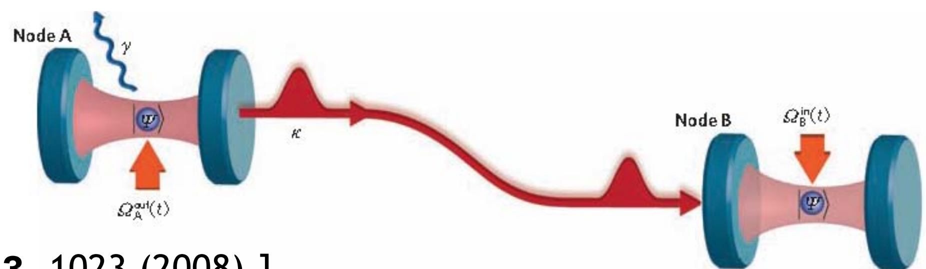
Photon generation and frequency conversion for quantum information science

C. J. McKinstrie
Bell Laboratories, Alcatel-Lucent

Collaborators: S. Radic (UCSD), M. Raymer (Oregon) and K. Rottwitt (DTU)

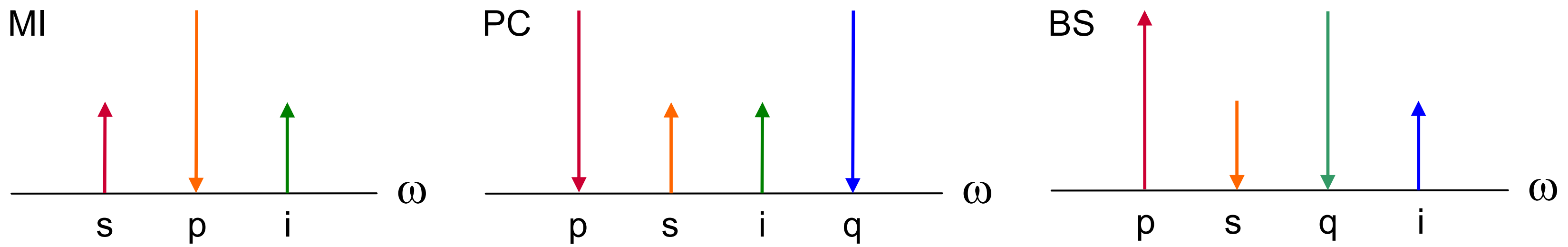
Parametric devices for quantum information science

- In conventional optical communication systems, quantum effects are responsible for noise inherent in the signal, noise added during transmission (by amplifiers) and noise in the detectors.
- There are many schemes for quantum communication and computation that rely on the quantum properties of light.
- Examples include quantum key distribution, entanglement generation or transfer and linear optical quantum computation.
- Common requirements include photon generators (singlet or pair) and (distortionless) photon frequency-convertors.



[H. Kimble, Nature **453**, 1023 (2008).]

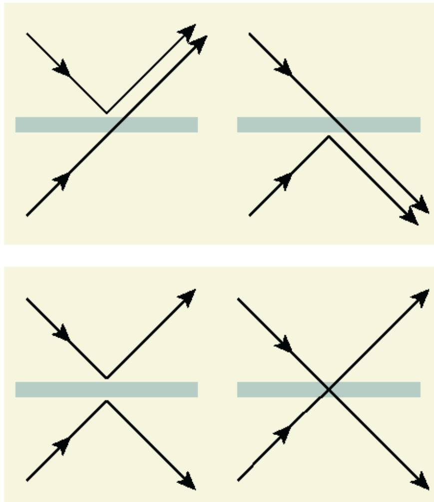
Parametric devices are enabled by four-wave mixing

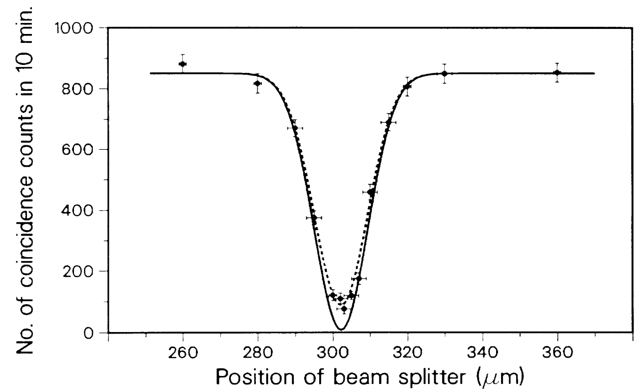


- In four-wave mixing (FWM), weak sidebands (s and i) are driven by strong pumps (p and q).
- Modulation instability (MI): $2\pi_p \rightarrow \pi_s + \pi_i$ (π_j is a photon with frequency ω_j).
- Phase conjugation (PC): $\pi_p + \pi_q \rightarrow \pi_s + \pi_i$.
- Bragg scattering (BS), or frequency conversion (FC): $\pi_s + \pi_q \rightarrow \pi_p + \pi_i$.
- Classical: MI (inverse MI) and PC amplify signals, but add excess noise, whereas BS frequency converts signals without adding noise.
- Quantum: MI (inverse MI) and PC generate photons, whereas BS frequency converts photons.

[C. McKinstrie, J. Sel. Top. Quantum Electron. **8**, 538 and 956 (2002).]

Hong-Ou-Mandel interferometry





- For a two-mode BS, $U(\theta) = \exp[i\theta(a_1^+ a_2 + a_1 a_2^+)]$, where $\theta = \tan^{-1}(r/t)$.
- If the input is $|1,1\rangle$, the output is $(t^2 - r^2)|1,1\rangle + i\sqrt{2}tr(|2,0\rangle + |0,2\rangle)$.
- For a balanced BS ($t^2 = r^2$), the output is $(|2,0\rangle + |0,2\rangle)/\sqrt{2}$.
- In experiments photons are wave-packets, not continuous waves (CWs).
- If the wave-packets are identical and pure, and there is no distinguishing information, the wave-packets interfere like discrete modes.

[C. Hong, PRL **59**, 2044 (1987); I. Walmsley, Science **307**, 1733 (2005).]

Photon pair generation by MI or PC

- In the Schrodinger picture a_r, a_s are constant and $d_z|\psi\rangle = iH|\psi\rangle$.
- The two-mode Hamiltonian $H = \delta(a_r^+a_r + a_s^+a_s) + \gamma a_r^+a_s^+ + \gamma^* a_r a_s$ (CW pumps).
- The output state $|\psi(z)\rangle = \exp(iHz)|0,0\rangle$.
- Recall that $a^+|n\rangle = (n+1)^{1/2}|n+1\rangle$ and $a|n\rangle = n^{1/2}|n-1\rangle$.

- In the low-gain regime ($\delta \approx 0$), $|\psi(z)\rangle \approx (1 + iHz)|0,0\rangle = |0,0\rangle + i\gamma z|1,1\rangle$.
  MI and PC produce pairs of signal and idler photons.

