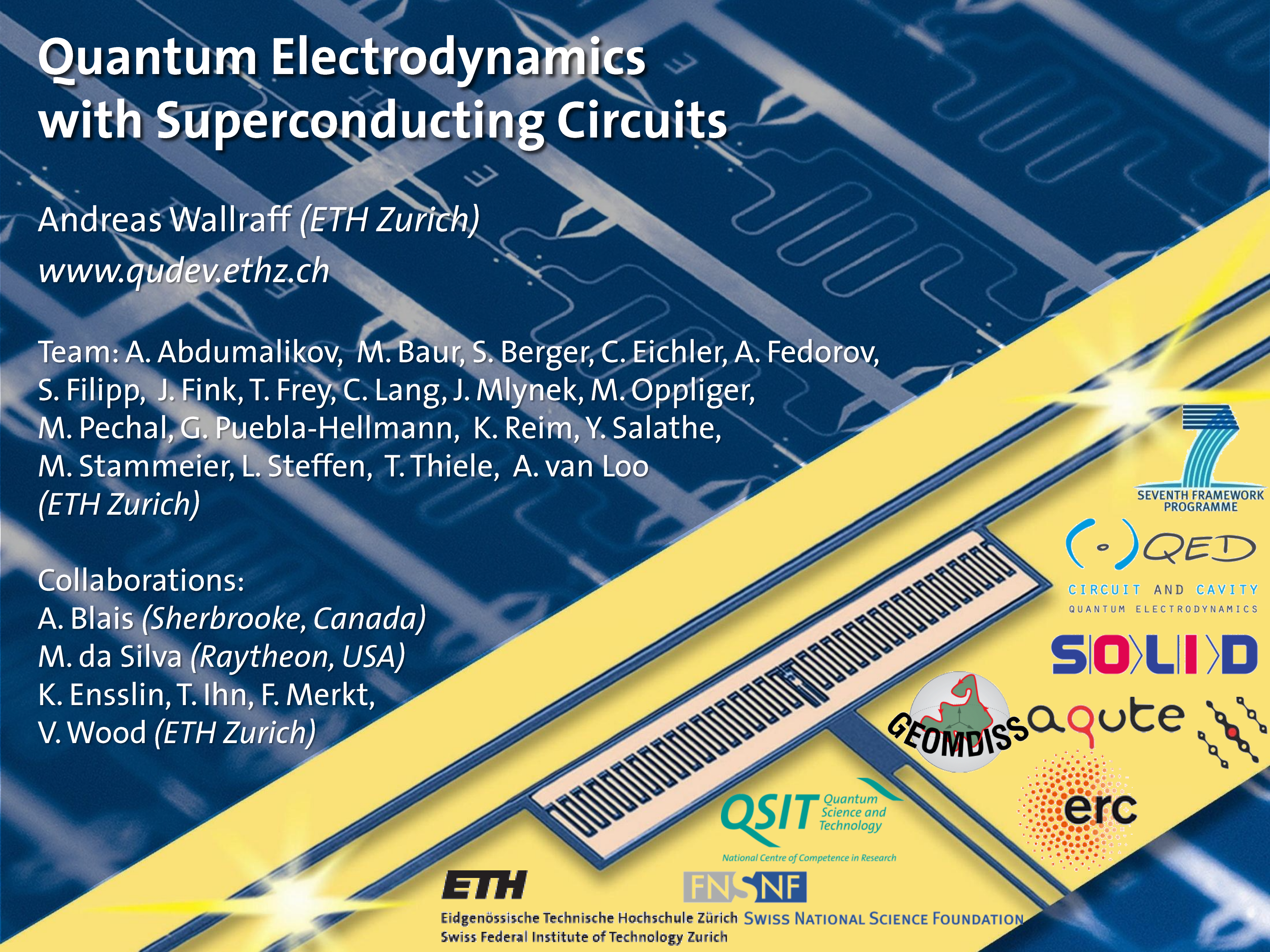

Quantum Electrodynamics with Superconducting Circuits

Andreas Wallraff *(ETH Zurich)*
www.qudev.ethz.ch

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Collaborations:
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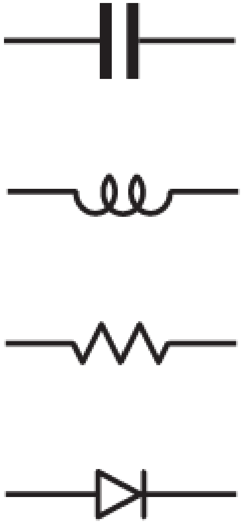
V. Wood *(ETH Zurich)*

SEVENTH FRAMEWORK PROGRAMME
(·)QED
CIRCUIT AND CAVITY QUANTUM ELECTRODYNAMICS
SOLID
GEOMDISS
aqute
erc
QSIT Quantum Science and Technology
National Centre of Competence in Research
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
FNSNF
Swiss National Science Foundation

Conventional Electronic Circuits

basic circuit elements:

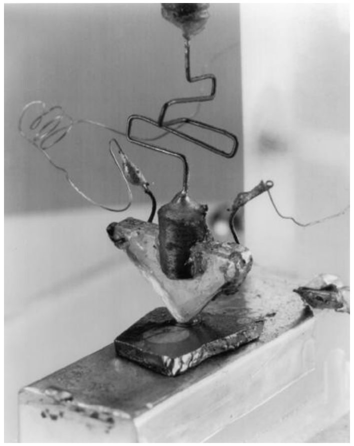


basis of modern information and communication technology

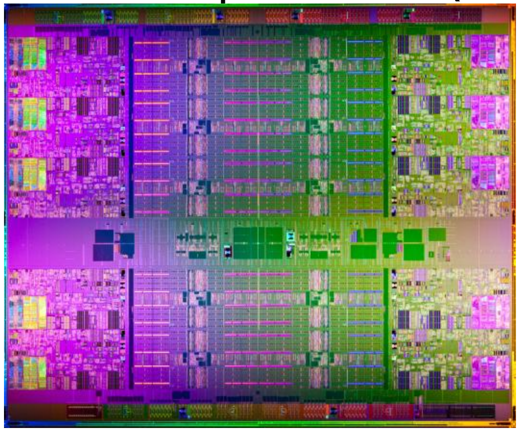
properties :

- classical physics
- no quantum mechanics
- no superposition principle
- no quantization of fields

first transistor at Bell Labs (1947)



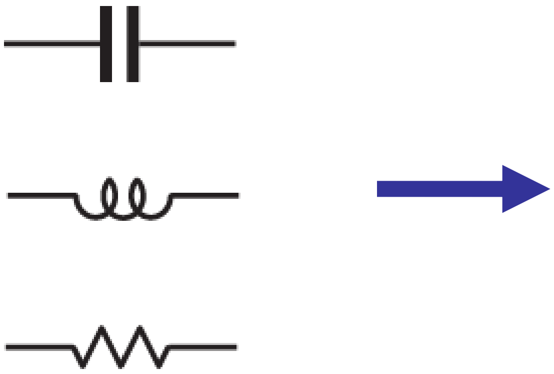
intel xeon processors (2011)



3.000.000.000 transistors
smallest feature size 32 nm
clock speed ~ 3 GHz
power consumption 10 W

Classical and Quantum Electronic Circuit Elements

basic circuit elements:



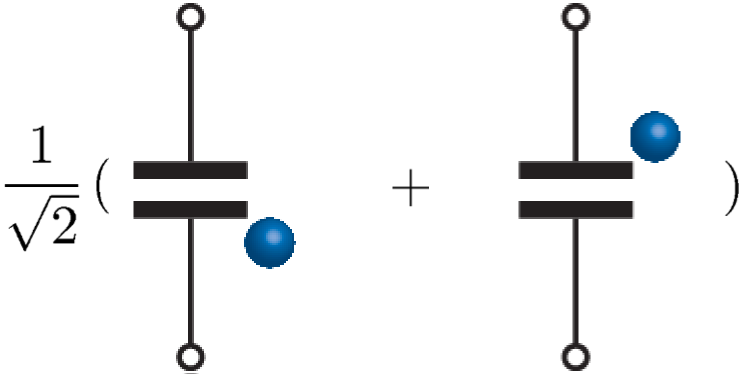
quantum superposition states:

