if gate photon present

D. E. Chang, A. S. Sørensen, E. A. Demler, and M. D. Lukin, Nat. Phys. **3**, 807 (2007),

Transistor



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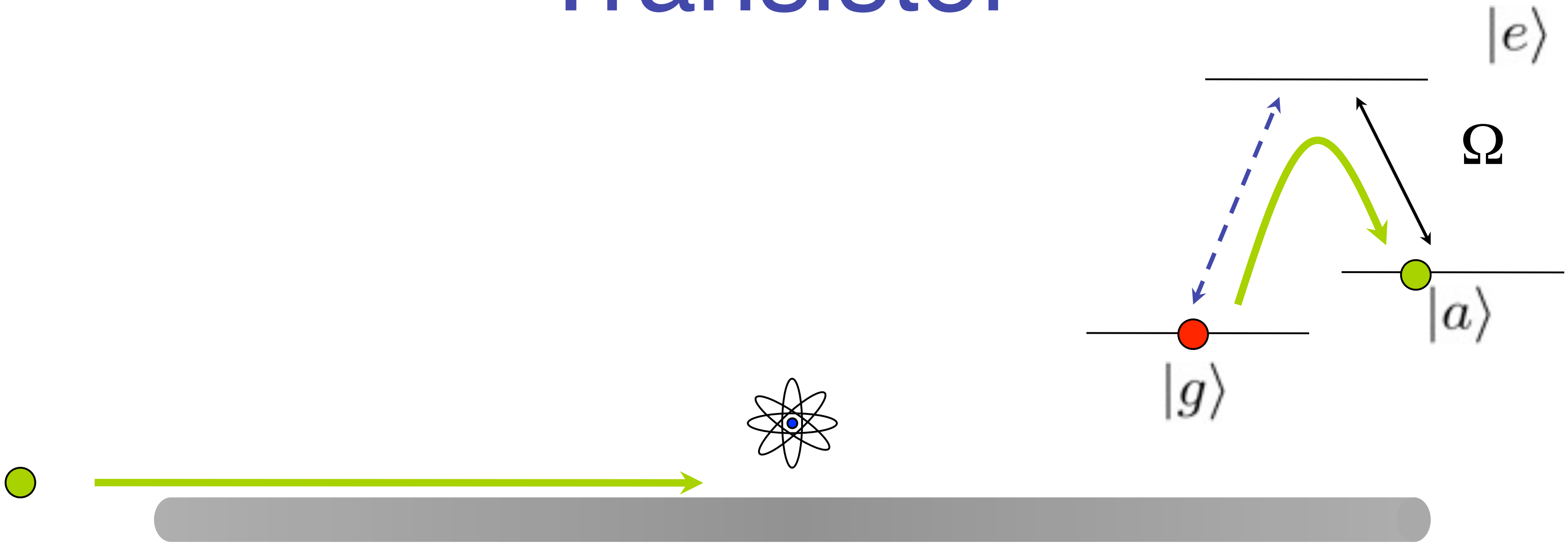