- In the high-gain regime ($\delta \neq 0$), use the operator-ordering theorem

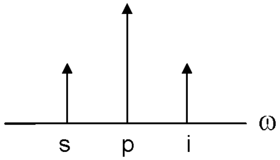
$$\exp(iHz) = \exp(c_1 a_r^+ a_s^+)\exp[c_2(a_r^+a_r + a_s a_s^+)]\exp(c_3 a_r a_s).$$

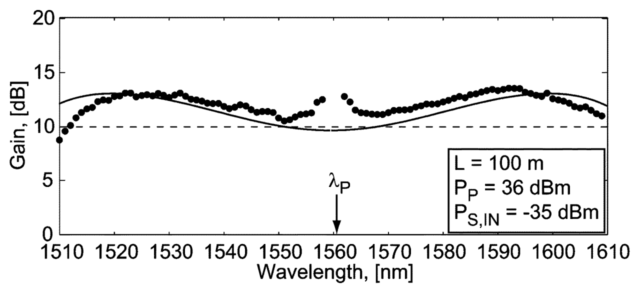
- The two-mode squeezed state $|\psi(z)\rangle = (\mu^*)^{-1}\Sigma_n(\nu/\mu^*)^n|n,n\rangle/n!$

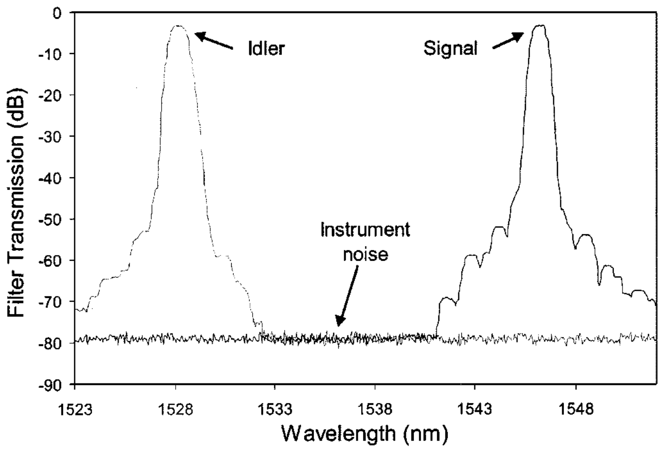
- The output state is determined by the Heisenberg transfer functions μ and ν!
- In the high-gain regime, there are many different multiple-photon states.
  One cannot generate specific states preferentially ($|\nu|^2 \geq 0$).

[R. Loudon, QTL (2000); C. McKinstrie, OC **282**, 2155 (2009).]

Photon-pair generation by MI (or PC)







- MI driven by a CW pump produces the frequency-entangled state

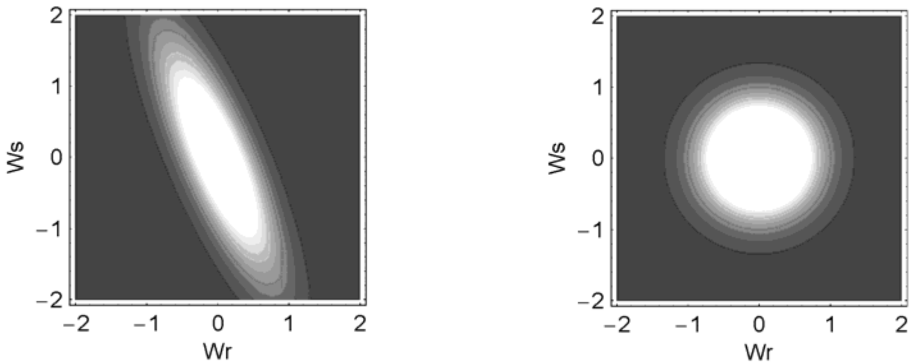
  $|\psi\rangle = c_0|0,0\rangle + c_1|\omega_1,-\omega_1\rangle + c_2|\omega_2,-\omega_2\rangle + \ldots$

- Non-frequency-resolving measurement: $|0,0\rangle$ is pure, but useless. $|\omega_1,-\omega_1\rangle$ with probability $|c_1|^2$, or $|\omega_2,-\omega_2\rangle$ with probability $|c_2|^2$, . . . , is not pure (mixed).
- Frequency-resolving measurement (filter): $|\omega_f,-\omega_f\rangle$ is pure, but the pair-production rate is low.
- It is better to generate pure states directly. Try a pulsed pump!
- Pure-state photons are produced for certain fiber and pump parameters.

[Sharping, OL **26**, 367 (2001); Fiorentino, PTL **14**, 983 (2002); Torounidis, PTL **19**, 650 (2007).]

Qualitative description of photon-pair generation (1)

- If pumps p and q are pulsed, each signal component interacts with many idler components, subject to the condition $\omega_r + \omega_s = \omega_p + \omega_q$.
- In the low-gain regime, $|\psi\rangle = \int\int f(\omega_r,\omega_s)|\omega_r,\omega_s\rangle d\omega_r d\omega_s$, where $f(\omega_r,\omega_s)$ is the two-photon amplitude and $|\omega_r,\omega_s\rangle = a^+(\omega_r)a^+(\omega_s)|0,0\rangle$ is a pair-state.
- The two-photon amplitude depends on the pump spectra, the mismatch $\beta(\omega_r) + \beta(\omega_s) - \beta(\omega_p) - \beta(\omega_q)$ and the fiber length (choose judiciously).
- $J(\omega_r,\omega_s) = \langle a_r^+(\omega_r)a_r(\omega_r)a_s^+(\omega_s)a_s(\omega_s)\rangle = |f(\omega_r,\omega_s)|^2$ is the joint probability.
- If $f(\omega_r,\omega_s) = f_r(\omega_r)\, f_s(\omega_s)$, then $|\psi\rangle = \int f_r(\omega_r)|\omega_r\rangle d\omega_r \int f_s(\omega_s)|\omega_s\rangle d\omega_s = |\psi\rangle_r\, |\psi\rangle_s$, which is a pure state! The photon wave-packets might (not) be identical.

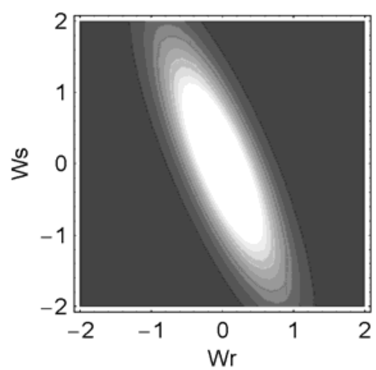


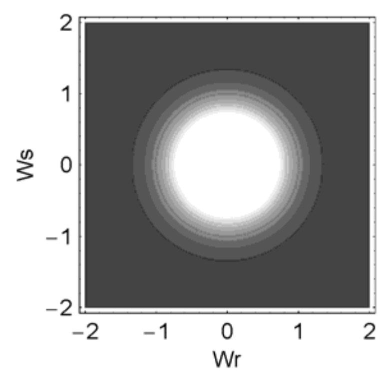
[R. Loudon, QTL (2000); W. Grice, PRA **56**, 1627 (1997) & **64**, 063815 (2001); K. Garay, OE **15**, 14870 (2007).]

