Cavity QED:
atom & photon in love

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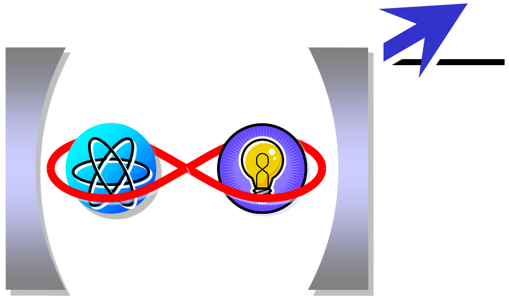
a century of cavity physics

two numbers:

- cavity finesse F
- atom number N

} F×N=const



cavity QED:

high F, low N

light & matter

ongly coupled

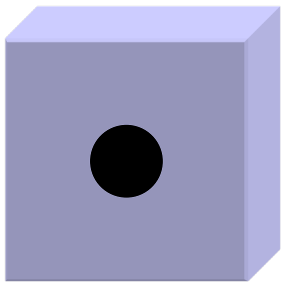
increase light-matter coupling

laser:

moderate F, N

light & matter

weakly coupled



blackbody:

low F, high N

light & matter

hardly coupled

quantum physics …

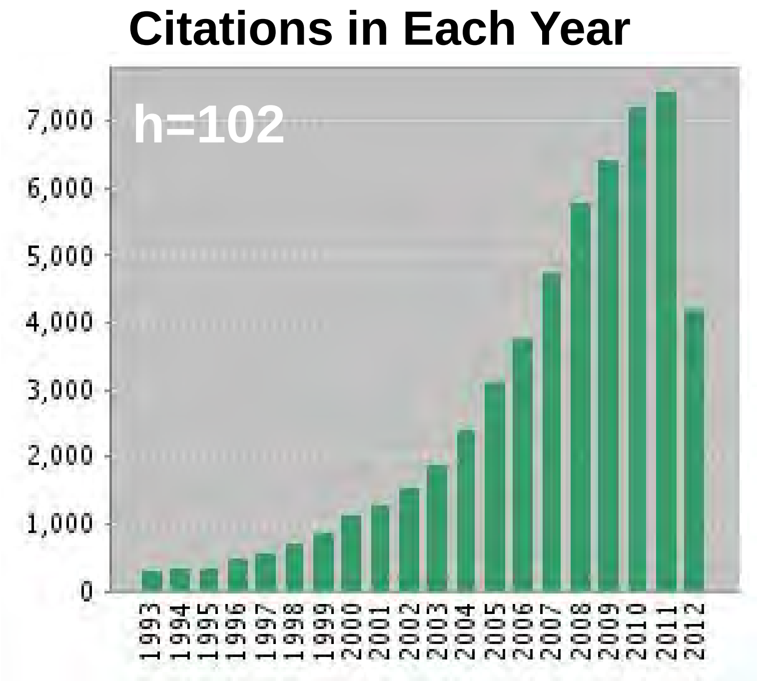
… classical physics …

… quantum technology

a growing research field

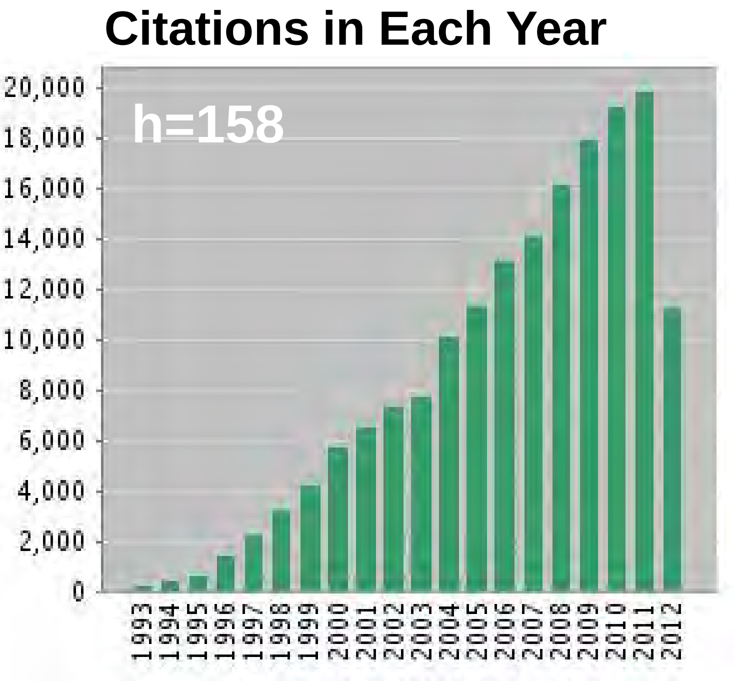
* topic ISI Web of Knowledge (August 2012)

cavity QED*



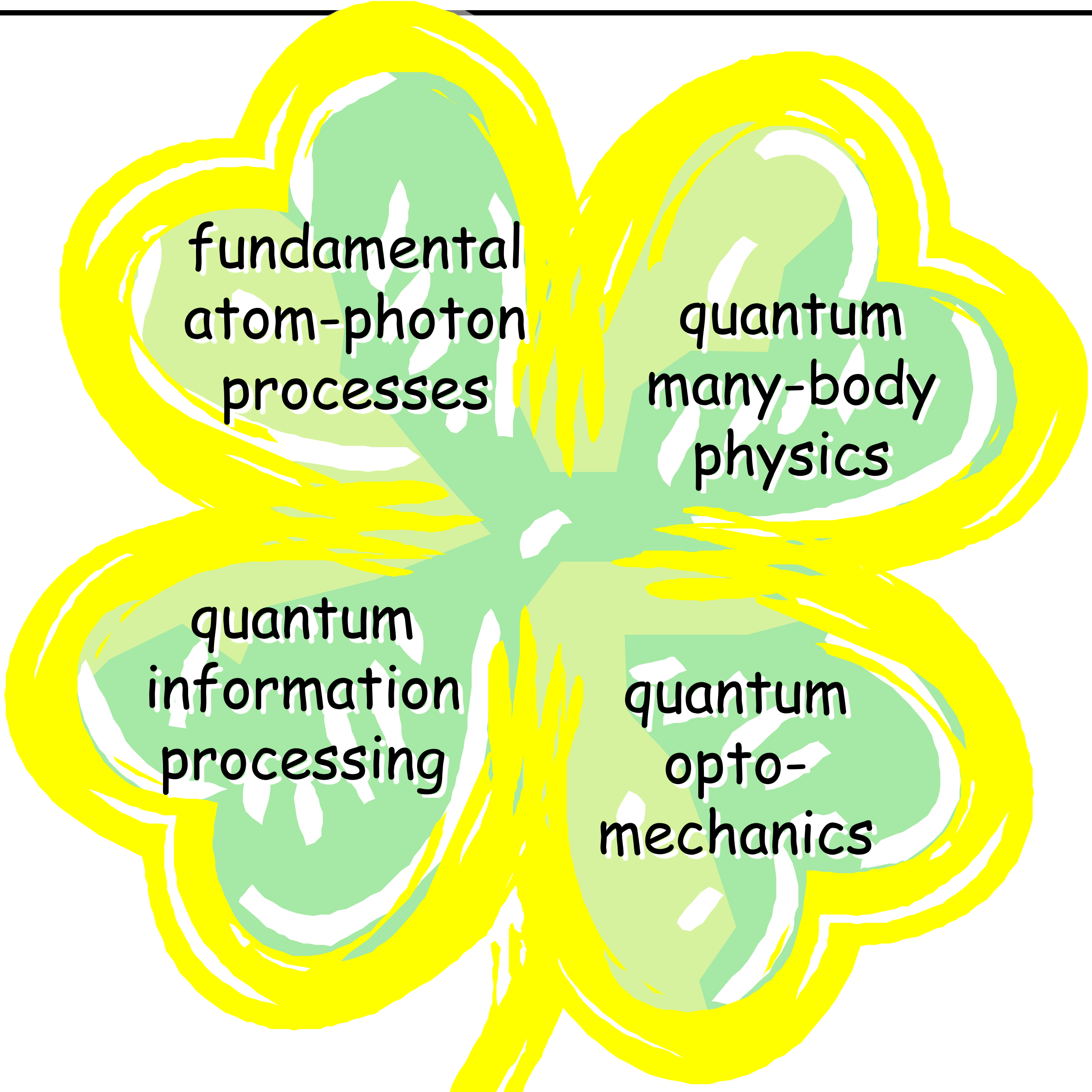
~16 years of
exponential growth

BEC*



~16 years of
linear growth

cavity QED as a tool box



fundamental atom-photon processes

quantum many-body physics

quantum information processing

quantum opto-mechanics

outline

today: fundamentals = it's all different

1) classical linear optics:
   - introduction & real time experiments
2) quantum nonlinear optics:
   - photon statistics & field fluctuations

tonight: applications = it's pretty useful

1) quantum information:
   - single photons & quantum networks

the challenge of lecturing

Wieman (Physics Nobel Prize 2001): teach students how wooden back of violin is what produces sound they hear.

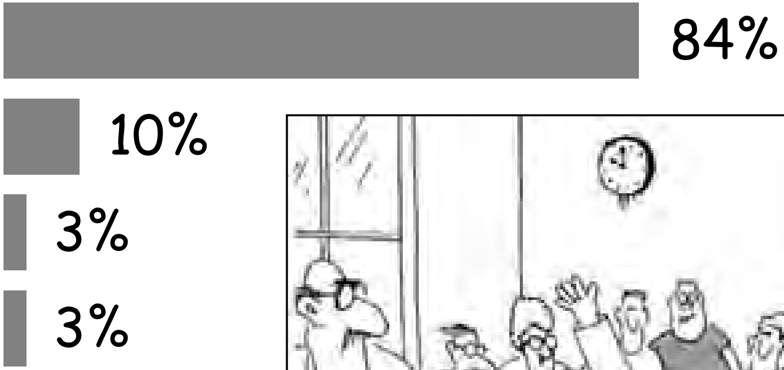
15 minutes later: ask student whether sound from violin is produced …

a. … mostly by strings,

b. … mostly by wood,

c. … both equally,

d. … none of above?



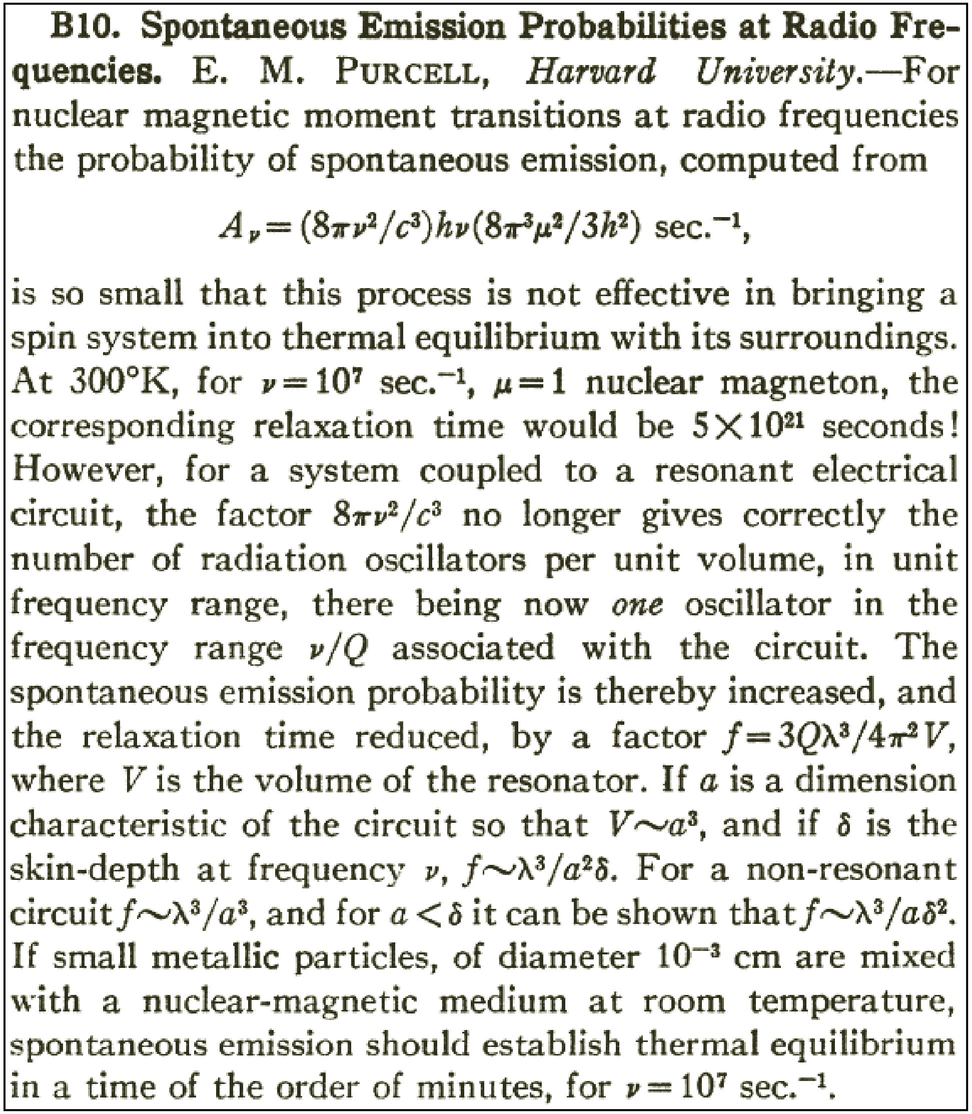
so please ask questions!

classical

picture

the role of the mode density

Purcell, Phys. Rev. **69**, 681 (1946)



B10. Spontaneous Emission Probabilities at Radio Frequencies. E. M. Purcell, *Harvard University.*—For nuclear magnetic moment transitions at radio frequencies the probability of spontaneous emission, computed from

$$A_\nu = (8\pi\nu^2/c^3)h\nu(8\pi^3\mu^2/3h^2)\ \text{sec.}^{-1},$$

is so small that this process is not effective in bringing a spin system into thermal equilibrium with its surroundings. At 300°K, for $\nu = 10^7$ sec.$^{-1}$, $\mu = 1$ nuclear magneton, the corresponding relaxation time would be 5×10^{21} seconds! However, for a system coupled to a resonant electrical circuit, the factor $8\pi\nu^2/c^3$ no longer gives correctly the number of radiation oscillators per unit volume, in unit frequency range, there being now *one* oscillator in the frequency range ν/Q associated with the circuit. The spontaneous emission probability is thereby increased, and the relaxation time reduced, by a factor $f = 3Q\lambda^3/4\pi^2V$, where V is the volume of the resonator. If a is a dimension characteristic of the circuit so that $V \sim a^3$, and if δ is the skin-depth at frequency ν, $f \sim \lambda^3/a^2\delta$. For a non-resonant circuit $f \sim \lambda^3/a^3$, and for $a < \delta$ it can be shown that $f \sim \lambda^3/a\delta^2$. If small metallic particles, of diameter 10^{-3} cm are mixed with a nuclear-magnetic medium at room temperature, spontaneous emission should establish thermal equilibrium in a time of the order of minutes, for $\nu = 10^7$ sec.$^{-1}$.

the spontaneous emission rate is proportional to the mode density of the radiation field:

in free space:

$$\rho(\omega) = (2/\pi)\,\omega^2/c^3$$

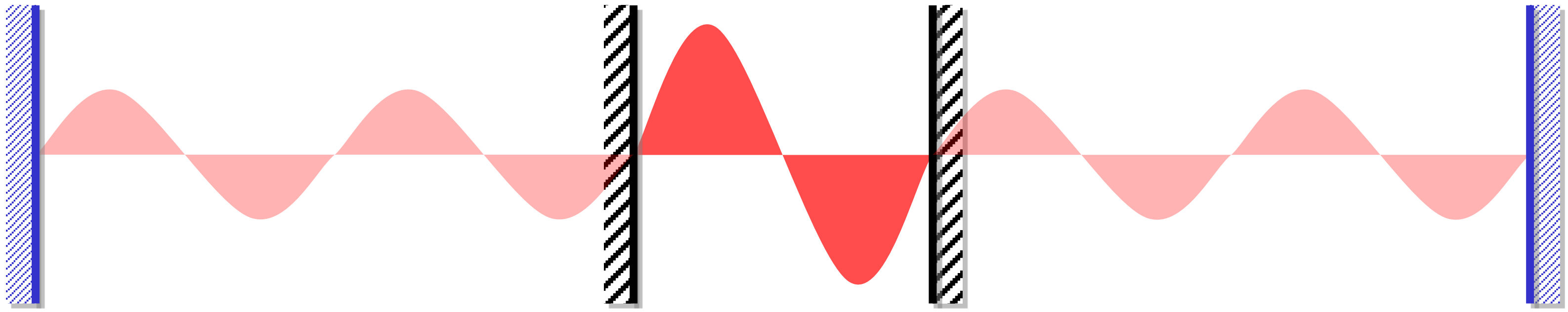
in a cavity with quality factor Q and volume V:

$$\rho(\omega) = 1/\Delta\omega V = Q/\omega V$$

Purcell enhancement factor:

$$f = (3/4\pi^2)\,Q\,(\lambda^3/V)$$

what's wrong with this mode density?

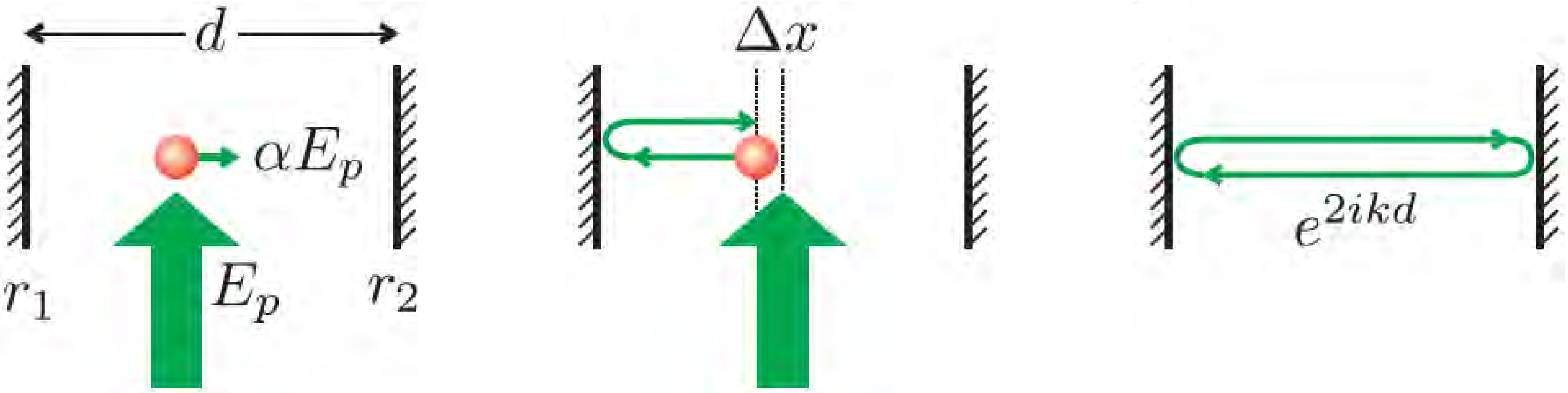


local mode density is changed

but:
environment is part of the system

global mode density is not changed

light interference

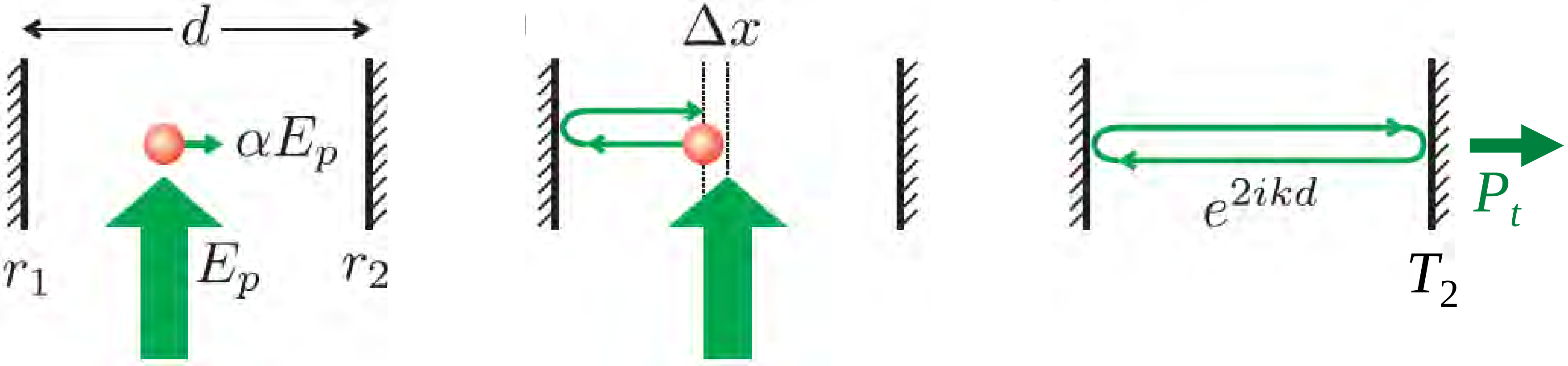


$$E_c = \alpha E_p + r_1 e^{ik(d+2\Delta x)} \alpha E_p + r_1 r_2 e^{2ikd} E_c$$

$$\Rightarrow \quad E_c = \alpha E_p \frac{1 + r_1 e^{ikd} e^{2ik\Delta x}}{1 - r_1 r_2 e^{2ikd}}$$

$$\Rightarrow \quad P_c \underset{\substack{\text{resonance} \\ + \\ \text{position} \\ \text{averaging}}}{=} \alpha^2 P_p \frac{1 + r_1^2}{\left(1 - r_1 r_2\right)^2} \underset{\substack{\text{high} \\ \text{reflectivity} \\ r_1 \approx r_2 \approx 1}}{\approx} 2\alpha^2 P_p \left(F/\pi\right)^2$$

light interference



$$E_c = \alpha E_p + r_1 e^{ik(d+2\Delta x)} \alpha E_p + r_1 r_2 e^{2ikd} E_c$$

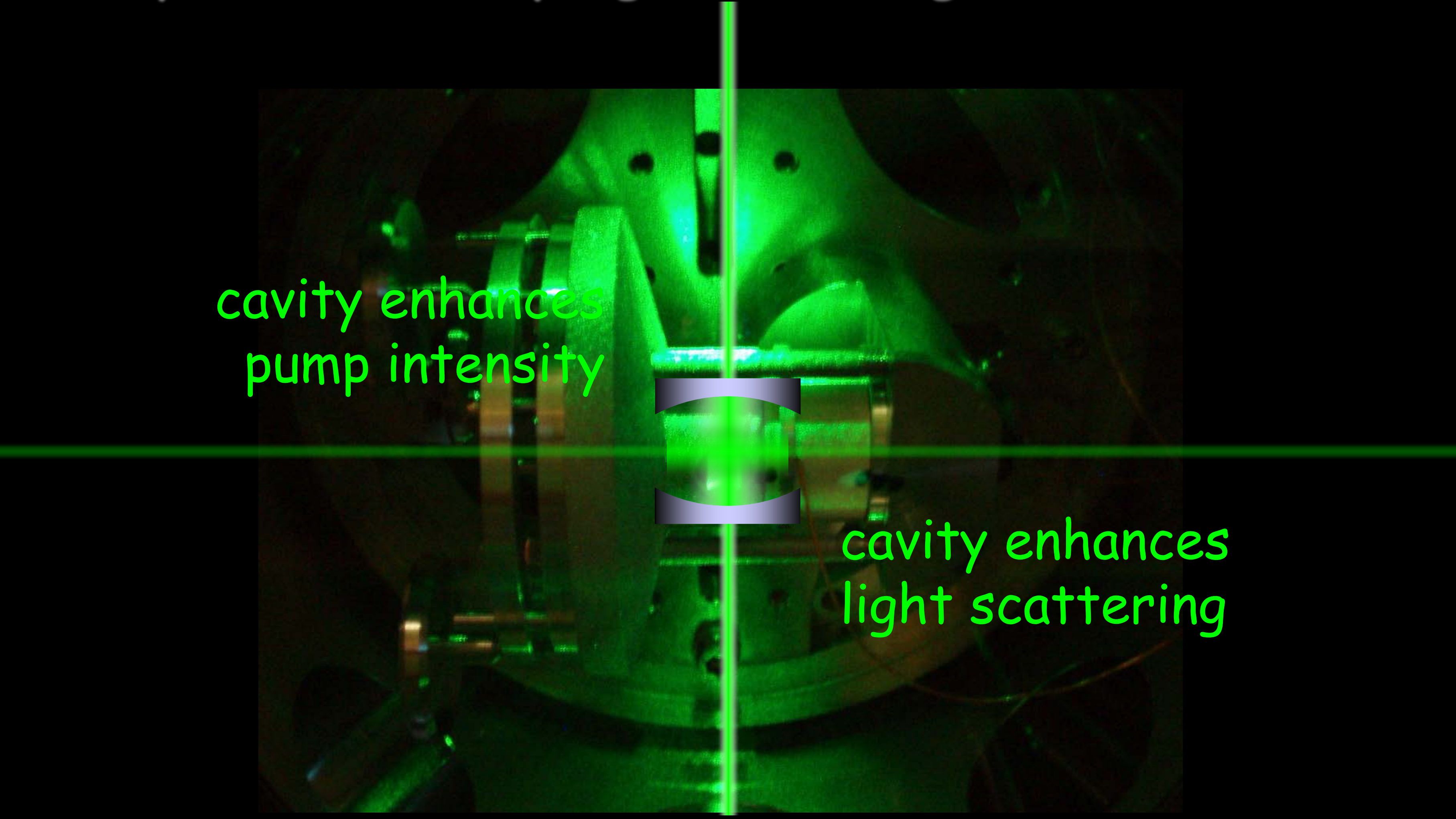
$$\Rightarrow E_c = \alpha E_p \frac{1 + r_1 e^{ikd} e^{2ik\Delta x}}{1 - r_1 r_2 e^{2ikd}}$$

$$\Rightarrow P_t = T_2 \times P_c \approx 2\alpha^2 P_p \times F/\pi$$

symmetric cavity

number of reflections

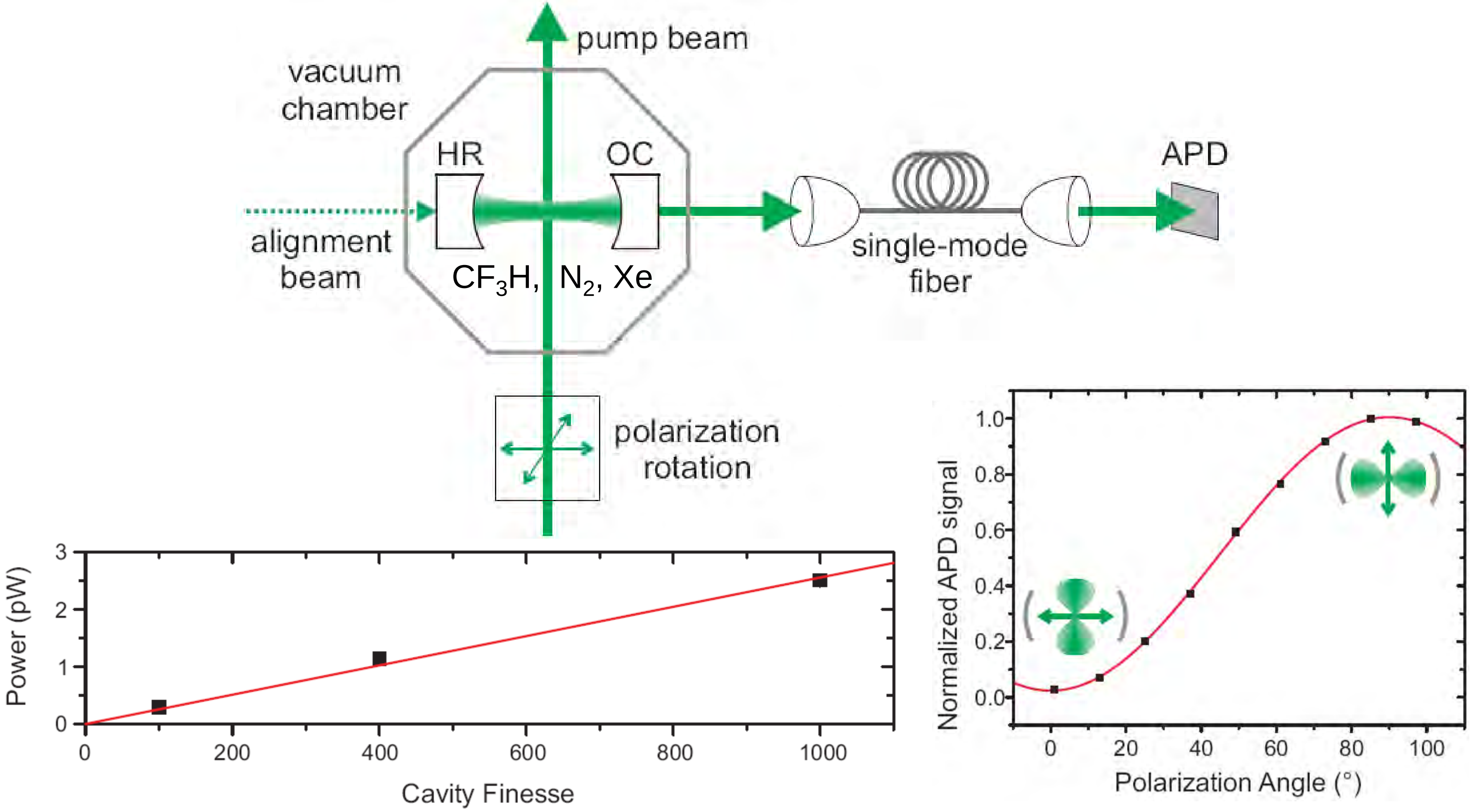
cavity enhanced Rayleigh scattering



- rotated geometry, same rate
- benefit: small laser intensity

cavity enhanced Rayleigh scattering

Motsch et al., NJP **12**, 063022 (2010)

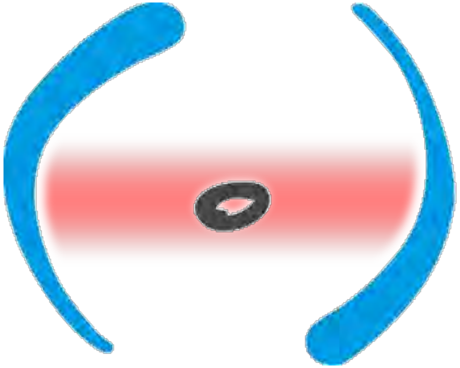


38× more light into mode defined by the cavity quantum

description

cavity QED = QED for pedestrians

- no renormalization & no divergences
- only one mode of the radiation field



perturbative regime:

(Purcell 1946, …)

- dissipation » coupling

non-perturbative regime:

(Jaynes & Cummings 1963, …)

- dissipation « coupling

one atom & one cavity mode

coupling constant: $g = \frac{\vec{d} \cdot \vec{E}}{\hbar}$

dipole moment: $\vec{d} = e \langle \psi_e | \vec{r} | \psi_g \rangle$

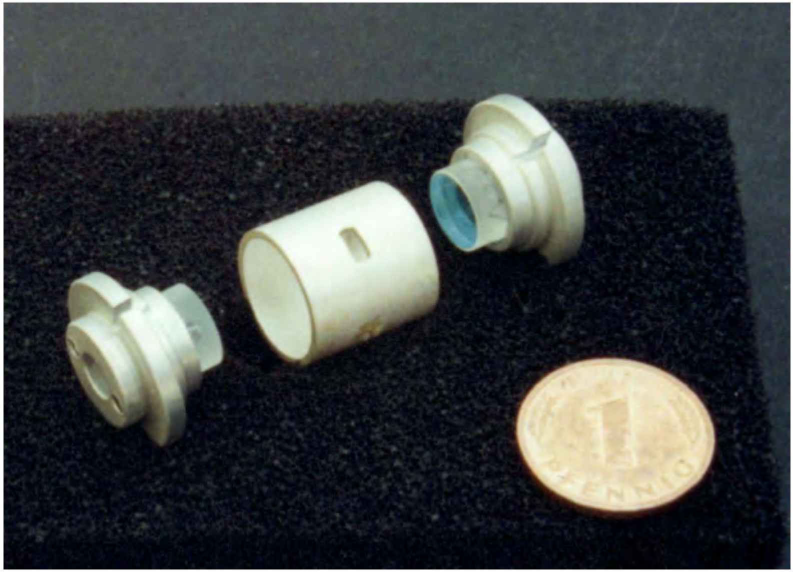
$\approx 10^0 - 10^1 \; ea_0$ resonance line

$\approx 10^3 - 10^4 \; ea_0$ Rydberg atom

$\approx 10^4 - 10^5 \; ea_0$ mesoscopic system

field per photon: $\left| \vec{E} \right| = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$

how large is the electric field of a photon ?



10 μm mirror separation:
$E \approx 100$ V/cm

it can be measured !
it has dramatic effects !