- charge q
- flux ϕ

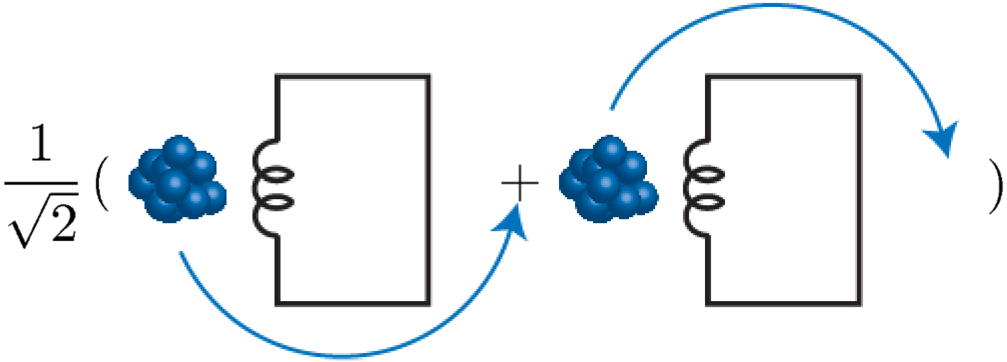
commutation relation (c.f. *x*, *p*):

$$\left[\hat{\phi}, \hat{q}\right] = i\hbar$$

charge on a capacitor:

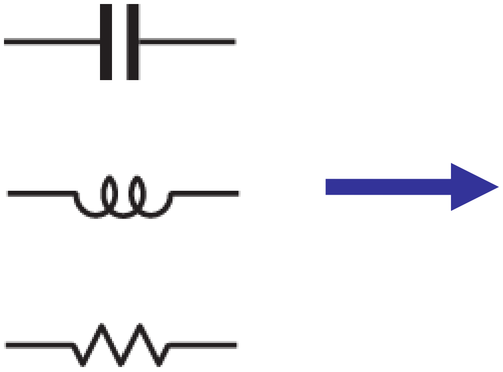


current or magnetic flux in an inductor:

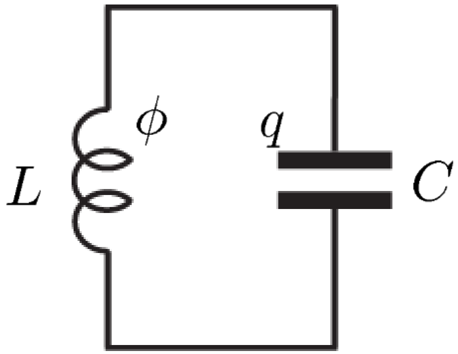


Constructing Linear Quantum Electronic Circuits

basic circuit elements:



harmonic LC oscillator:



$$\omega = \frac{1}{\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

classical physics:

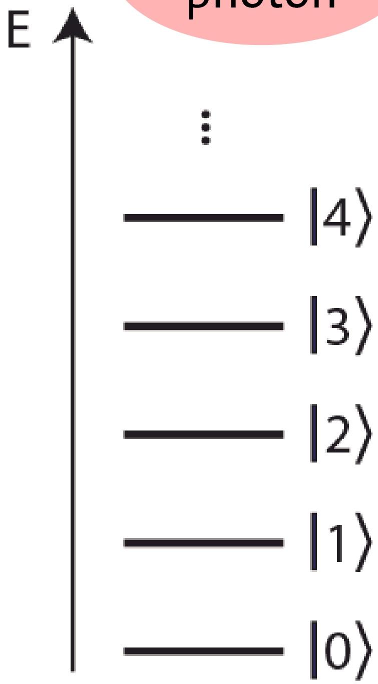
$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

quantum mechanics:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \qquad \left[\hat{\phi}, \hat{q}\right] = i\hbar$$

energy:

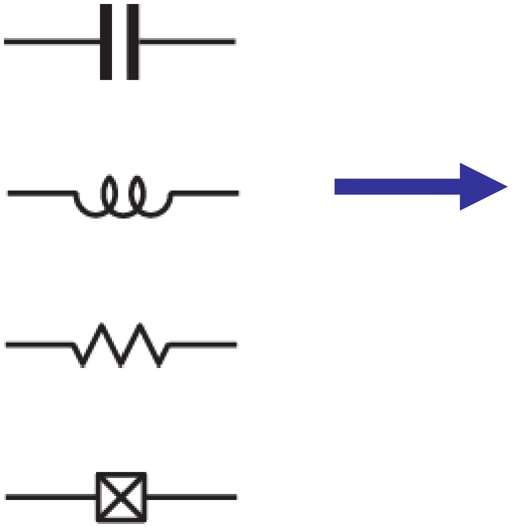
electronic photon



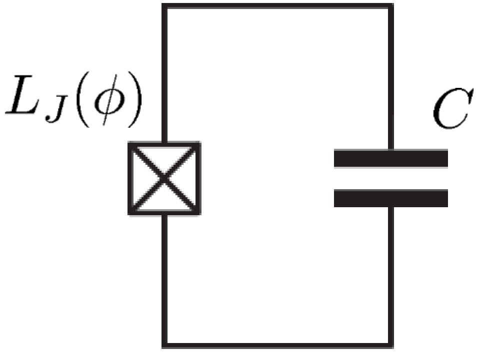
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Constructing Non-Linear Quantum Electronic Circuits

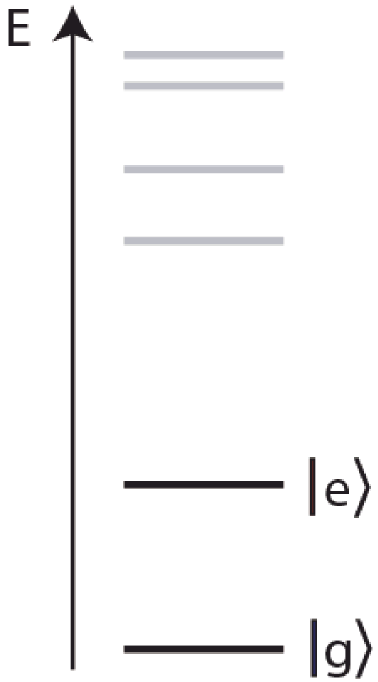
circuit elements:



anharmonic oscillator:



non-linear energy level spectrum:



Josephson junction:

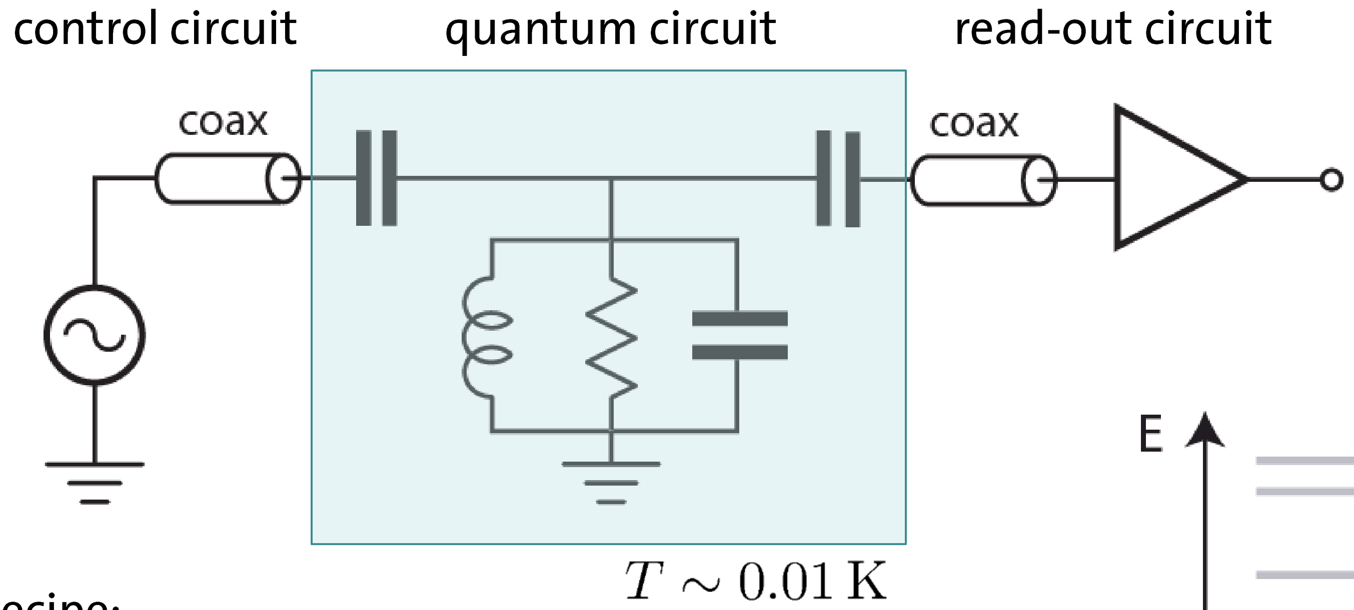
a non-dissipative nonlinear element (inductor)