Qualitative description of photon-pair generation (2)

- If $f(\omega_r,\omega_s) = f_r(\omega_r)\, f_s(\omega_s)$, then $|\psi\rangle = \int f_r(\omega_r)|\omega_r\rangle d\omega_r \int f_s(\omega_s)|\omega_s\rangle d\omega_s = |\psi\rangle_r\, |\psi\rangle_s$, which is a pure state! The photon wave-packets might (not) be identical.
- Schmidt decomposition: $f(\omega_r,\omega_s) = \Sigma_j\, \phi_{jr}(\omega_r)\, \sigma_j\, \phi_{js}(\omega_s)$.
- Let $a_{jr}^+ = \int \phi_{jr}(\omega_r) a^+(\omega_r) d\omega_r$. Then $[a_{jr}, a_{kr}^+] = \delta_{jk}$. ($a_{js}$ has similar properties.)
- $|\psi\rangle = \int\int f(\omega_r,\omega_s) a^+(\omega_r) a^+(\omega_s) d\omega_r d\omega_s |0,0\rangle = \Sigma_j\, \sigma_j a_{jr}^+ a_{js}^+ |0,0\rangle = \Sigma_j\, \sigma_j |1_r,1_s\rangle_j$.
- Photons with frequency e-modes $\phi_{jr}(\omega_r)$ and $\phi_{js}(\omega_s)$ are generated with probability σ_j^2.
- Relate σ_j to physical parameters and design experiment so that only $\sigma_1 \neq 0$.

many $\sigma_j \neq 0$



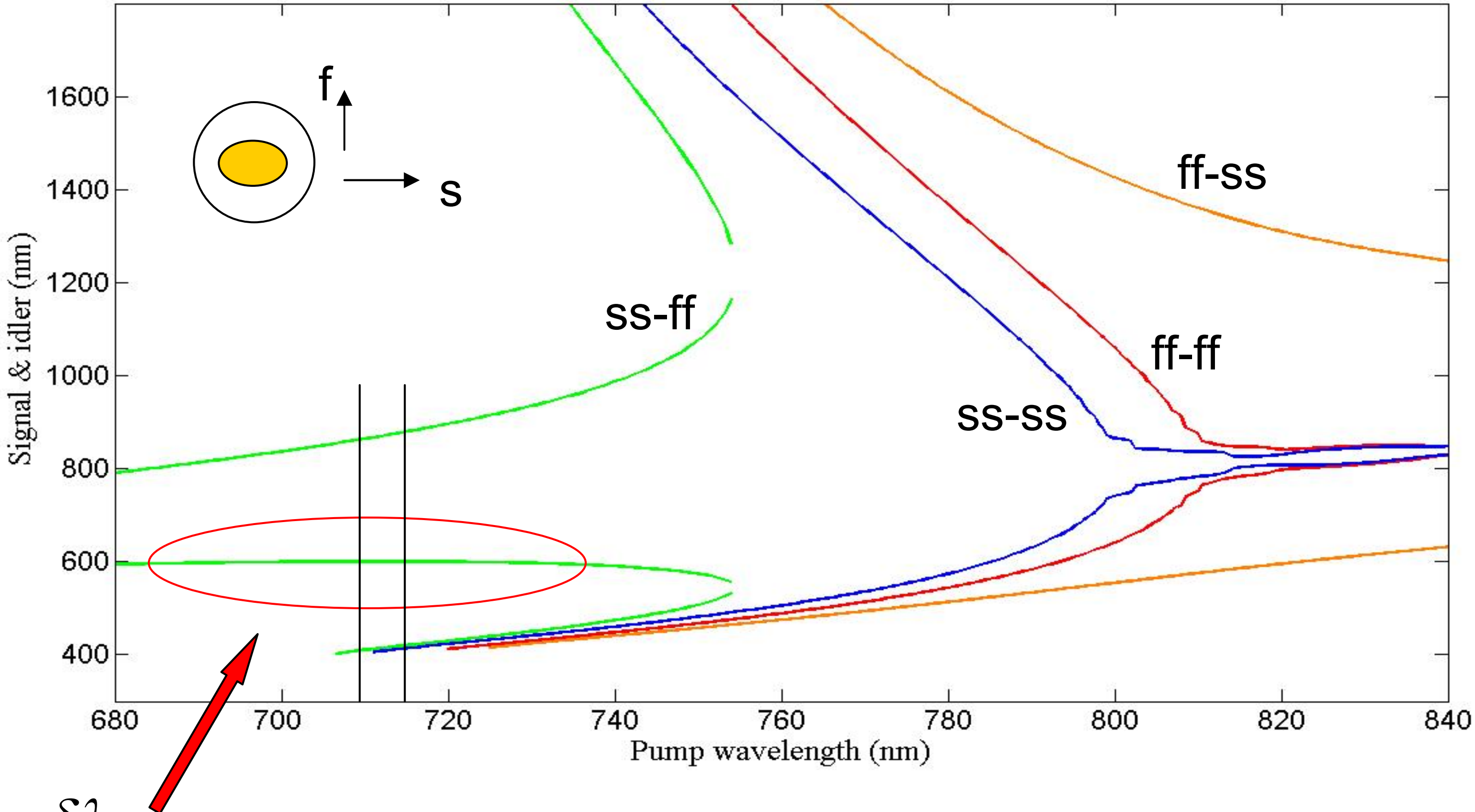


$\sigma_1 \neq 0$,
other $\sigma_j = 0$

SD works for high gain

[C. Law, PRL **84**, 5304 (2000), W. Grice, PRA **64**, 063815 (2001); K. Garay, OE **15**, 14870 (2007).]