Jaynes-Cummings molecule

Jaynes and Cummings, Proc. IEEE **51**, 89 (1963)

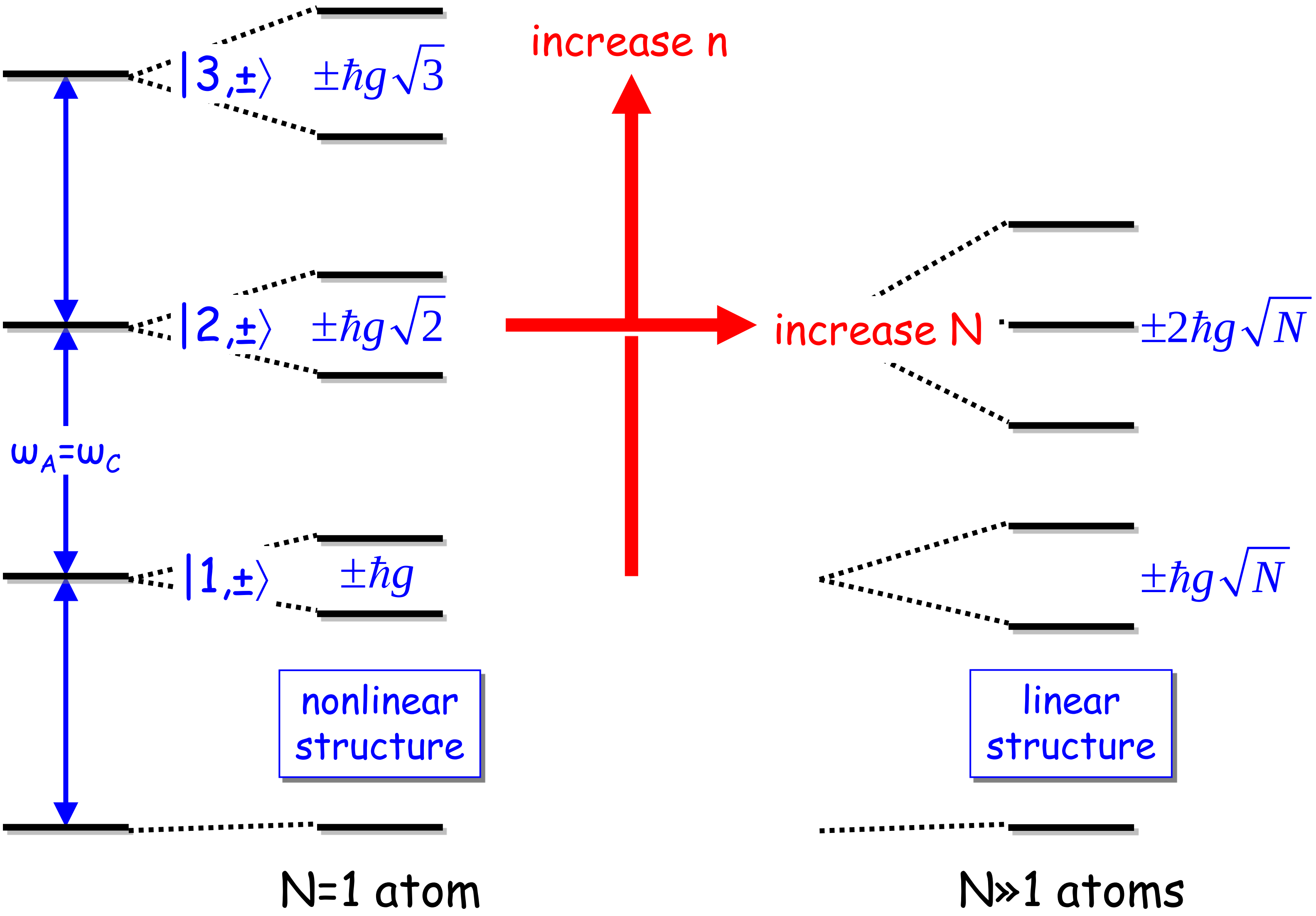
microwave cavity QED ($\omega=\omega_A=\omega_C$):
without dissipation: $\gamma=\kappa=0$
without driving: $\eta=0$

Hamiltonian: $$\hat{H}_{JC}=\hbar\omega_A\hat{\sigma}_+\hat{\sigma}_-+\hbar\omega_C\hat{a}^+\hat{a}+\hbar g\left(\hat{a}^+\hat{\sigma}_-+\hat{\sigma}_+\hat{a}\right)$$

eigenenergies: $$E_{n,\pm}=\hbar\omega n\pm\hbar g\sqrt{n}\qquad n=0,1,2,\ldots$$

eigenstates: $$\left|n,\pm\right\rangle=\frac{\left|g,n\right\rangle\pm\left|e,n-1\right\rangle}{\sqrt{2}}$$ dressed states

energy-level structure



driven atom-cavity system

Alsing et al., Phys. Rev. A **45**, 5135 (1992)

optical cavity QED ($\omega=\omega_L=\omega_A=\omega_C$):
without dissipation: $\gamma=\kappa=0$
with driving: $\eta\neq0$

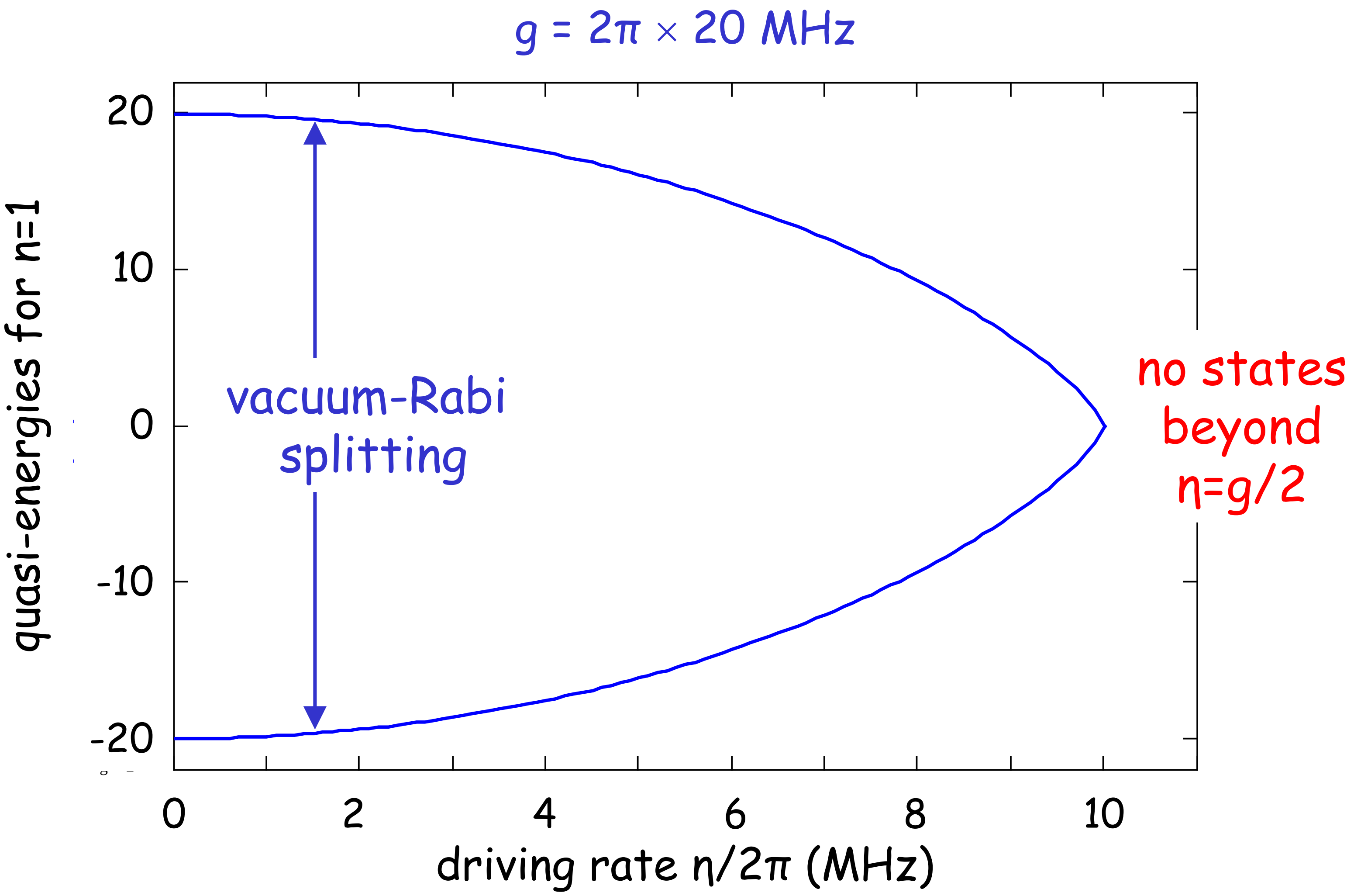
Hamiltonian: $\hat{H}=\hat{H}_{JC}+\hbar\eta\left(\hat{a}^{+}+\hat{a}\right)$

quasienergies: $E_{n,\pm}=\hbar\omega n\pm\hbar g\sqrt{n}\sqrt[\frac{4}{3}]{1-\left(\frac{2\eta}{g}\right)^2}$

Stark-shifted energy levels

Stark-shifted energy levels

Alsing et al., Phys. Rev. A **45**, 5135 (1992)



weakly driven dissipative atom-cavity system

optical cavity QED ($\omega_L = \omega_A = \omega_C$):
polarization decay $\gamma \neq 0$, field decay $\kappa \neq 0$, driving $\eta \neq 0$

master equation:

$$\dot{\rho} = -\frac{i}{\hbar}\left[\hat{H}_{JC} + \hat{H}_P, \rho\right] - \kappa\left(\hat{a}^+\hat{a}\rho + \rho\hat{a}^+\hat{a} - 2\hat{a}\rho\hat{a}^+\right)$$
$$-\gamma\left(\hat{\sigma}_+\hat{\sigma}_-\rho + \rho\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\rho\hat{\sigma}_+\right)$$

state vector: $\left|\psi\right\rangle = \left|g,0\right\rangle + c_g\left|g,1\right\rangle + c_e\left|e,0\right\rangle \qquad c_{g,e} \ll 1$

equations of motion:

$$\dot{c}_g = -\kappa c_g + g c_e + \eta$$

$$\dot{c}_e = -\gamma c_e - g c_g$$

steady-state solution:

$$c_g = \frac{\eta / \kappa}{1 + 2C_1} \equiv \alpha$$

$$c_e = -\frac{g}{\gamma} c_g \equiv \beta$$

cooperativity $C_1 = \dfrac{g^2}{2\gamma\kappa}$

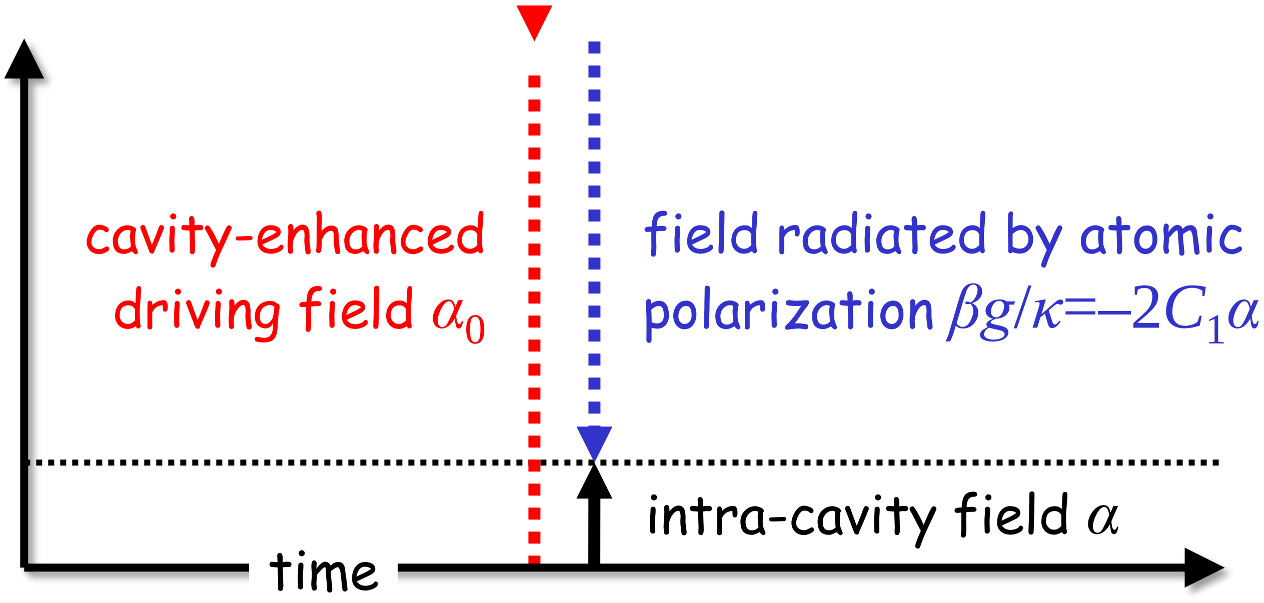
physical interpretation

amplitude of intracavity field:

$$\alpha = \frac{\eta / \kappa}{1 + 2C_1} = \alpha_0 - 2C_1\alpha \qquad \alpha_0 = \frac{\eta}{\kappa}$$

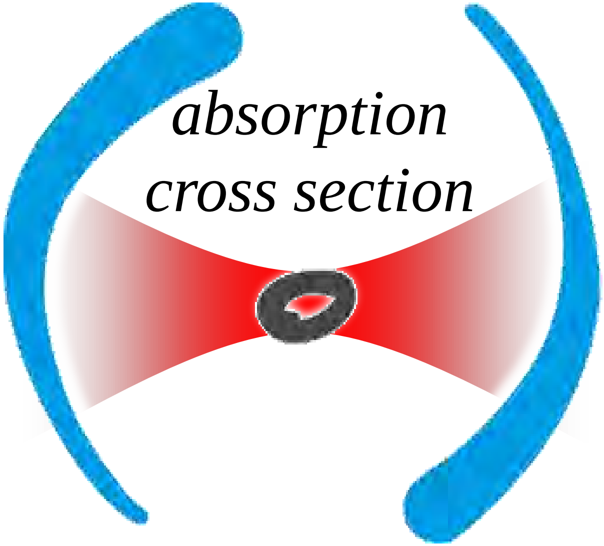
excited-state amplitude:

$$\beta = -\alpha \frac{g}{\gamma}$$



no absorption, only interference

the famous cooperativity parameter



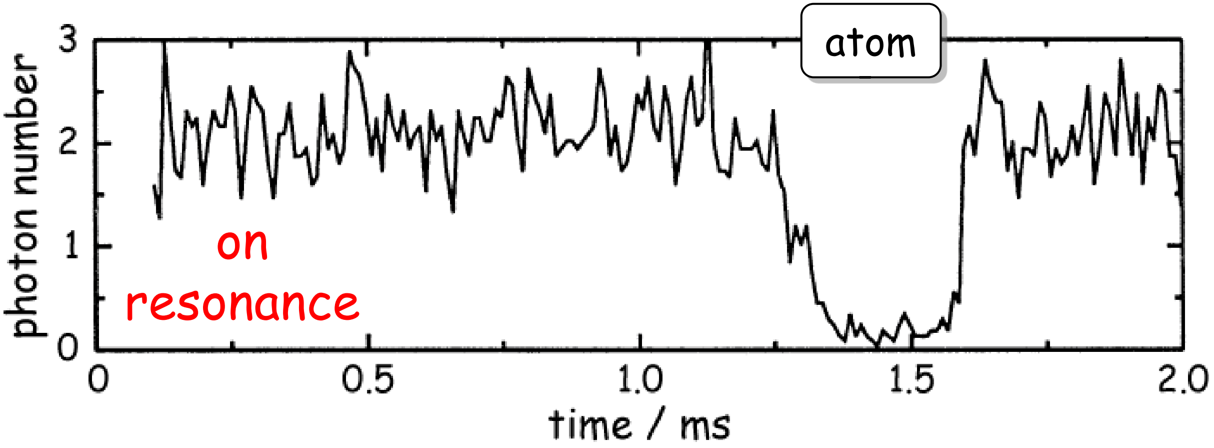
mode waist w_0

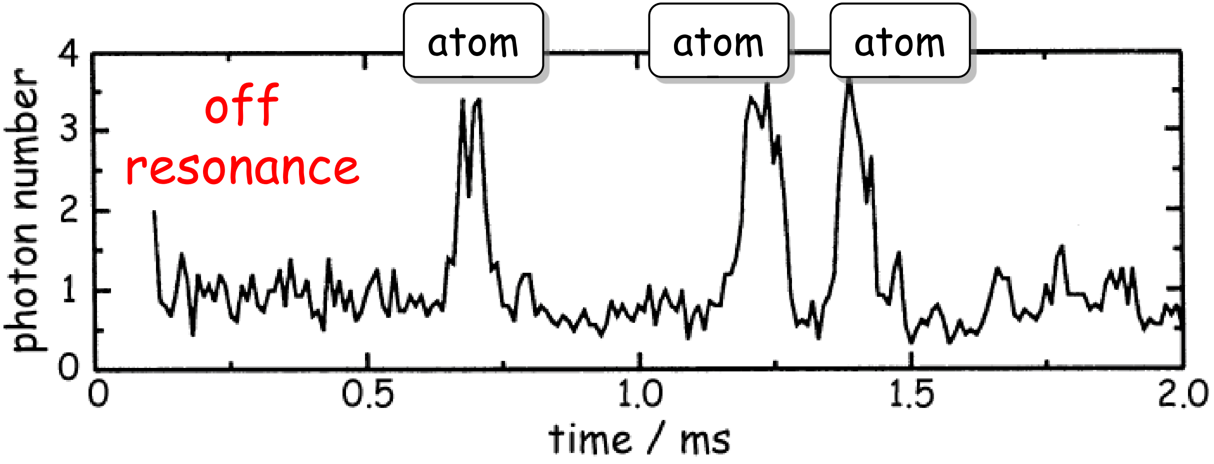
finesse F

$$\boxed{2C_N = \underbrace{N}_{\text{number of atoms}} \times \underbrace{\frac{3\lambda^2}{2\pi}}_{\text{absorption cross section}} \times \underbrace{\frac{1}{\frac{1}{4}\pi w_0^2}}_{\text{mode area}} \times \underbrace{\frac{F}{\pi}}_{\text{number of reflections}} \gg 1}$$

single-atom watching

Münstermann et al., Opt. Commun. **159**, 63 (1999)
see also Chapman, Esslinger, Kimble, Mabuchi, Meschede, Orozco, Stamper Kurn, …





single-atom watching

Pinkse et al., Nature **404**, 365 (2000)

single-atom watching

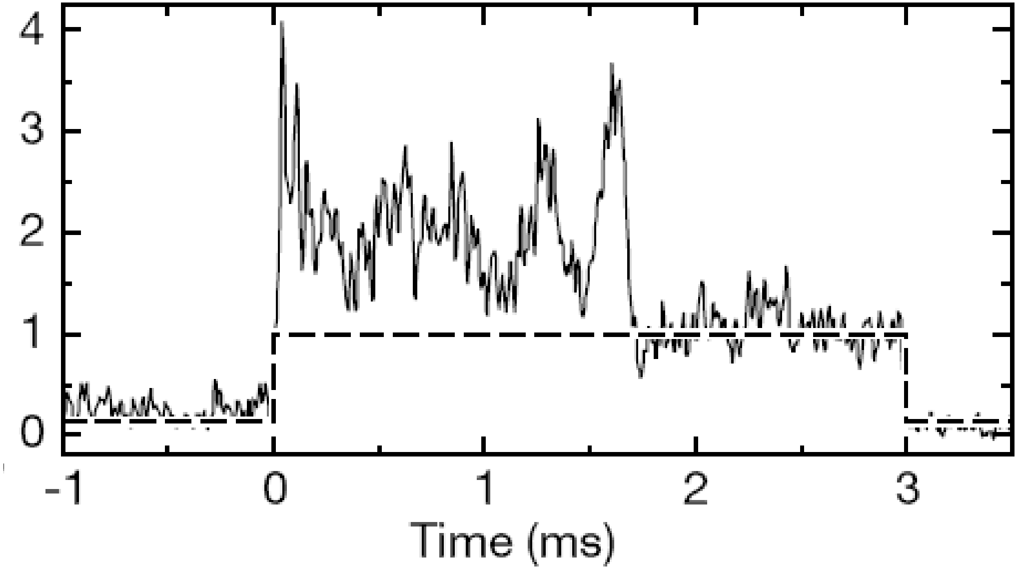
Trapping an atom with single photons

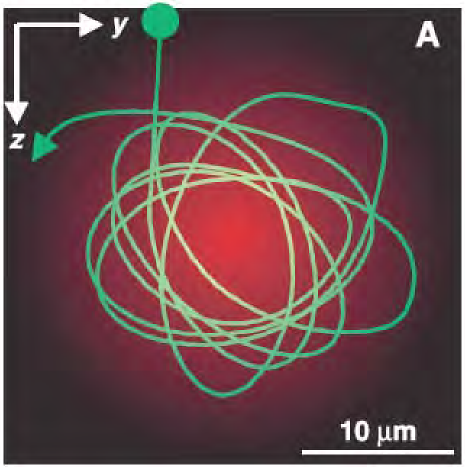
P. W. H. Pinkse, T. Fischer, P. Maunz & G. Rempe

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, 85748 Garching, Germany

The Atom-Cavity Microscope: Single Atoms Bound in Orbit by Single Photons

C. J. Hood,[1] T. W. Lynn,[1] A. C. Doherty,[2] A. S. Parkins,[2] H. J. Kimble[1]*



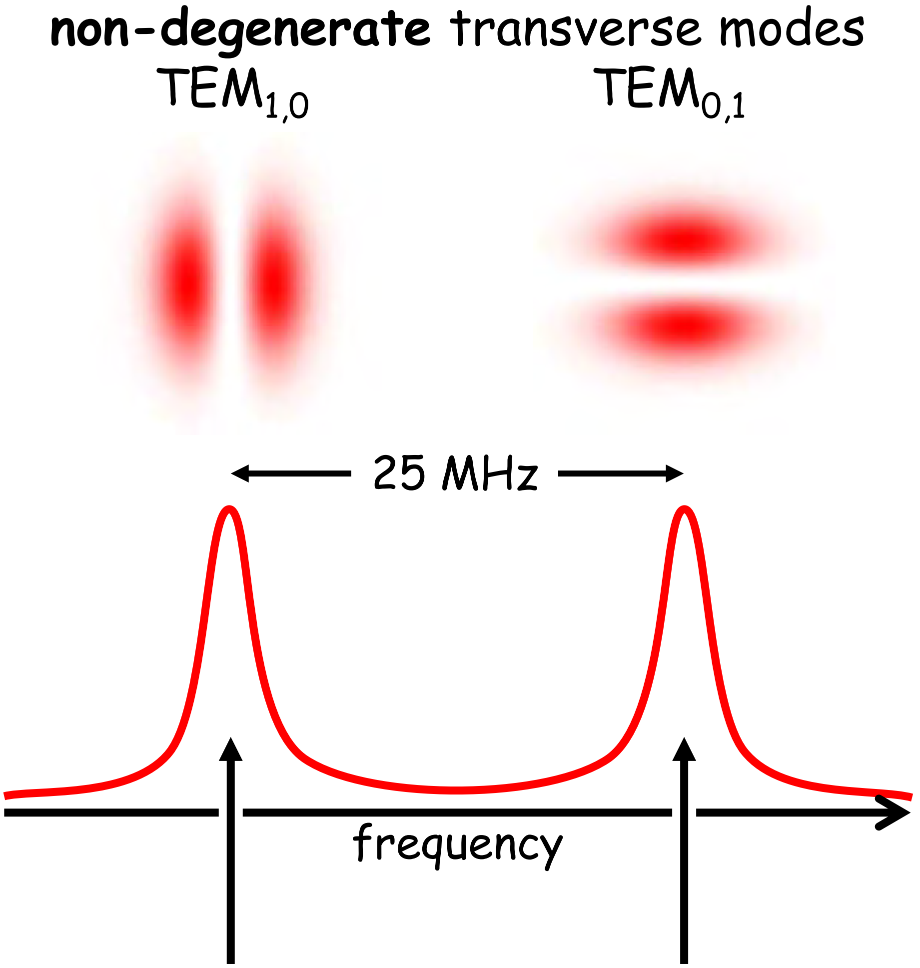


reconstructed trajectory, not direct imaging

spatial imaging

Puppe et al., Physica Scripta **T112** (2004) 7

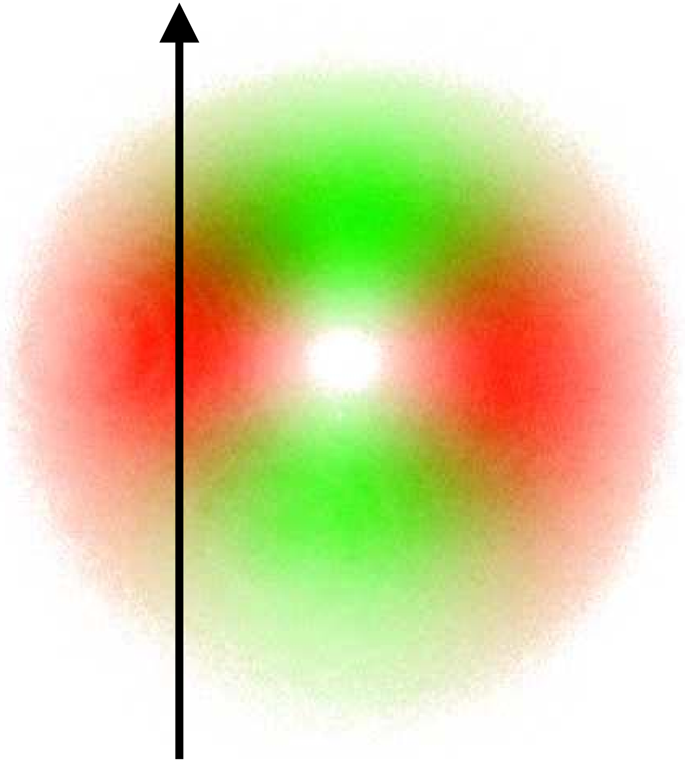
non-degenerate transverse modes

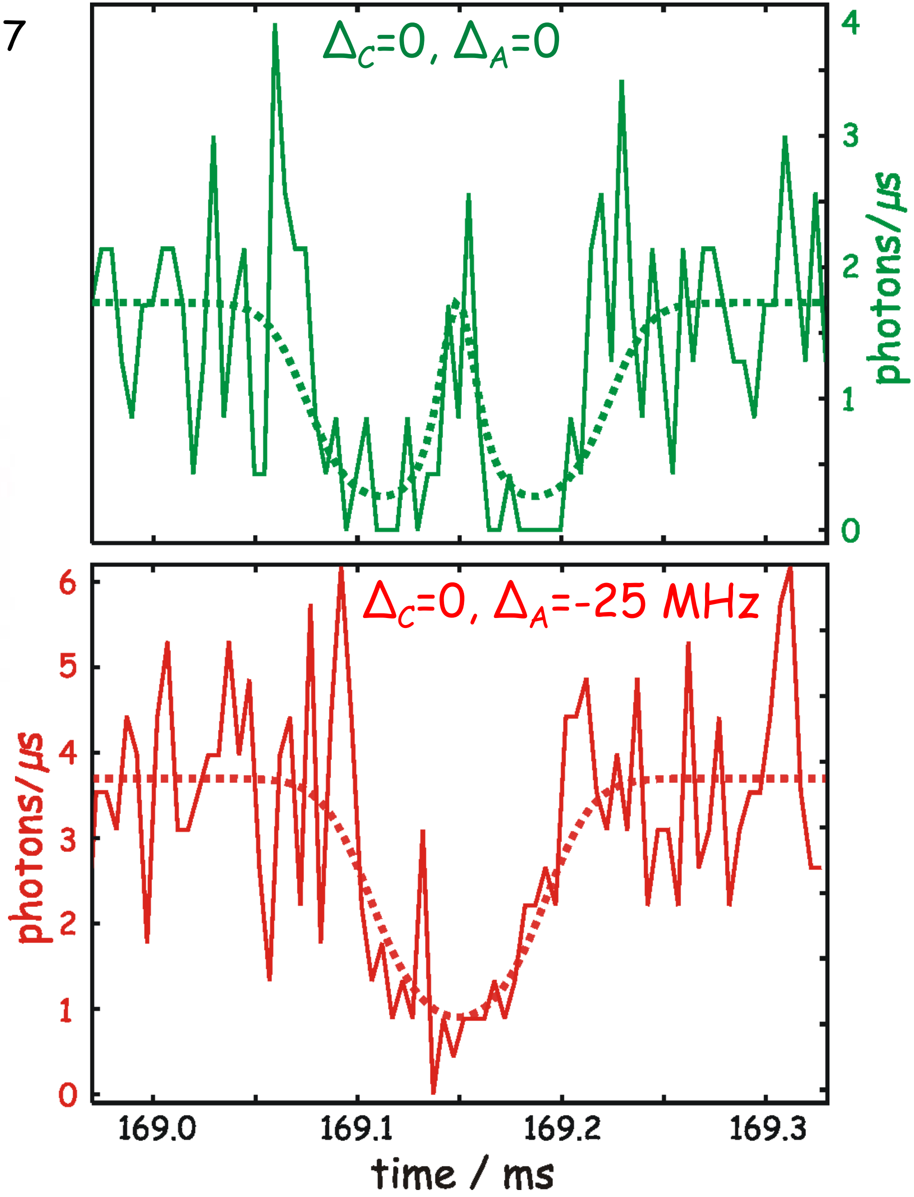


probe-laser frequency jumps to and fro between modes every 1 μs

spatial imaging

Puppe et al., Physica Scripta **T112** (2004) 7





v = 0.8(1) m/s; δx = 24(6) μm

limited by shot-noise

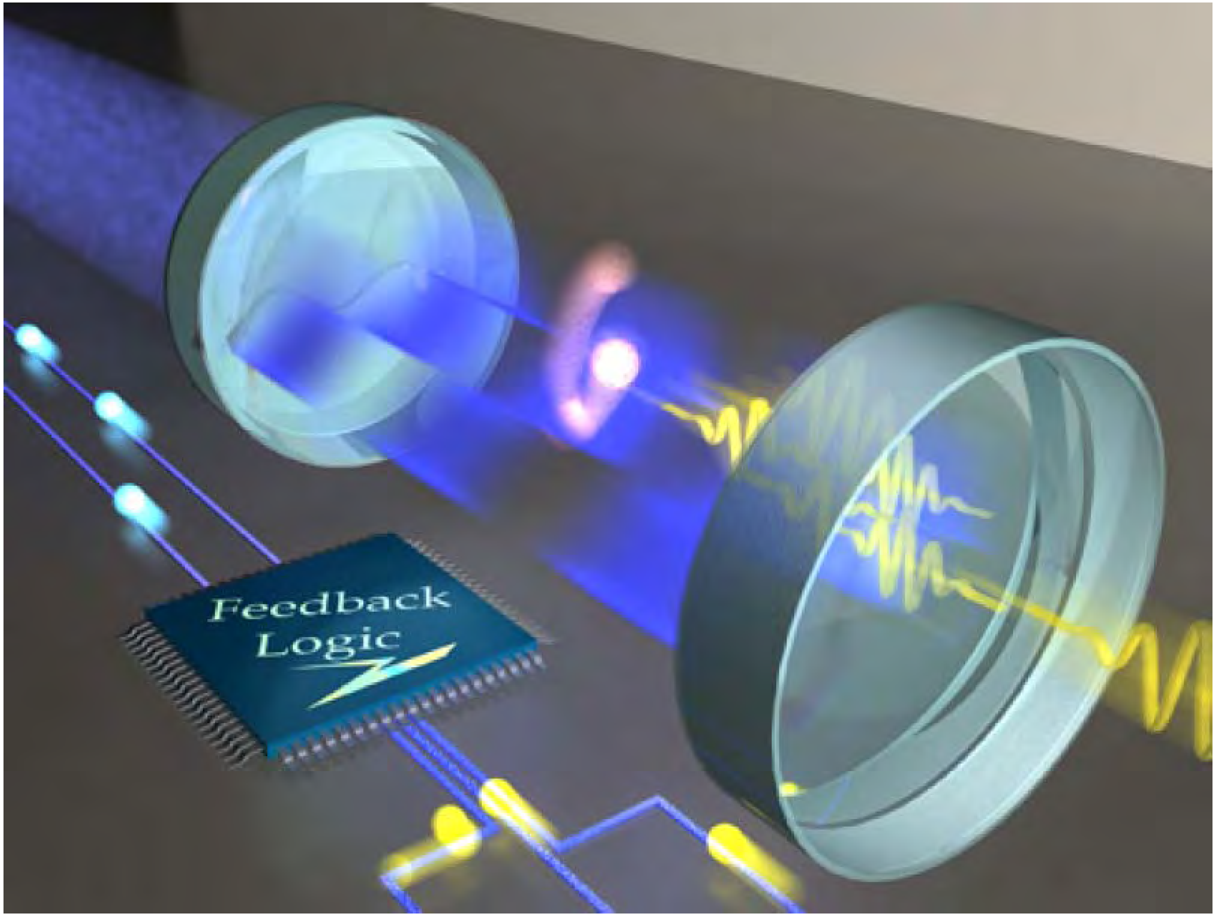
real-time feedback

based on the result of an observation,
is it possible to …

1) … maneuver an individual atom, thus validating its "measured" trajectory?
2) … cool a randomly moving particle?
3) … reach the standard quantum limit (Heisenberg's uncertainty relation)?
4) … explore tailored servo loops?