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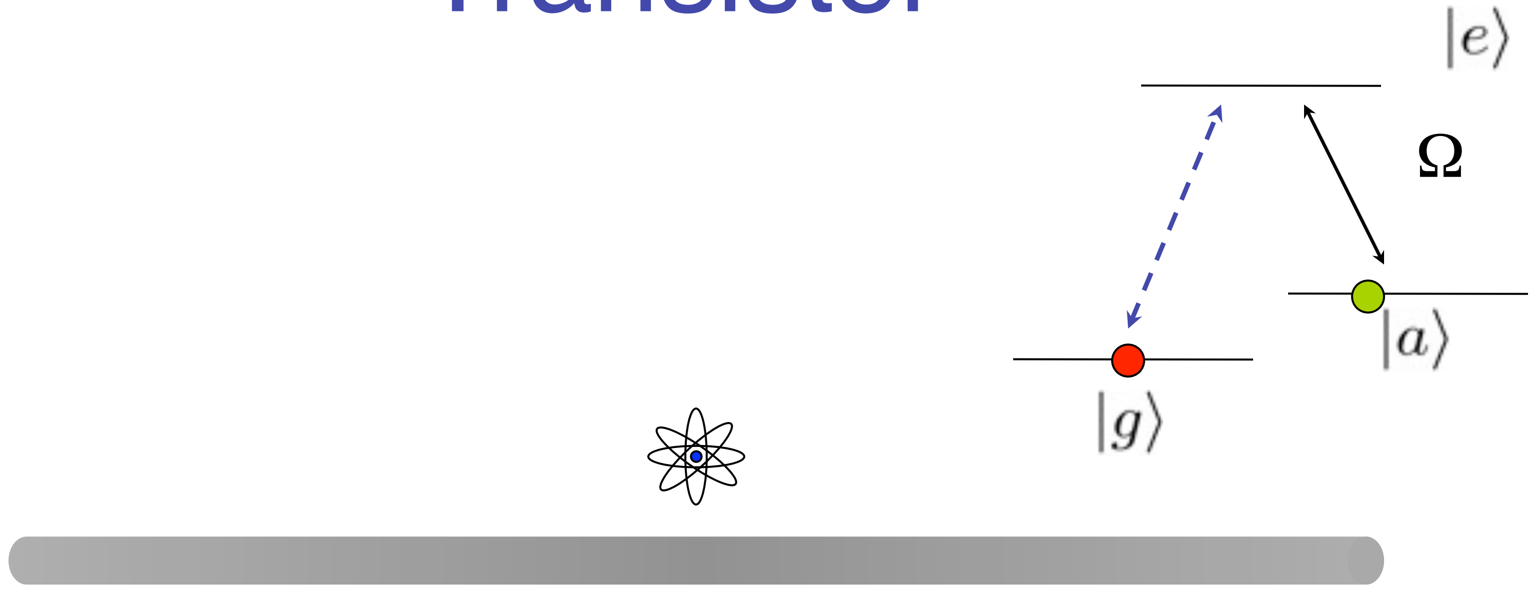

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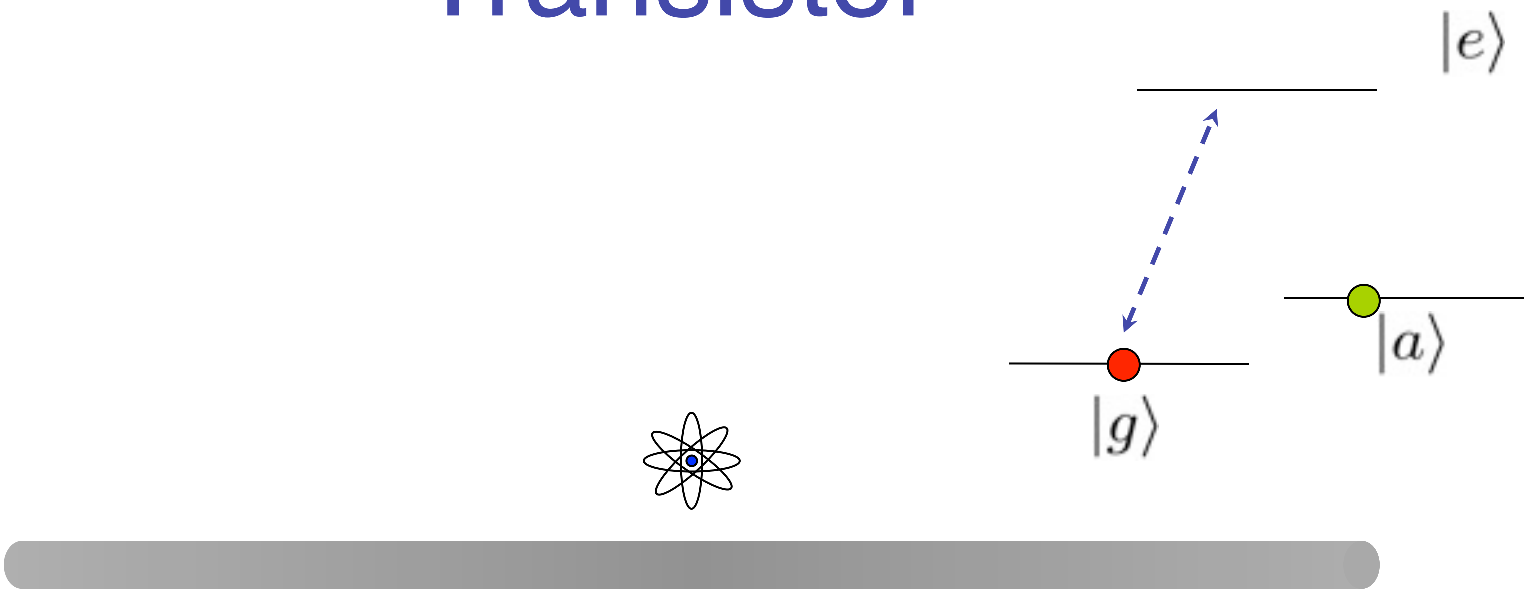