$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1} = \frac{\phi_0}{2\pi I_c}\frac{1}{\cos(2\pi\phi/\phi_0)}$$

electronic artificial atom

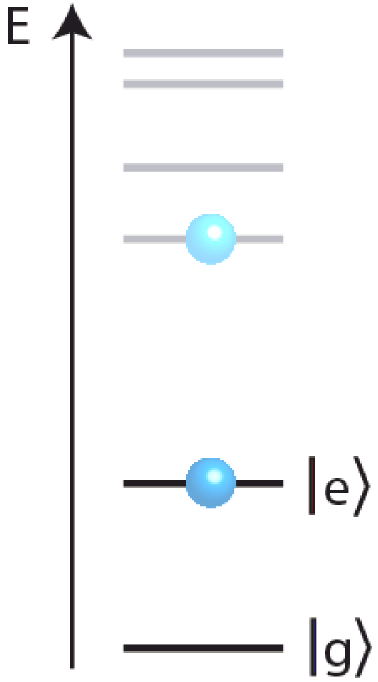
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

How to Operate Circuits in the Quantum Regime?



recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment



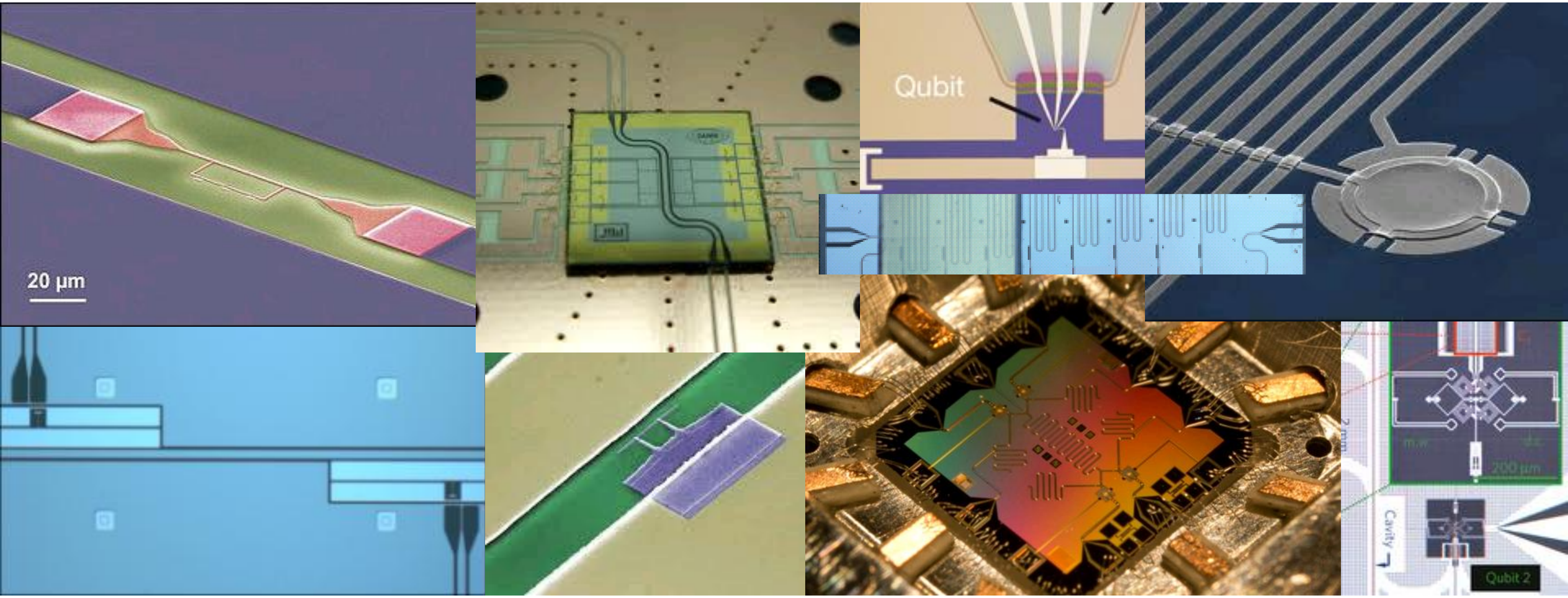
Can one actually build and operate such circuits?

Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Superconducting Quantum Electronic Circuits

single or multiple superconducting qubits coupled to harmonic oscillators

- investigated in a few dozen labs around the world
- for basic science and applications

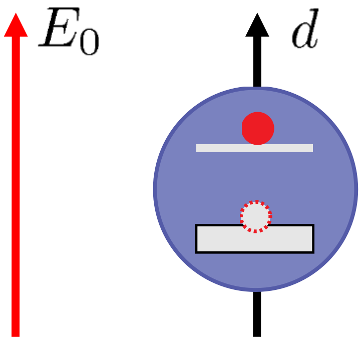


reviews:

R. J. Schoelkopf, S. M. Girvin, *Nature* **451**, 664 (2008)
J. Clarke and F. Wilhelm, *Nature* **453**, 1031 (2008)
J. Q. You and F. Nori, *Nature* **474**, 589 (2011)

Controlling the Interaction of Light and Matter

challenging on the level of single (artificial) atoms and single photons



- dipole moment *d* (usually small ~ ea_0)
- single photon fields E_0 (small in 3D)
- photon/atom interaction $\hbar g \sim dE_0$ (usually small)

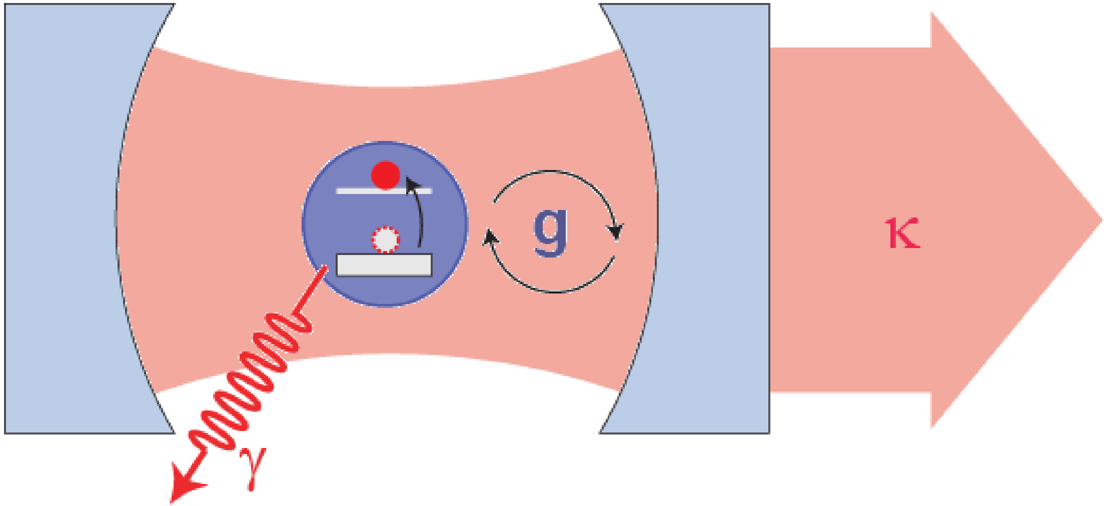
What to do?