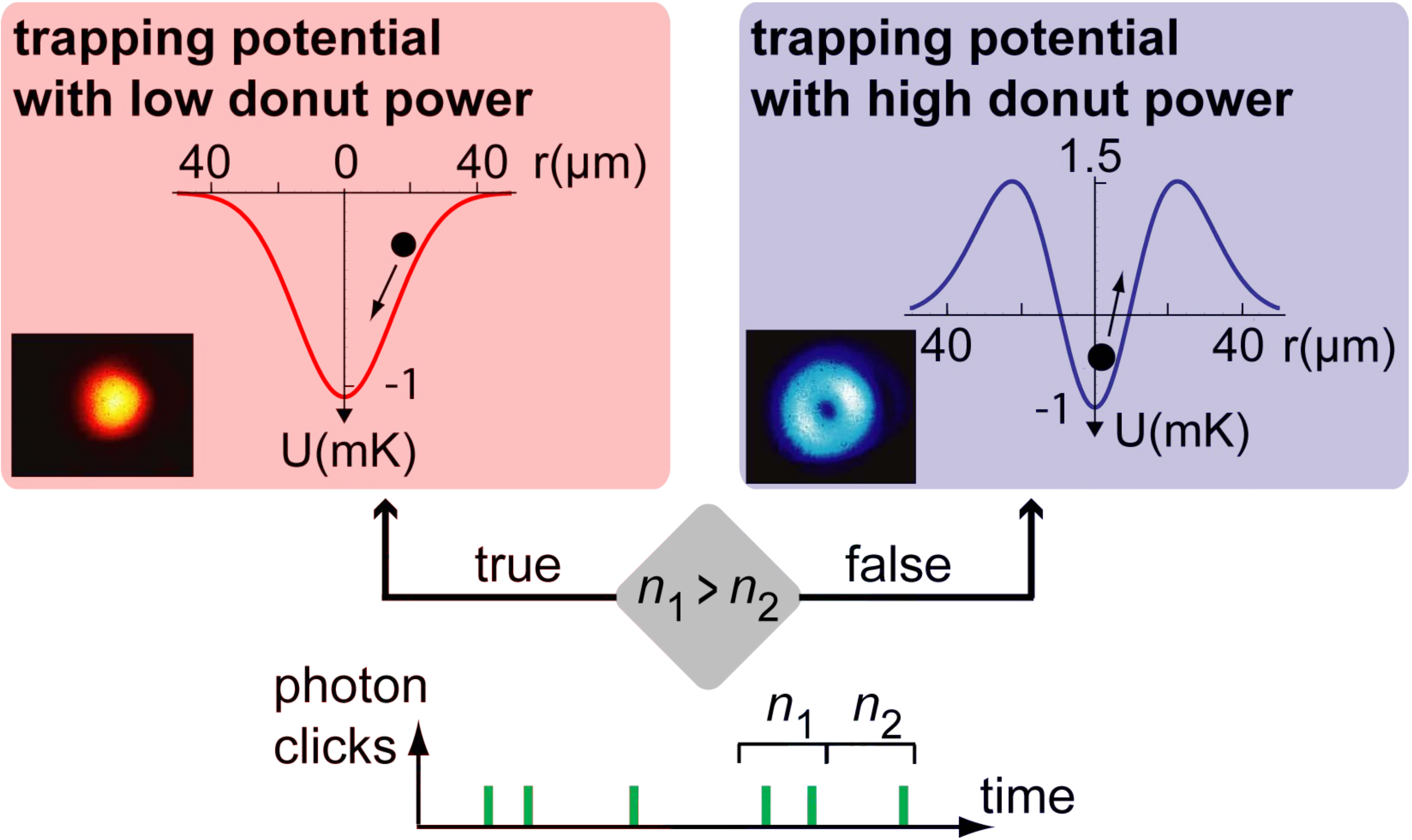
experimental scheme

Kubanek et al., Nature **462**, 898 (2009)
Koch et al., PRL **105**, 173003 (2010)

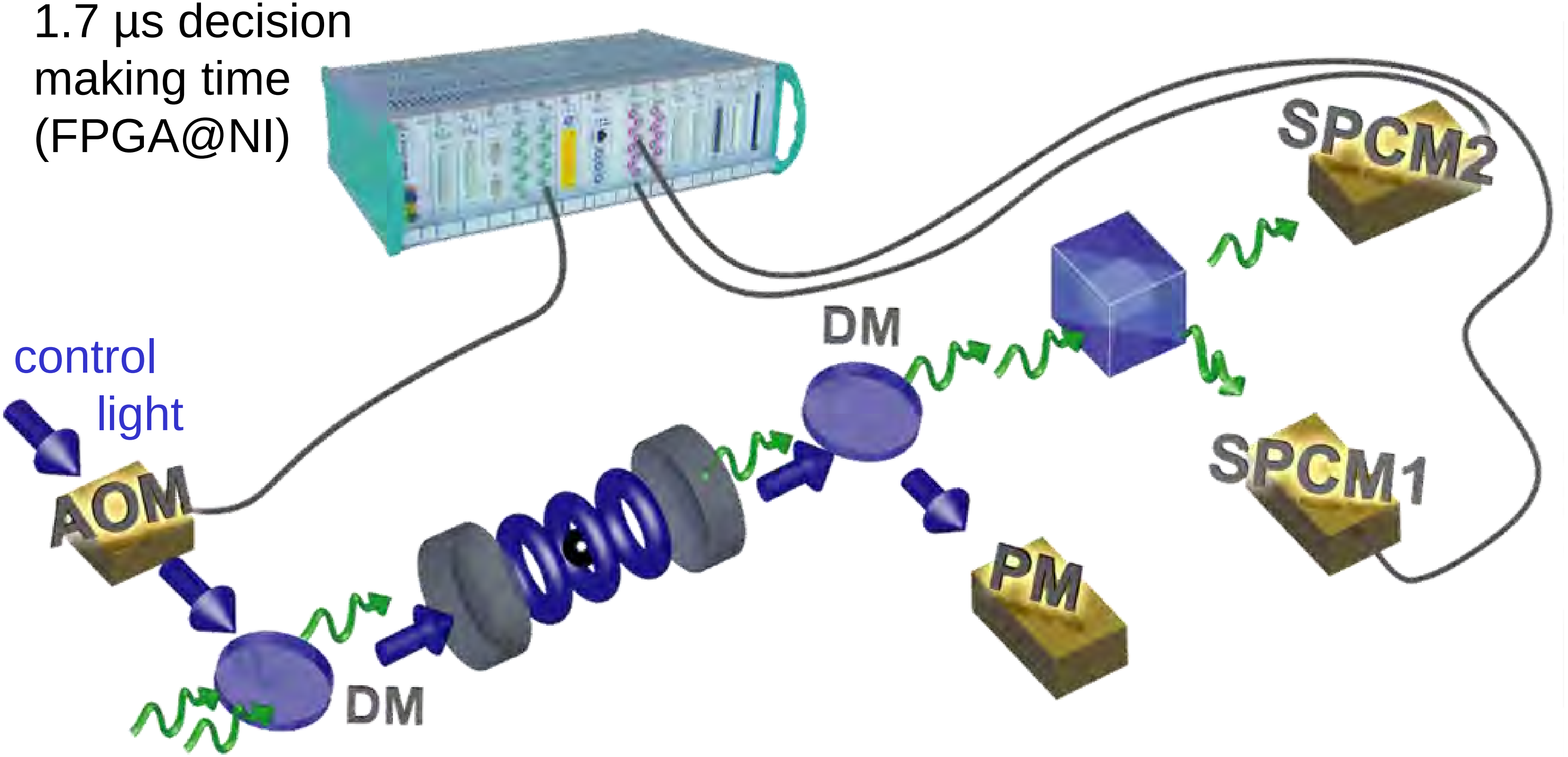


experimental protocol

Kubanek et al., Appl. Phys. B **102**, 433 (2011)



experimental setup



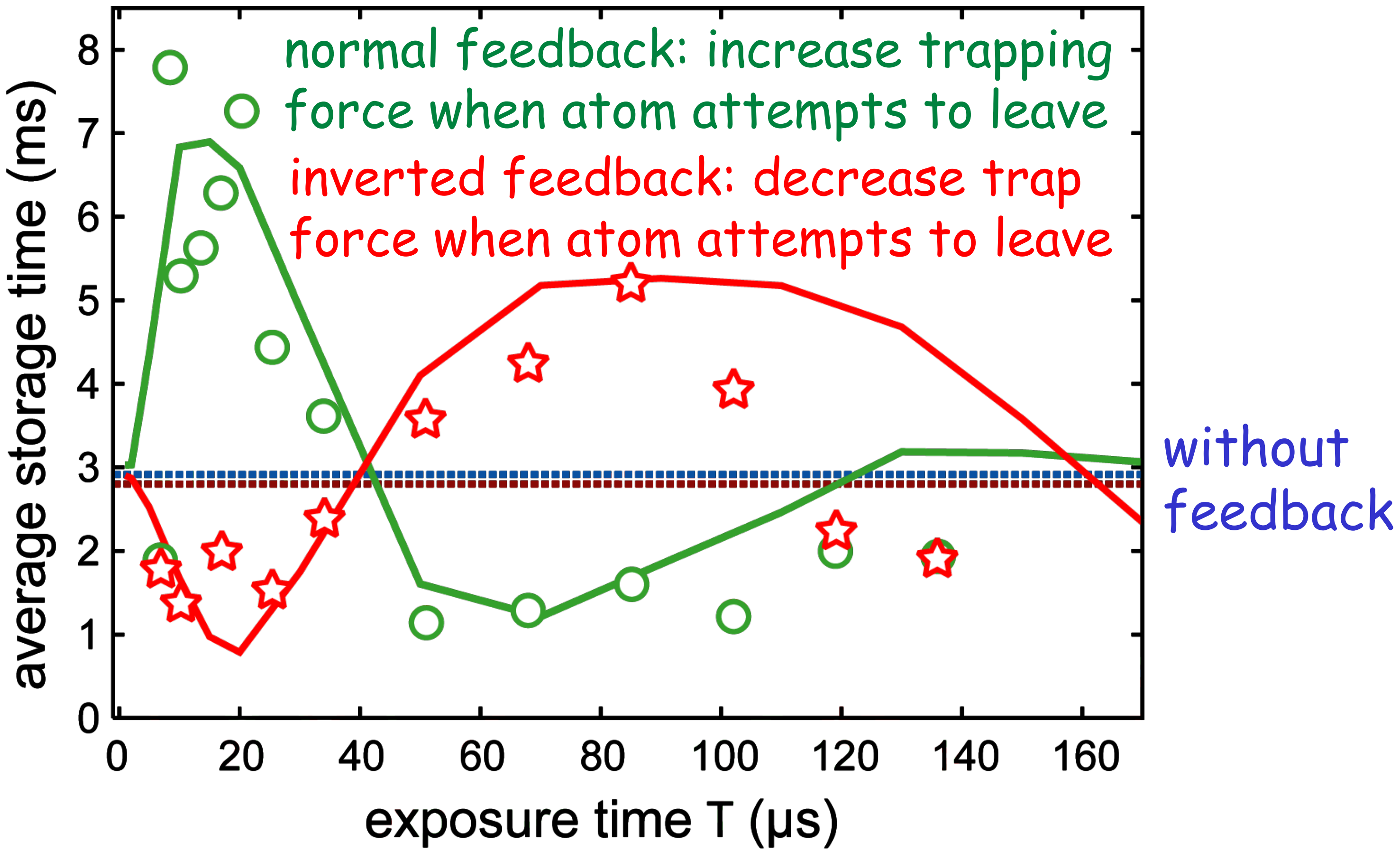
1.7 µs decision making time (FPGA@NI)

probe light

strategic decisions are made upon detection of single photons

experimental results

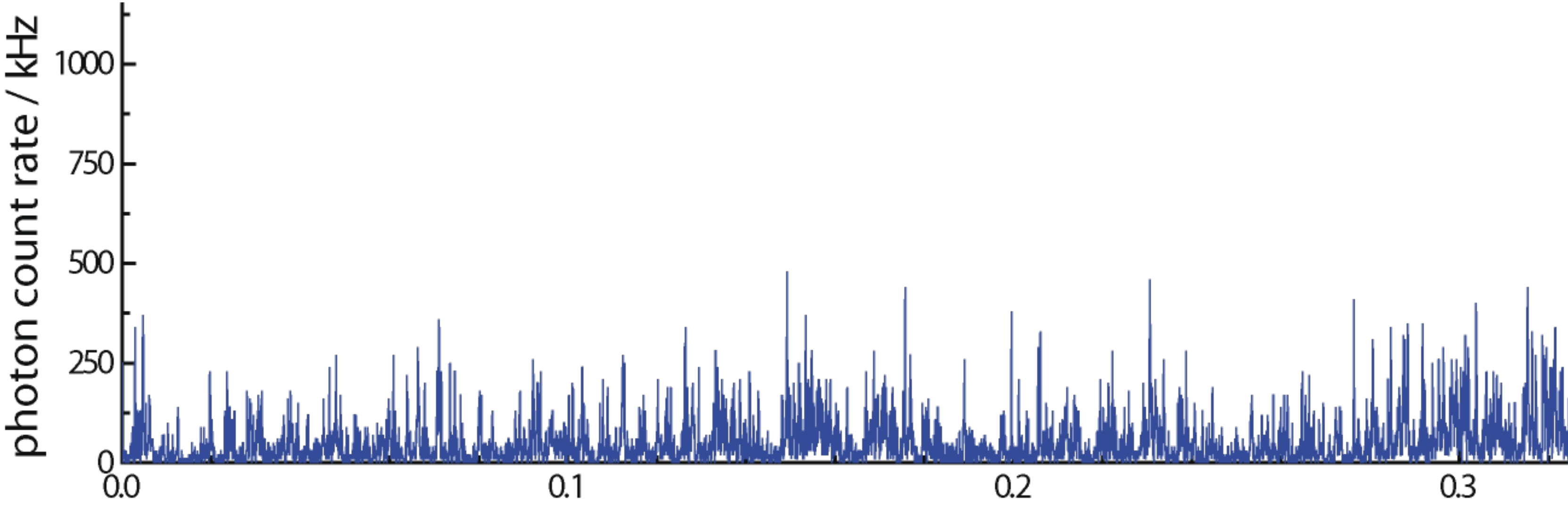
Kubanek et al., Nature **462**, 898 (2009)



solid lines = simulations

experimental results: improved system

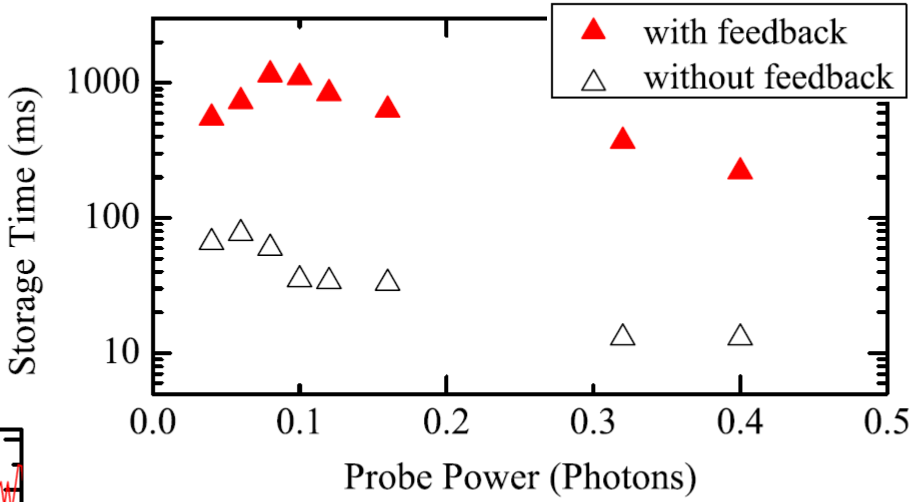
Koch et al., PRL **105**, 173003 (2010)



typical trace, can be 10× longer

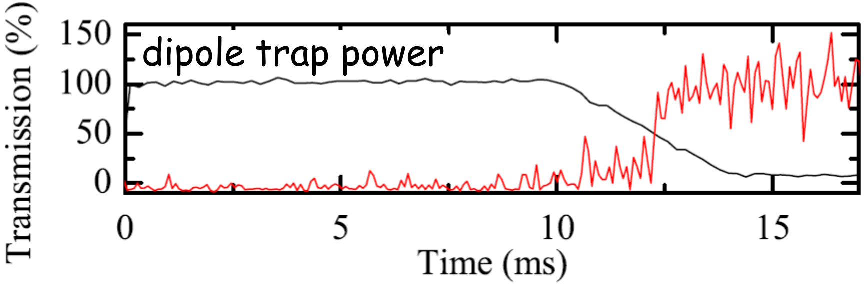
feedback cooling

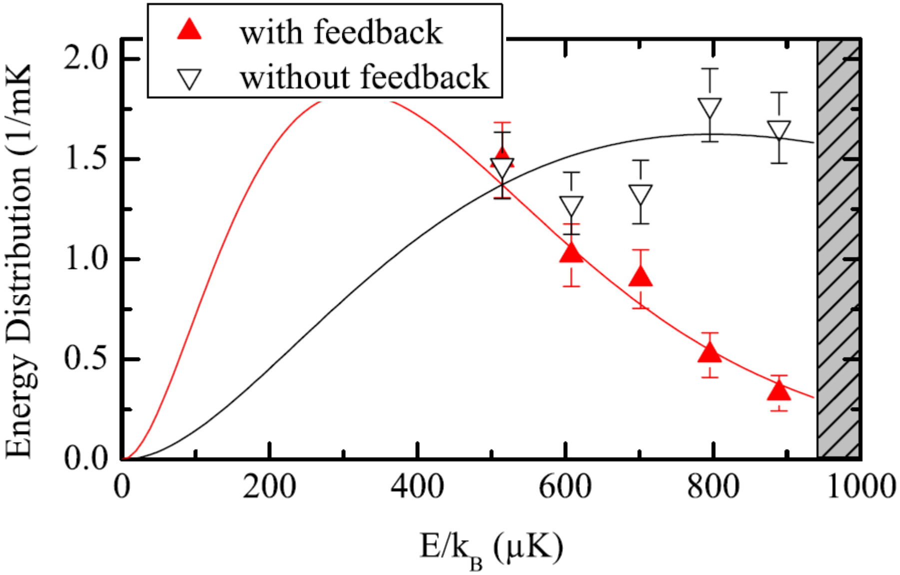
Koch et al., PRL **105**, 173003 (2010)



feedback helps:

- 20× longer trapping





feedback cools:

- 3× lower energy

feedback simplifies:

- 2D with 1D laser

question

ask student whether intensity of the light field, after passing by the atom, is reduced by …

a. … mostly absorption,

b. … mostly interference,

c. … both equally,

d. … none of above?

weakly driven dissipative atom-cavity system

optical cavity QED (ω_L=ω_A=ω_C):
polarization decay $\gamma\neq 0$, field decay $\kappa\neq 0$, driving $\eta\neq 0$

master equation:
$$\dot{\rho} = -\frac{i}{\hbar}\left[\hat{H}_{JC} + \hat{H}_P, \rho\right] - \kappa\left(\hat{a}^+\hat{a}\rho + \rho\hat{a}^+\hat{a} - 2\hat{a}\rho\hat{a}^+\right)$$
$$-\gamma\left(\hat{\sigma}_+\hat{\sigma}_-\rho + \rho\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\rho\hat{\sigma}_+\right)$$

state vector:
$$|\psi\rangle = |g,0\rangle + c_g|g,1\rangle + c_e|e,0\rangle \qquad c_{g,e} \ll 1$$

equations of motion:

$$\dot{c}_g = -\kappa c_g + g c_e + \eta$$

$$\dot{c}_e = -\gamma c_e - g c_g$$

steady-state solution:

$$c_g = \frac{\eta/\kappa}{1+2C_1} \equiv \alpha$$

$$c_e = -\frac{g}{\gamma} c_g \equiv \beta$$

cooperativity $C_1 = \frac{g^2}{2\gamma\kappa}$

spectral response

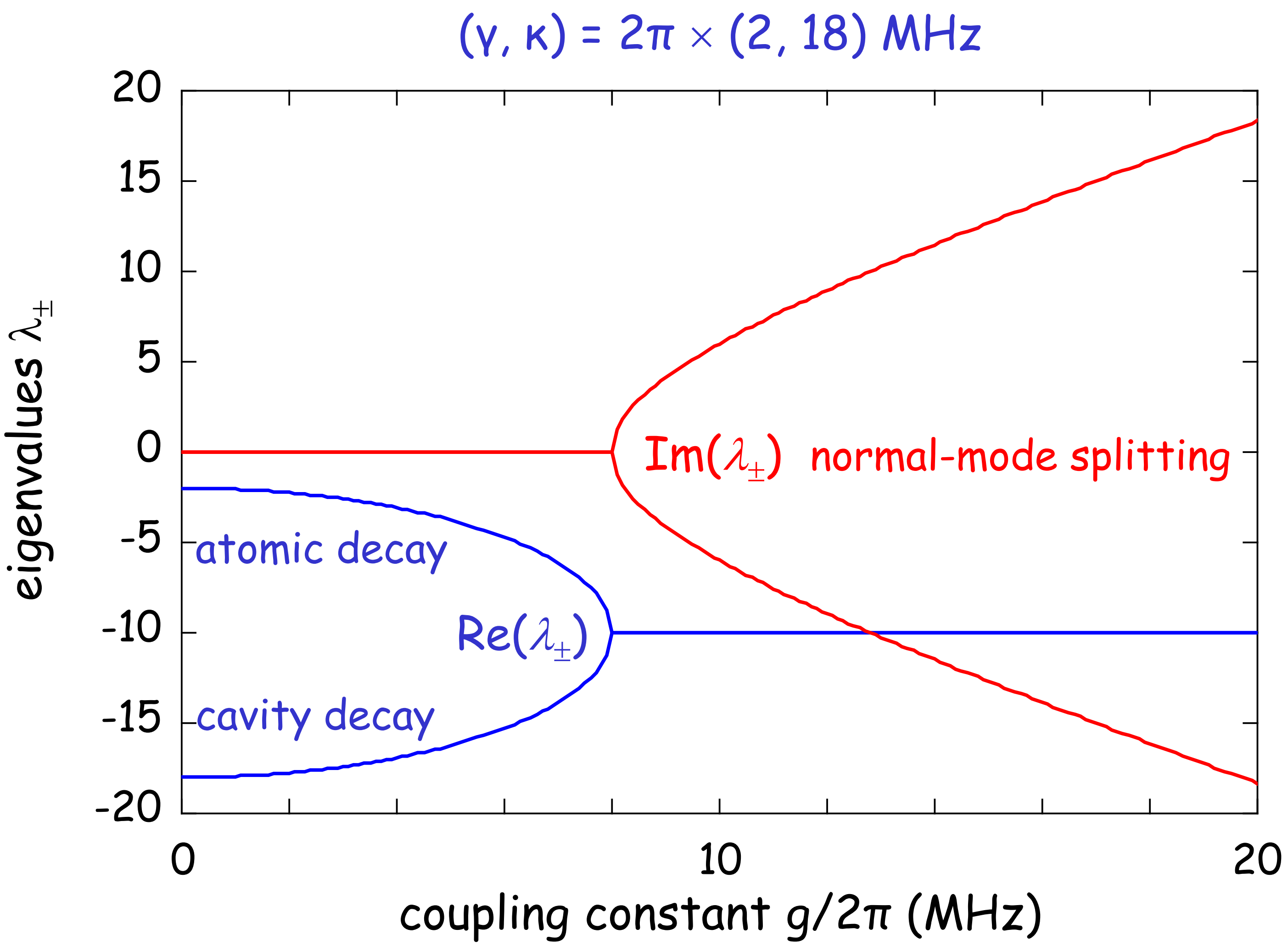
eigenvalues of the coupled system:

$$\lambda_{\pm} = -\frac{\gamma+\kappa}{2} \pm \sqrt{\left(\frac{\gamma-\kappa}{2}\right)^2 - g^2}$$

cavity transmission ($\Delta=\omega_L-\omega_{A,C}$):

$$T(\Delta) = T_0 \left| \frac{\kappa(\gamma - i\Delta)}{(\lambda_+ + i\Delta)(\lambda_- + i\Delta)} \right|^2$$

eigenvalue structure



fast-cavity limit κ » g²/κ » γ

real (negative) eigenvalues:

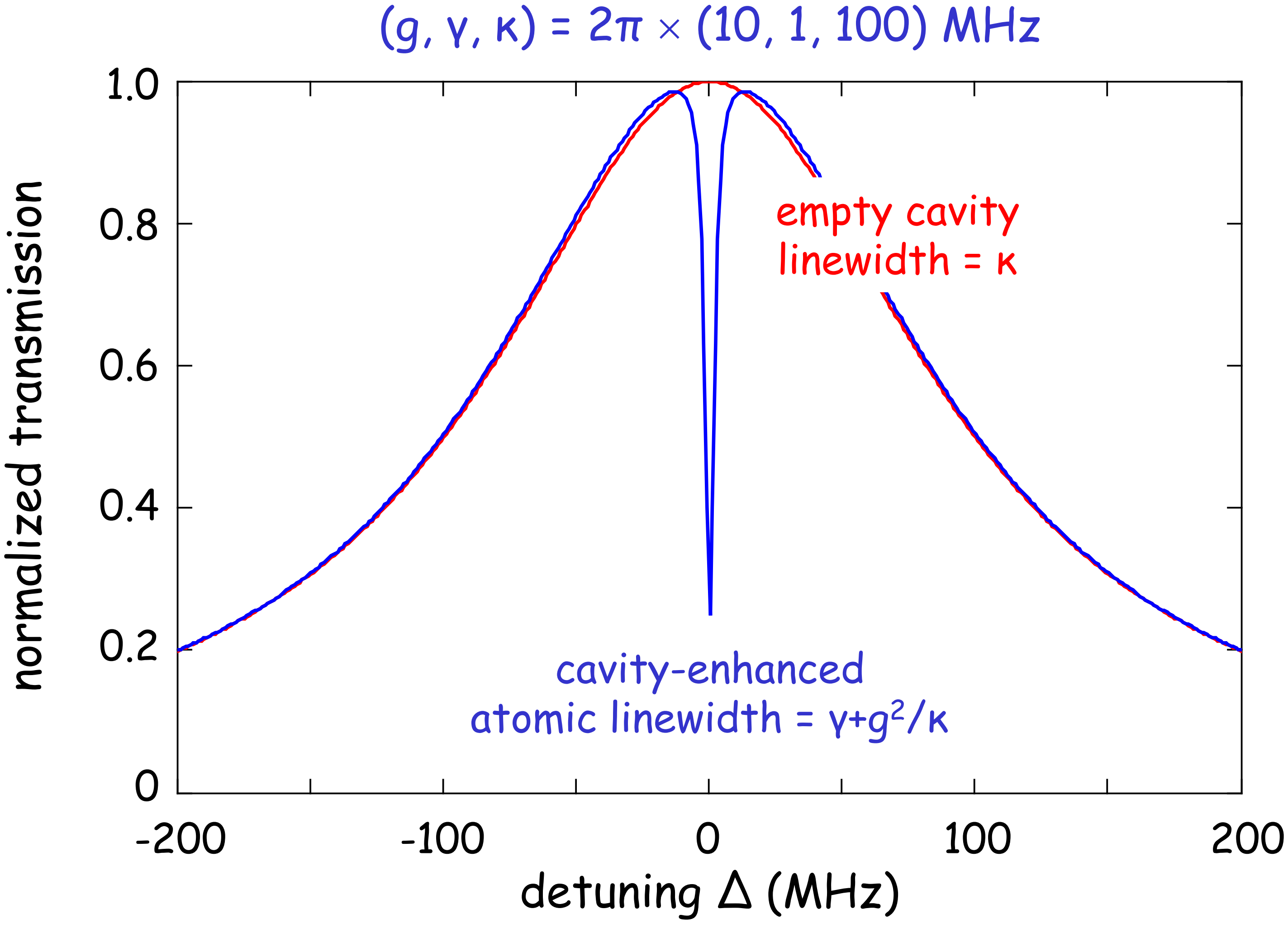
$$\lambda_+ = -\gamma\left(1+2C_1\right) = -\left(\gamma + \frac{g^2}{\kappa}\right)$$

cavity-enhanced atomic decay

$$\lambda_- = -\kappa\left(1-2C_1\frac{\gamma}{\kappa}\right) = -\left(\kappa - \frac{g^2}{\kappa}\right)$$

atom-reduced cavity decay

fast-cavity limit κ » $g^2/κ$ » γ



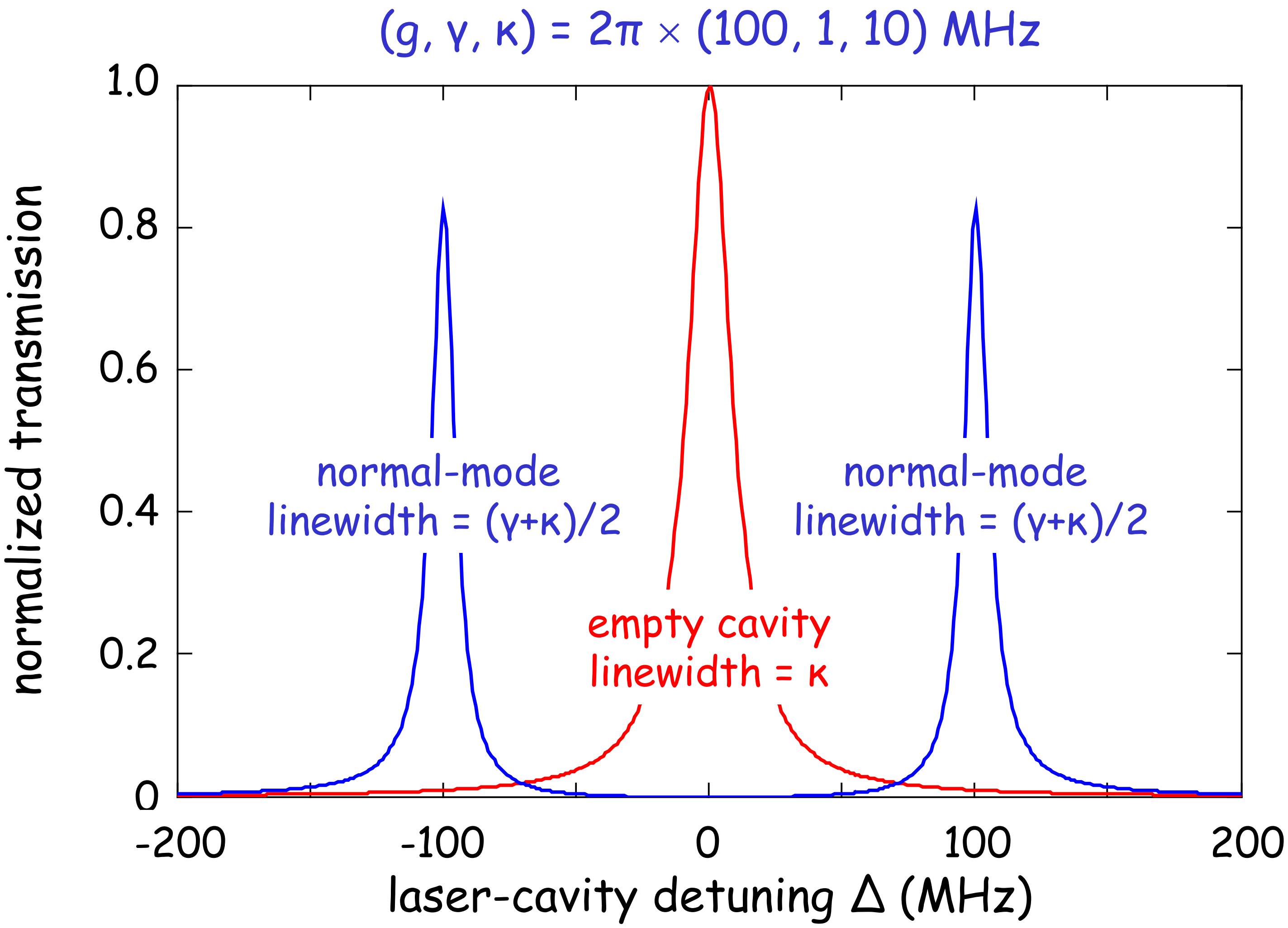
strong-coupling limit $g \gg (\gamma, \kappa)$

complex eigenvalues:

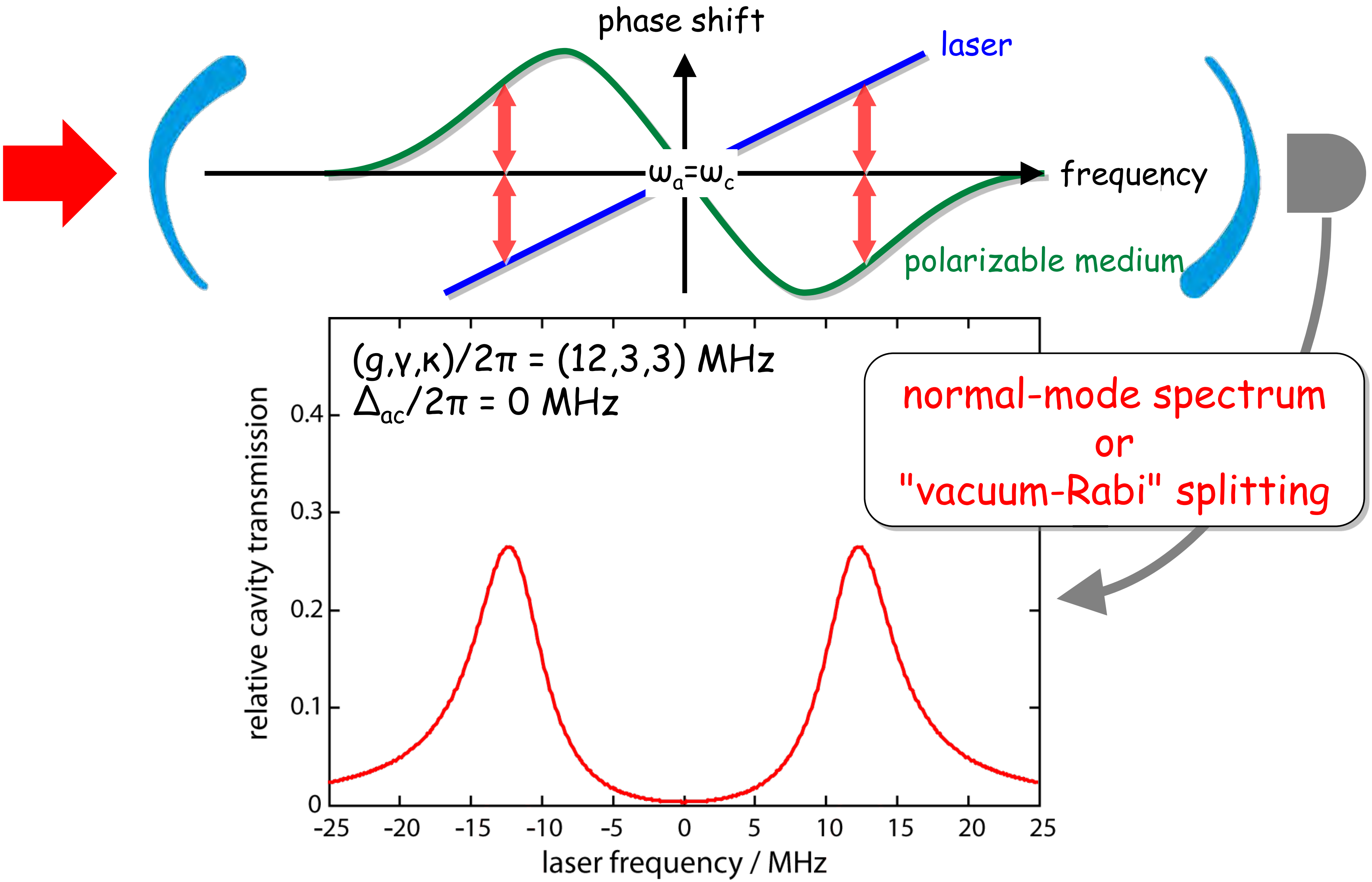
$$\lambda_{\pm} = -\frac{\gamma + \kappa}{2} \pm ig$$

- normal-mode (vacuum-Rabi) splitting
- linewidth averaging

strong-coupling limit $g \gg (\gamma, \kappa)$



linear absorption & dispersion

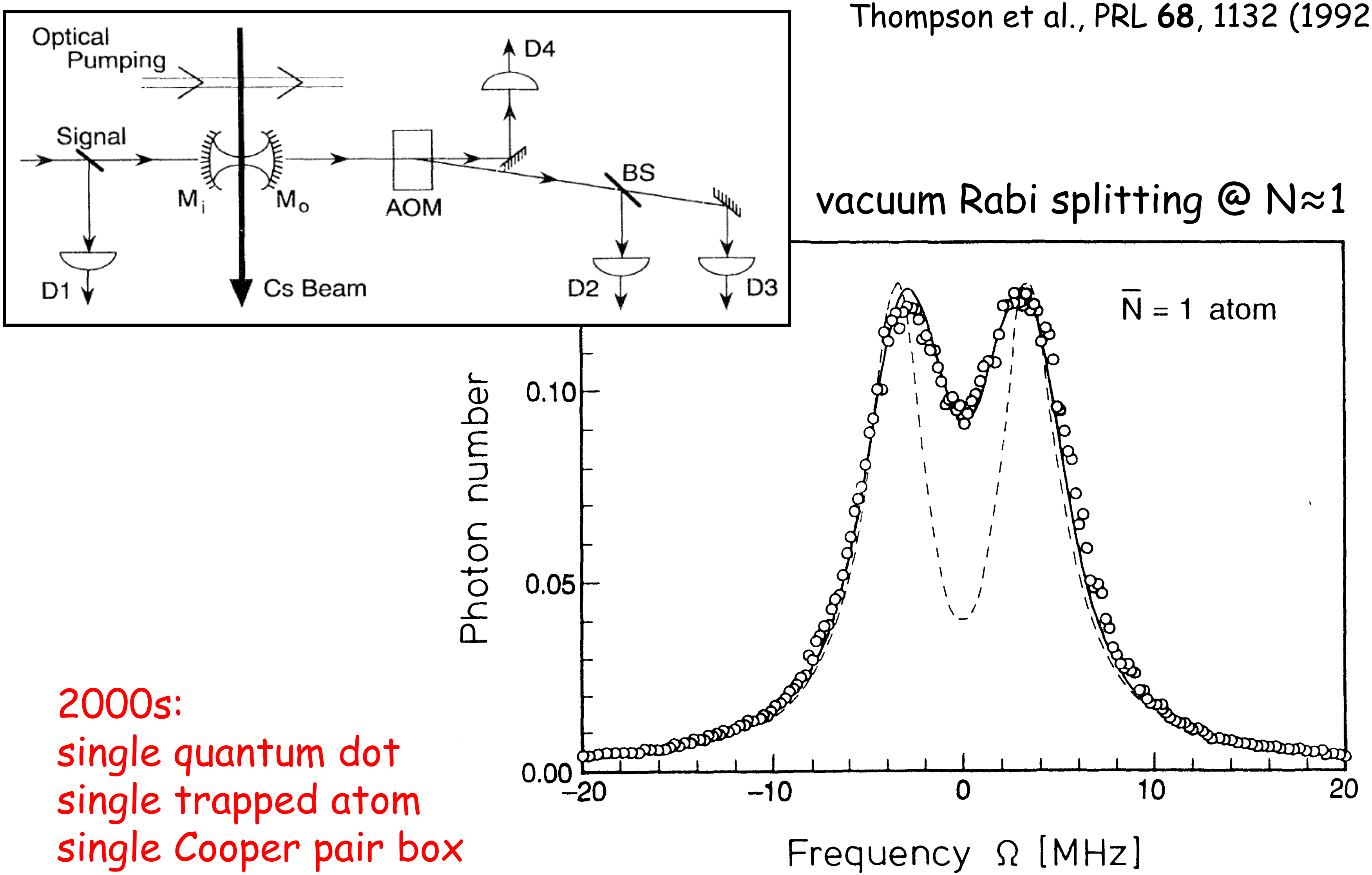


normal-mode spectrum
or
"vacuum-Rabi" splitting

normal-mode splitting

Thompson et al., PRL **68**, 1132 (1992)

vacuum Rabi splitting @ N≈1



2000s:
single quantum dot
single trapped atom
single Cooper pair box

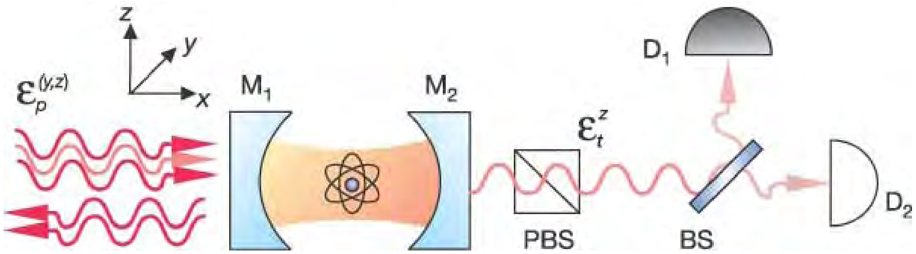
is cavity QED quantum ?