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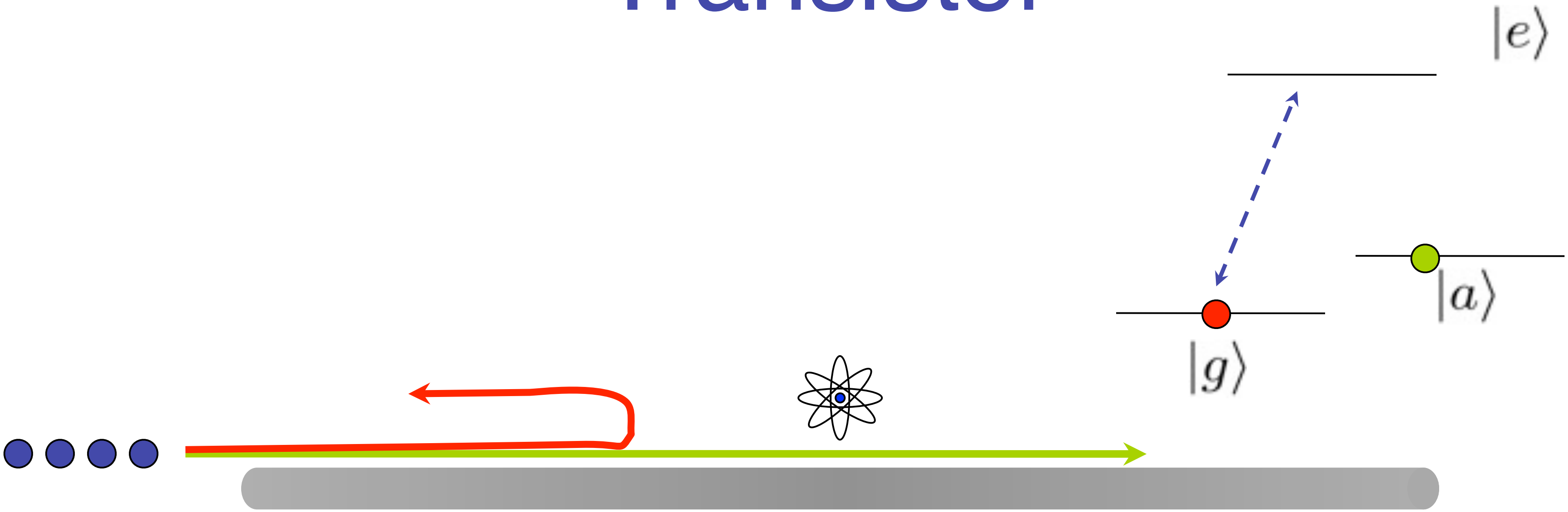

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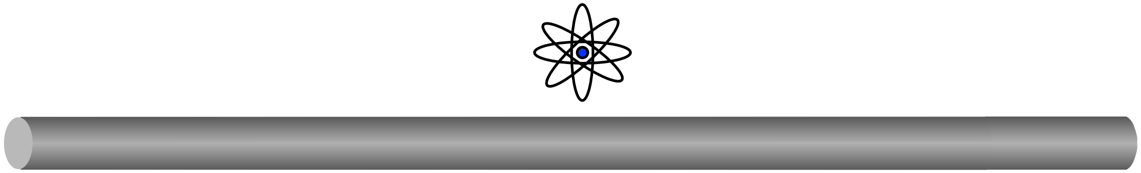
Conclusion (2)



- Strong coupling enables single photon non-linear optics

- Can be used to create single photon transistor

- Only important parameter is the probability to collect a single photon from an atom

- Can be done for any system with good collection efficiency
  Ex. atoms in cavities, atoms surrounded by big lenses, atoms in specially designed nanostructures