- confine atom and photon in a cavity (cavity QED)
- engineer matter/light interactions, e.g. in solid state circuits

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Cavity Quantum Electrodynamics

interaction of atom and photon in a cavity



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit: $g = dE_0/\hbar > \gamma,\, \kappa\,,\, 1/t_{\text{transit}}$

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Dressed States Energy Level Diagram

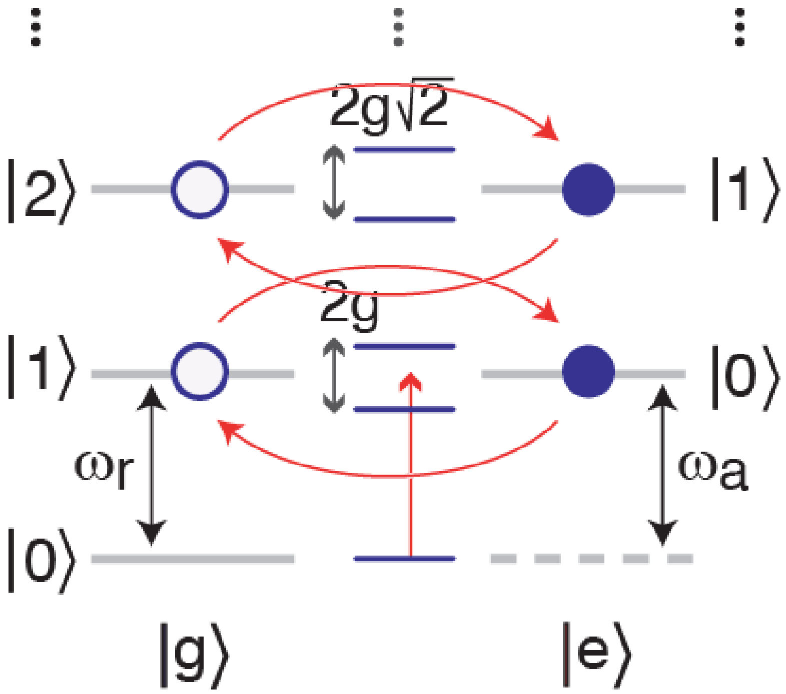
$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma, \kappa$$



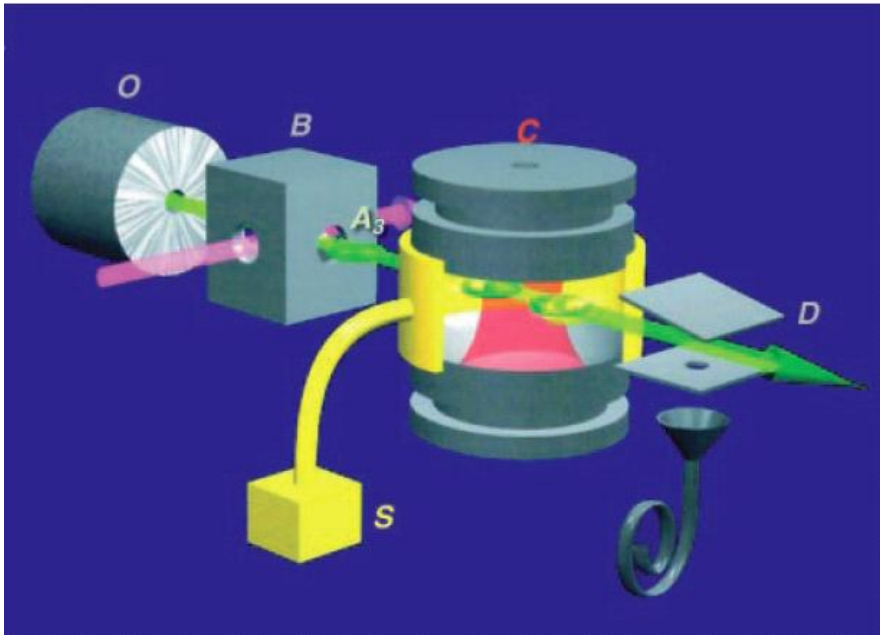
Jaynes-Cummings Ladder

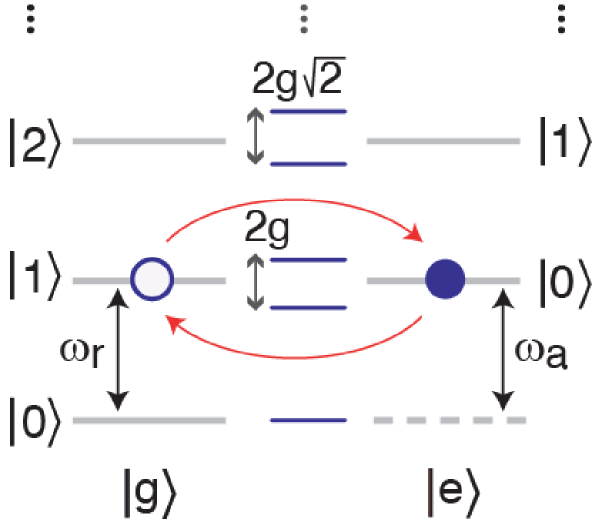
atomic cavity QED reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734-1738 (2008)

S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

Vacuum Rabi Oscillations with Rydberg Atoms





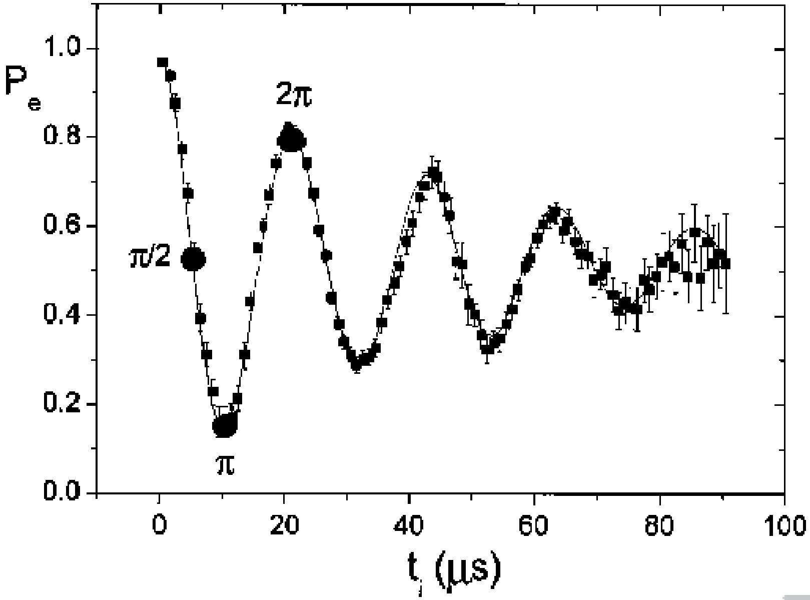
with Rydberg atoms in microwave domain:

- large d

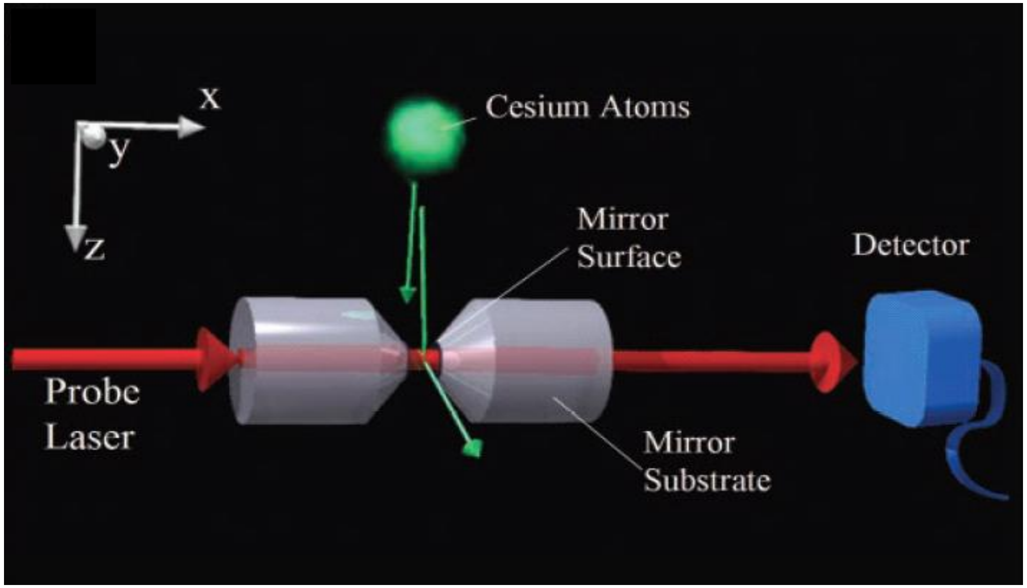
reviews:

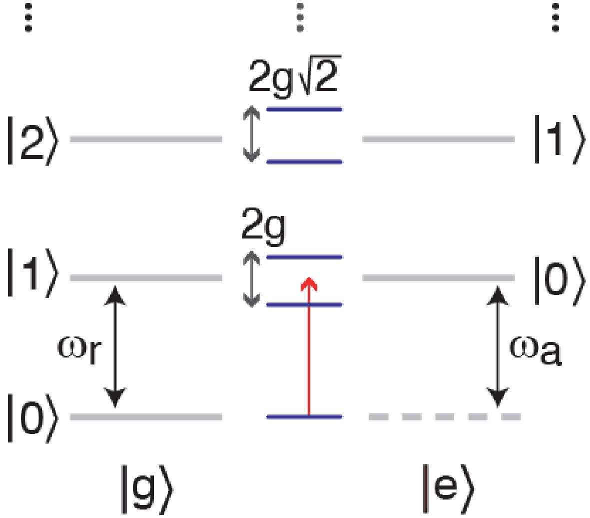
S. Haroche & J. Raimond, *OUP Oxford* (2006)

J. M. Raimond, M. Brune, and S. Haroche *Rev. Mod. Phys.* 73, 565 (2001)



Vacuum Rabi Mode Splitting with Alkali Atoms





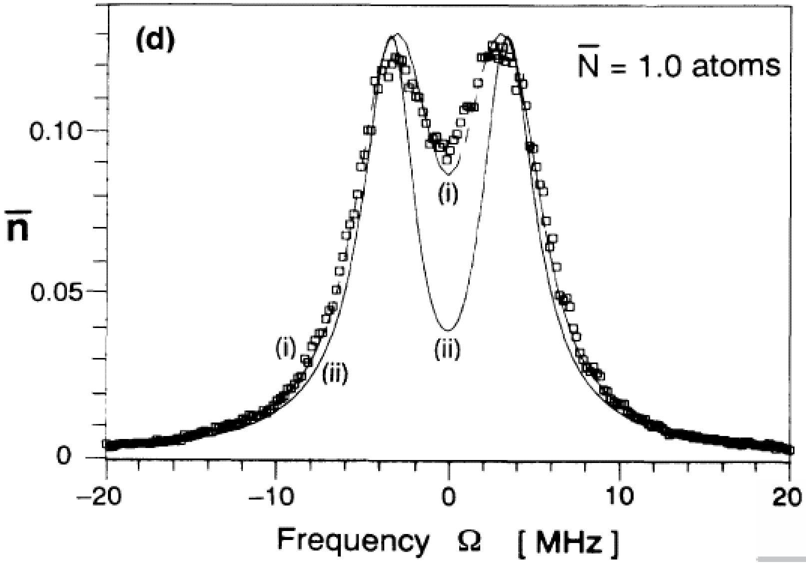
with alkali atoms in optical domain:

- large E_0

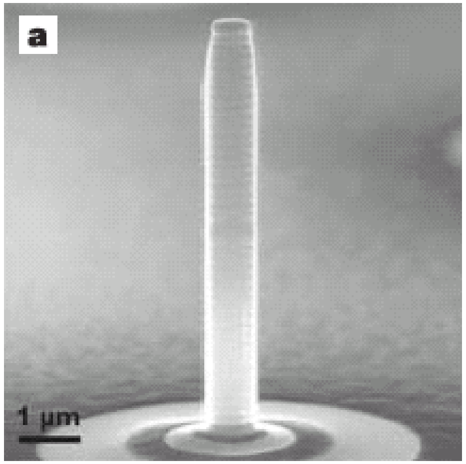
reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* 320, 1734 (2008)

H. Mabuchi, A. C. Doherty, *Science* 298, 1372 (2002)

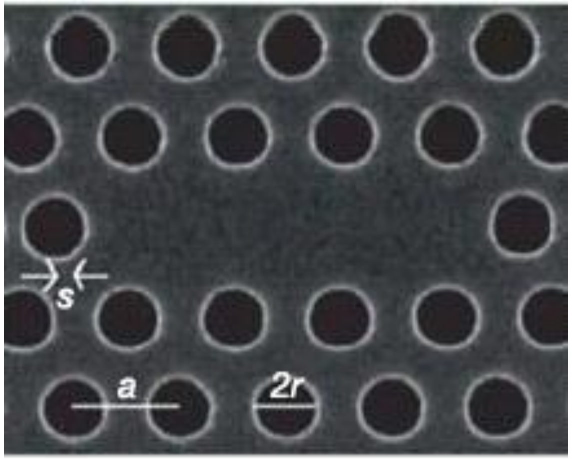


… and also with Semiconductors



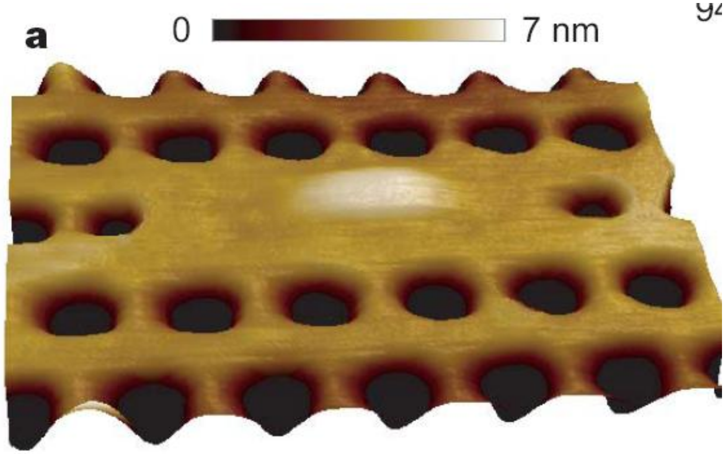
Wurzburg

*Nature **432**, 197 (2004)*



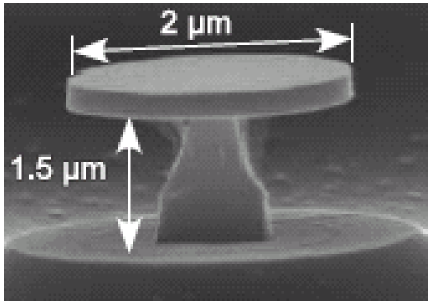
Arizona

*Nature **432**, 200 (2004)*



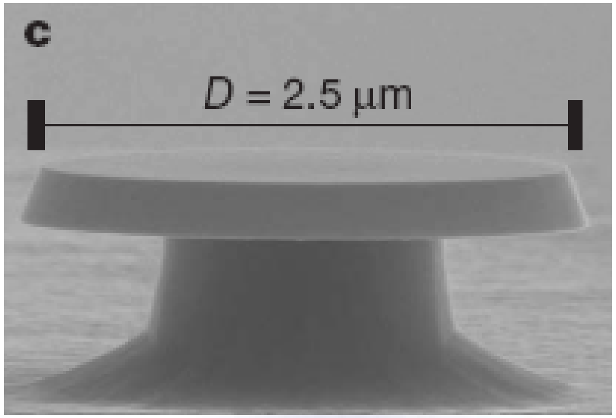
ETH Zurich

*Nature **445**, 896 (2007)*



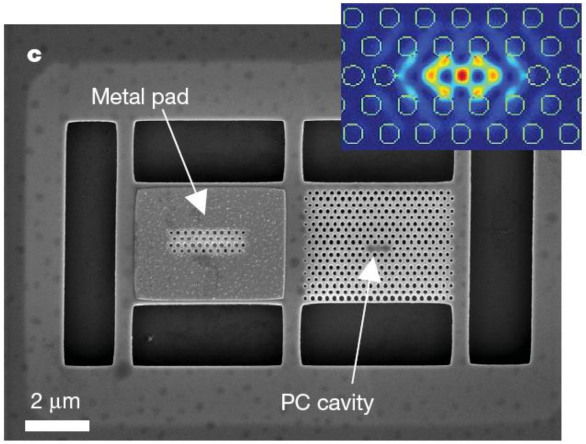
Paris

PRL (2004)