«classical» cavity ~~Q~~ED:

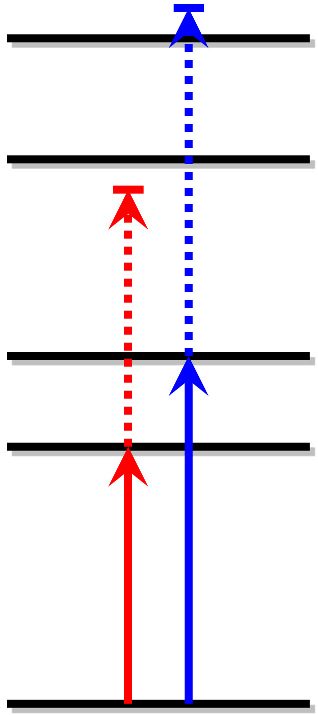
1) linear intensity response = classical
2) double-peaked spectrum = classical

cavity QED at its best: quantum anharmonicity

Birnbaum et al., Nature **436**, 87 (2005)



photon blockade



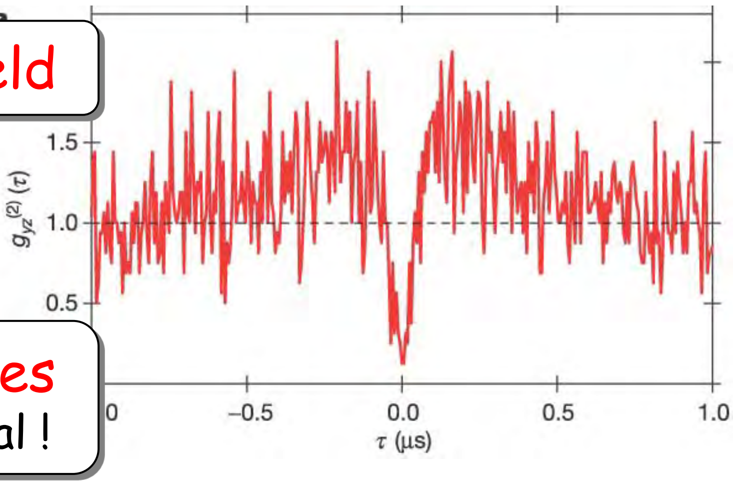
$\pm g\sqrt{2}$ $\quad \frac{1}{\sqrt{2}}\left(|e,1\rangle \pm |g,2\rangle\right)$

quantum field

$\pm g$ $\quad \frac{1}{\sqrt{2}}\left(|e,0\rangle \pm |g,1\rangle\right)$

normal modes

purely classical !

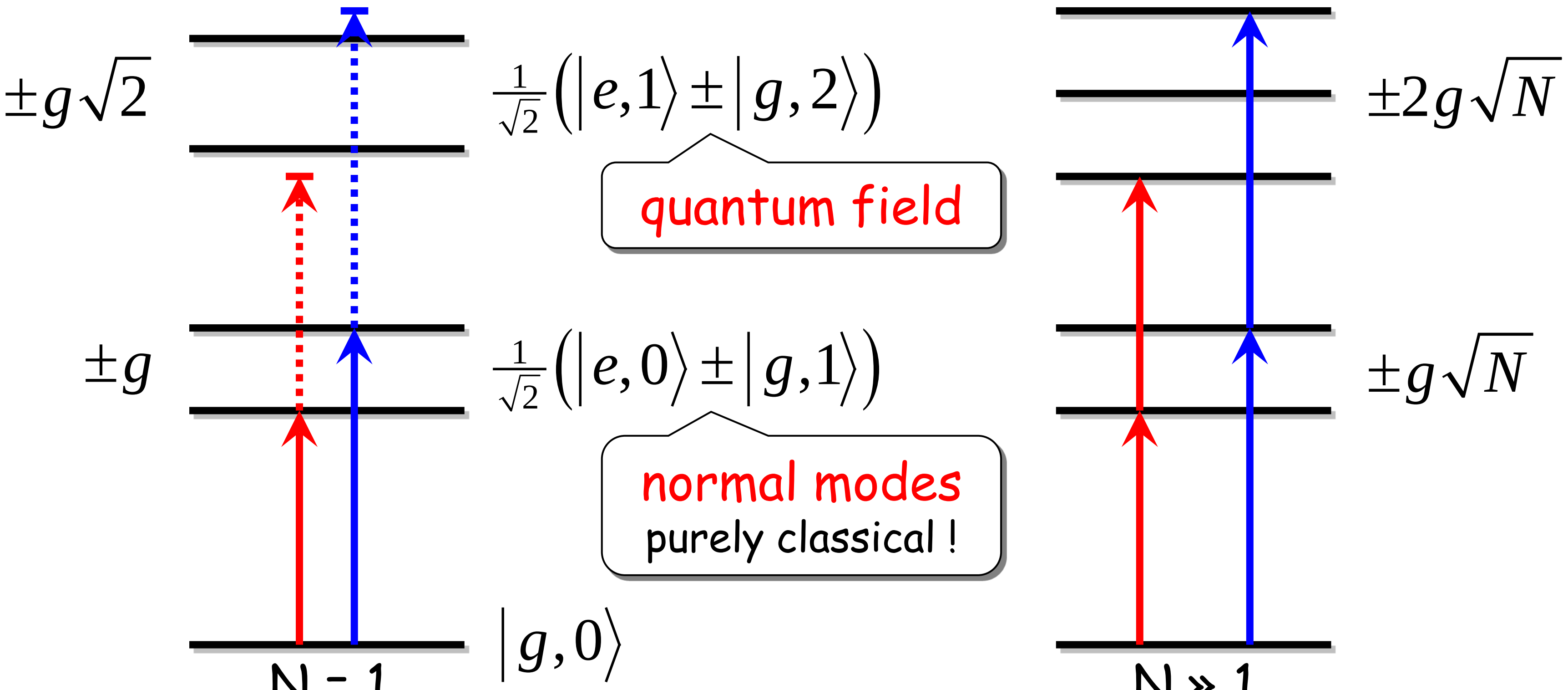


$|g,0\rangle$

N = 1

anharmonic

cavity QED at its best: quantum anharmonicity



photon blockade

$\pm g\sqrt{2}$

$\frac{1}{\sqrt{2}}\left(|e,1\rangle \pm |g,2\rangle\right)$

quantum field

$\pm g$

$\frac{1}{\sqrt{2}}\left(|e,0\rangle \pm |g,1\rangle\right)$

normal modes

purely classical !

$|g,0\rangle$

N = 1

anharmonic

$\pm 2g\sqrt{N}$

$\pm g\sqrt{N}$

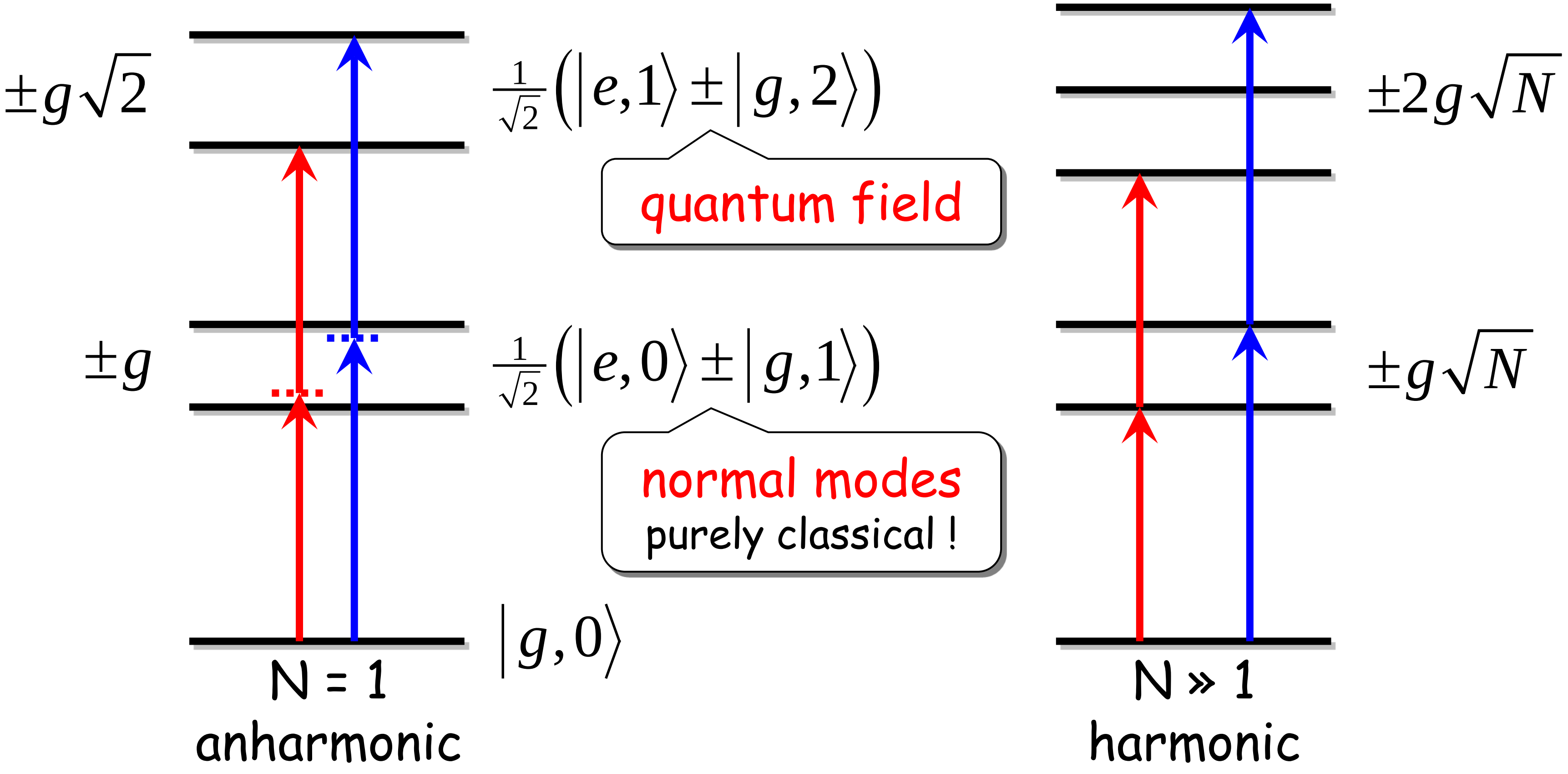
N » 1

harmonic

cavity QED at its best: quantum anharmonicity

stepwise excitation:
Carmichael et al., PRL **77**, 631 (1996)

two-photon excitation !



$\pm g\sqrt{2}$

$\frac{1}{\sqrt{2}}\left(|e,1\rangle \pm |g,2\rangle\right)$

quantum field

$\pm g$

$\frac{1}{\sqrt{2}}\left(|e,0\rangle \pm |g,1\rangle\right)$

normal modes
purely classical !

$|g,0\rangle$

N = 1
anharmonic

$\pm 2g\sqrt{N}$

$\pm g\sqrt{N}$

N » 1
harmonic

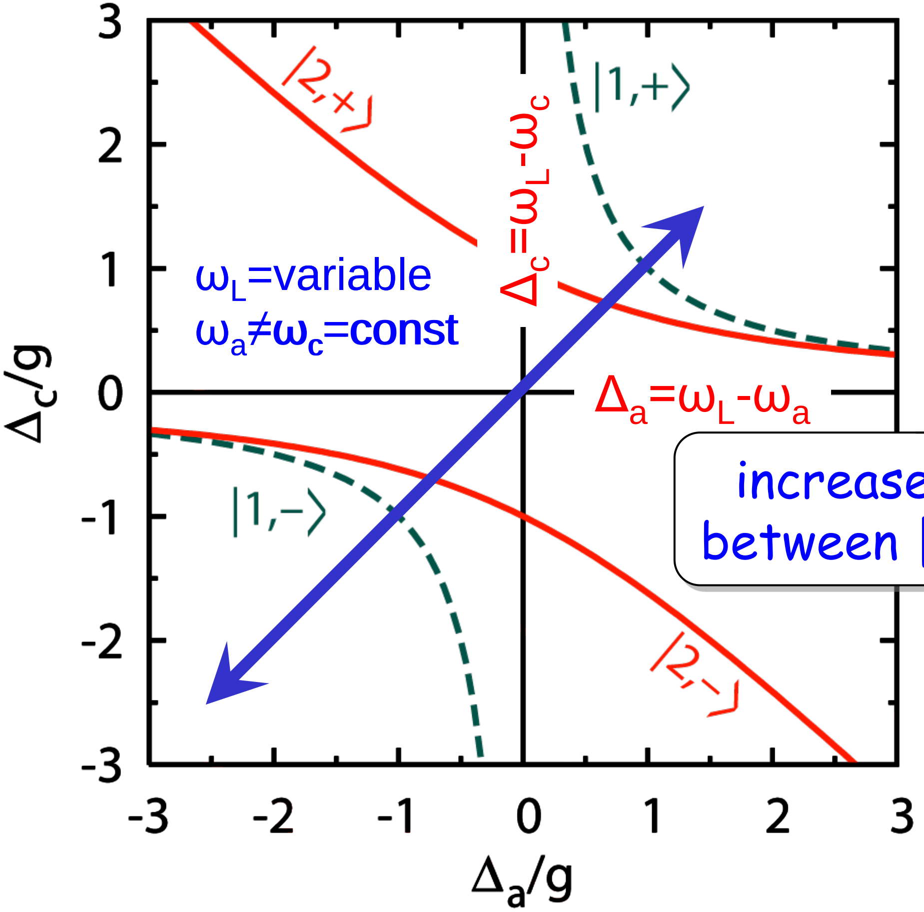
no simple scaling with atom number

«quantum» cavity QED:

1) many atoms = harmonic oscillator
2) one atom = anharmonic oscillator

two-photon excitation: resonance frequencies

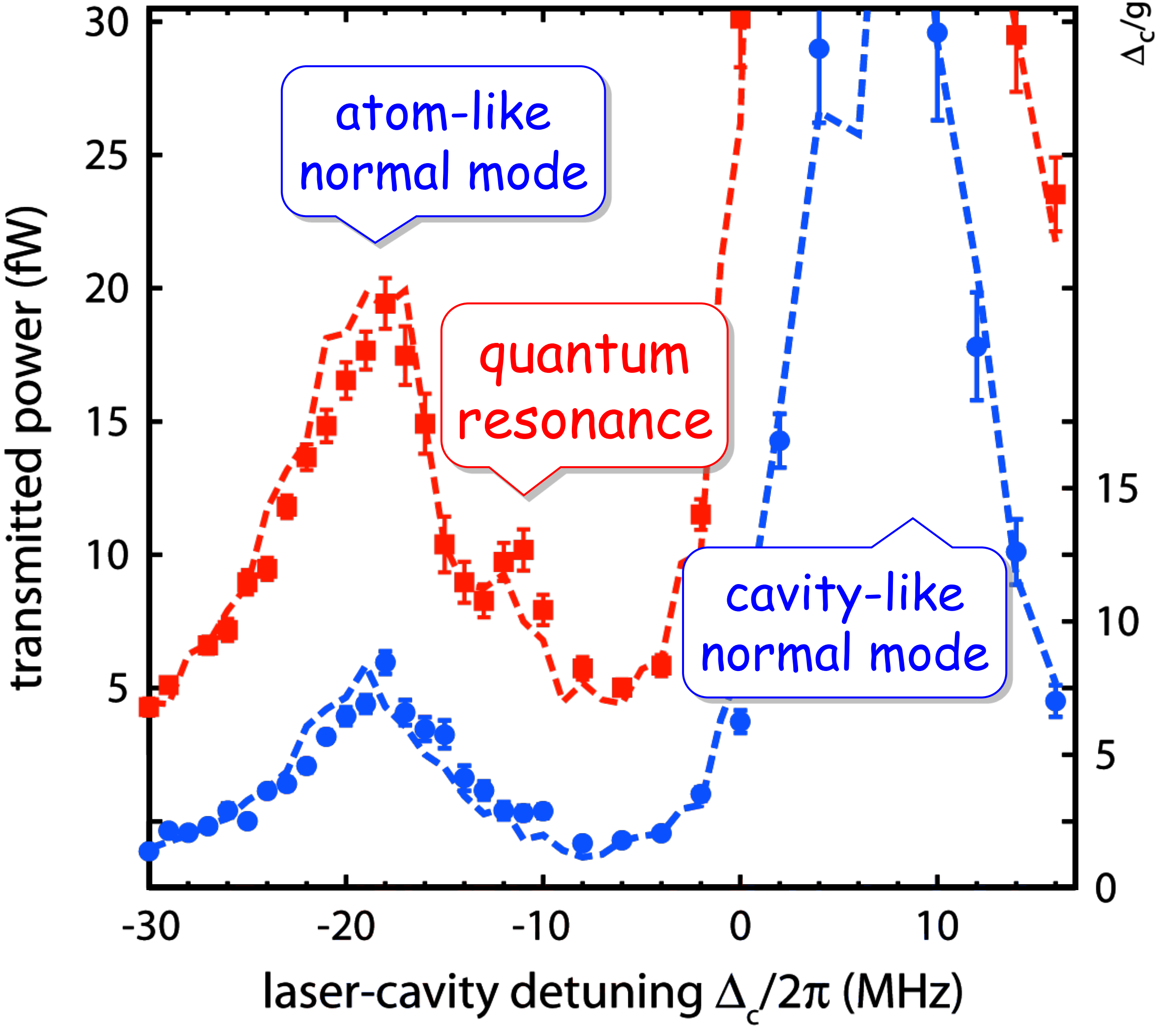
$$\omega_{n+1,\pm} = n\omega_c + \frac{1}{2}(\omega_a + \omega_c) \pm \frac{1}{2}\sqrt{4g^2(n+1) + (\omega_a - \omega_c)^2} = (n+1)\omega_L$$

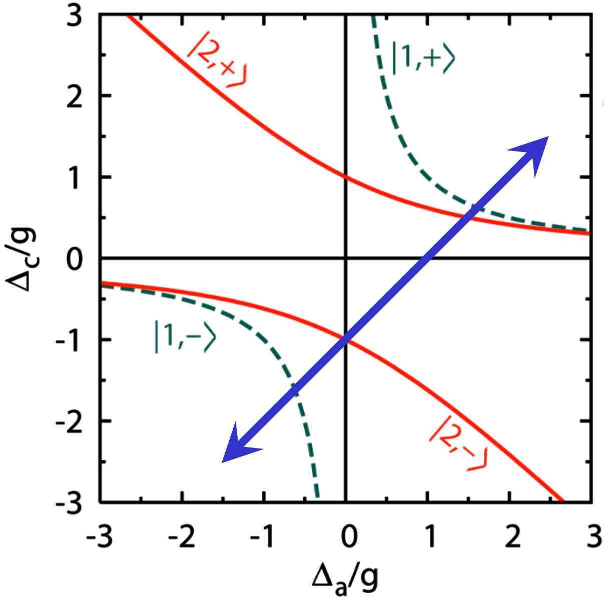


increase separation between |1,-⟩ and |2,-⟩

quantum anharmonicity

Schuster et al., Nature Phys. **4**, 382 (2008)





ω_L=variable

ω_a=const ≠ ω_c=const

«high» power

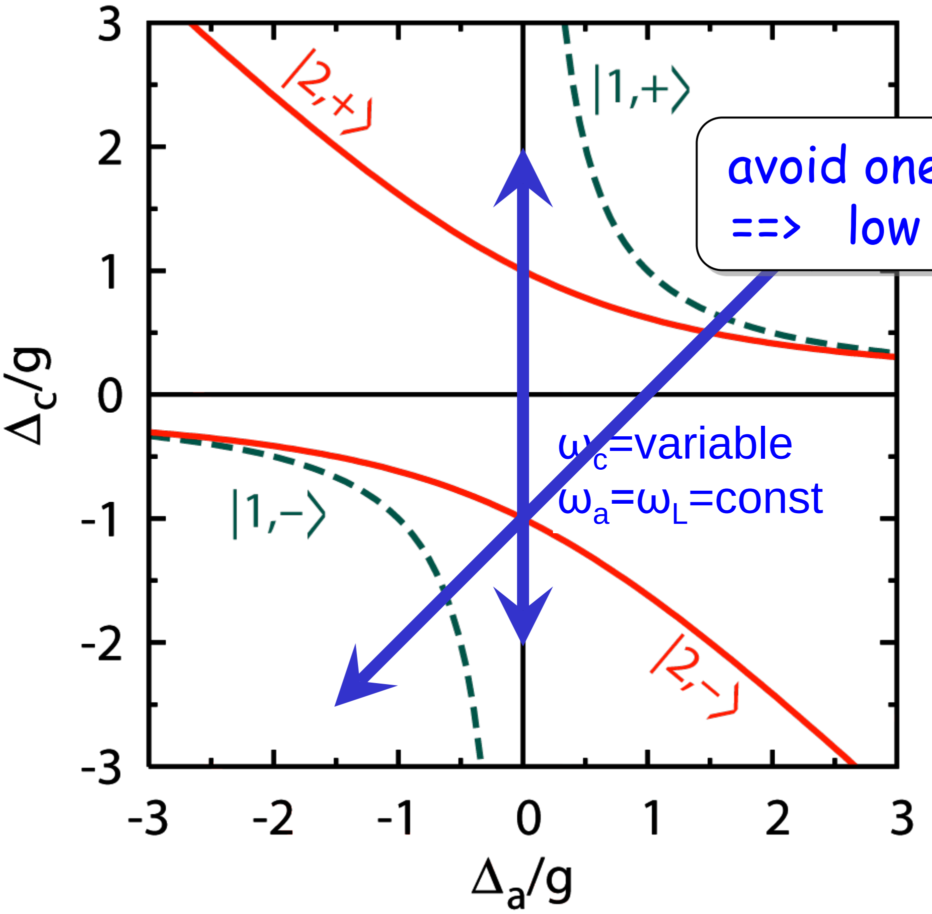
P_{in} = 1.5 pW

«low» power

P_{in} = 0.5 pW

how to avoid the classical response ?

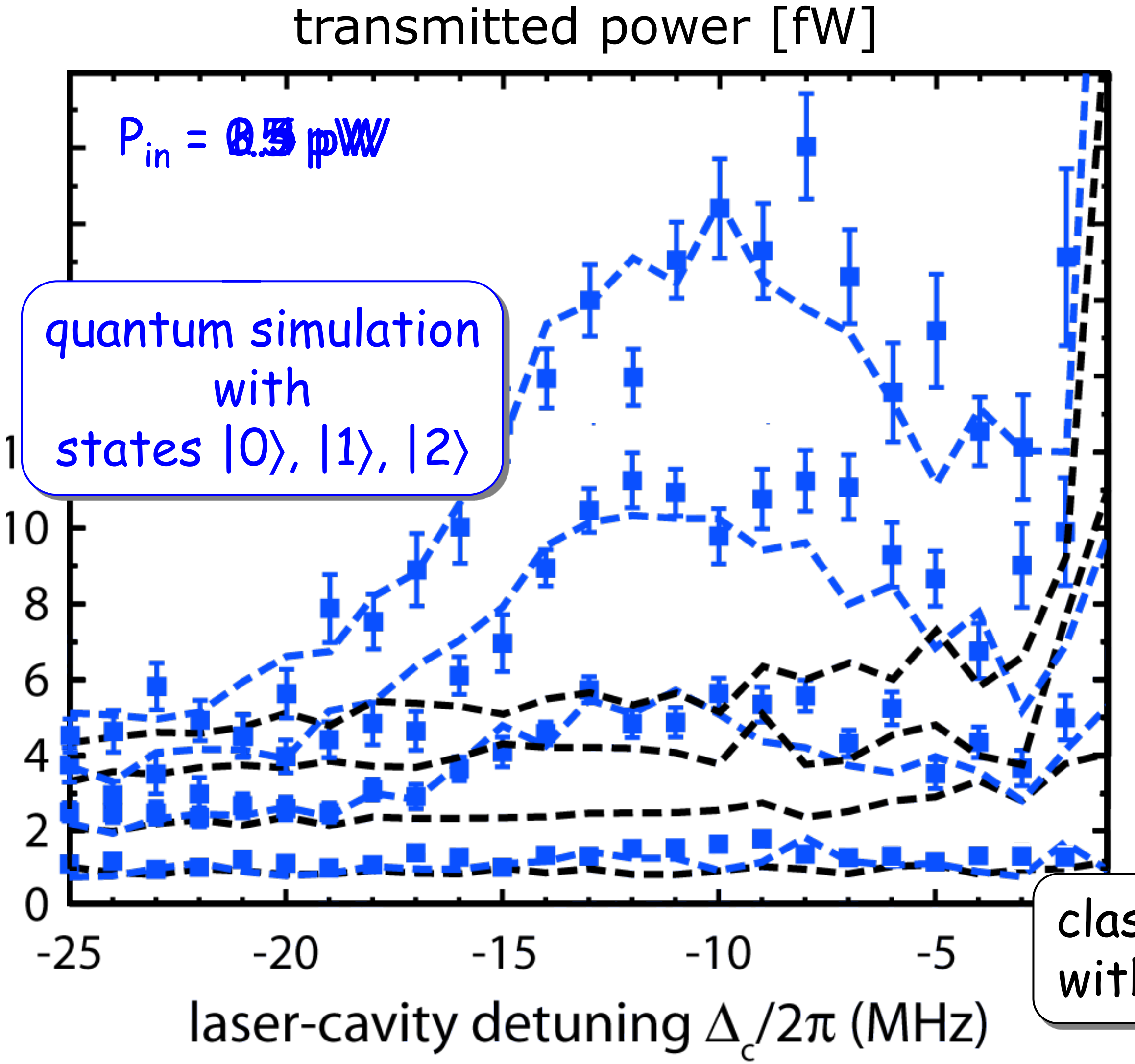
$$\omega_{n+1,\pm} = n\omega_c + \frac{1}{2}(\omega_a + \omega_c) \pm \frac{1}{2}\sqrt{4g^2(n+1) + (\omega_a - \omega_c)^2} = (n+1)\omega_L$$



avoid one-photon resonance
==> low atomic excitation !

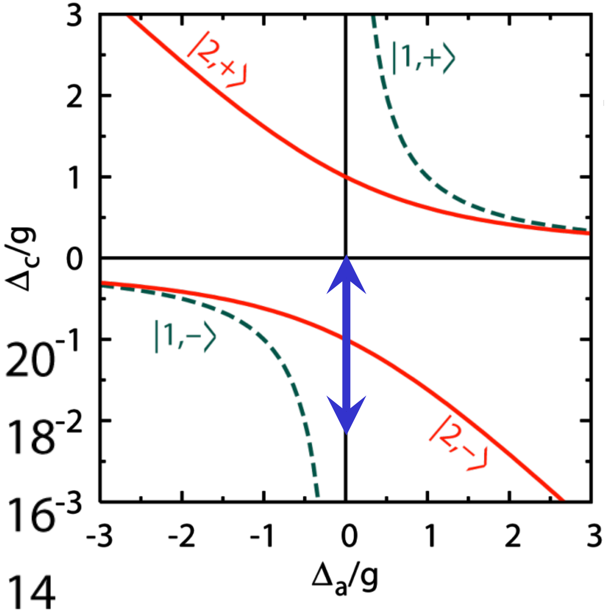
multi-photon resonances

Schuster et al., Nature Phys. **4**, 382 (2008)



quantum simulation with states |0⟩, |1⟩, |2⟩

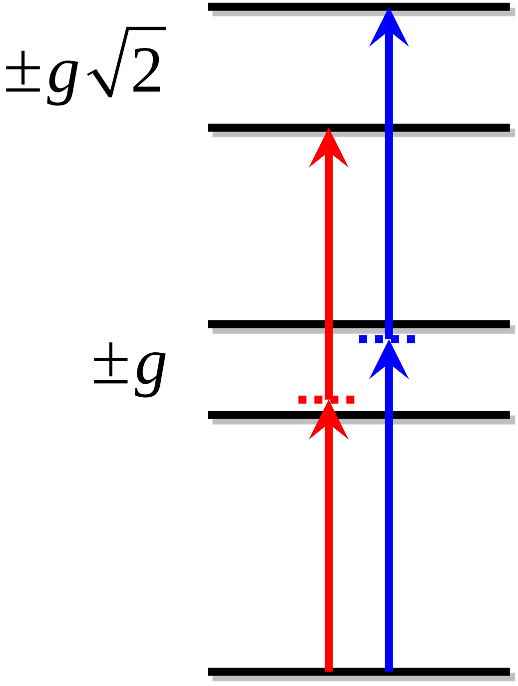
classical simulation with states |0⟩, |1⟩



ω_c=variable

$\omega_a=\omega_L$=const

cavity QED at its best: quantum anharmonicity



N = 1
anharmonic

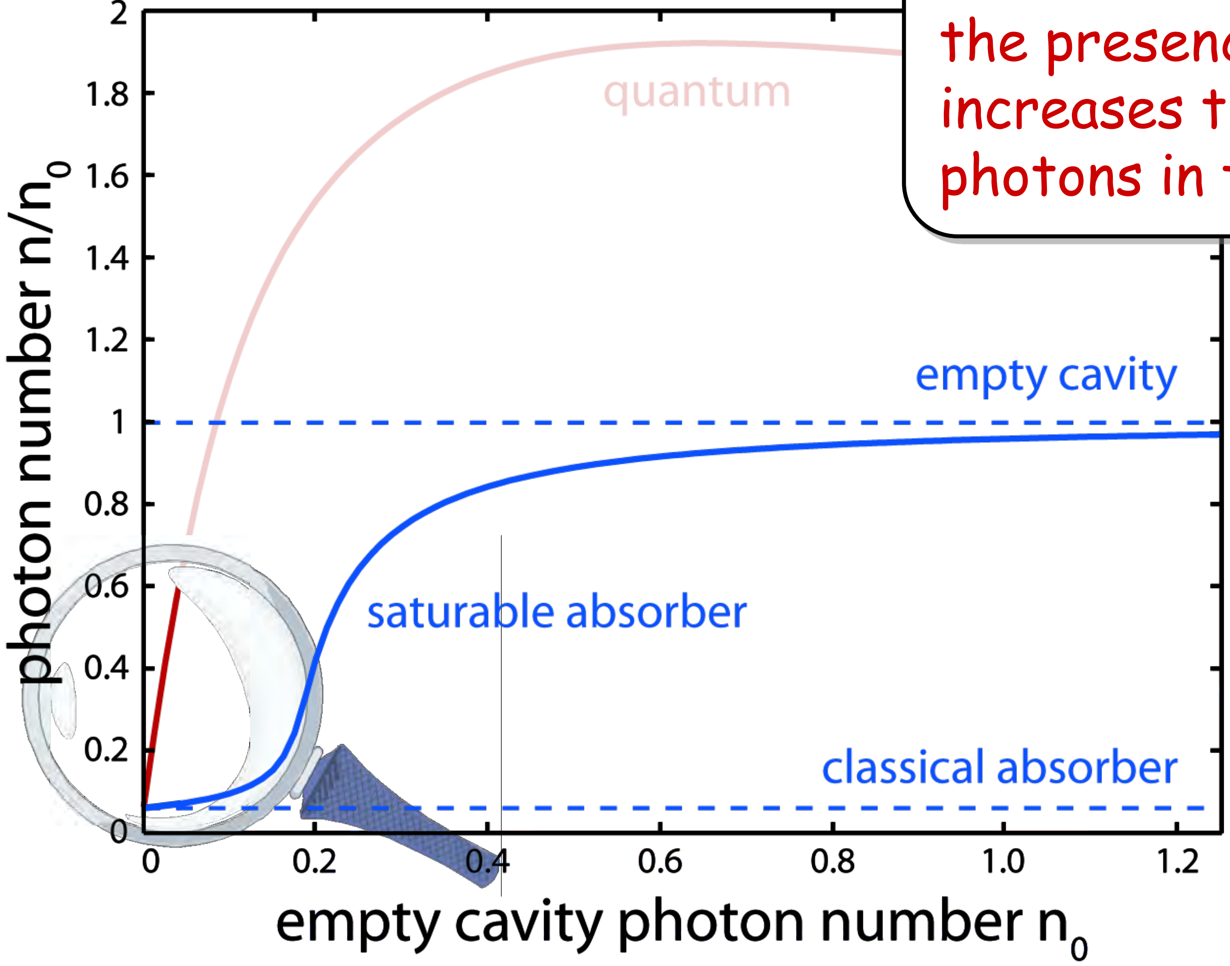
quantum response:

$$I_{out} = c_1 I_{in} + \underbrace{q_2 I_{in}^2 + q_3 I_{in}^3 + \ldots}_{\text{nonlinear due to anharmonicity}}$$

classical response:

$$I_{out} = c_1 I_{in} + \underbrace{c_2 I_{in}^2 + c_3 I_{in}^3 + \ldots}_{\text{nonlinear due to atomic saturation}}$$

quantum nonlinearity

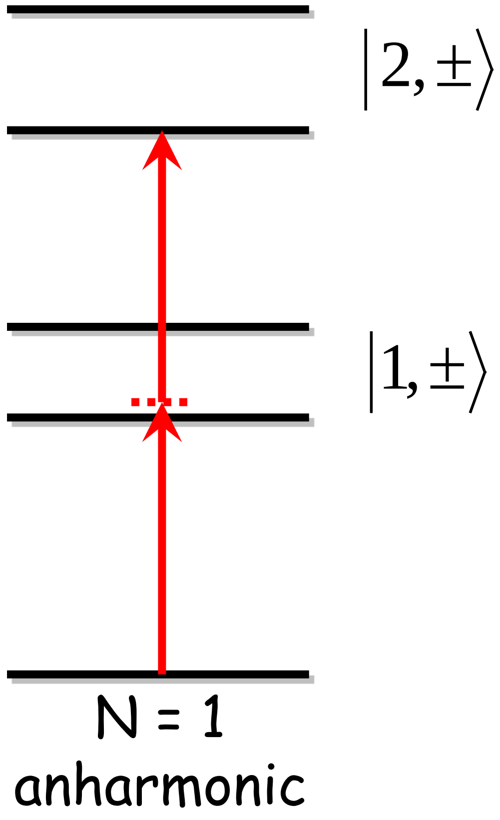


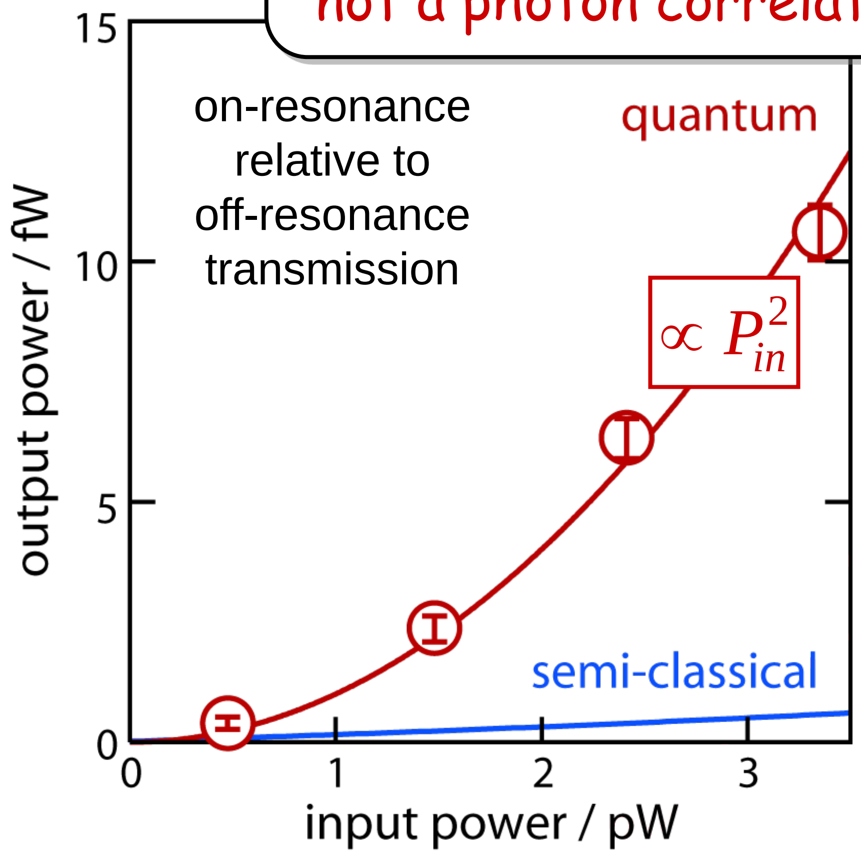
counterintuitive:
the presence of an atom increases the number of photons in the cavity

quantum nonlinearity

Schuster et al., Nature Phys. **4**, 382 (2008)
see also: Bishop et al., Nature Physics **5**, 105 (2009)

quantum effect observed in the average intensity, not a photon correlation





photon correlations

Carmichael et al., Opt. Comm. **82**, 73 (1991)

state vector:

$$|\psi\rangle = |g,0\rangle + c_g|g,1\rangle + c_e|e,0\rangle + \varepsilon_g|g,2\rangle + \varepsilon_e|e,1\rangle + \ldots \qquad \varepsilon_{g,e} \ll c_{g,e} \ll 1$$

equations of motion:

$$\dot{c}_g = -\kappa c_g + g c_e + \eta \qquad \dot{\varepsilon}_g = -2\kappa\varepsilon_g + g\sqrt{2}\varepsilon_e + \eta\sqrt{2}c_g$$

$$\dot{c}_e = -\gamma c_e - g c_g \qquad \dot{\varepsilon}_e = -(\kappa+\gamma)\varepsilon_e - g\sqrt{2}\varepsilon_g + \eta c_e$$

steady-state solution (strong coupling):

$$|\psi\rangle = |g,0\rangle + \alpha|g,1\rangle - \alpha\tfrac{g}{\gamma}|e,0\rangle - \tfrac{\alpha^2}{\sqrt{2}}\tfrac{g^2}{\gamma^2}|g,2\rangle - \alpha^2\tfrac{g}{\gamma}\left(1+\tfrac{\kappa}{\gamma}\right)|e,1\rangle + \ldots$$

probability for simultaneous transmission of two photons:

$$\left|\langle g,0|\hat{a}^2|\psi\rangle\right|^2 \xrightarrow{g\gg(\gamma,\kappa)} \left|\tfrac{\alpha^2}{2n_S}\right|^2 \gg \left|\alpha^2\right|^2$$