Co- and cross-polarized phase-matching curves

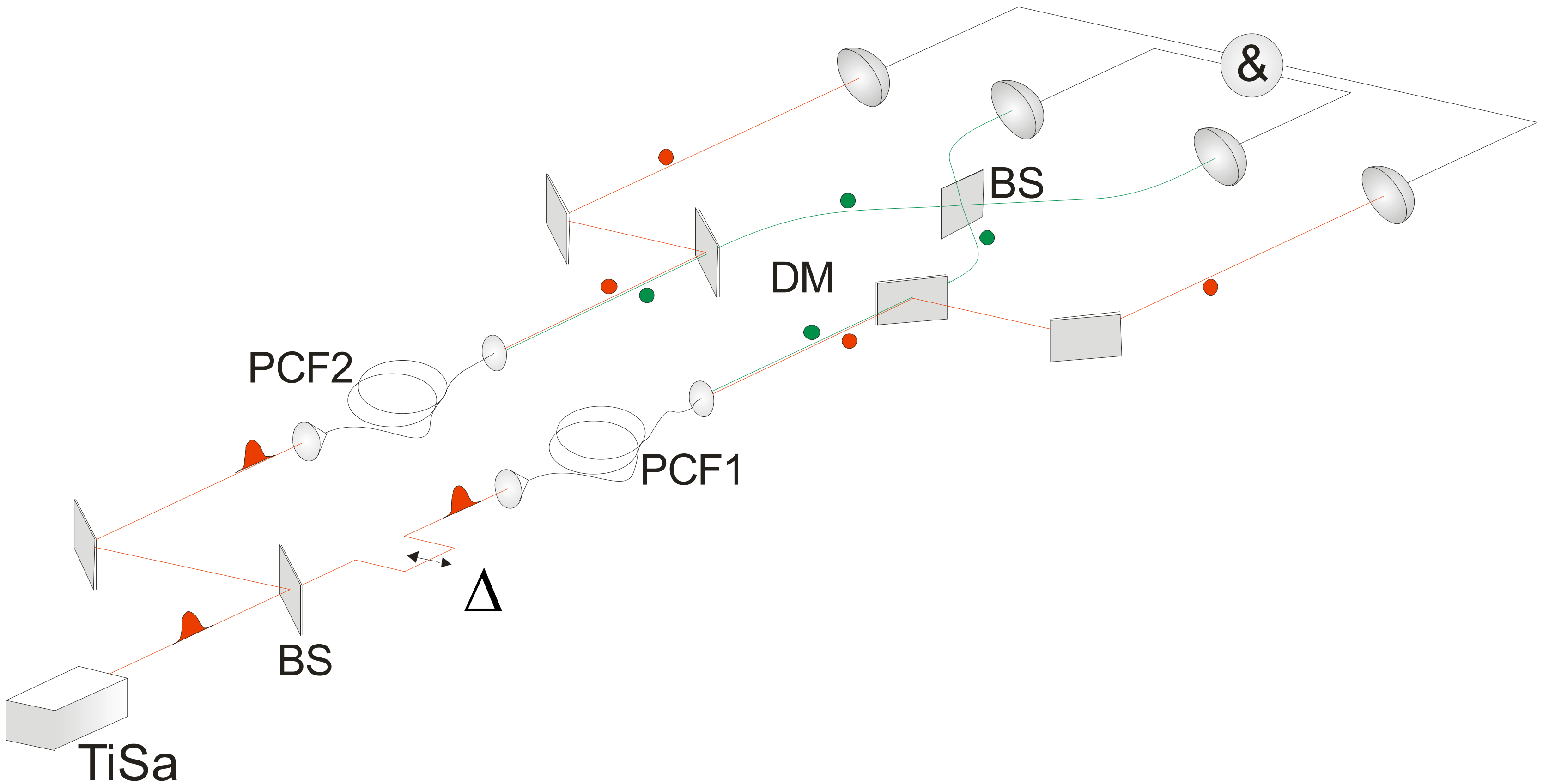


$$\frac{\delta\lambda_s}{\delta\lambda_p} = 0$$

[J. Rarity, FiO 2008, paper FMH4 ; M. Halder, OE **17**, 4670 (2009).]

Experimental setup with two sources

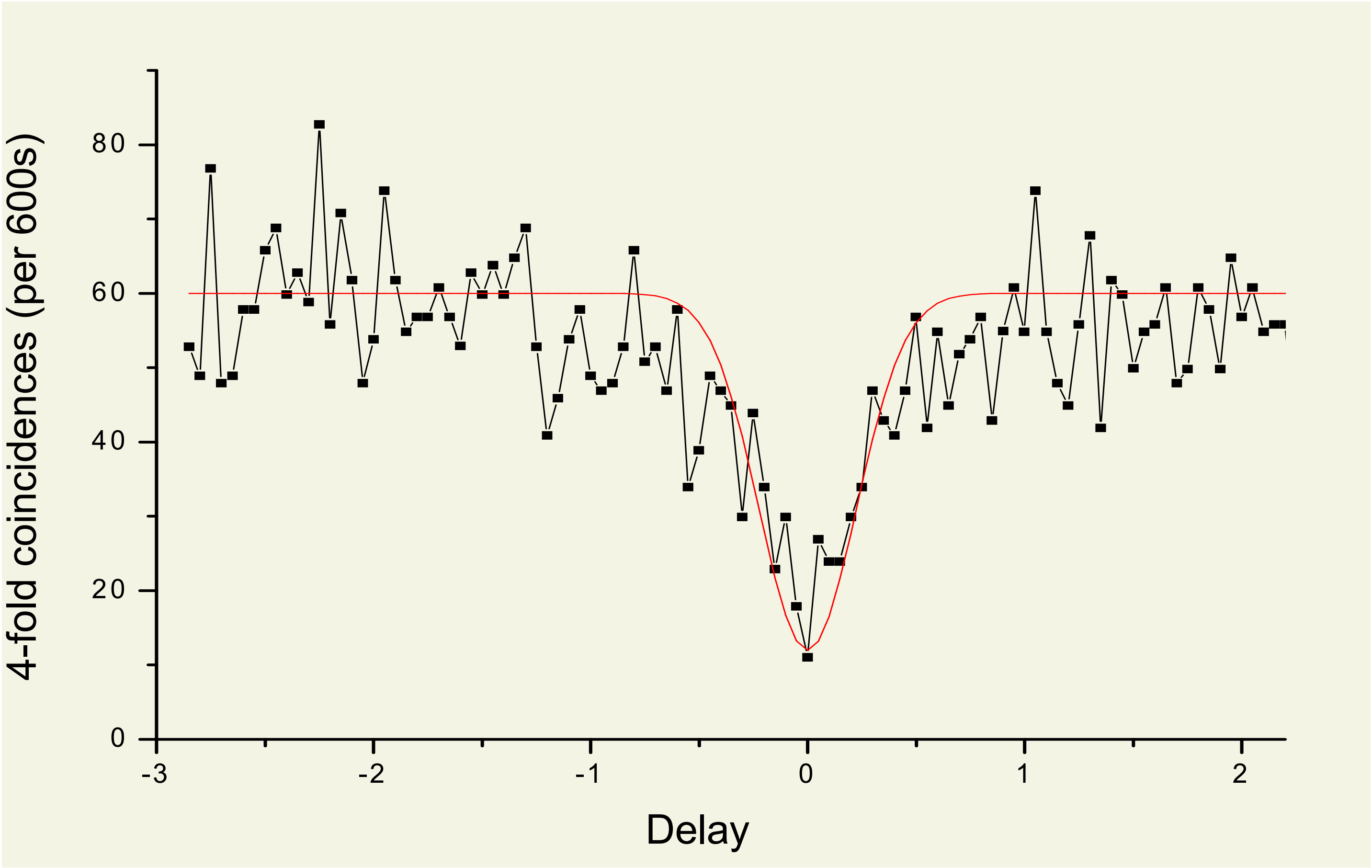
$$|1_A, 1_B\rangle \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(|2_A, 0_B\rangle + |0_A, 2_B\rangle\right)$$



[J. Rarity, FiO 2008, paper FMH4 ; M. Halder, OE **17**, 4670 (2009).]

The Hong-Ou-Mandel dip was observed

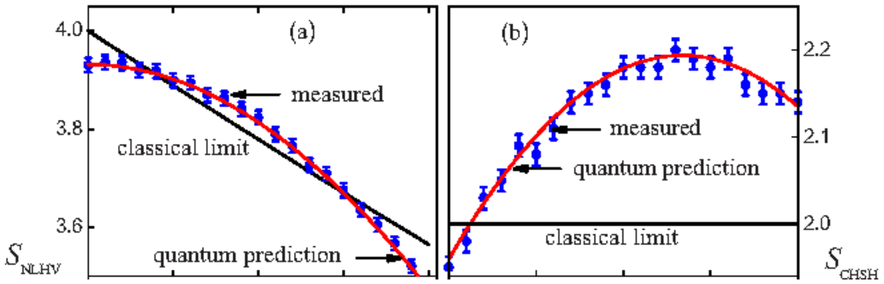
- Collection efficiency >20% and visibility > 80%.



[J. Rarity, FiO 2008, paper FMH4; M. Halder, OE **17**, 4670 (2009).]