Caltech

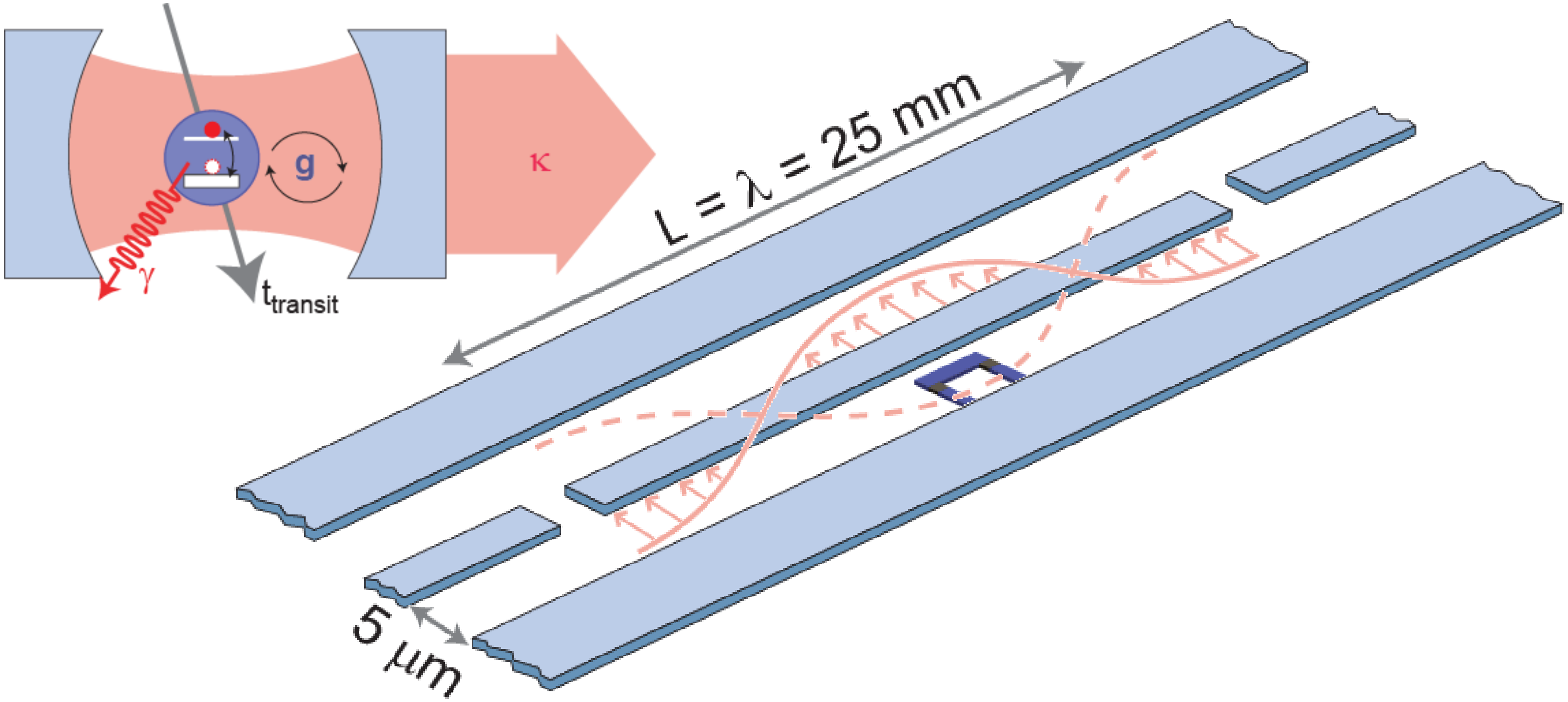
*Nature **450**, 862 (2007)*



Stanford

*Nature **450**, 857 (2007)*

Circuit Quantum Electrodynamics



elements:

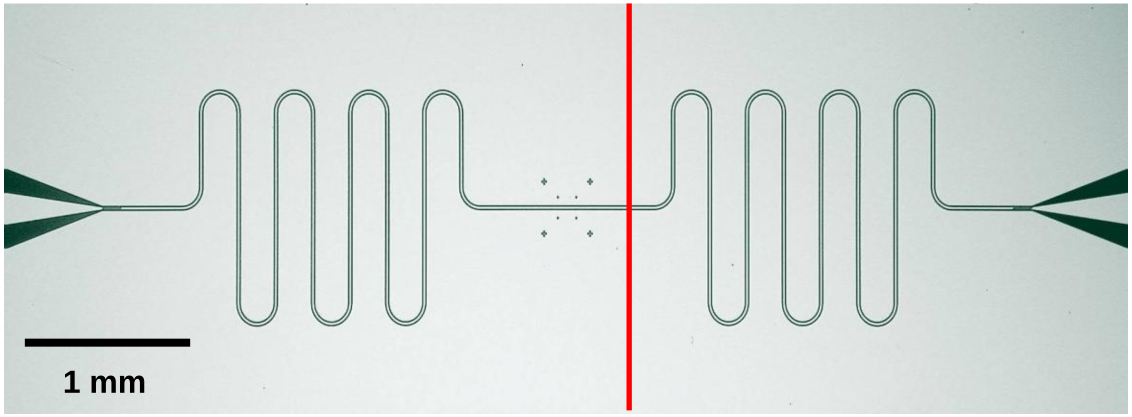
- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a superconducting qubit with large dipole moment d and long coherence time $1/\gamma$ and fixed position

A. Blais, *et al.*, *PRA* **69**, 062320 (2004)

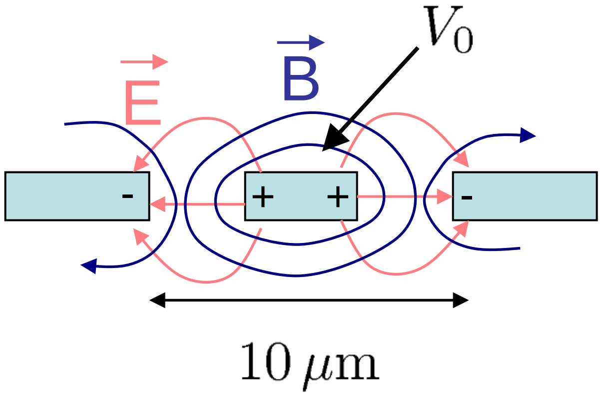
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Large Vacuum Field in 1D Cavity



optical microscope image of strip line resonator



electric field across resonator in vacuum state (*n*=*0*):

$$\int \epsilon_0 E_{0,\mathrm{rms}}^2 \, dV_{\mathrm{mod}} = \frac{\hbar\omega_r}{2}$$

harmonic oscillator

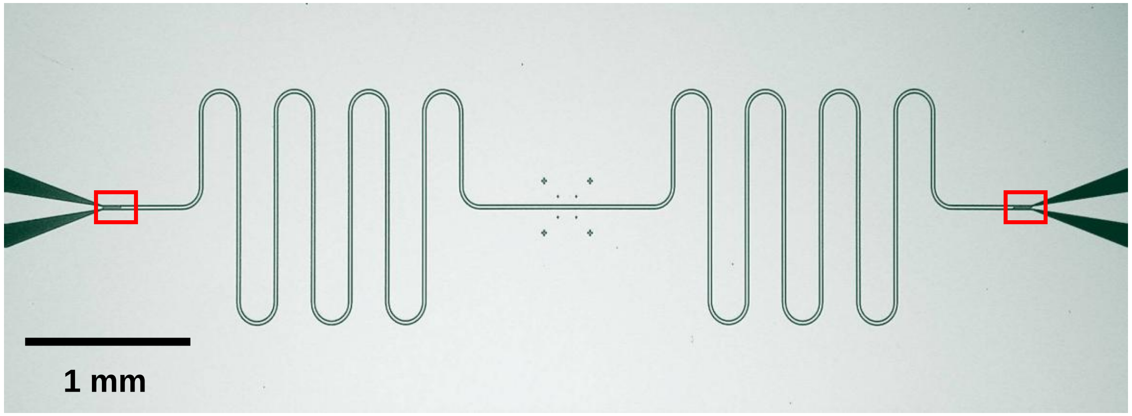
$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