$n_S = \frac{\gamma^2}{2g^2}$ saturation photon number

physical interpretation

Carmichael et al., Opt. Comm. **82**, 73 (1991)

amplitude for two-photon transmission as a product:

$$\langle g,0|\hat{a}^2|\psi\rangle = \langle g,0|\hat{a}|\psi\rangle\langle g,0|\hat{a}|\overline{\psi}\rangle$$

with the collapsed state:

$$|\overline{\psi}\rangle = \frac{\hat{a}|\psi\rangle}{\langle g,0|\hat{a}|\psi\rangle}$$

amplitude for photon emission from steady state $|\psi\rangle$:

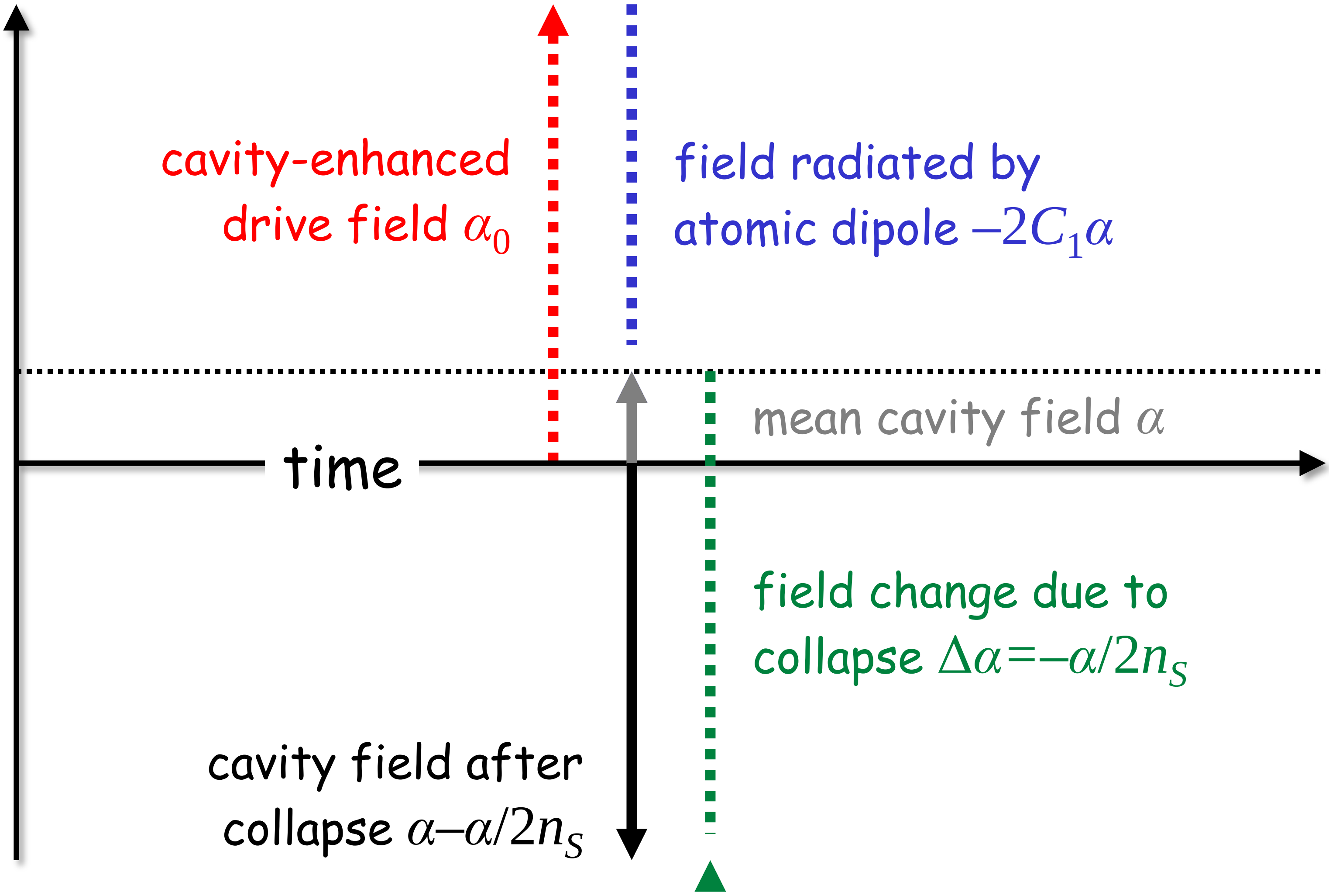
$$\langle g,0|\hat{a}|\psi\rangle = \alpha = \alpha_0 - 2C_1\alpha$$

amplitude for photon emission from collapsed state $|\overline{\psi}\rangle$:

$$\langle g,0|\hat{a}|\overline{\psi}\rangle = \alpha + \Delta\alpha \xrightarrow{g\gg(\gamma,\kappa)} \alpha - \frac{\alpha}{2n_S} \qquad \Rightarrow \qquad \Delta\alpha \gg -\alpha$$

physical interpretation

Carmichael et al., Opt. Comm. **82**, 73 (1991)



cavity-enhanced drive field α_0

field radiated by atomic dipole $-2C_1\alpha$

mean cavity field α

time

field change due to collapse $\Delta\alpha = -\alpha/2n_S$

cavity field after collapse $\alpha - \alpha/2n_S$

cavity QED: two dimensionless numbers

single-atom cooperativity $C_1 = \frac{g^2}{2\gamma\kappa}$

large C_1: one atom has a big effect

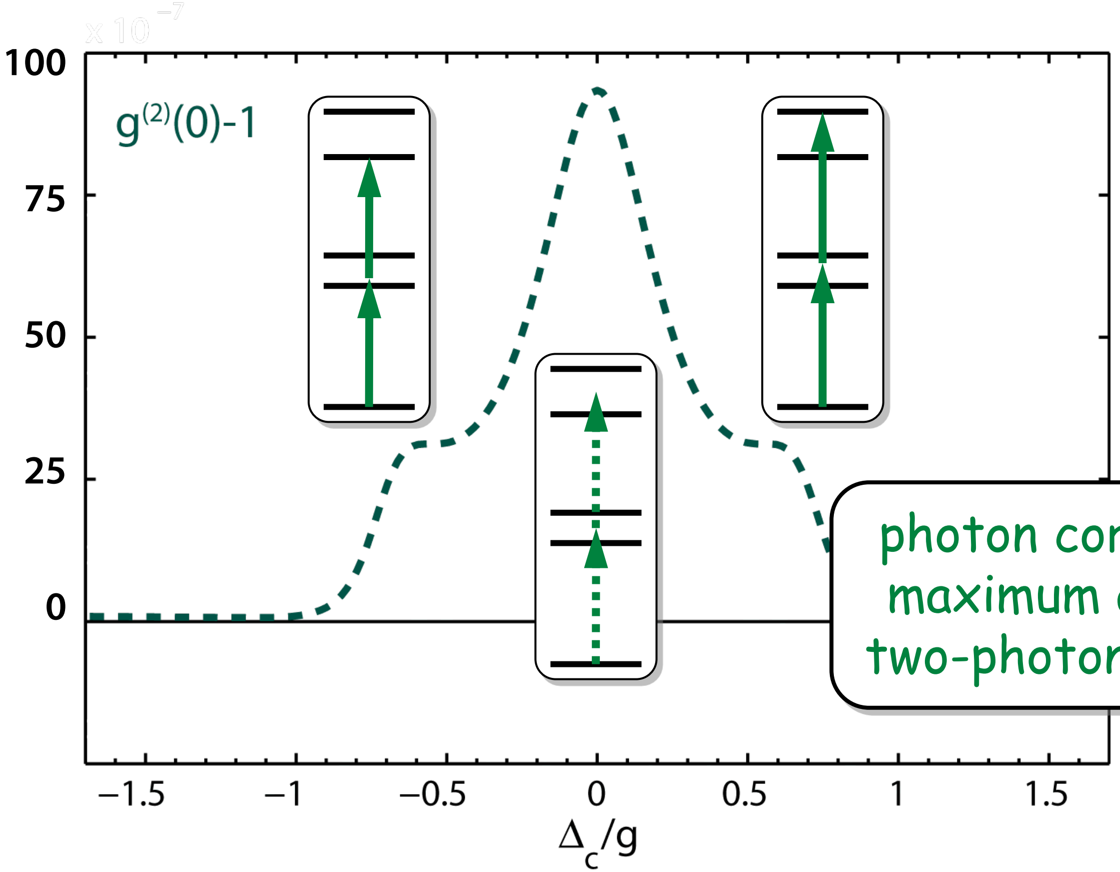
saturation photon number $n_S = \frac{\gamma^2}{2g^2}$

small n_S: one photon has a big effect

photon correlations

normalized correlation function:

$$g^{(2)}(0)=\left\langle a^{\dagger 2}a^{2}\right\rangle \Big/ \left\langle a^{\dagger}a\right\rangle^{2}=2P_{\text{two photons}}\Big/P_{\text{one photon}}^{2}$$

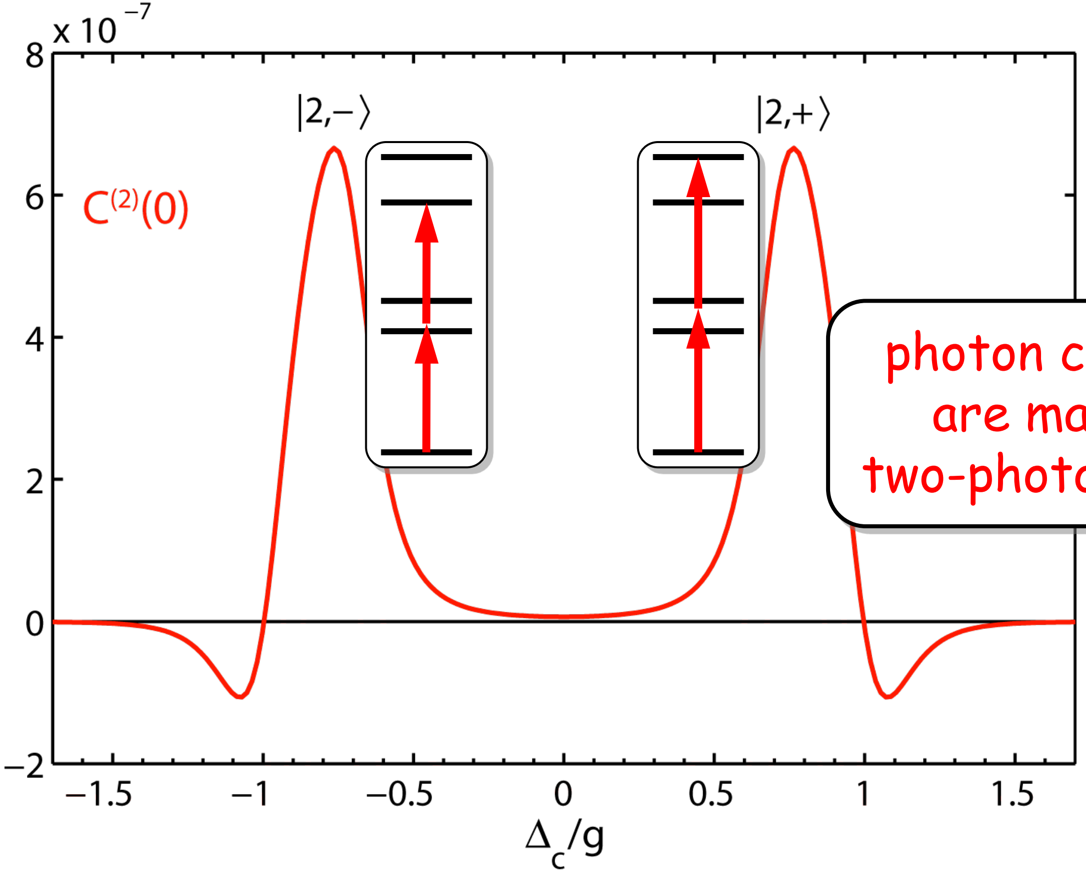


photon correlation is maximum away from two-photon resonance

photon coincidences

differential correlation function:

$$C^{(2)}(0) = \left\langle a^{\dagger 2} a^{2} \right\rangle - \left\langle a^{\dagger} a \right\rangle^{2} = 2P_{\text{two photons}} - P^{2}_{\text{one photon}}$$

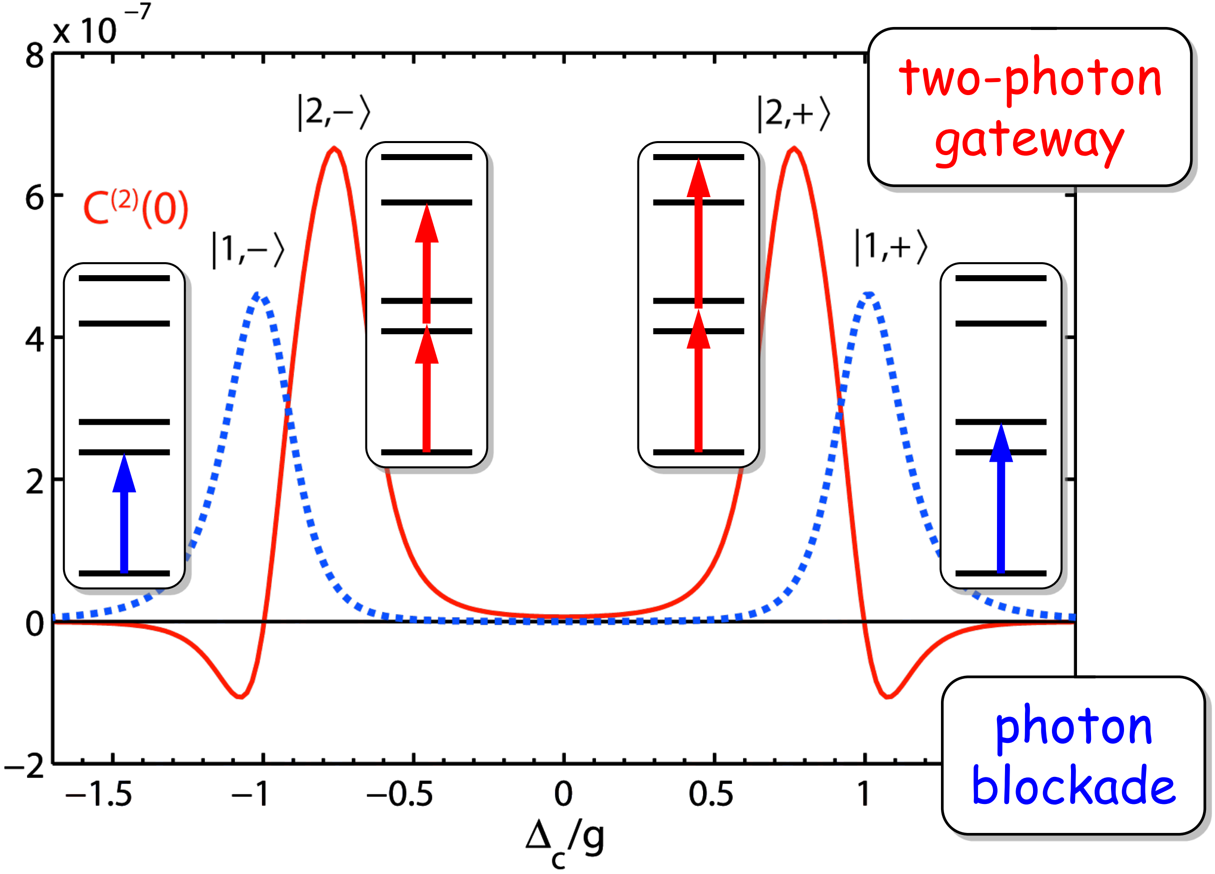


photon coincidences are maximum on two-photon resonance

photon coincidences

differential correlation function:

$$C^{(2)}(0) = \langle a^{\dagger 2} a^2 \rangle - \langle a^\dagger a \rangle^2 = 2P_{\text{two photons}} - P^2_{\text{one photon}}$$

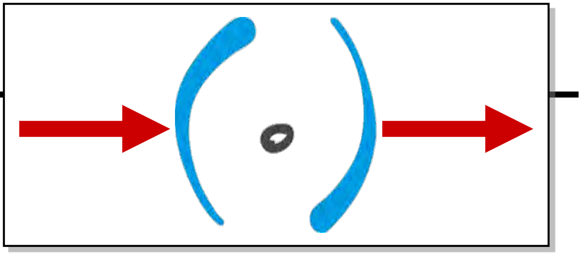


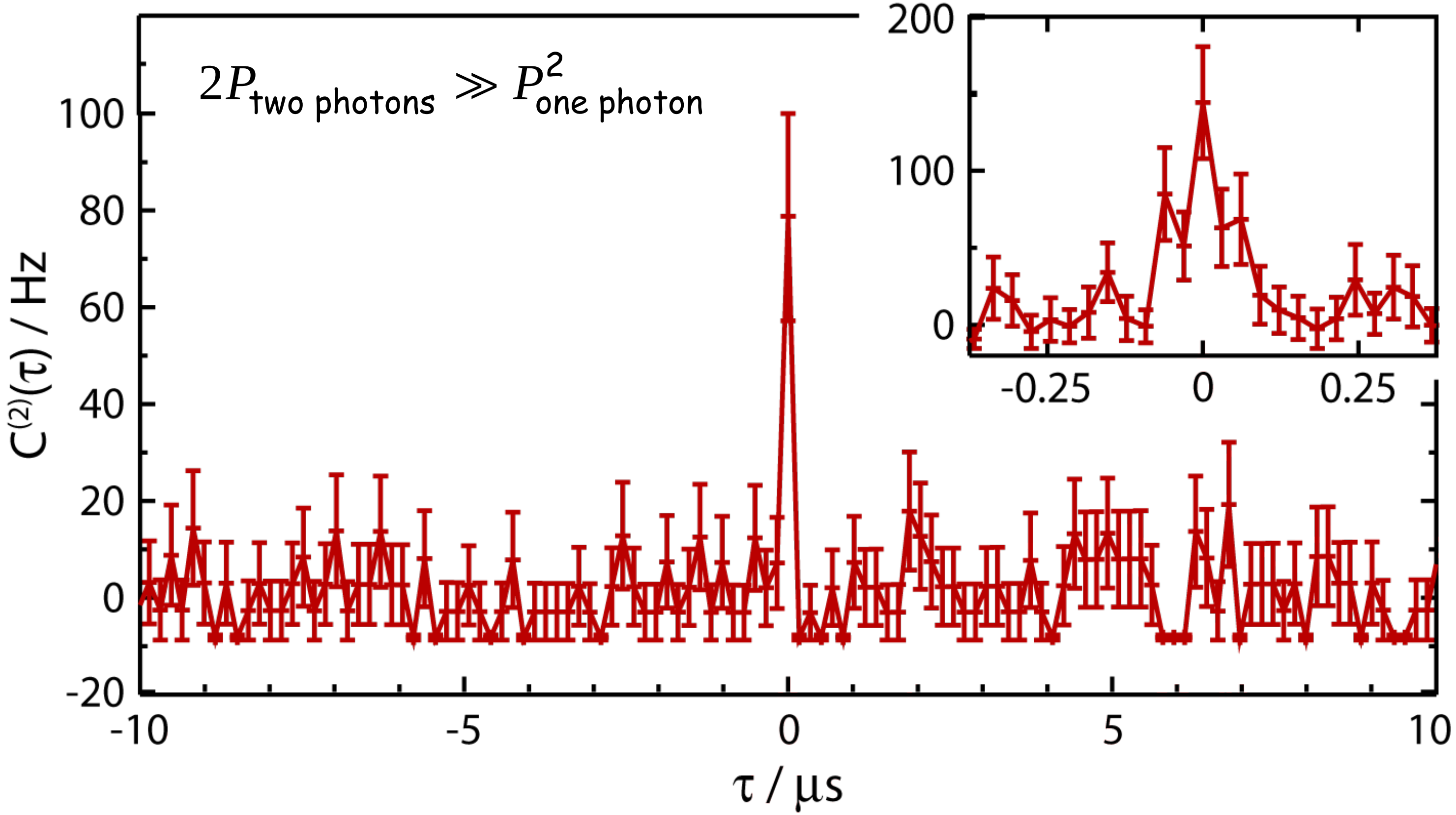
two-photon gateway

photon blockade

two-photon gateway

Kubanek et al., PRL **101**, 203602 (2008)

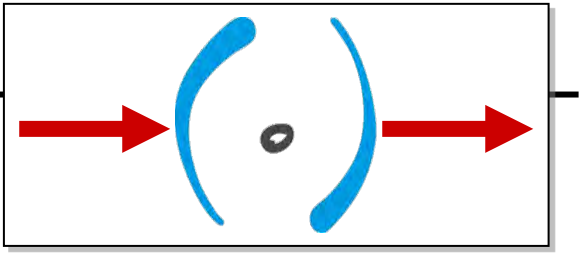


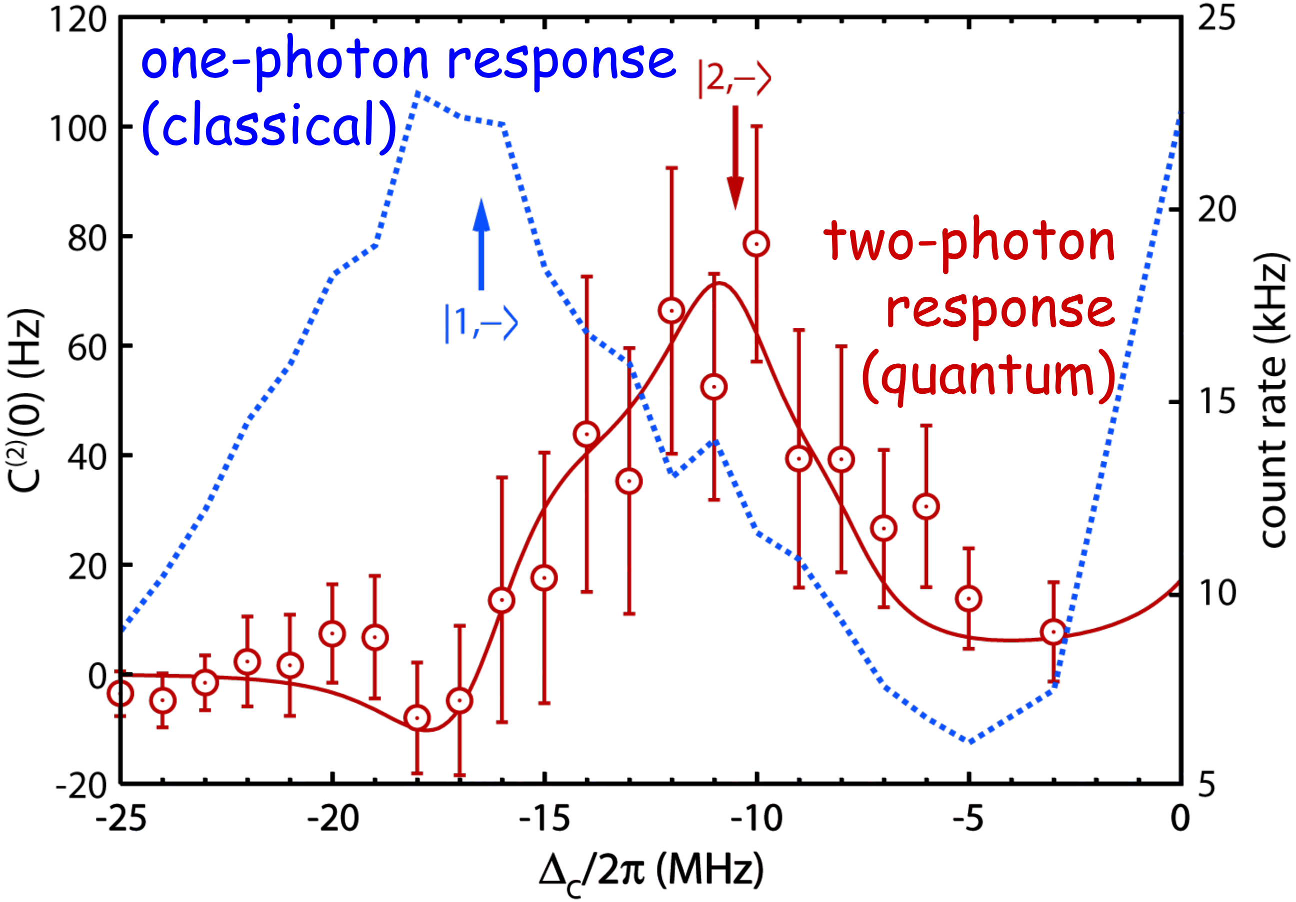


observed coincidence rate ~10 times higher than for a coherent field of equal intensity

photon coincidence spectroscopy

Kubanek et al., PRL **101**, 203602 (2008)





~20 000 atoms, ~127 hours measurement time

question

ask student whether photons transmitted through resonant cavity containing a single resonant atom arrive …

a. … regularly = anti-bunched:
one atom can emit only one photon,

b. … randomly:
laser beam is a random stream of photons,

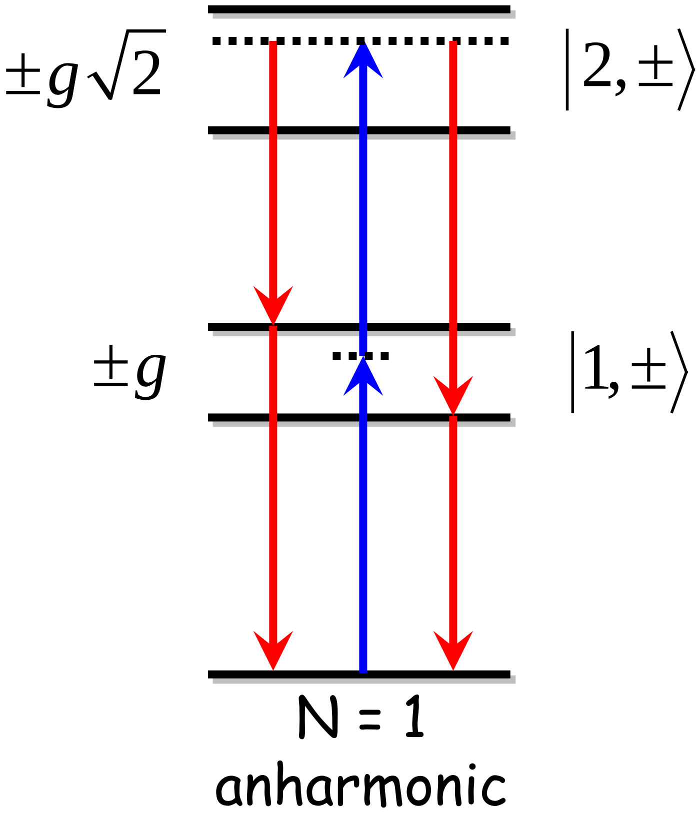
c. … chaotic = bunched:
coherent laser interferes with incoherent spontaneous emission,

d. … none of the above?

cavity QED at its best: quantum anharmonicity

from incoherent decay …

… to coherent scattering



quadrature squeezing:

$$X_\Theta = \tfrac{1}{2}\left(e^{-i\Theta}a + e^{+i\Theta}a^\dagger\right)$$

$$\Delta X_\Theta = X_\Theta - \langle X_\Theta \rangle$$

from free space …

Walls & Zoller, Phys. Rev. Lett. **47**, 709 (1981)

single-atom in free-space:

$$\sigma = |g\rangle\langle e|$$

$$\left\langle \left(\sigma - \langle\sigma\rangle\right)^2 \right\rangle = -\langle\sigma\rangle^2$$

The squeezing generated by one atom is at least an order of magnitude more difficult to observe than anti-bunching

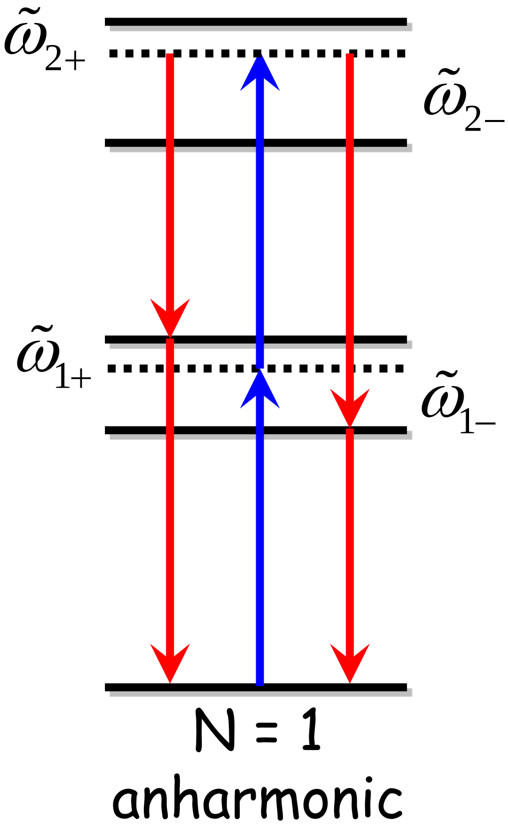
L Mandel, PRL 1982

but:

- small squeezing predicted
  low driving, photon number « 1
- detection solid angle « 4π
- mode matching is difficult

… to confined space

Ourjoumtsev et al., Nature **474**, 623 (2011)



N = 1
anharmonic

single-atom cavity QED:

$$\left\langle \left(a - \langle a \rangle\right)^2 \right\rangle = -K \langle \sigma \rangle^2$$

atomic coherence (1st rung):

$$\langle \sigma \rangle = \frac{\varepsilon g}{\tilde{\omega}_{1+} \tilde{\omega}_{1-}}$$

enhancement (2nd rung):

$$K = \frac{2g^2}{\tilde{\omega}_{2+} \tilde{\omega}_{2-}} \gg 1$$

quadrature squeezing

quadrature electric field:

$$X_\Theta = \tfrac{1}{2}\left(e^{-i\Theta}a + e^{i\Theta}a^\dagger\right)$$

variance:

$$\left\langle \Delta X_\Theta^2 \right\rangle = \left\langle \left(X_\Theta - \left\langle X_\Theta \right\rangle \right)^2 \right\rangle = \underbrace{\tfrac{1}{2}\Re\left(e^{-2i\Theta}\left\langle \Delta a^2 \right\rangle\right)}_{\text{coherent part}} + \underbrace{\tfrac{1}{2}\left\langle \Delta a^\dagger \Delta a \right\rangle}_{\text{incoherent part}} + \tfrac{1}{4}$$

normal ordering (removes $\frac{1}{4}$) and

weak excitation (removes incoherent part):

$$\left\langle : \Delta X_\Theta^2 : \right\rangle = \tfrac{1}{2}\Re\left(e^{-2i\Theta}\left\langle \Delta a^2 \right\rangle\right)$$

master equation

master equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[H_{JC} + H_P, \rho\right] + \kappa L_a \rho + \gamma L_\sigma \rho$$

$$L_a \rho = 2a\rho a^\dagger - \rho a^\dagger a - a^\dagger a \rho$$

$$L_\sigma \rho = 2\sigma\rho\sigma^\dagger - \rho\sigma^\dagger\sigma - \sigma^\dagger\sigma\rho$$