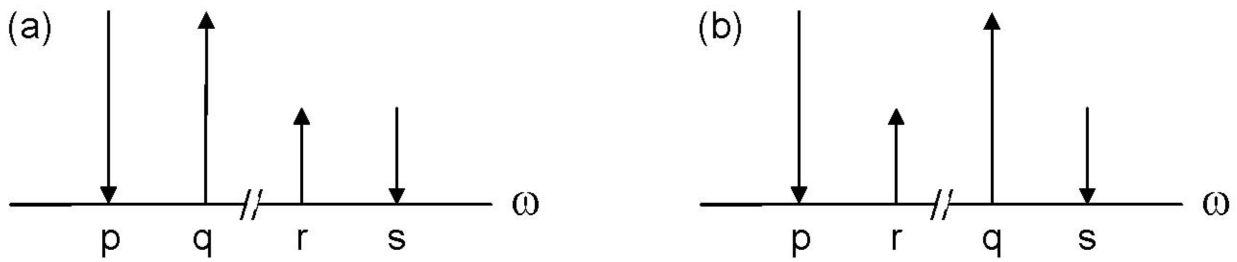
Entangled photons were used for Q-physics experiments

- Differential-phase quantum key distribution experiment using a series of quantum entangled photon pairs
- Nonclassical interference and entanglement generation using a photonic crystal fiber pair photon source.
- Deterministic quantum beam splitter based on time-reversed Hong-Ou-Mandel interference.
- Experimental test of nonlocal realism using a fiber-based source of polarization-entangled photon pairs (figure).
- Demonstration of a quantum controlled-NOT gate in the telecommunication band.



[T. Honjo, OL **32**, 1165 (2007); J. Fulconis, PRL **99**, 120501 (2007); J. Chen; PRA **76**, 031804 (2007); M. Eisaman, PRA **77**, 032339 (2008); J. Chen, PRL **100**, 133603 (2008).]

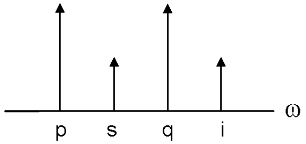
Frequency conversion by Bragg scattering

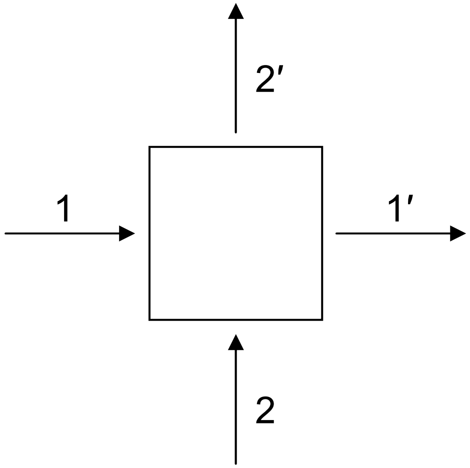


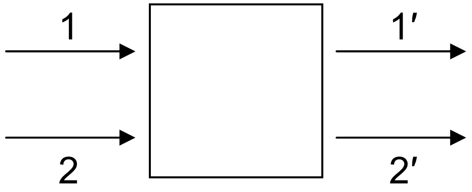
- Asymmetric four-wave mixing (FWM), also called Bragg scattering (BS), can frequency convert an optical pulse without adding excess noise.
- Two strong pumps (p and q) couple weak signal (s) and idler (r) sidebands.
- In BS, $\pi_s + \pi_q \rightarrow \pi_p + \pi_i$, where π_j is a photon with frequency ω_j. Input signal photons are converted to output idler photons.
- If ω_p, ω_q and ω_s are specified, energy conservation determines ω_r.
- Momentum conservation (efficient conversion) is possible if the frequencies are symmetric with respect to the zero-dispersion frequency of the fiber.

[K. Inoue, PTL **6**, 1451 (1994); C. McKinstrie, OE **13**, 9131 (2005); A. Gnauck, OE **14**, 8989 (2006).]

BS can translate arbitrary quantum states!







- Frequency conversion by BS is modeled by beam-splitter IO relations:

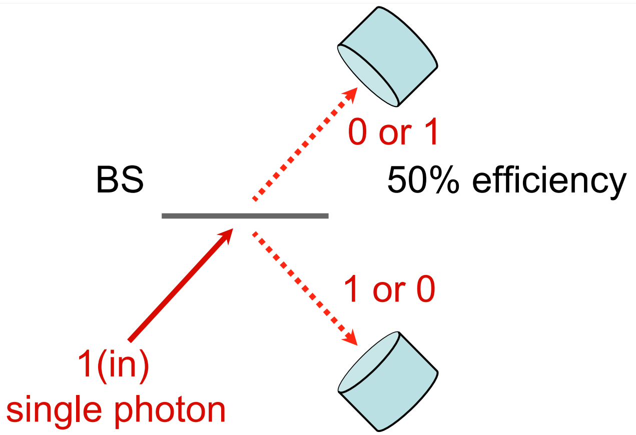
  $a_1(z) = \tau(z)a_1(0) + \rho(z)a_2(0), \quad a_2(z) = -\rho^*(z)a_1(0) + \tau^*(z)a_2(0).$

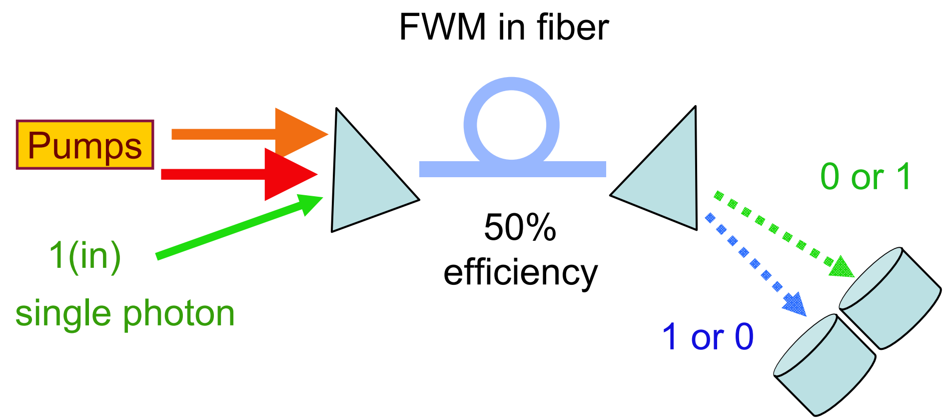
- For complete conversion, $|\rho(z)| = 1$, so $\langle m[a_2(z)]\rangle = \langle m[a_1(0)]\rangle$!
- Frequency conversion does not add excess noise (NF is 0 dB).
- Arbitrary (entangled) quantum states can be frequency shifted without distortion! Key function for quantum communication.
- Entanglement generation or HOM interference between photons with different frequencies!

[C. McKinstrie, Opt. Express **13**, 9131 (2005); M. Raymer, Opt. Commun. **283**, 747 (2010).]