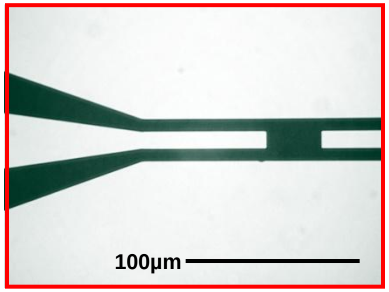
$$E_{0,\mathrm{rms}} \approx 0.2\,\mathrm{V/m}$$

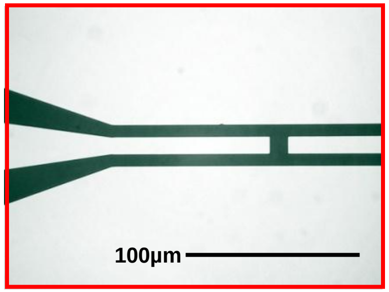
for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$

$\times 10^6$ larger than E_0
in 3D microwave cavity

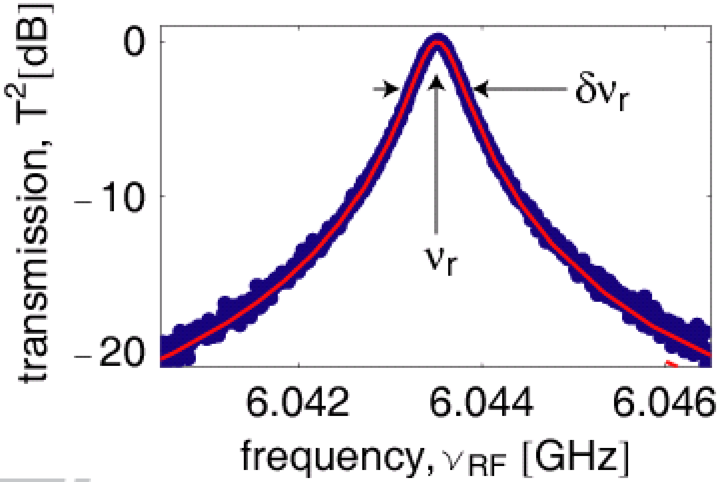
Long Controllable Photon Life Time







measuring the life time:



quality factor:

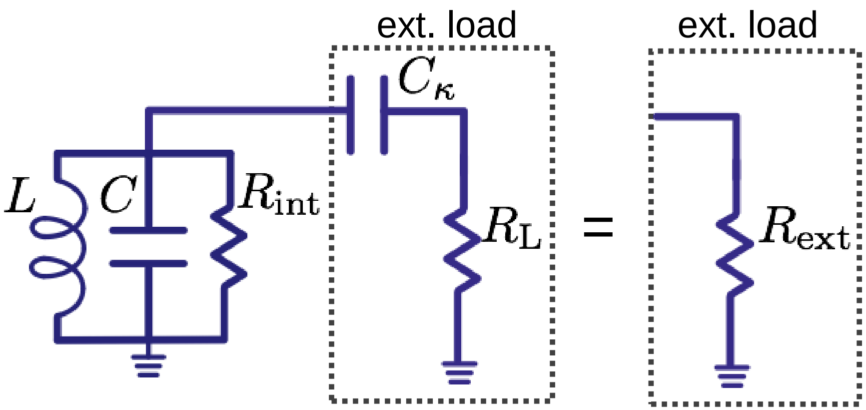
$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^2 - 10^5$$

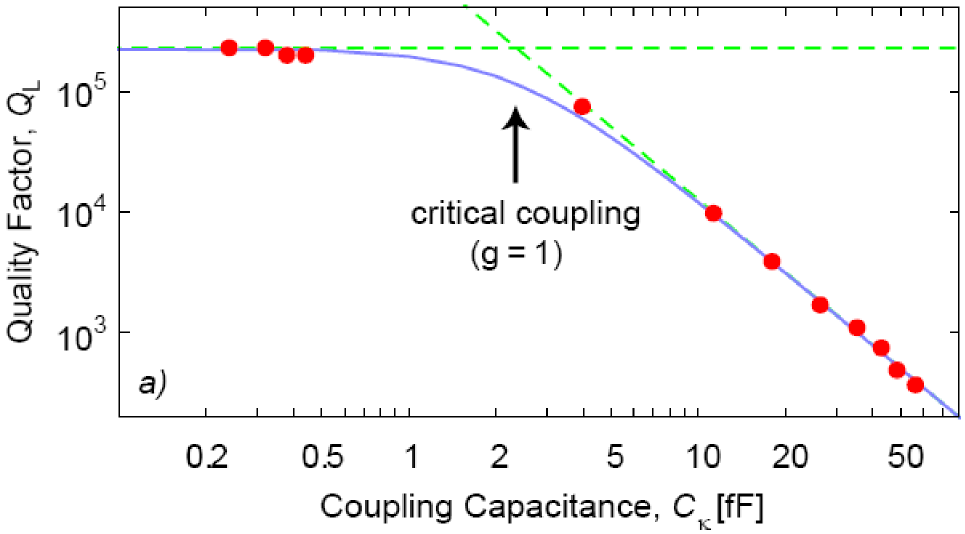
photon lifetime:

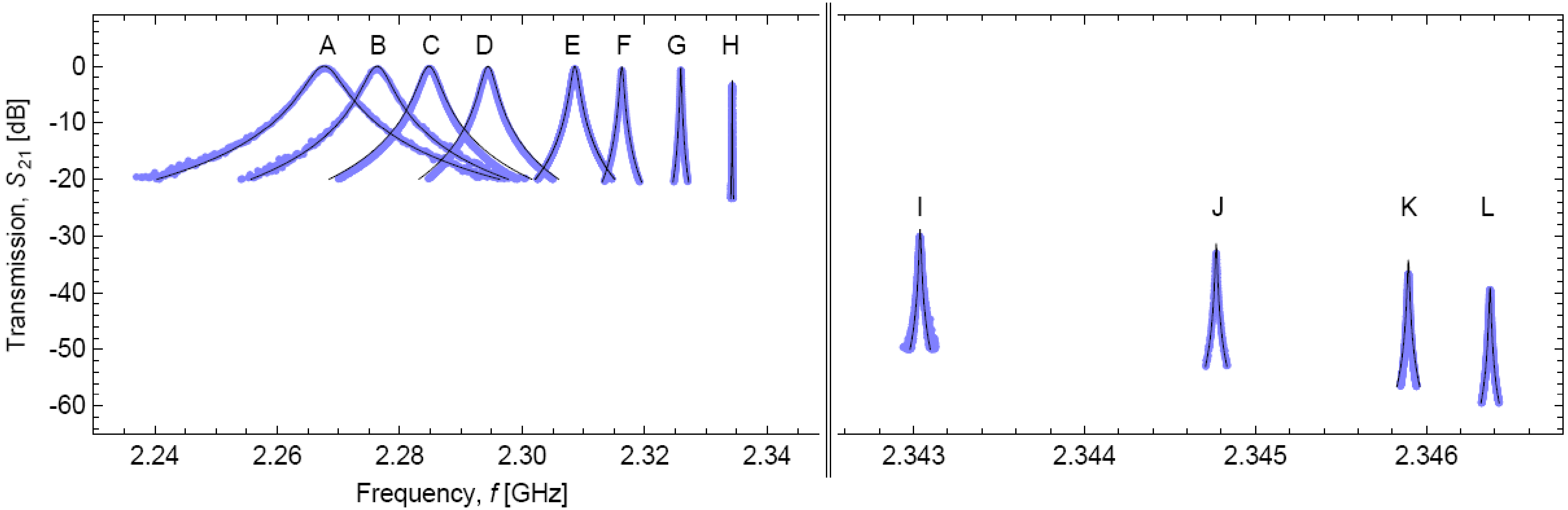
$$T_\kappa = 1/\kappa \approx 10\,\mathrm{ns} - 10\,\mu\mathrm{s}$$