Jaynes-Cummings Hamiltonian:

$$H_{JC}/\hbar = -\Delta_c a^\dagger a - \Delta_a \sigma^\dagger \sigma + g\left(a^\dagger \sigma + a\sigma^\dagger\right)$$

drive Hamiltonian:

$$H_P/\hbar = \varepsilon\left(a^\dagger + a\right)$$

equations of motion

equations of motion:

$$\frac{d}{dt}\left\langle a\right\rangle = i\left(\tilde{\omega}_c\left\langle a\right\rangle - g\left\langle \sigma\right\rangle - \varepsilon\right) \qquad \tilde{\omega}_c = \Delta_c + i\kappa \qquad \Delta_c = \omega - \omega_c$$

$$\frac{d}{dt}\left\langle \sigma\right\rangle = i\left(\tilde{\omega}_a\left\langle \sigma\right\rangle + g\left\langle a\sigma_z\right\rangle\right) \qquad \tilde{\omega}_a = \Delta_a + i\gamma \qquad \Delta_a = \omega - \omega_a$$

$$\frac{d}{dt}\left\langle a^2\right\rangle = 2i\left(\tilde{\omega}_c\left\langle a^2\right\rangle - g\left\langle a\sigma\right\rangle - \varepsilon\left\langle a\right\rangle\right)$$

$$\frac{d}{dt}\left\langle a\sigma\right\rangle = i\left(\left(\tilde{\omega}_a + \tilde{\omega}_c\right)\left\langle a\sigma\right\rangle + g\left\langle a^2\sigma_z\right\rangle - \varepsilon\left\langle \sigma\right\rangle\right)$$

steady-state solutions

steady-state solutions:

$$\langle a\rangle = \varepsilon\frac{\tilde{\omega}_a}{\tilde{\omega}_a\tilde{\omega}_c - g^2} \qquad \langle a^2\rangle = \varepsilon^2\frac{\tilde{\omega}_a\left(\tilde{\omega}_a + \tilde{\omega}_c\right) + g^2}{\left(\tilde{\omega}_c\left(\tilde{\omega}_a + \tilde{\omega}_c\right) - g^2\right)\left(\tilde{\omega}_a\tilde{\omega}_c - g^2\right)}$$

$$\langle\sigma\rangle = \varepsilon\frac{g}{\tilde{\omega}_a\tilde{\omega}_c - g^2} \qquad \langle a\sigma\rangle = \varepsilon^2\frac{g\left(\tilde{\omega}_a + \tilde{\omega}_c\right)}{\left(\tilde{\omega}_c\left(\tilde{\omega}_a + \tilde{\omega}_c\right) - g^2\right)\left(\tilde{\omega}_a\tilde{\omega}_c - g^2\right)}$$

fluctuations:

$$\langle\Delta a^2\rangle = \frac{-\varepsilon^2 g^4}{\left(\tilde{\omega}_c\left(\tilde{\omega}_a + \tilde{\omega}_c\right) - g^2\right)\left(\tilde{\omega}_a\tilde{\omega}_c - g^2\right)^2} \qquad \langle\Delta a\Delta\sigma\rangle = \frac{\tilde{\omega}_c}{g}\langle\Delta a^2\rangle$$

quadrature squeezing

Ourjoumtsev et al., Nature **474**, 623 (2011)

detunings:

$$\tilde{\omega}_{n\pm} = (n-1)\tilde{\omega}_c + \tfrac{1}{2}(\tilde{\omega}_a + \tilde{\omega}_c) \pm \tfrac{1}{2}\sqrt{4ng^2 + (\tilde{\omega}_a - \tilde{\omega}_c)^2}$$

rewrite:

$$\langle\sigma\rangle = \frac{\varepsilon g}{\tilde{\omega}_{1+}\tilde{\omega}_{1-}} \quad \text{and} \quad \langle\Delta a^2\rangle = K\left(-\langle\sigma\rangle^2\right) \quad \text{with} \quad K = \frac{2g^2}{\tilde{\omega}_{2+}\tilde{\omega}_{2-}}$$

squeezing:

$$\langle :\Delta X_\Theta^2:\rangle = -\tfrac{1}{2}\Re\left(e^{-2i\Theta}K\langle\sigma\rangle^2\right)$$

quadrature autocorrelations

Ourjoumtsev et al., Nature **474**, 623 (2011)

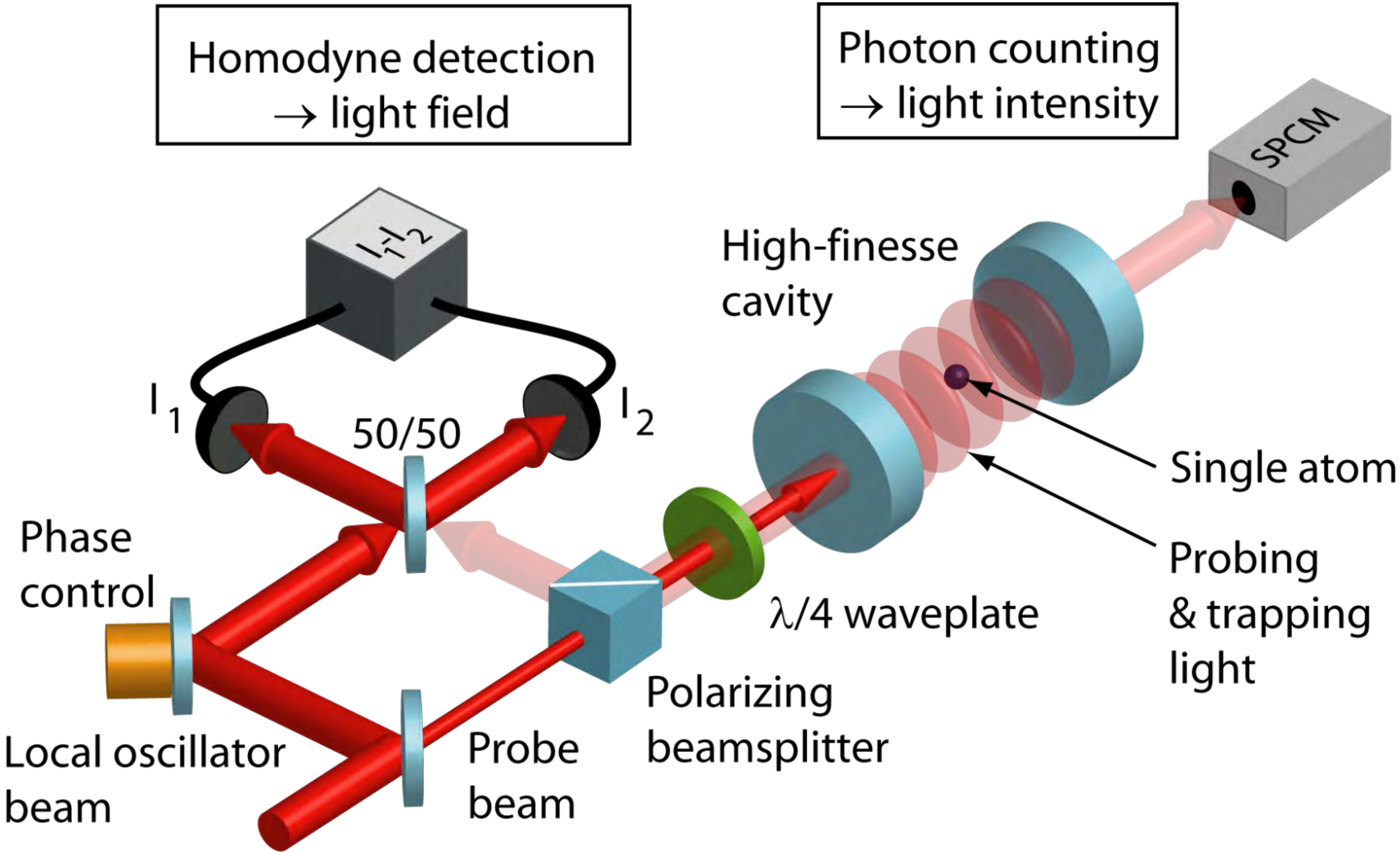
quadrature autocorrelation:

$$\langle : \Delta X_\Theta(\tau)\Delta X_\Theta(0) : \rangle = -\tfrac{1}{2}\Re\left(e^{-2i\Theta} K \langle\sigma\rangle^2 \, f(\tau)\right)$$

regression to steady state:

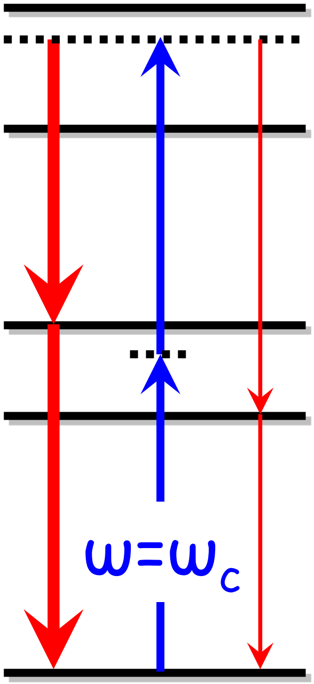
$$f(\tau) = \frac{\exp(i\tilde{\omega}_{1-}\tau)}{1-\tilde{\omega}_{1-}/\tilde{\omega}_{1+}} + \frac{\exp(i\tilde{\omega}_{1+}\tau)}{1-\tilde{\omega}_{1+}/\tilde{\omega}_{1-}}$$

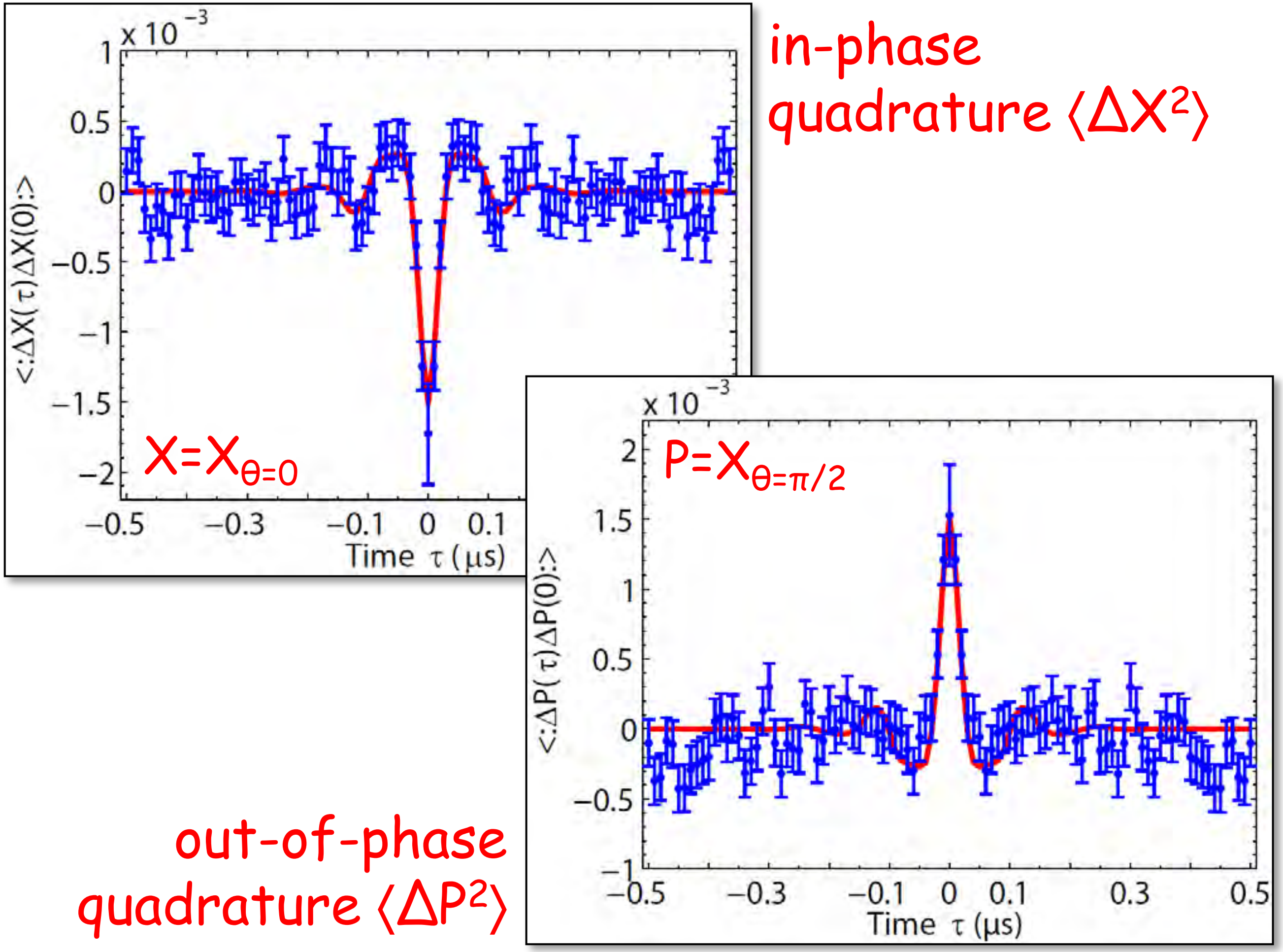
experiment


Homodyne detection
→ light field
Photon counting
→ light intensity
SPCM
I_1-I_2
High-finesse cavity
I_1
I_2
50/50
Single atom
Phase control
Probing & trapping light
λ/4 waveplate
Polarizing beamsplitter
Local oscillator beam
Probe beam

autocorrelation of homodyne signal

Ourjoumtsev et al., Nature **474**, 623 (2011)





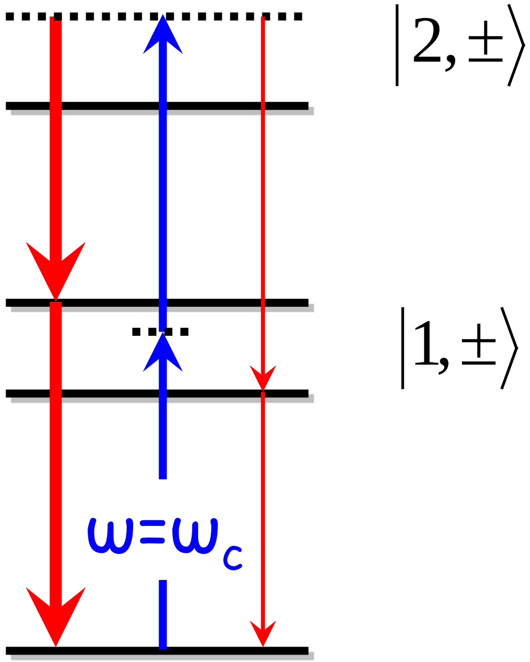
in-phase quadrature $\langle\Delta X^2\rangle$

out-of-phase quadrature $\langle\Delta P^2\rangle$

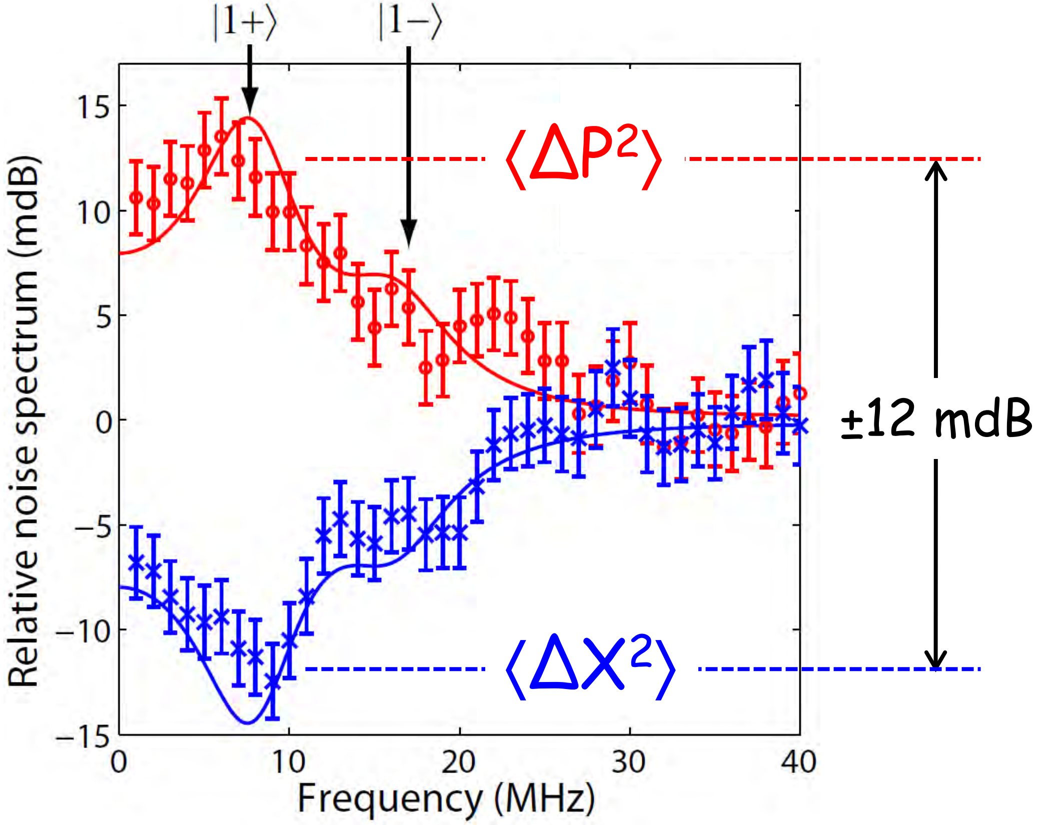
* empty cavity reference subtracted

Fourier transform of autocorrelation

Ourjoumtsev et al., Nature **474**, 623 (2011)



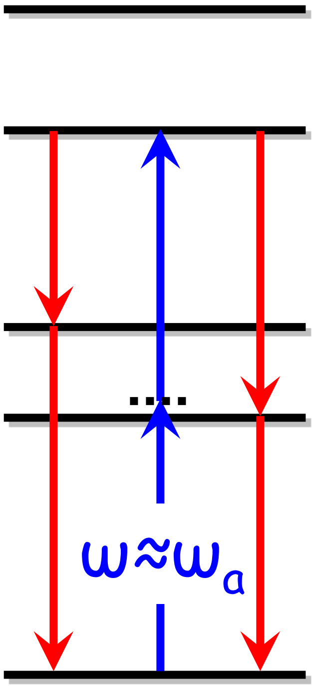
squeezing spectrum

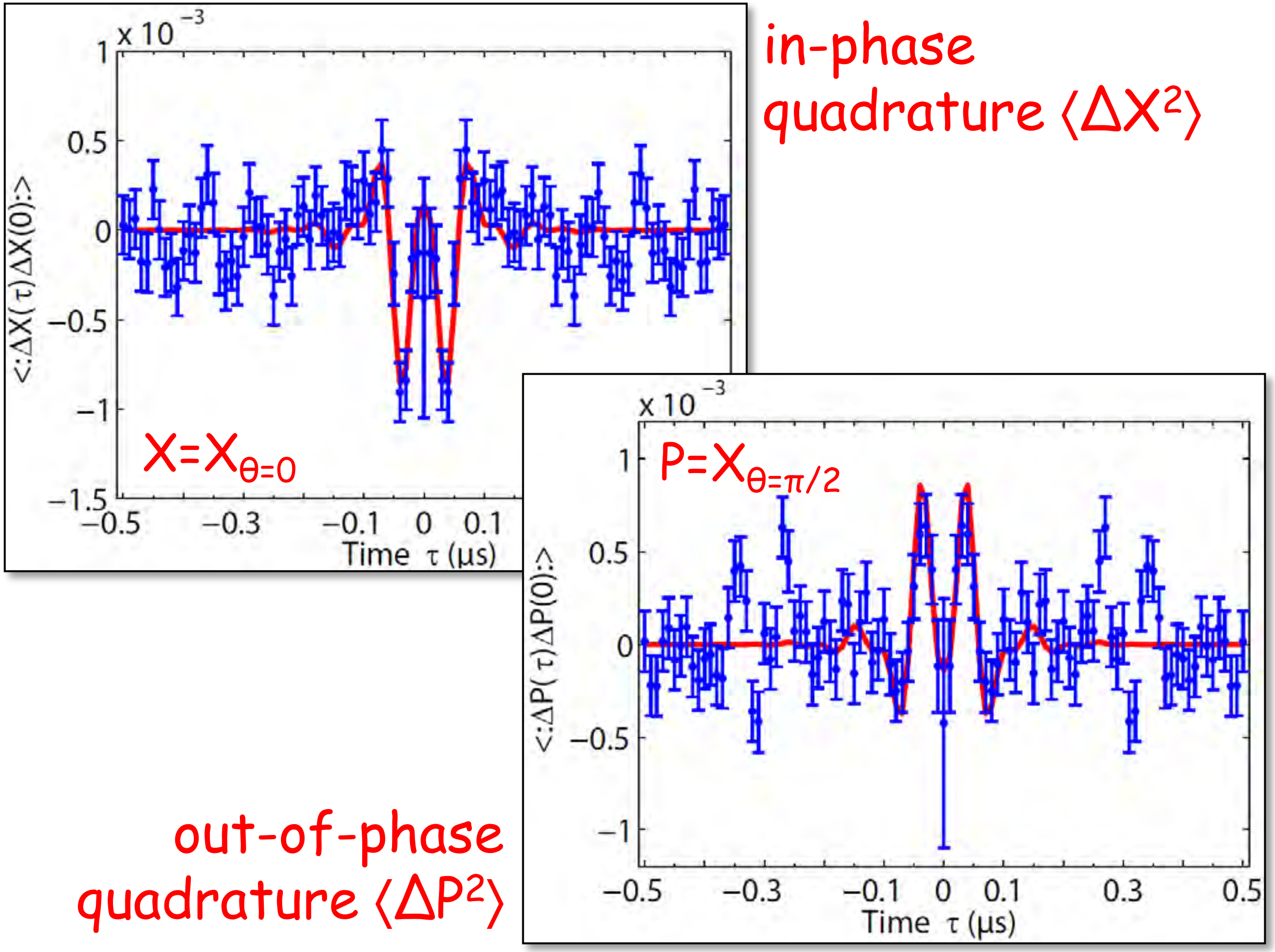


* empty cavity reference subtracted

autocorrelation of homodyne signal

Ourjoumtsev et al., Nature **474**, 623 (2011)





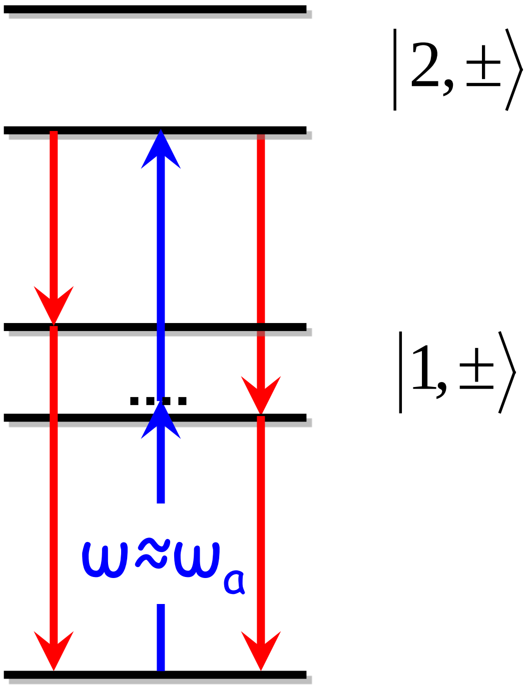
in-phase
quadrature $\langle \Delta X^2 \rangle$

out-of-phase
quadrature $\langle \Delta P^2 \rangle$

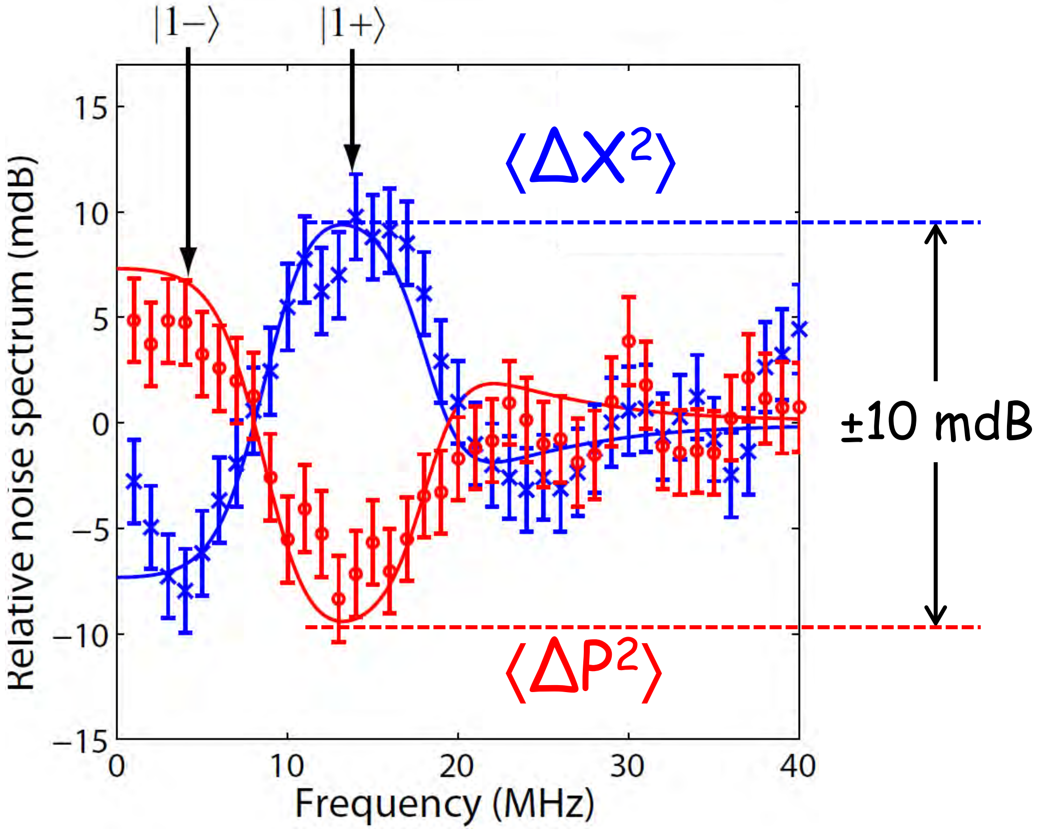
* empty cavity reference subtracted

Fourier transform of autocorrelation

Ourjoumtsev et al., Nature **474**, 623 (2011)



squeezing spectrum

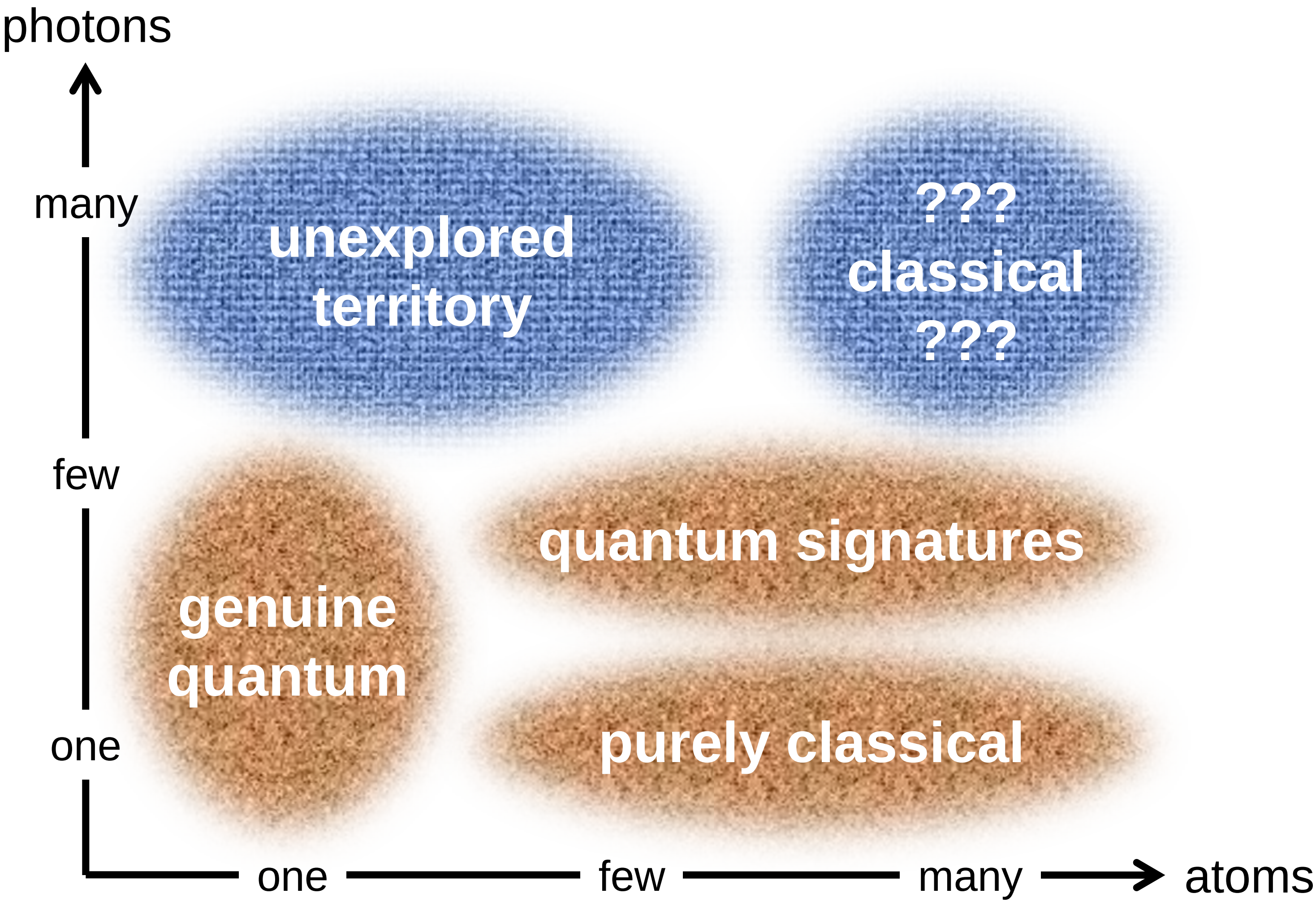


* empty cavity reference subtracted

quantum nonlinear optics

- single atom emits light with reduced amplitude fluctuations (not intensity)
- small absolute squeezing: ~10^{-2} dB, limited by losses & two-sided cavity
- large relative squeezing: pump with 2 photons/60 ns cavity-decay time
- parametric down-conversion at 10 pW pump would give squeezing of 10^{-9} dB
- non-linearity is 10^{7} times larger than Kerr non-linearity of optical fibers
- non-linearity 10^{4} times larger than for four-wave mixing in atomic ensembles

a general perspective


photons

many

few

one

one

few

many

atoms

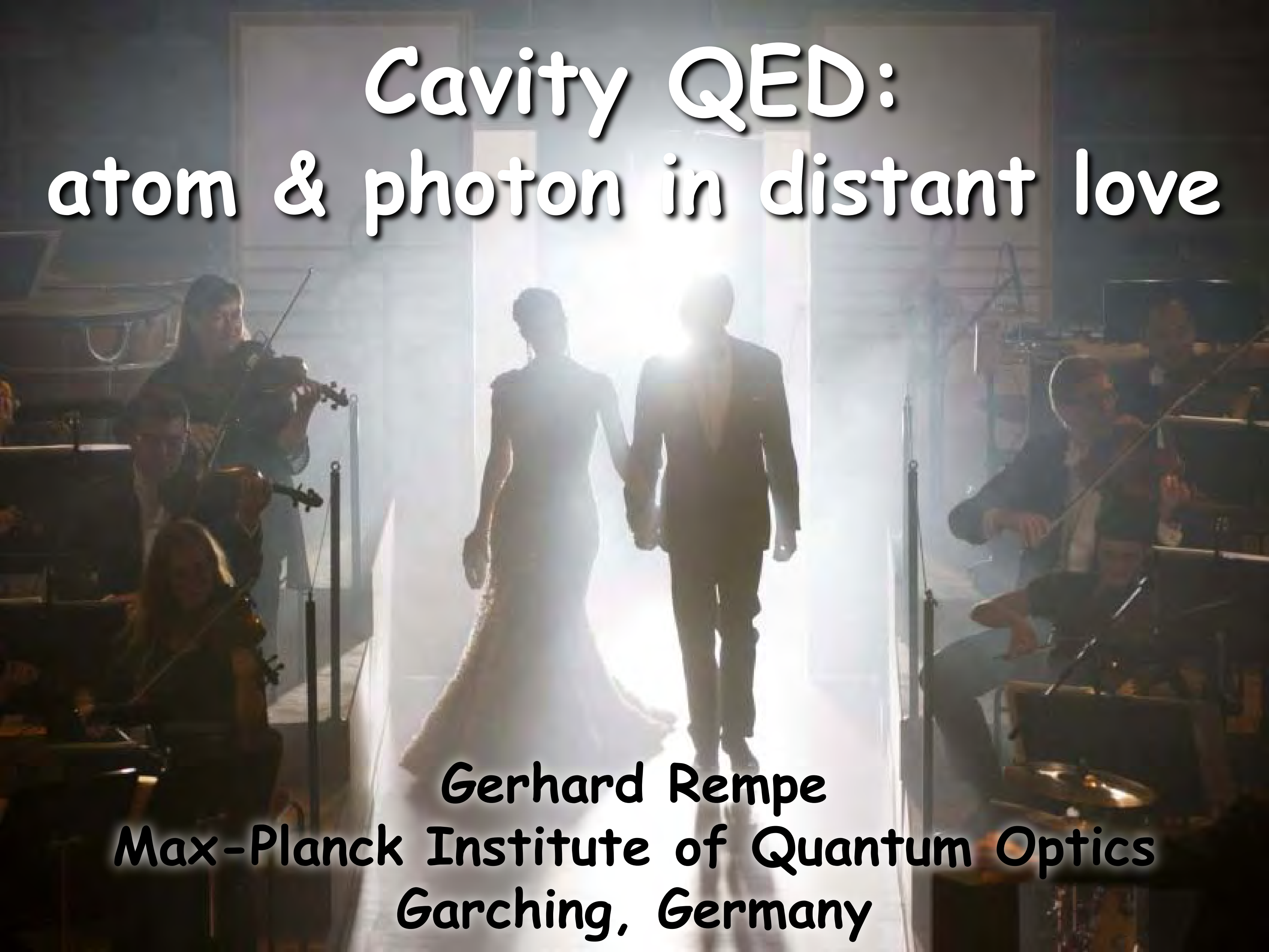
unexplored territory

??? classical ???

genuine quantum

quantum signatures

purely classical



Cavity QED: atom & photon in distant love

Gerhard Rempe

Max-Planck Institute of Quantum Optics

Garching, Germany

quantum information technology: quo vadis?

Unless one is an archaeologist, it is unlikely that one has found the Holy Grail.