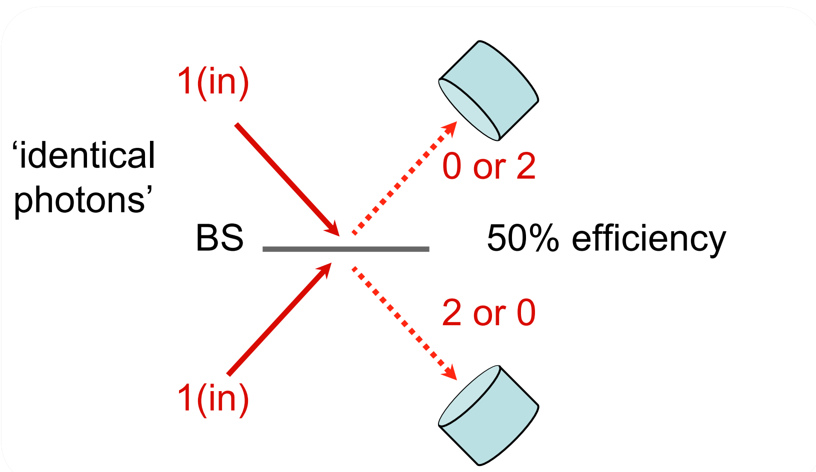
Quantum FC is mathematically equivalent to beam splitting

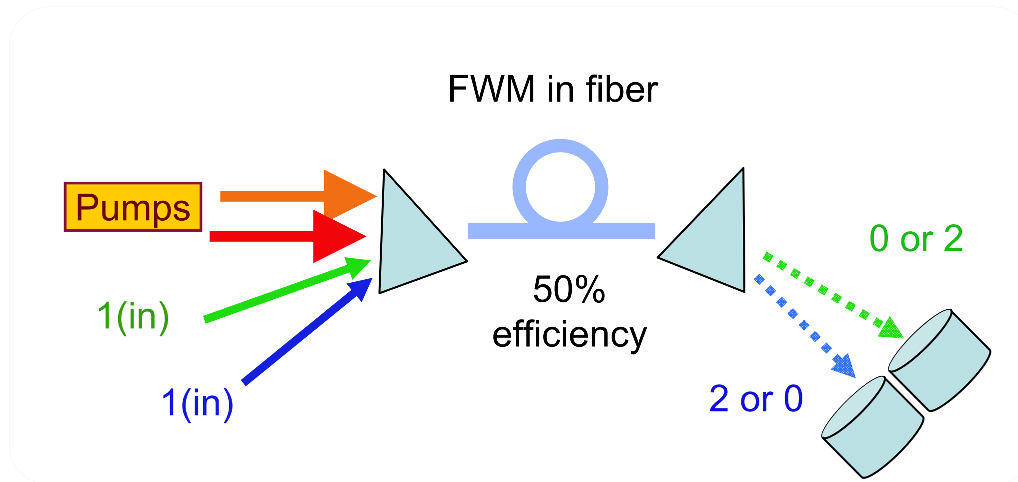
- Two-color entanglement generation





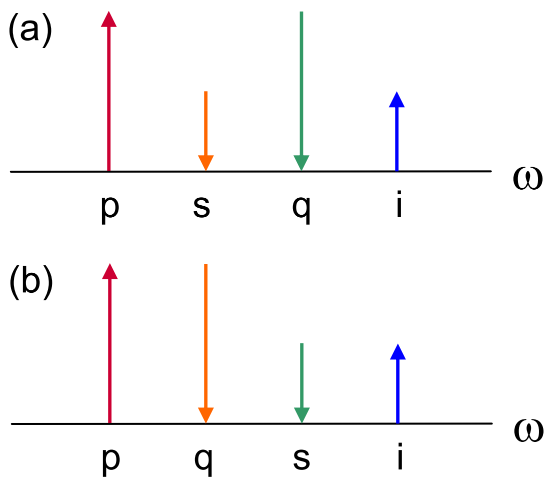
- Two-color HOM interference

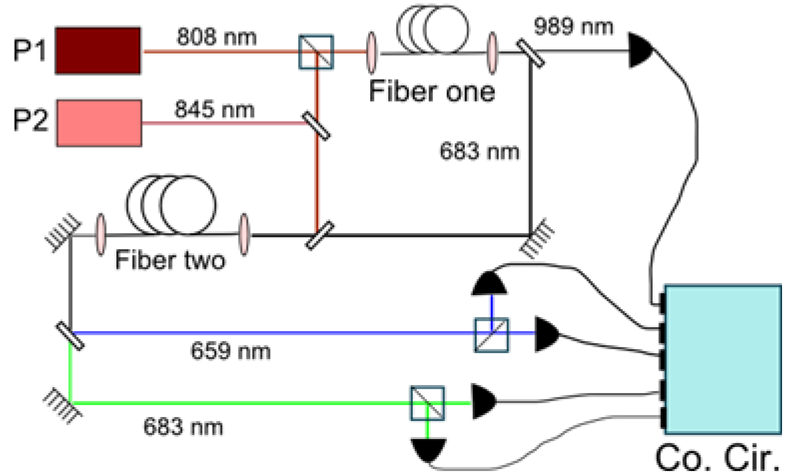




[M. Raymer, Opt. Commun. **238**, 747 (2010).]

Single-photon frequency conversion



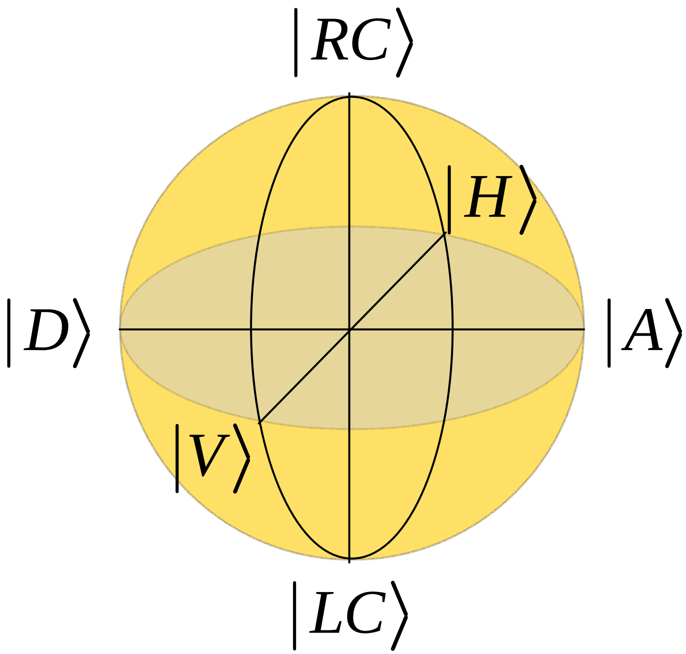


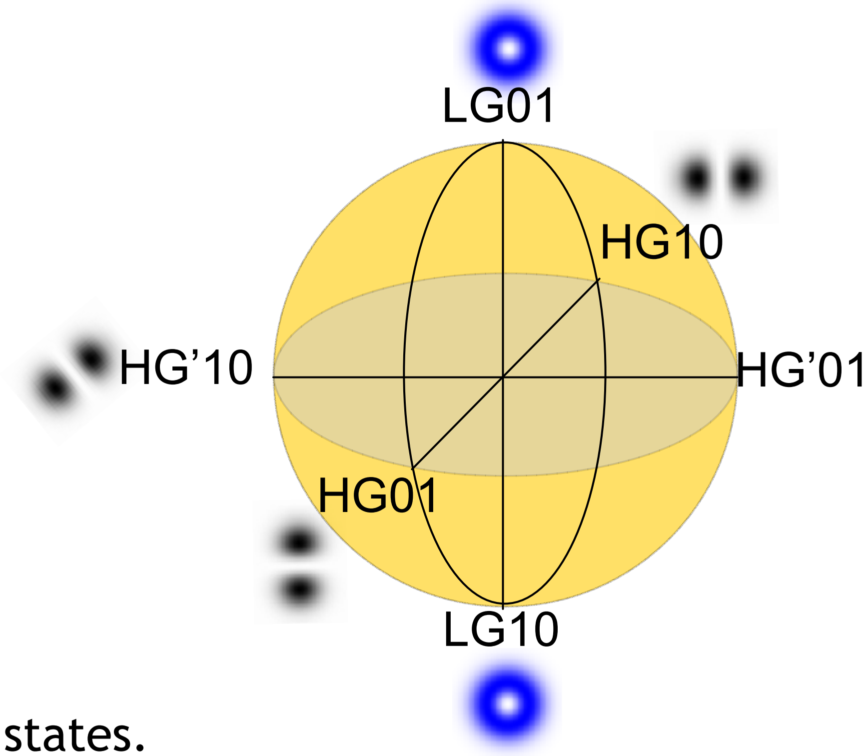
- In FC by FWM (BS), $\gamma_s + \gamma_q \rightarrow \gamma_p + \gamma_i$. The signal is FC without excess noise (NF = 0 dB).
- (a) Gnauck demonstrated FC of a classical signal, with 99% conversion efficiency. The output idler had a 3-dB higher SNR than the idler produced by PA (MI or PC).
- (b) McGuinness demonstrated single-photon FC, with 28% conversion efficiency.
- Quantumness is measured by second-order correlation coefficient $G^{(2)}$. Classical signals have $G^{(2)} \geq 1$, whereas perfect photon pairs have $G^{(2)} = 0$.
- Correlation counting showed that $G_{683} = 0.21$, $G_{659} = 0.19$. More to come!

[A. Gnauck, Opt. Express **14**, 8989 (2006); H. McGuinness, Phys. Rev. Lett. **105**, 093604 (2010).]

Photonic qubits for quantum information

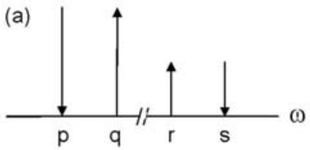
- Describe the states using Stokes/Poincare/Bloch spheres.





- Basis states are polarization or OAM states.
- One could also use different pulse shapes as the basis states (Silberhorn).
- How does pulsed-pump FWM work and how does it affect the pulse shapes?

The details (1)



- FC by BS is governed by the coupled-mode equations (CMEs)

$$(\partial_z + \beta_r \partial_t) A_r = i\gamma(z,t) A_s, \quad (\partial_z + \beta_s \partial_t) A_s = i\gamma^*(z,t) A_r$$

where $\gamma(z,t) = \gamma_K A_p(t - \beta_p z) A_q^*(t - \beta_q z)$.

- If ω_p and ω_s are symmetric with respect to ω_0, $\beta_p = \beta_s$ and $\beta_q = \beta_r$.
- The solutions of the CMEs can be written in the input-output (IO) forms

$$\begin{bmatrix} A_r(l,t) \\ A_s(l,t) \end{bmatrix} = \int dt' \begin{bmatrix} G_{rr}(t,t') & G_{rs}(t,t') \\ G_{sr}(t,t') & G_{ss}(t,t') \end{bmatrix} \begin{bmatrix} A_r(0,t') \\ A_s(0,t') \end{bmatrix}$$

- Each Green function has the Schmidt decomposition $G(t,t') = \Sigma_n v_n(t) \sigma_n u_n^*(t')$, where $u_n(t')$ and $v_n(t)$ are input and output S-modes, and σ_n is a S-coefficient.
- Because FC is a unitary process, the G-functions are related:

$$\begin{bmatrix} G_{rr} & G_{rs} \\ G_{sr} & G_{ss} \end{bmatrix} = \sum_n \begin{bmatrix} v_{rn} \tau_n u_{rn}^* & v_{rn} \rho_n u_{sn}^* \\ -v_{sn} \rho_n^* u_{rn}^* & v_{sn} \tau_n^* u_{sn}^* \end{bmatrix} \qquad |\tau_n|^2 + |\rho_n|^2 = 1$$

The details (2)

- Decompose the input and output fields:

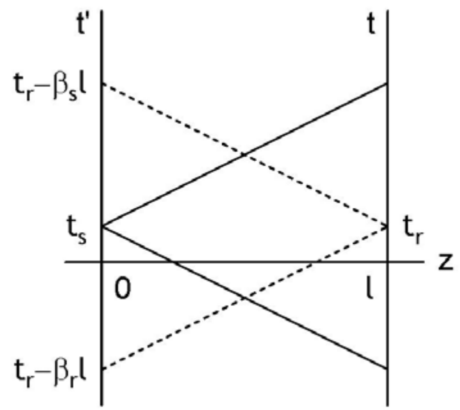
$$A(0,t') = \sum_n a_n(0)u_n(t'), \quad \mathrm{A}(l,t) = \sum_n a_n(l)v_n(t)$$

- The S-mode operators obey beam-splitter IO relations:

$$a_{rn}(l) = \tau_n a_{rn}(0) + \rho_n a_{sn}(0), \quad a_{sn}(l) = -\rho_n^* a_{rn}(0) + \tau_n^* a_{sn}(0)$$

- The G-functions can be determined analytically:

$$G_{rs}(t,t') = i\gamma_b A_q^*(t-\beta_r l)J_0\{2\gamma_b[\xi(t,t')\eta(t,t')]^{1/2}\}A_p(t')$$
$$\times H(t'+b_r l - t)H(t-t'-b_s l)$$



where

$$\xi(t,t') = \int_{t-\beta_r l}^{t'} |A_q(s)|^2\, ds, \qquad \eta(t,t') = \int_{t'}^{t-\beta_s l} |A_p(s)|^2\, ds$$

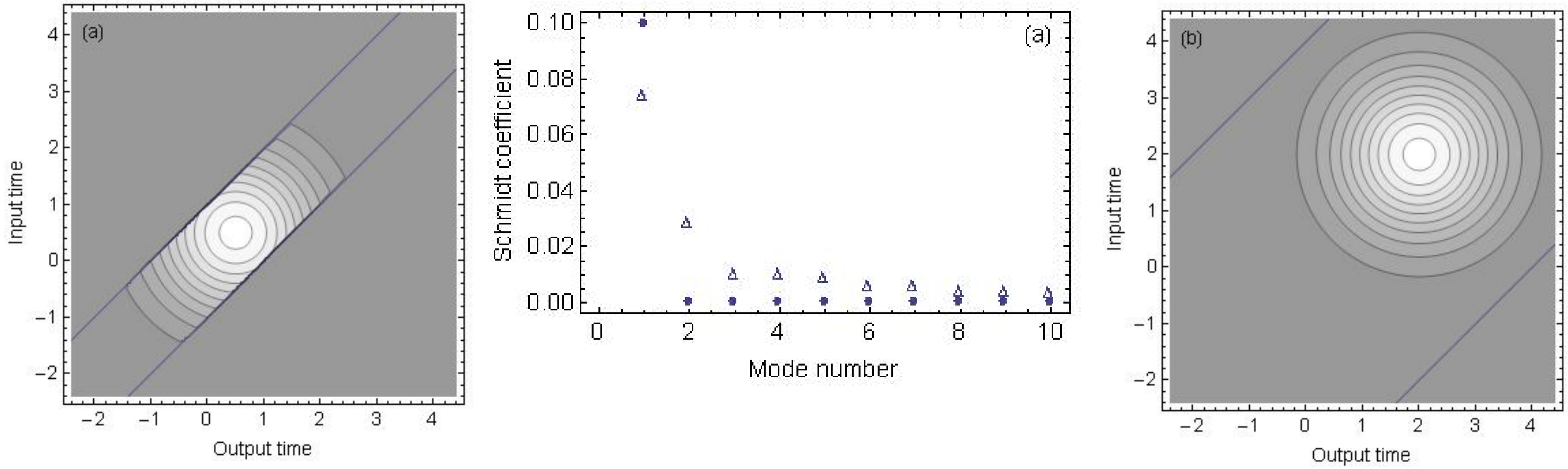
- In the low-conversion regime, the S-modes are the pump shape-functions.
- In the high-conversion regime the S-decomposition was done numerically.