Editorial, Nature Physics 3, 581 (2007)

… but quantum physics allows for powerful apps:

quantum simulator: simulates Hamiltonian systems not accessible to classical computers

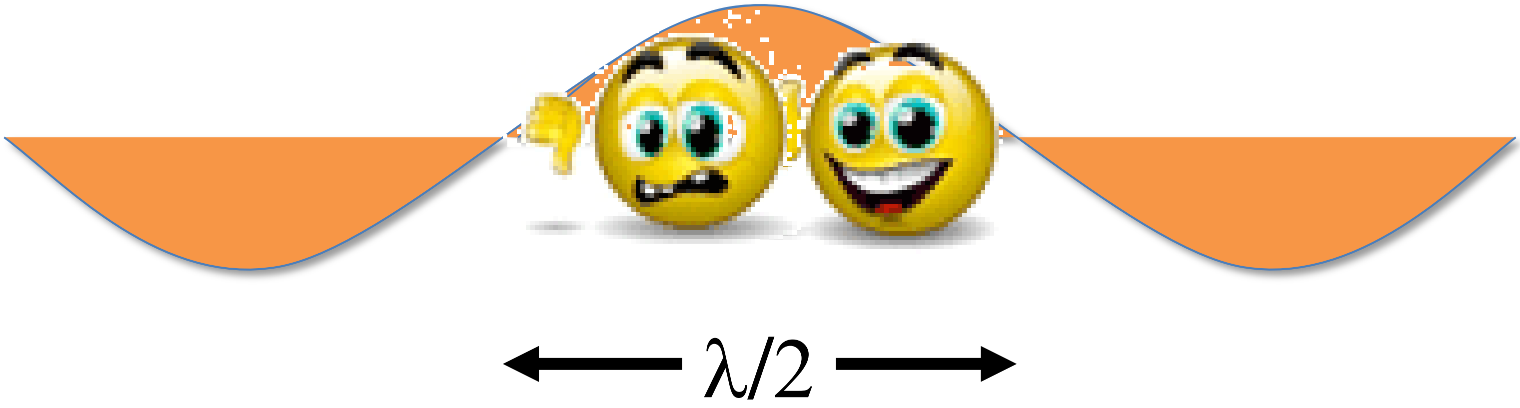
quantum computer: computes complex problems by exploiting the rules of quantum physics

quantum network: distributes information in a secure and controlled way over infinite distances

coupled quantum systems

a fundamental (almost textbook) system:

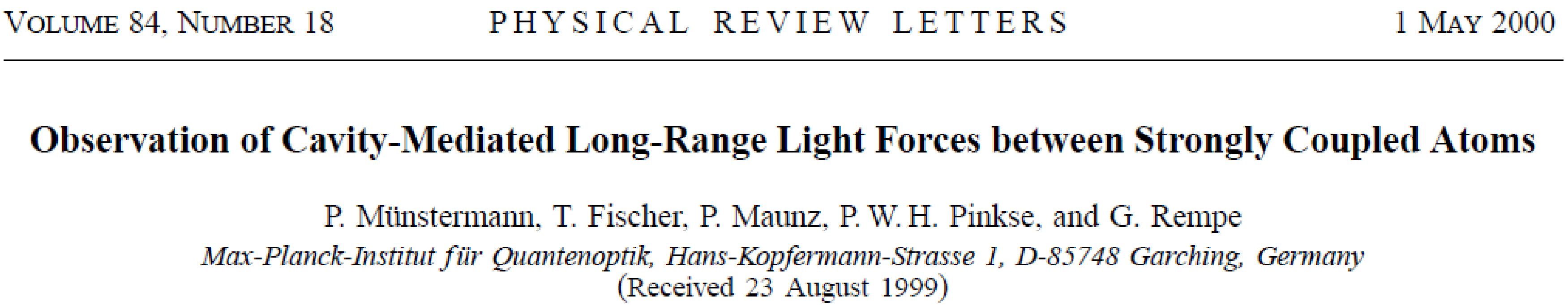
- two atoms exchanging energy & information



however:

- atoms are not individually addressable
- communication over short distance only

coupled quantum systems

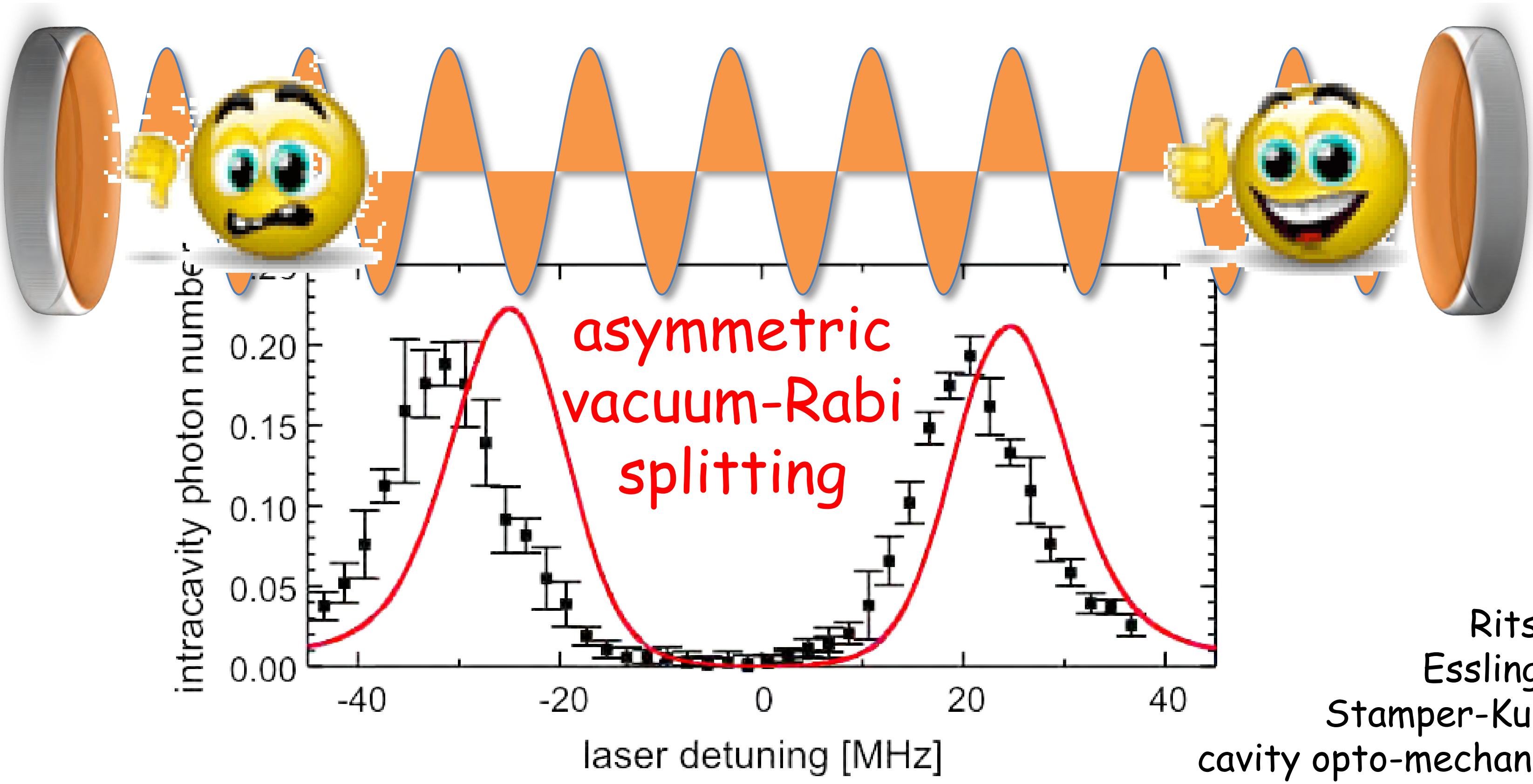


VOLUME 84, NUMBER 18 PHYSICAL REVIEW LETTERS 1 MAY 2000

Observation of Cavity-Mediated Long-Range Light Forces between Strongly Coupled Atoms

P. Münstermann, T. Fischer, P. Maunz, P. W. H. Pinkse, and G. Rempe
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany

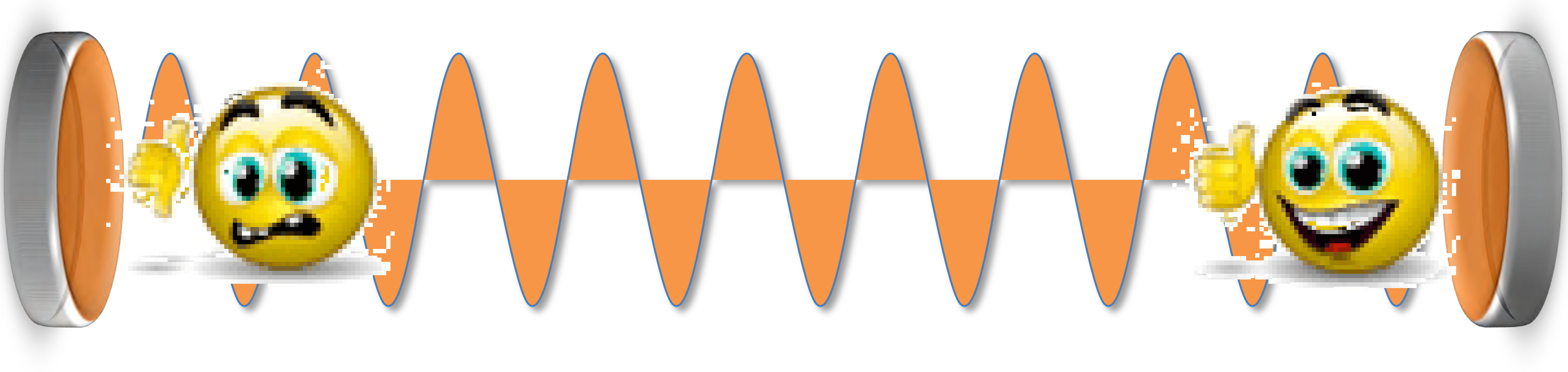
(Received 23 August 1999)



asymmetric
vacuum-Rabi
splitting

Ritsch
Esslinger
Stamper-Kurn:
cavity opto-mechanics

coupled quantum systems

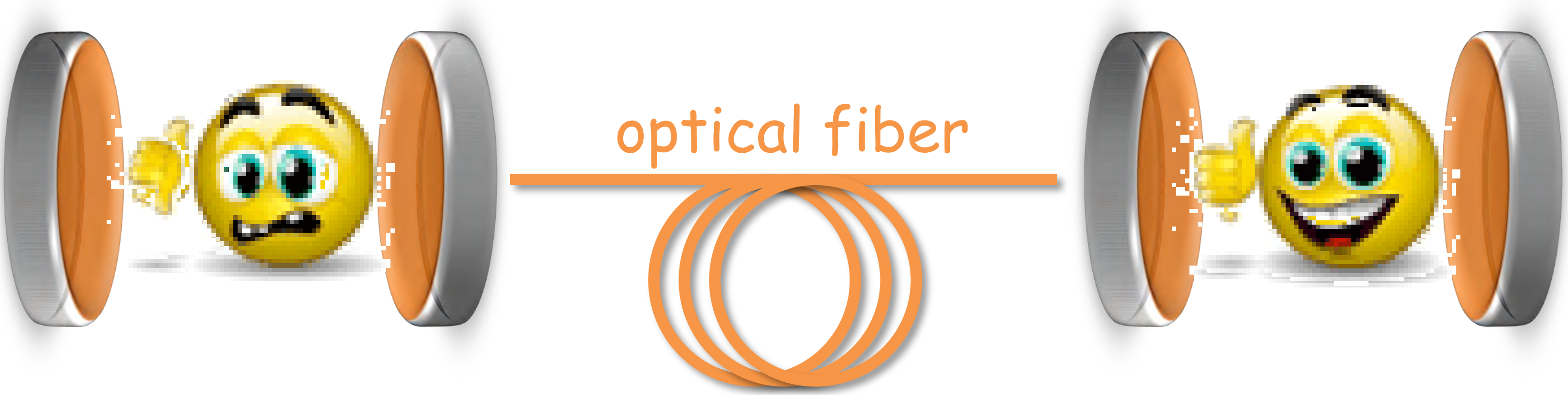


however:

- requires interferometric stability
- not practical for infinite distances

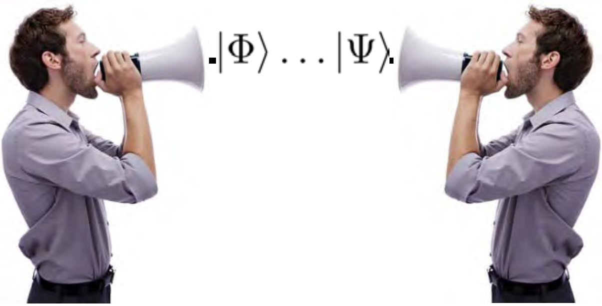
coupled quantum systems

- “infinite” distances
- controlled connectivity
- scalable to many atoms



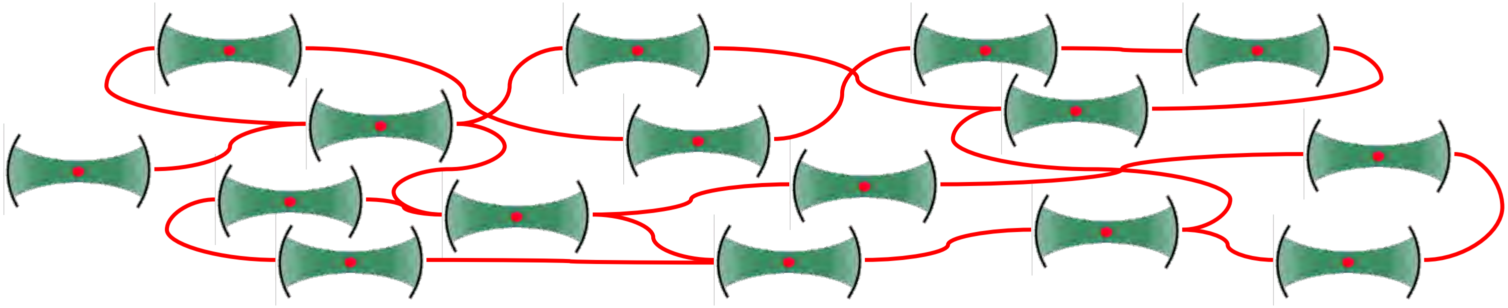
- in theory: one closed system
  → simple Hamiltonian $\hbar\Lambda(a^\dagger b+b^\dagger a)$
- in practice: two open systems
  → extremely challenging task

distributed quantum systems

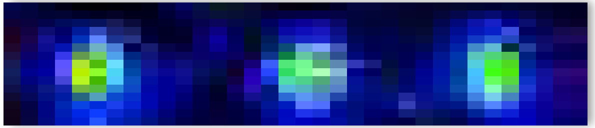


1) quantum communication & quantum computing

2) arbitrary topology = quantum internet: distributed quantum-many body system



3) single atoms = true quantum bits:

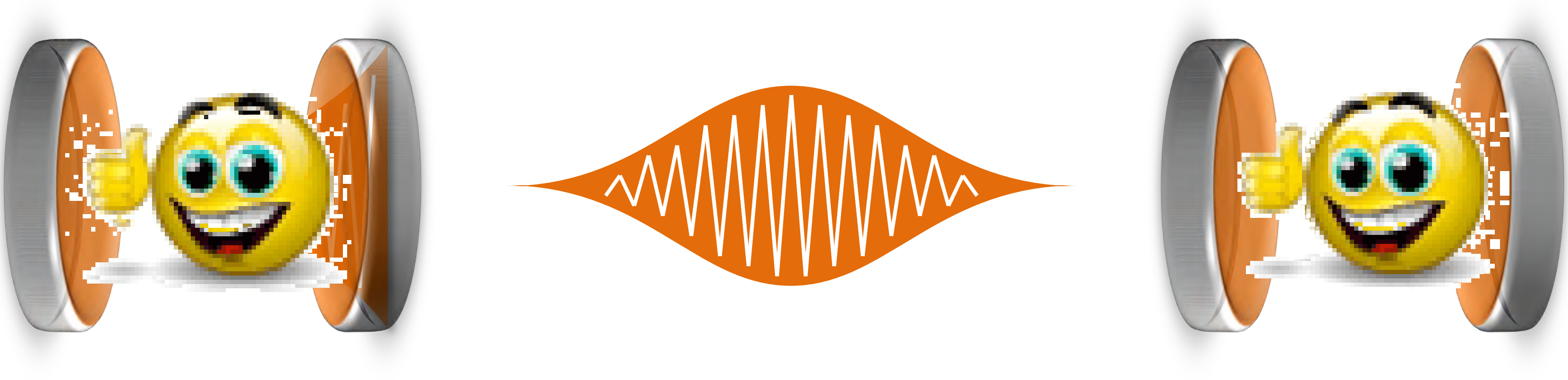


individually addressable & controllable quantum processors (localized information, no crosstalk)

the quest for symmetric photons

spontaneous emission:

- exponential decay → asymmetric photon



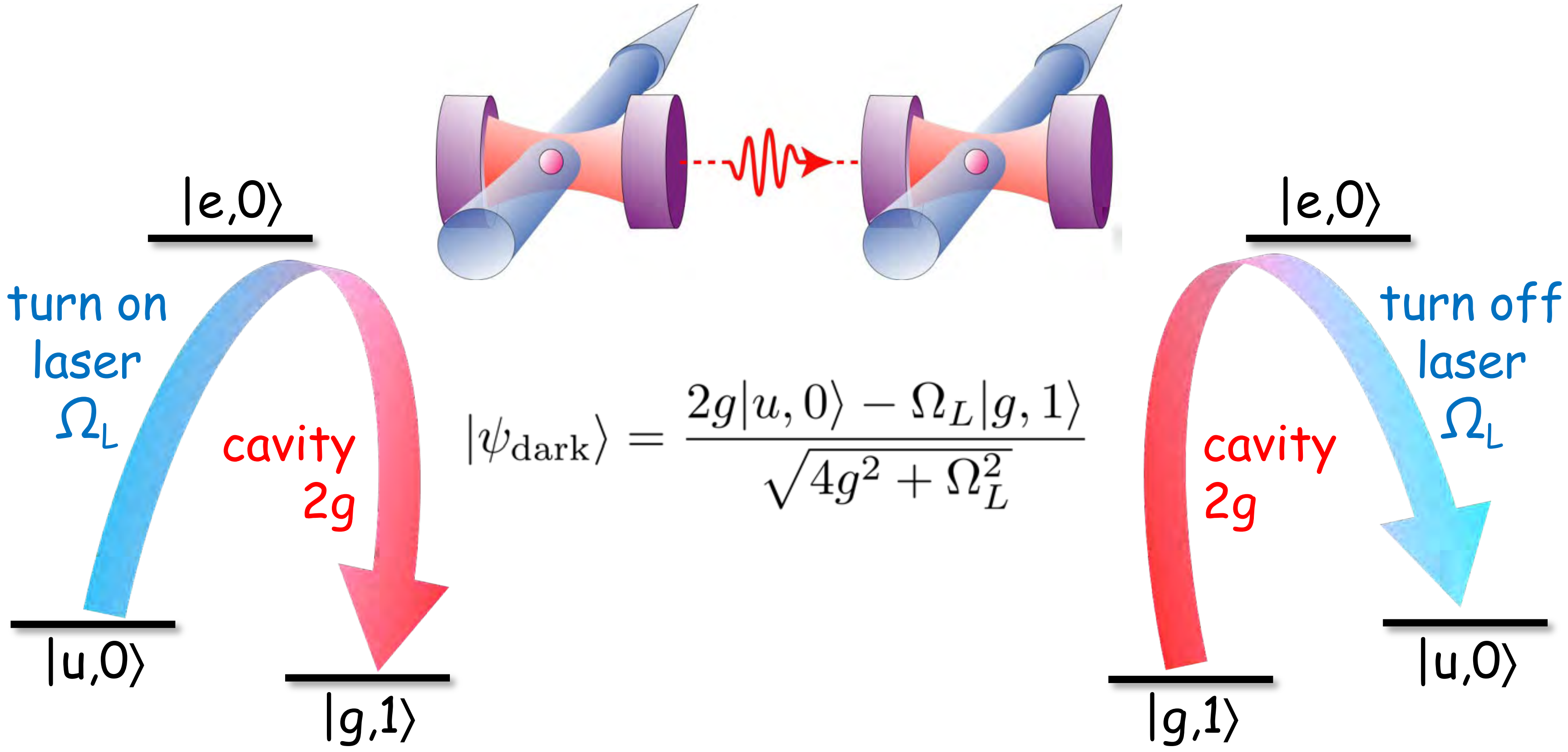
cavity QED:

- inside: controllable atom-photon interaction
- outside: single spatial and temporal mode

vacuum-stimulated Raman adiabatic passage

Hennrich et al., PRL **85**, 4872 (2000)
Kuhn et al., PRL **89**, 067901 (2002)

Mücke et al., Nature **465**, 755 (2010)
Specht et al., Nature **473**, 190 (2011)



$$|\psi_{\mathrm{dark}}\rangle = \frac{2g|u,0\rangle - \Omega_L|g,1\rangle}{\sqrt{4g^2+\Omega_L^2}}$$

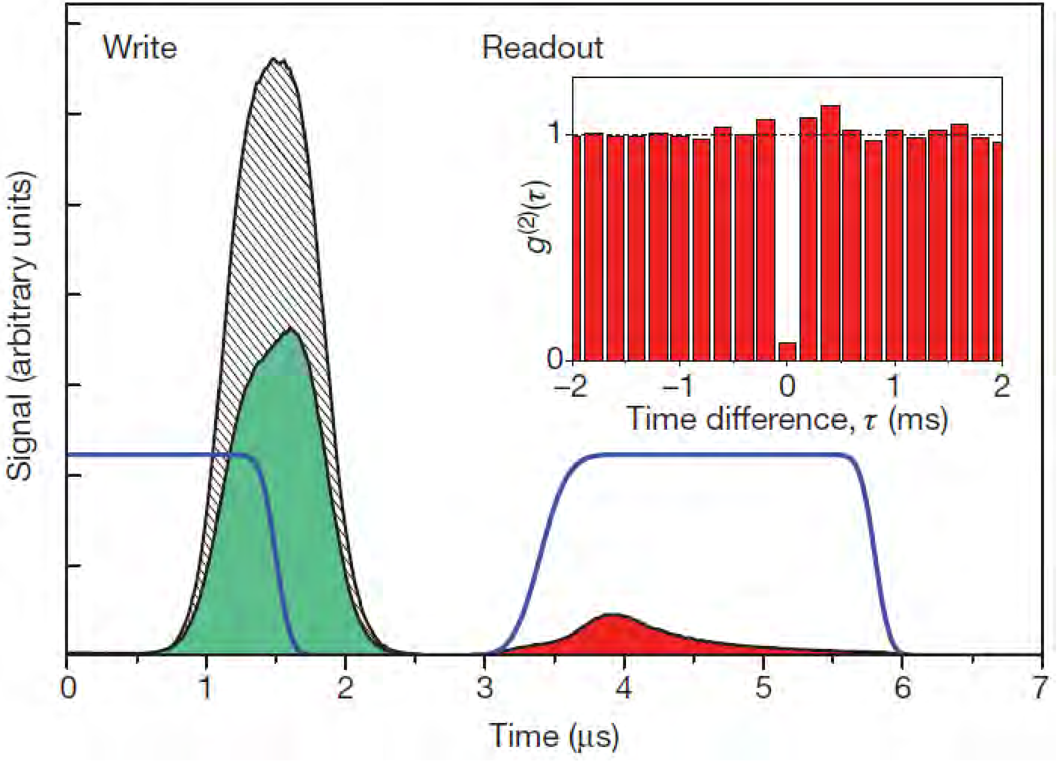
~60% emission probability

~15% absorption probability

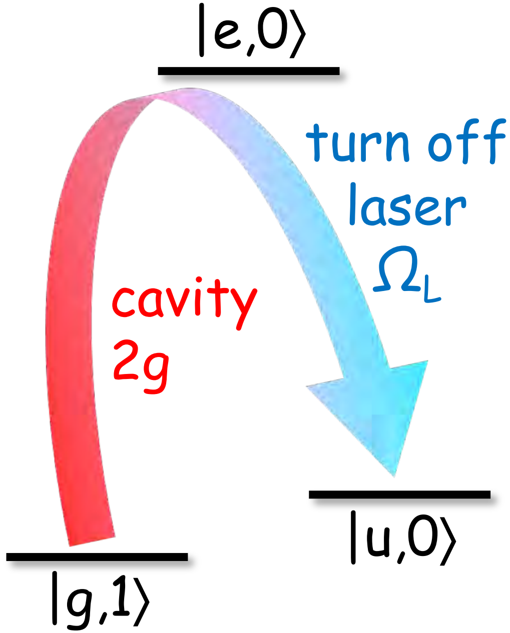
photon-stimulated Raman adiabatic passage

Mücke et al., Nature **465**, 755 (2010)
Specht et al., Nature **473**, 190 (2011)

here: coherent-state input @ <n>=1



10% read-write efficiency



~15% absorption probability

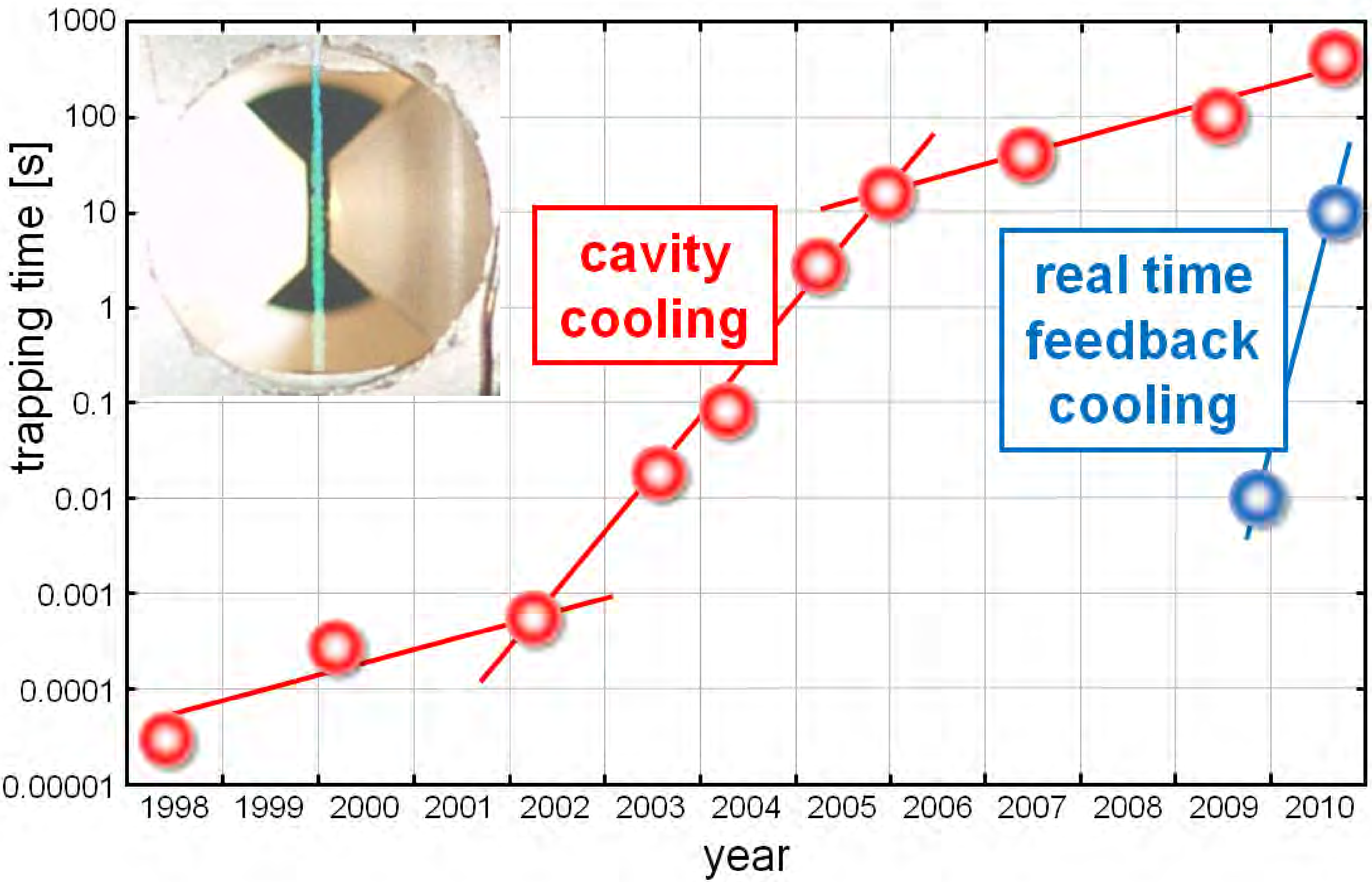
how to get single atoms?

Nußmann et al., PRL **95**, 173602 (2005); see also the work of Chapman and Meschede

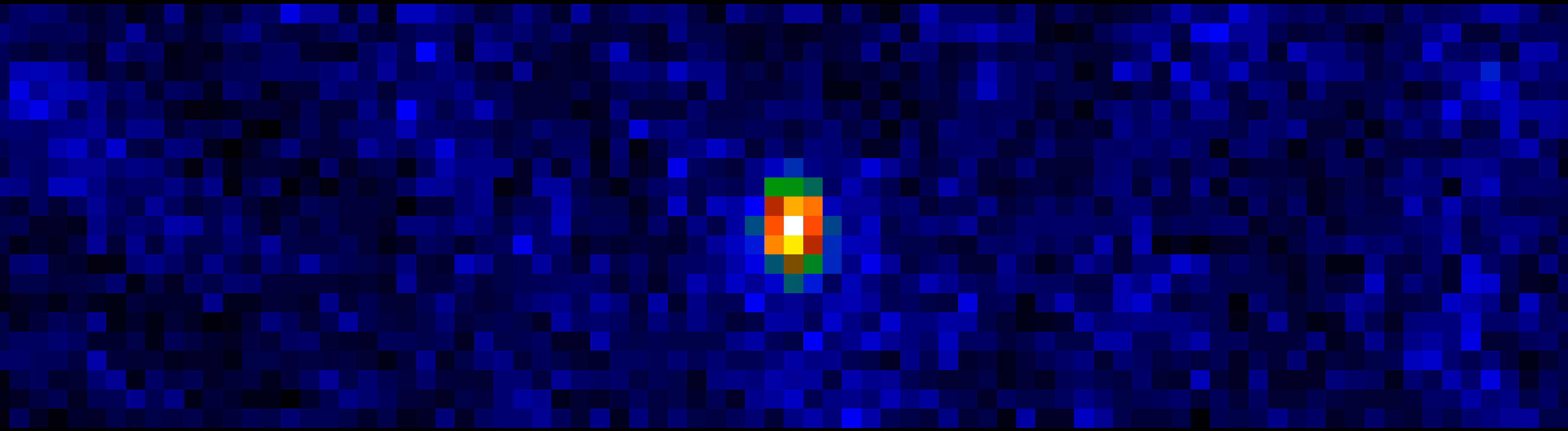
how to cool single atoms?

Pinkse et al., Nature **404**, 365 (2000)
Maunz et al., Nature **428**, 50 (2004)

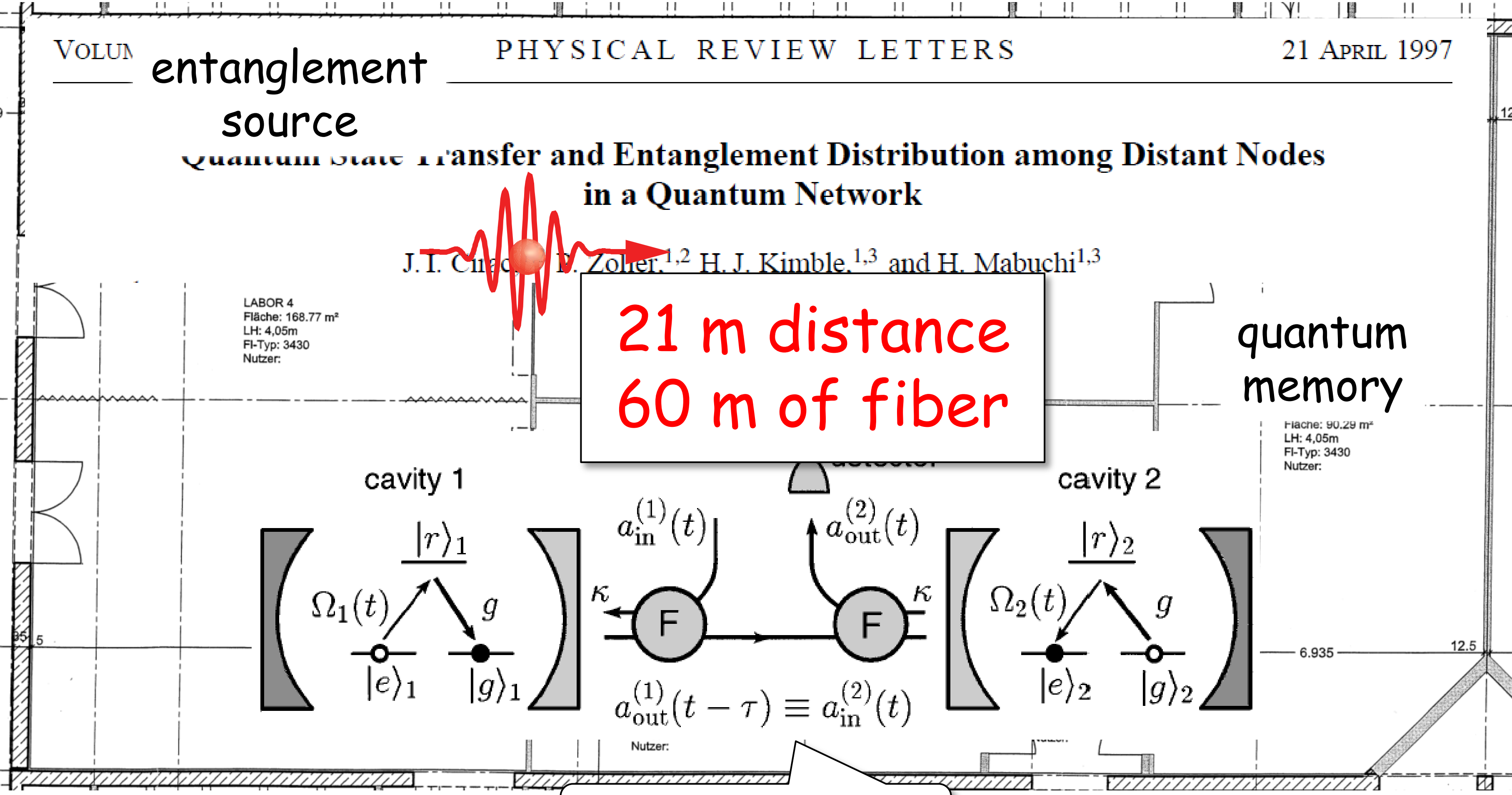
Kubanek et al., Nature **462**, 898 (2009)
Koch et al., PRL **105**, 173003 (2010)



a single atom trapped “forever”



two identical systems



VOLUM

PHYSICAL REVIEW LETTERS

21 APRIL 1997

Transfer and Entanglement Distribution among Distant Nodes in a Quantum Network

J. I. Cira P. Zoller,[1,2] H. J. Kimble,[1,3] and H. Mabuchi[1,3]

entanglement source

quantum memory

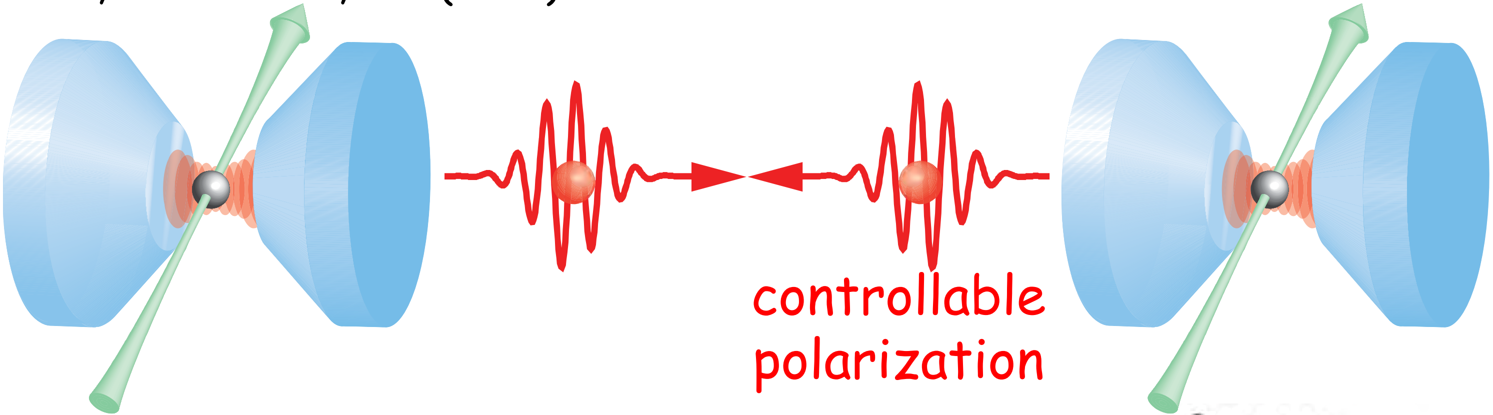
21 m distance
60 m of fiber

symmetric photon

NB: "photon length" exceeds "cavity distance"

quantum memory

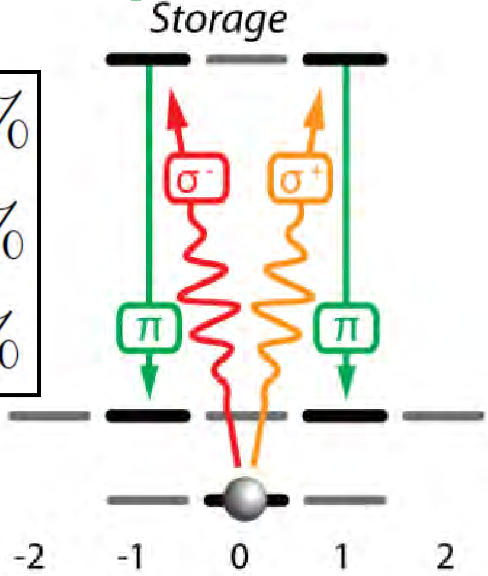
Ritter et al., Nature **484**, 195 (2012)



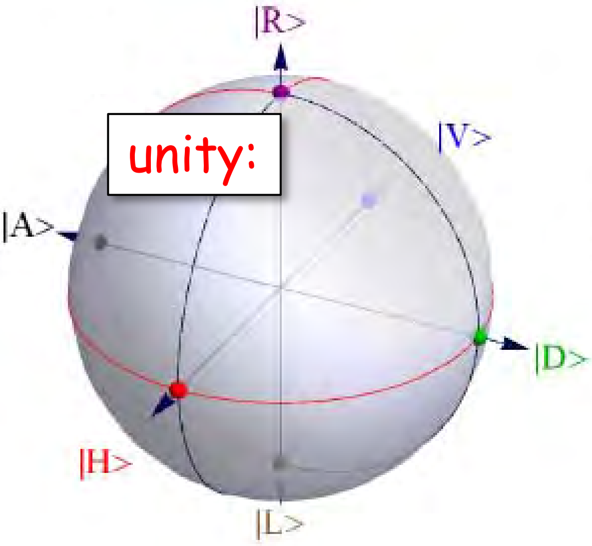
$$|H\rangle : \; 92.2\,(4)\,\%, \qquad |V\rangle : \; 92.0(4)\,\%$$

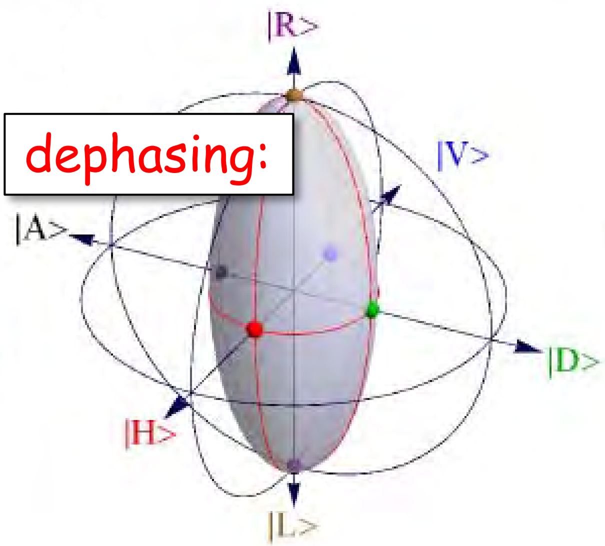
$$|D\rangle : \; 91.9\,(5)\,\%, \qquad |A\rangle : \; 90.9(4)\,\%$$

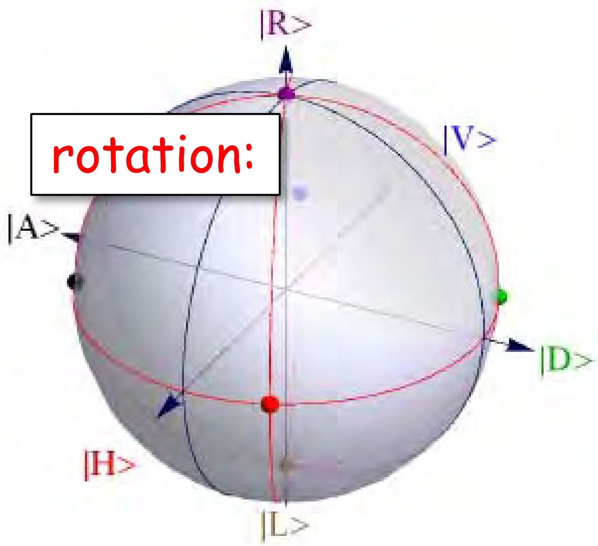
$$|R\rangle : \; 95.1\,(4)\,\%, \qquad |L\rangle : \; 94.2(4)\,\%$$

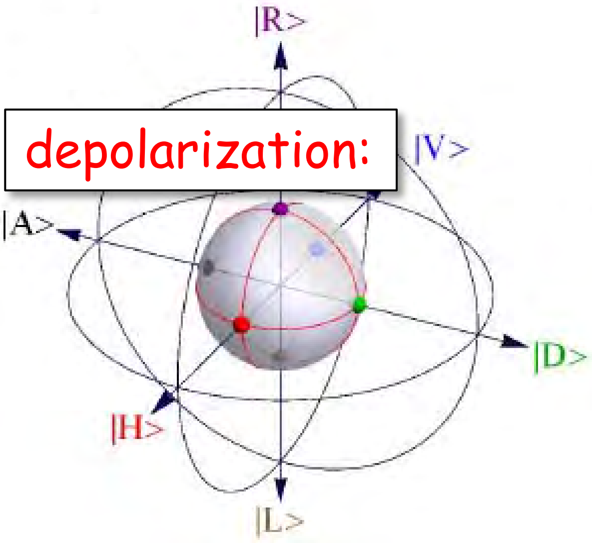


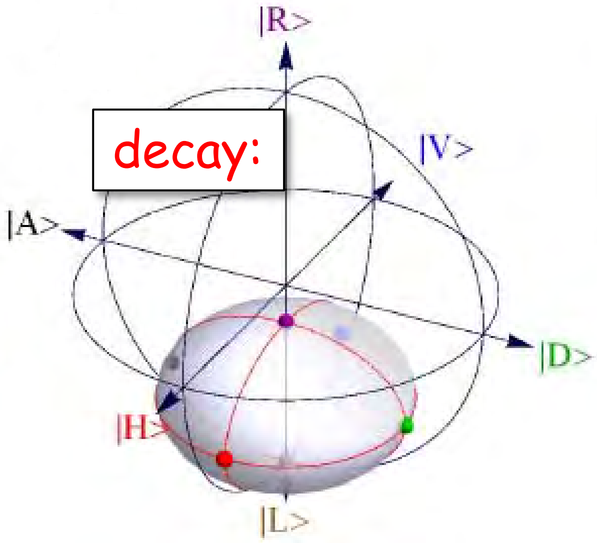
Poincaré “sphere”: examples

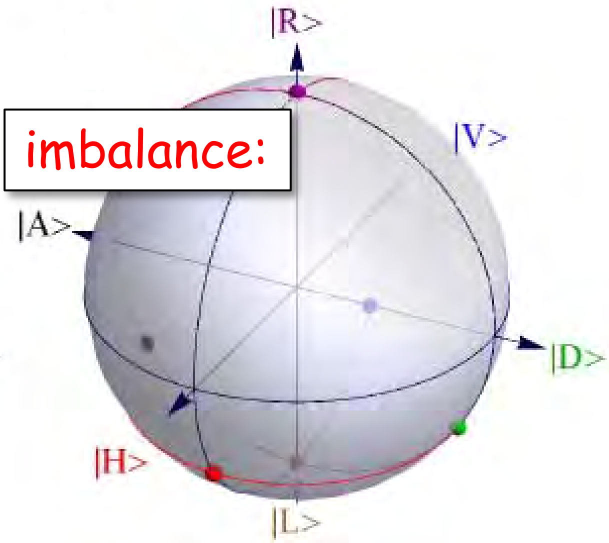

unity:


dephasing:


rotation:

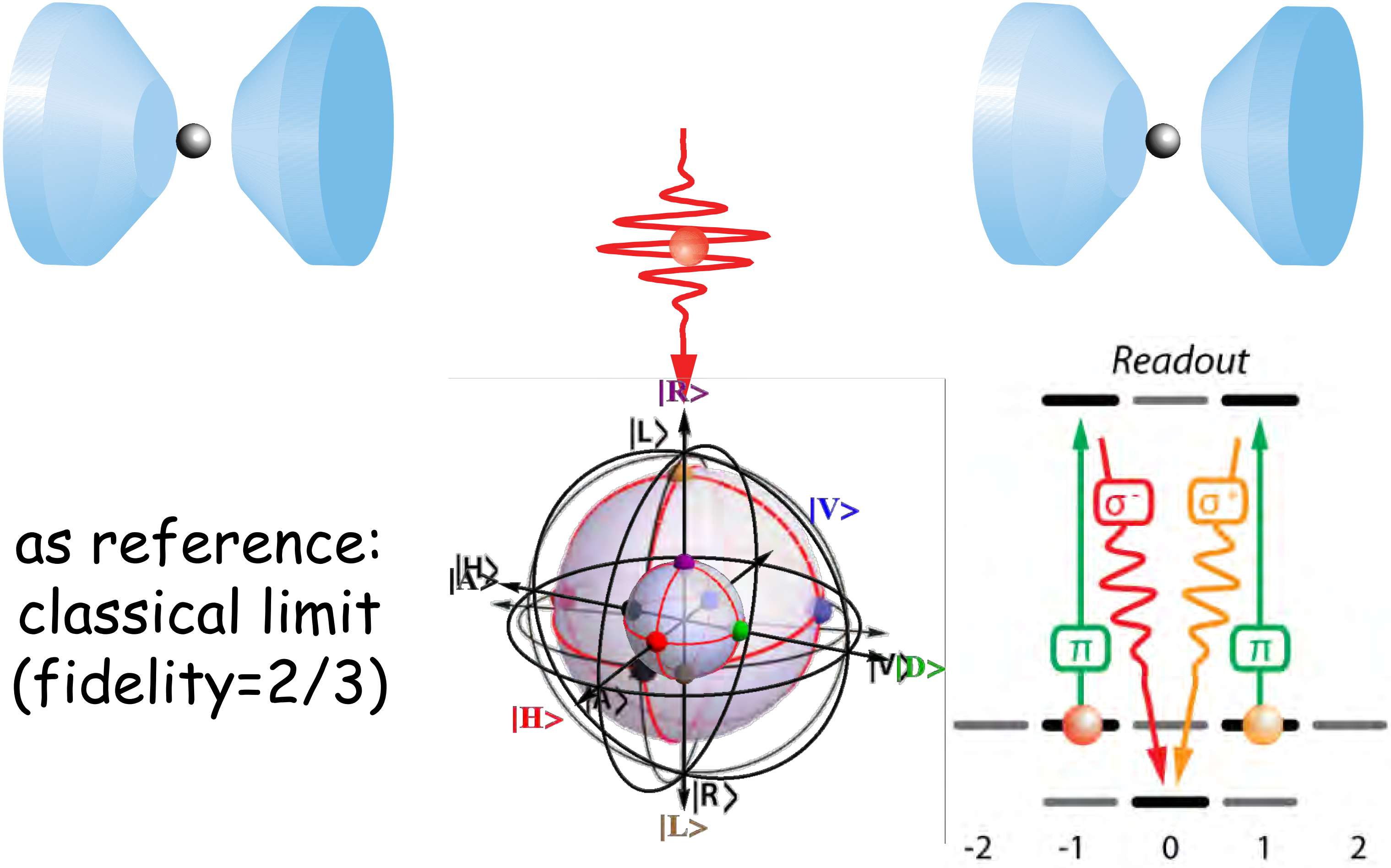

depolarization:


decay:


imbalance:

quantum memory

Ritter et al., Nature **484**, 195 (2012)

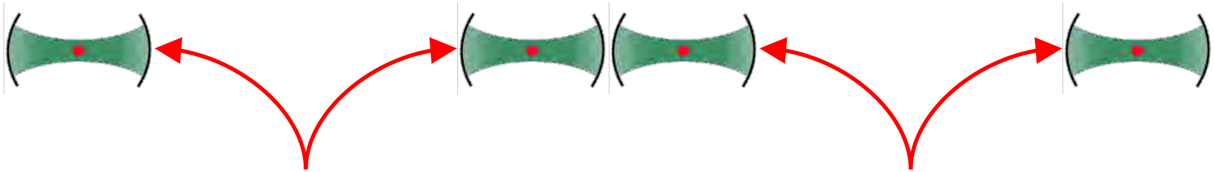


as reference:
classical limit
(fidelity=2/3)

write/read fidelity = 92.2(4)%

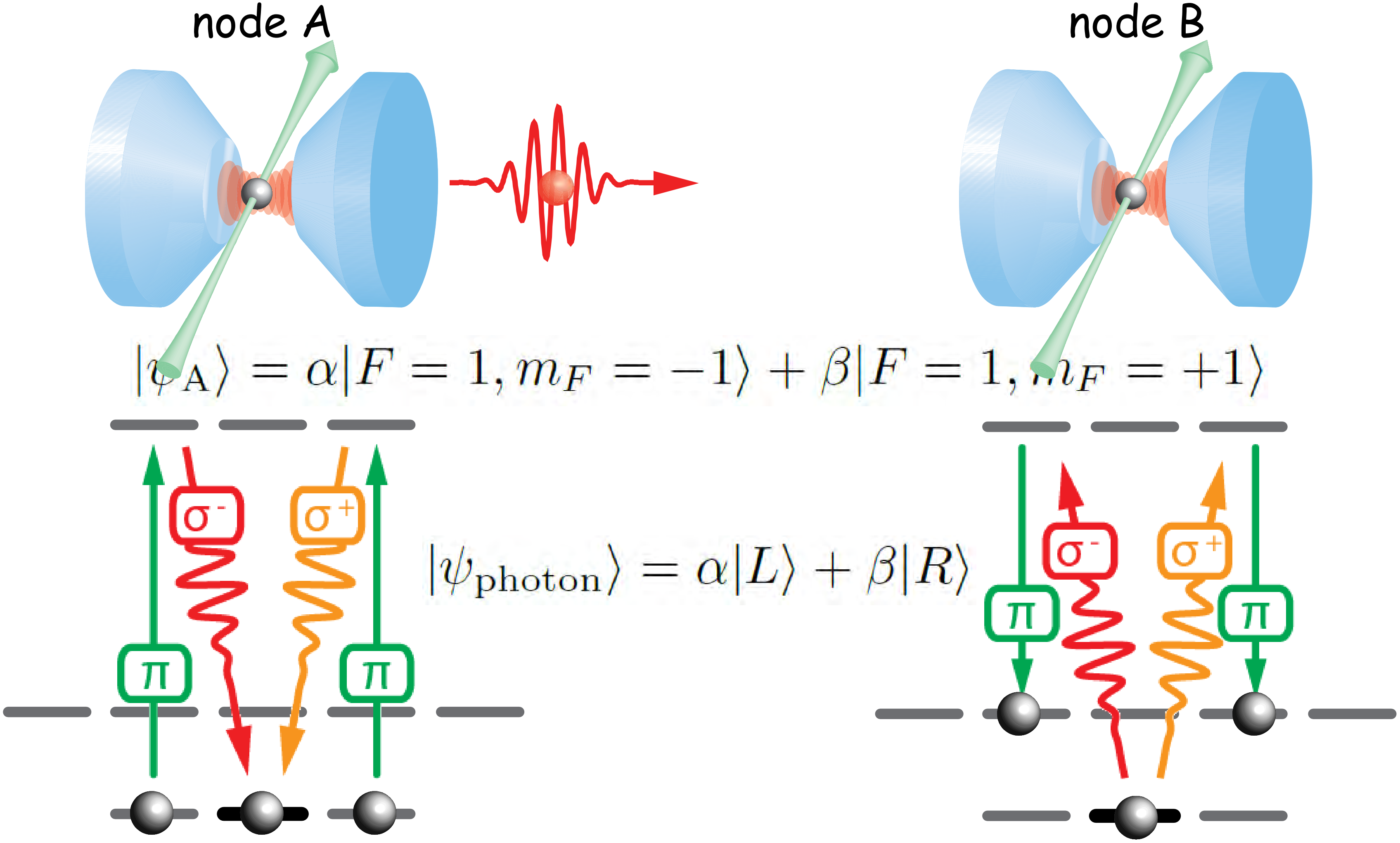
quantum memory: what is long ?

- in optical fibers, 780 nm photons travel 1 km in 5 µs before being lost (3 dB)
- for larger distances/longer times, one needs quantum repeaters
- quantum repeaters require quantum memories



- with 80 µs memory time (and everything else perfect), one could already build 1 repeater station over 8 km

quantum-state transfer



$$|\psi_{\rm A}\rangle = \alpha|F=1, m_F=-1\rangle + \beta|F=1, m_F=+1\rangle$$

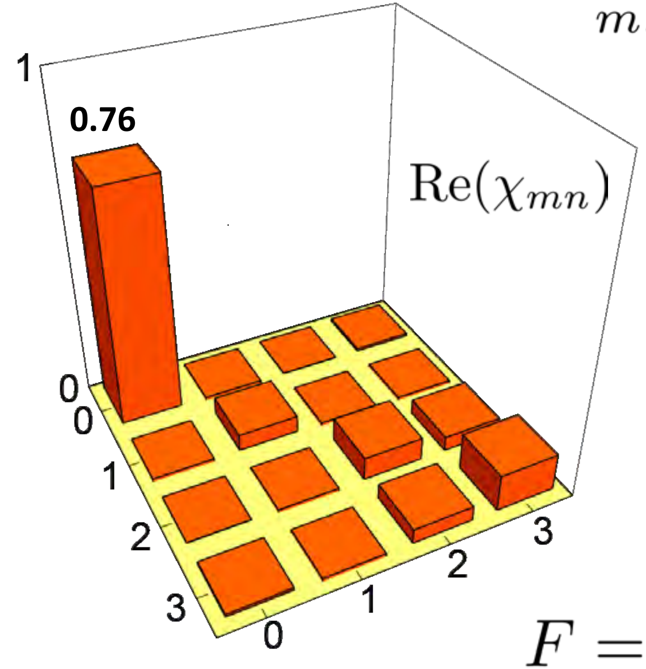
$$|\psi_{\rm photon}\rangle = \alpha|L\rangle + \beta|R\rangle$$

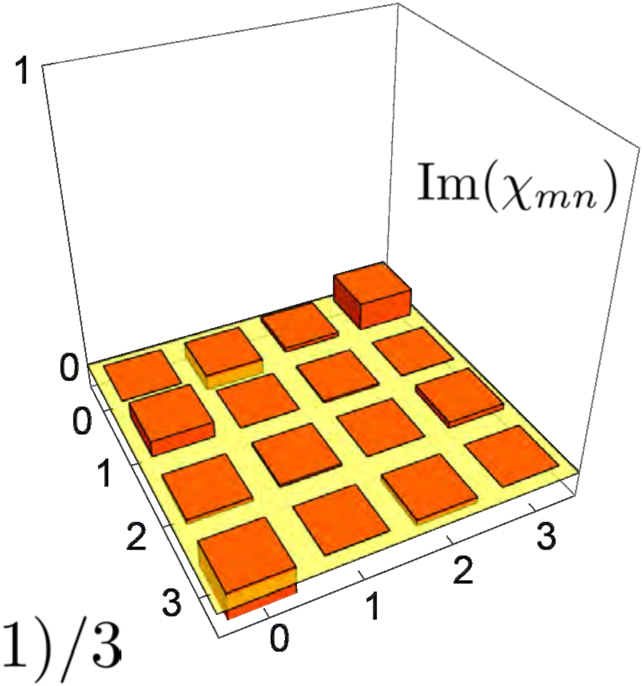
$$|\psi_{\rm B}\rangle = \alpha|F=2, m_F=-1\rangle + \beta|F=2, m_F=+1\rangle$$

quantum-state transfer: process matrix

Ritter et al., Nature **484**, 195 (2012)

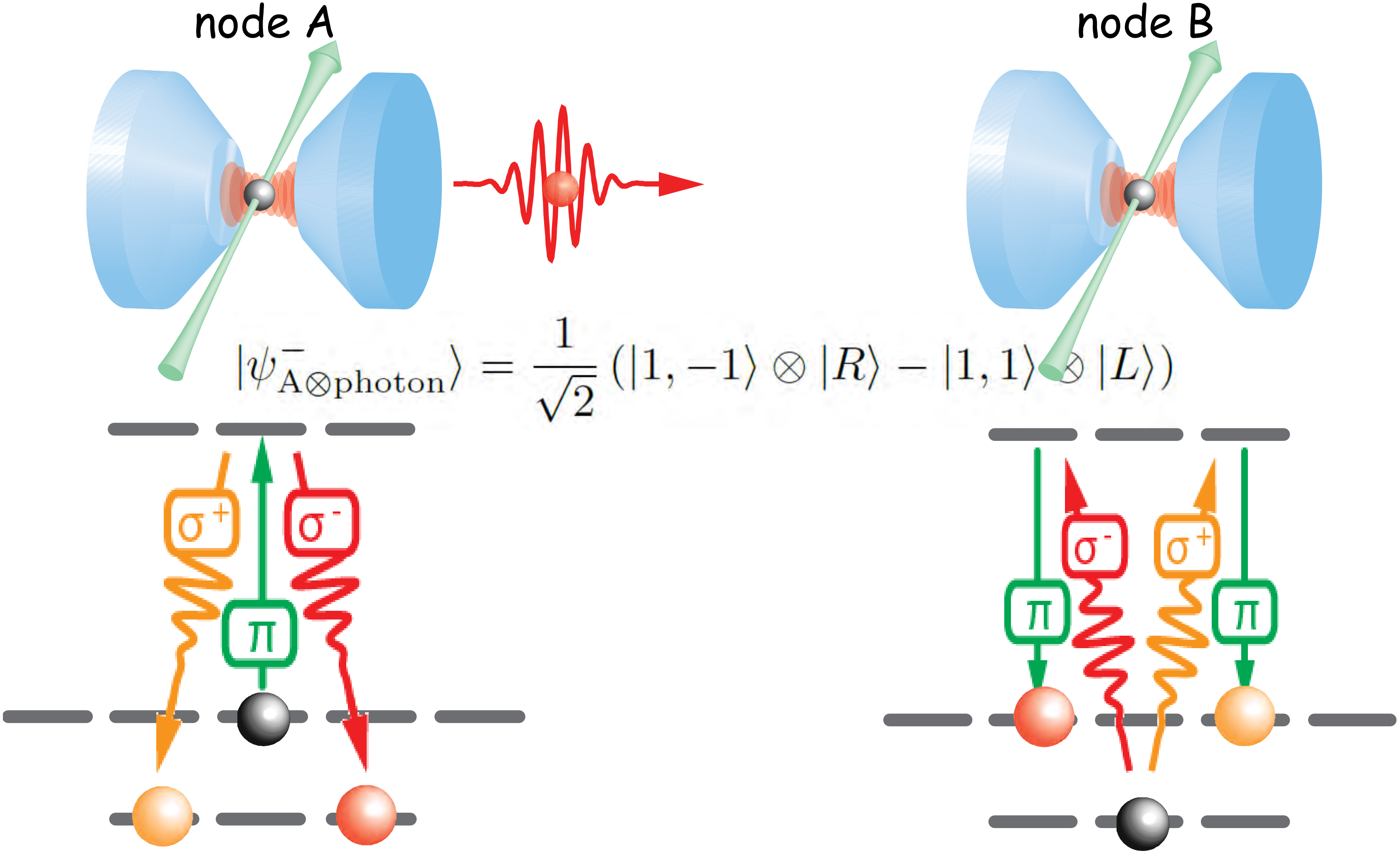
$$\rho_B = \sum_{m,n=0}^{3} \chi_{mn}\sigma_m\rho_A\sigma_n^\dagger$$





$$F = (2\chi_{00} + 1)/3$$
$$= (84 \pm 1)\,\%$$

atom-atom entanglement



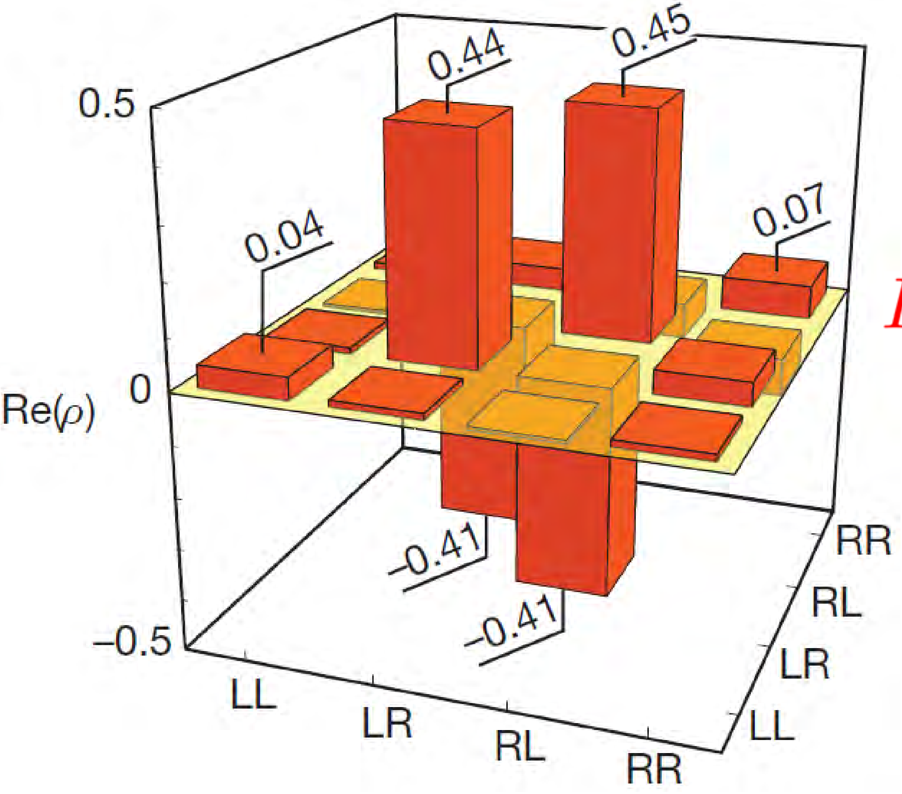
$$|\psi^-_{\mathrm{A}\otimes\mathrm{photon}}\rangle = \frac{1}{\sqrt{2}}\left(|1,-1\rangle \otimes |R\rangle - |1,1\rangle \otimes |L\rangle\right)$$

$$|\psi^-_{\mathrm{A}\otimes\mathrm{B}}\rangle = \frac{1}{\sqrt{2}}\left(|1,-1\rangle \otimes |2,1\rangle - |1,1\rangle \otimes |2,-1\rangle\right)$$

atom-atom entanglement: density matrix

Ritter et al., Nature **484**, 195 (2012)

$$|\psi^-_{\mathrm{A}\otimes\mathrm{B}}\rangle = \frac{1}{\sqrt{2}}\left(|1,-1\rangle \otimes |2,1\rangle - |1,1\rangle \otimes |2,-1\rangle\right)$$

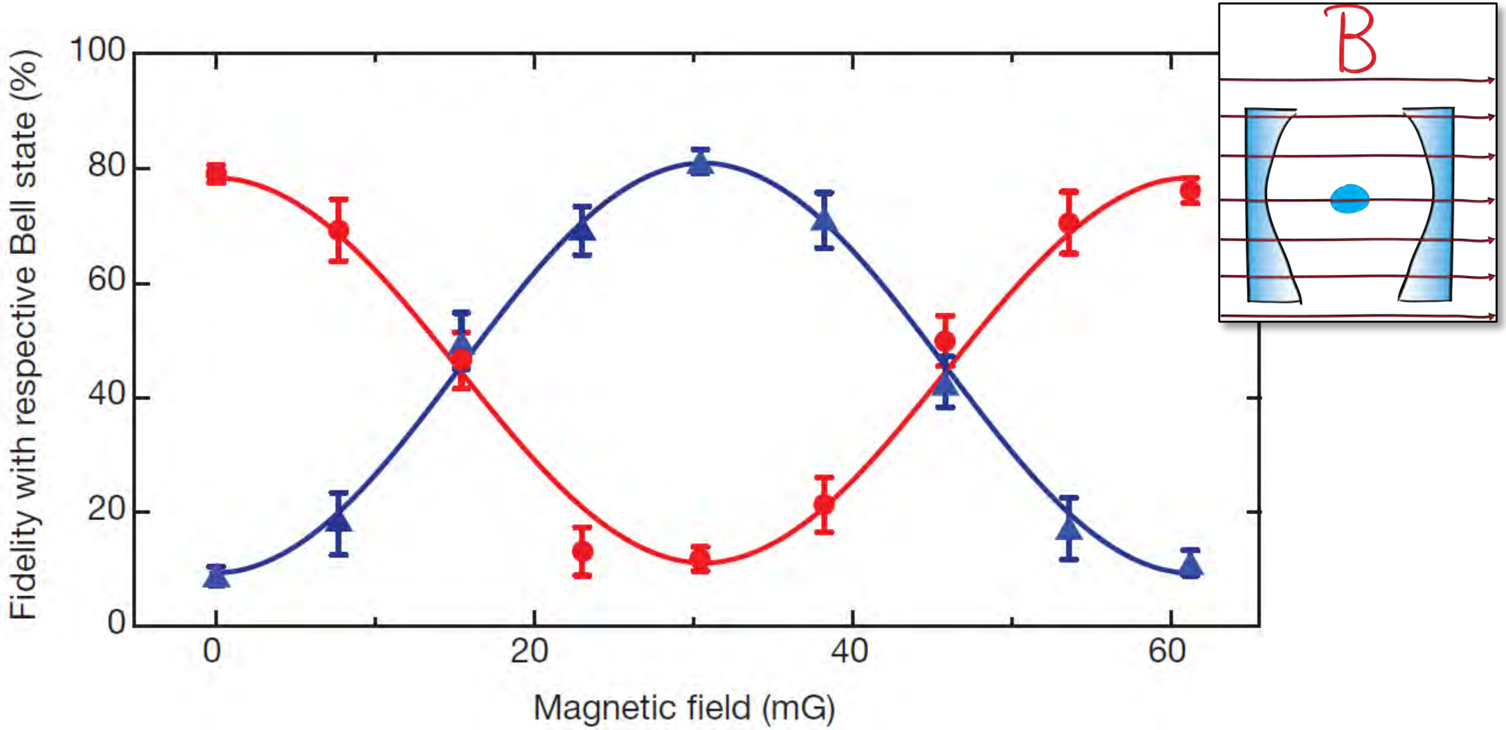


$$F_{|\psi^-\rangle} = (85 \pm 1.3)\%$$

atom-atom entanglement: nonlocality

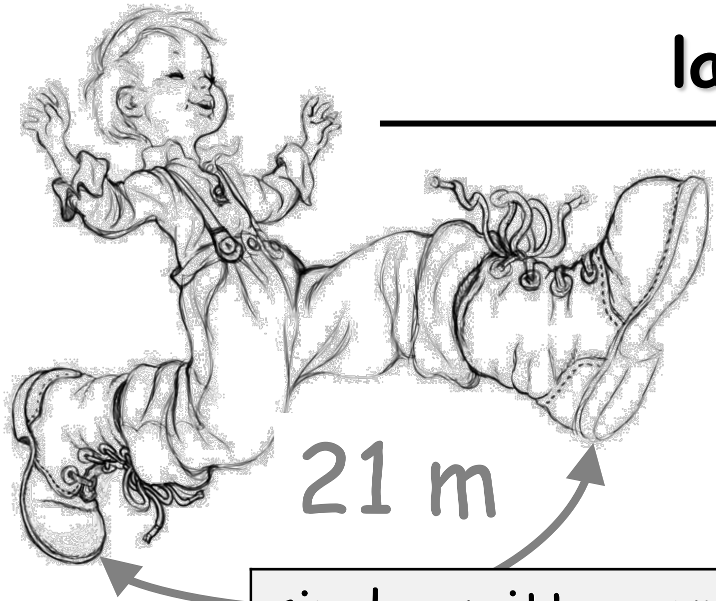
Ritter et al., Nature **484**, 195 (2012)

$$|\Psi^{-}_{\mathrm{A}\otimes\mathrm{B}}\rangle = \frac{1}{\sqrt{2}}\Big(|1,-1\rangle \otimes |2,1\rangle - e^{i\mu_B B\tau/\hbar}|1,1\rangle \otimes |2,-1\rangle\Big)$$



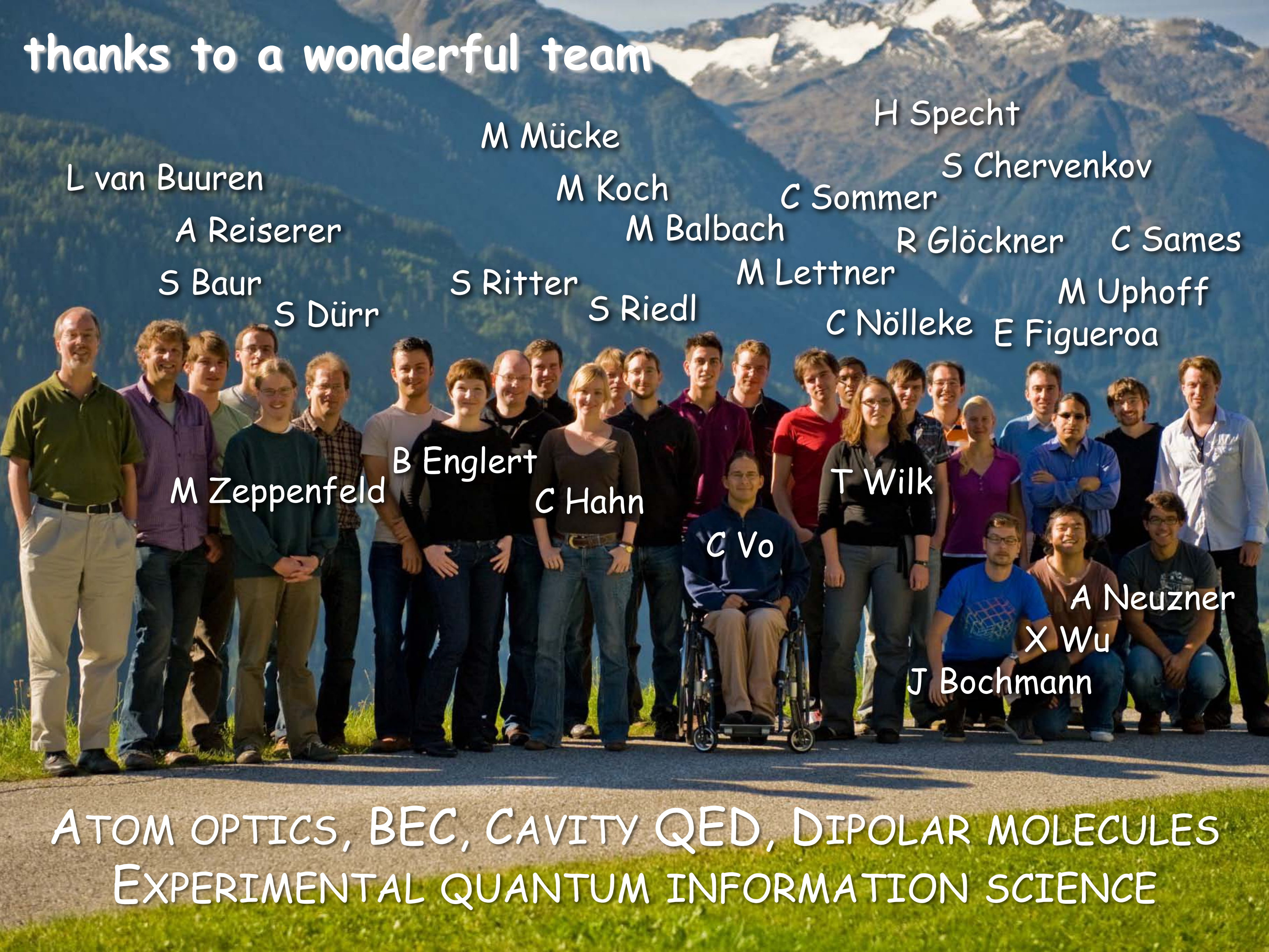
largest material quantum system*

* with a new twist:
locally = open system
globally = closed system



single-emitter experiments		fidelity	w/r efficiency
memory	cavity QED	0.92 (0.95)	0.3^2 (0.7^2)
	-----	-----	----
entanglement	cavity QED	0.85	0.02
	free space	0.63	0.000000004
state transfer	cavity QED	0.84	0.002
	-----	-----	-----
teleportation	-----	-----	-----
	free space	0.90	0.00000002

thanks to a wonderful team



ATOM OPTICS, BEC, CAVITY QED, DIPOLAR MOLECULES

EXPERIMENTAL QUANTUM INFORMATION SCIENCE