What does it all mean?

- The S-modes are the natural input and output modes of the FC process: Optimize experimental design (tailor pumps to signals or vice versa).
- The input and output modes are related to the pump shape-functions: Arbitrary pulse reshaping is possible!
- Different S-modes can carry different bits of information. This is called pulse-shape multiplexing or optical code-division multiplexing.
- If $\sigma_j = 1$ for some j and $\sigma_k = 0$, FC extracts or inserts signals in mode j: pulse-shape multiplexing.
- If $u_j(t) \neq v_j(t)$, one can exchange information between different pulse-shape channels.

- Related work on multiplexing by three-wave mixing has been done by the Silberhorn group [A. Eckstein, Opt. Express **19**, 13771 (2011)].

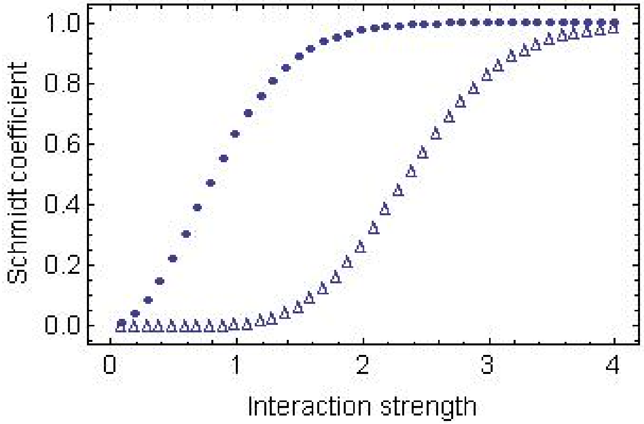
For long fibers, the Green function is separble

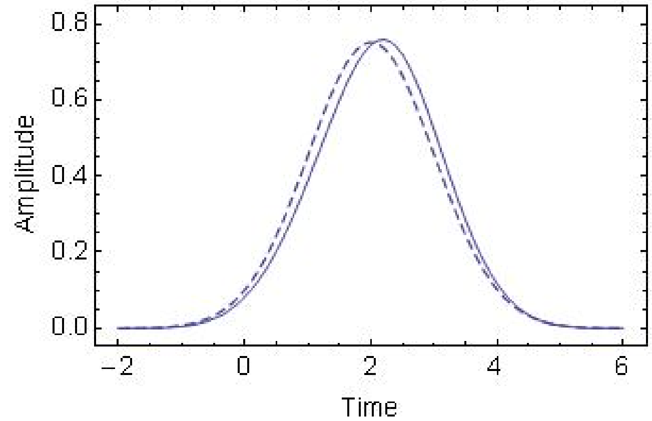


- For short fibers, the G-function is not separable, but for fibers longer than the pump-sideband collision length, it is always separable ($\neq$ TWM).
- The input and output S-modes are just the shapes of the co-propagating pumps ($\neq$ TWM).
- Results shown for low conversion efficiency (γ = 0.1 and CE = 0.01).

[L. Mejling, OE **20**, 8367 (2012), C. McKinstrie, PRA **85**, 053829 (2012).]

High conversion efficiencies are possible

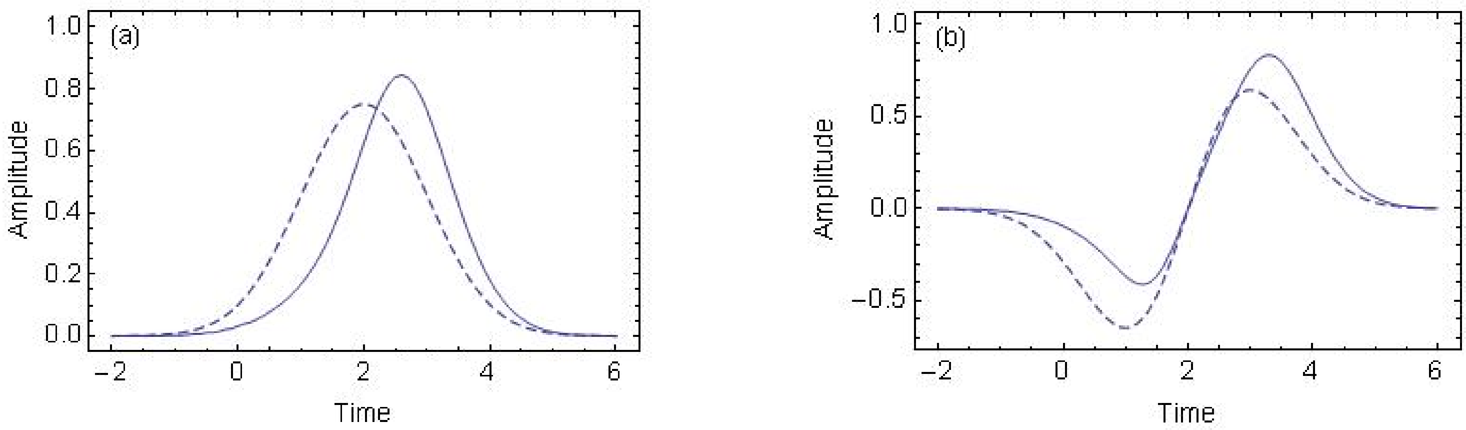




- The CEs $|\rho_n|^2$ increase as the interaction strength γ increases.
- There is no mode competition for $|\rho_1|^2 < 0.70$.
- For $|\rho_1|^2 = 0.50$ ($\gamma = 0.835$), and 2 Gauss pumps, the input- and output S-modes are mildly distorted versions of the (common) pump shape.

[H. McGuinness, OE **19**, 17876 (2011); C. McKinstrie, PRA **85**, 053829 (2012).]

Arbitrary pulse reshaping is possible



- For $|\rho_1|^2 = 0.90$ ($\gamma = 1.54$), the S-modes are distorted versions of the pump shapes.
- In the example, the pump shapes were zeroth- and first-order Hermite-Gauss functions.
- The input S-mode is related to one pump shape, and the output S-mode is related to the other.
- By varying the pump shapes, one can reshape the input pulse arbitrarily.

[L. Mejling, OE **20**, 8367 (2012), C. McKinstrie, PRA **85**, 053829 (2012).]

Summary 3

- MI and PC in fibers can generate photons and BS can frequency-convert photons for quantum information experiments.
- Pure photon-pair states were generated in birefringent micro-structured fibers (Rarity, Walmsley).
- Fiber-based photon-pair sources were used in quantum logic gates and a fundamental test of quantum mechanics (Migdall, Takesue).
- Single-photon frequency conversion was demonstrated (Raymer). Two-frequency entanglement and HOM experiments are underway.
- To model photon generation and frequency conversion, the main tools are Green functions and Schmidt decompositions. BS can shape the input and output modes arbitrarily.
- This is an exciting time to be doing quantum parametric-device research!

[J. Fan, OPN **18** (3), 26 (2007), A. U'Ren, OPN **22** (11), 36 (2011), K. Srinivasan, OPN **22** (12), 39 (2011).]