M. Goeppl, *et al. J. Appl. Phys.* **104**, 113904 (2008)

The Quality Factor

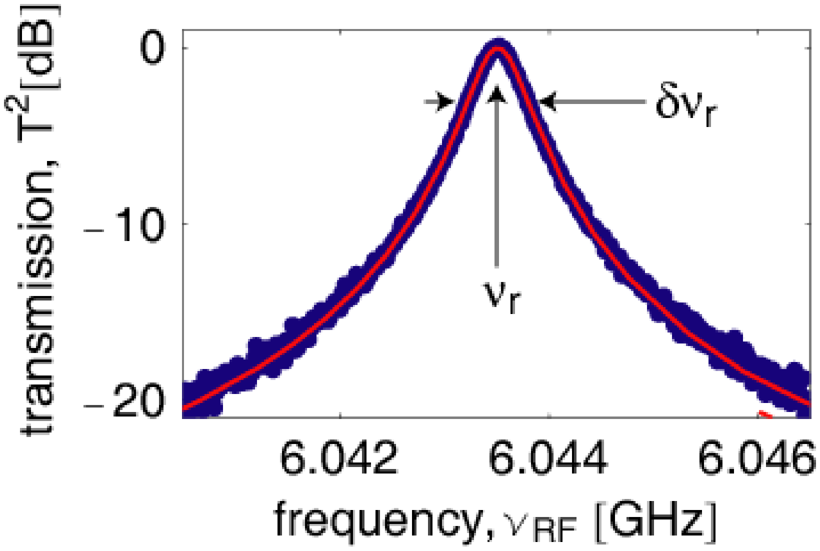


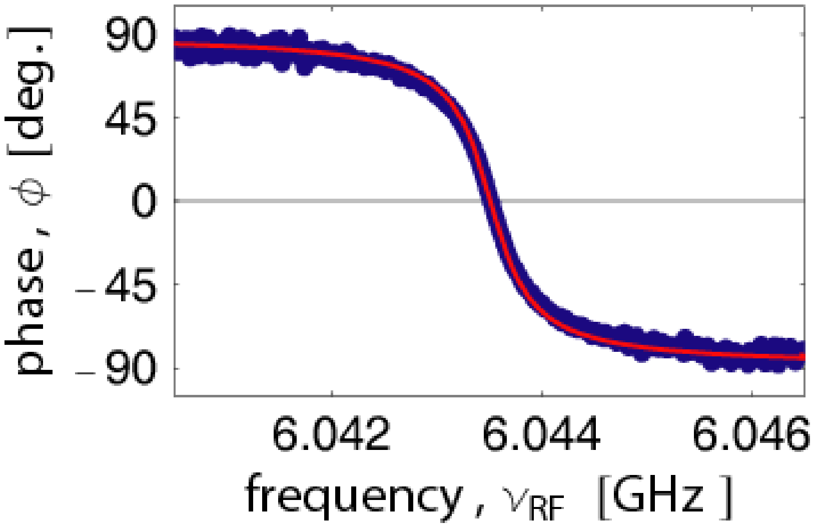




M. Goeppl, *et al. J. Appl. Phys.* **104**, 113904 (2008)

Resonator Quality Factor and Photon Lifetime





resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

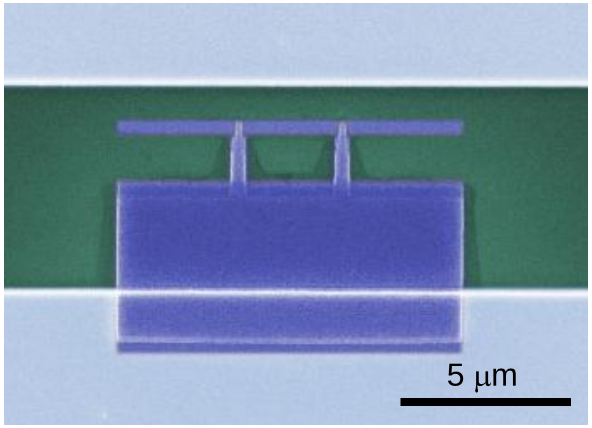
photon decay rate:

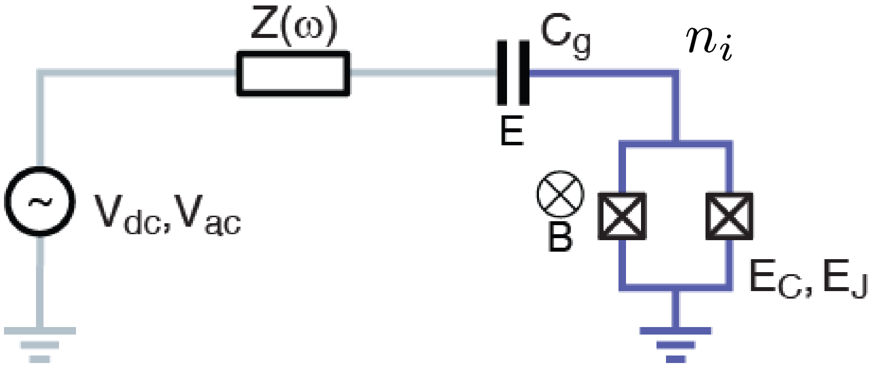
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

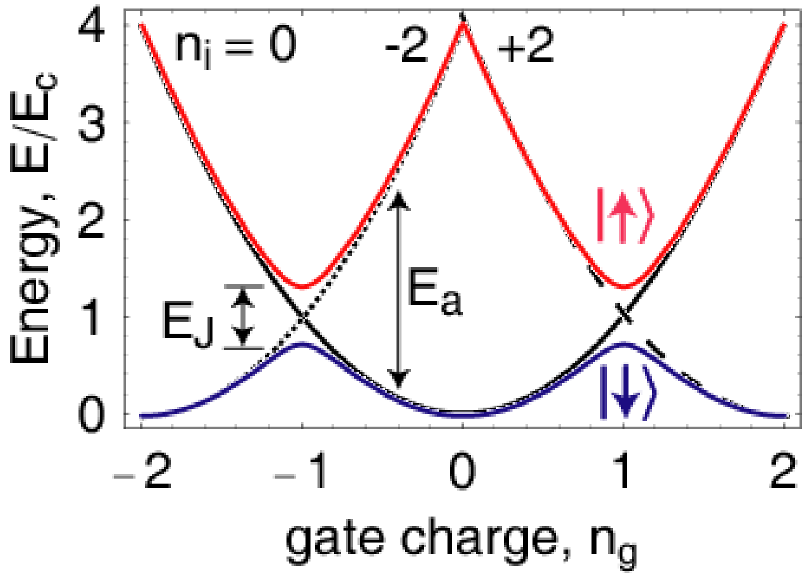
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

The Artificial Atom: A Cooper Pair Box







electrostatic energy

$$E_{\rm el} = \overset{E_C \; /\!/}{\frac{e^2}{2C_\Sigma}}\left(n_i - \overset{n_g \; /\!/}{\frac{C_gV_g}{e}}\right)^2$$

Josephson energy

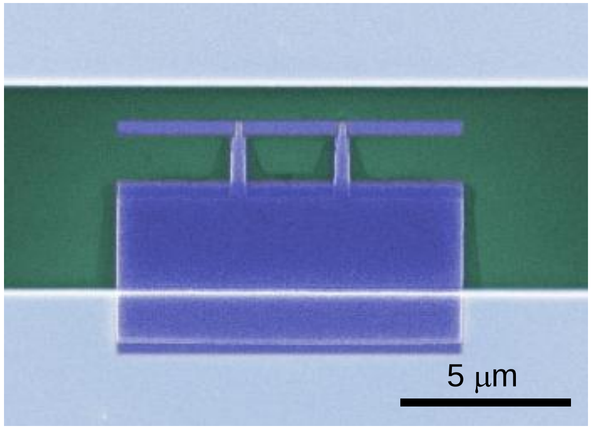
$$E_J = \overset{E_{J,\max} \; /\!/}{\frac{h\Delta}{8e^2R_J}}\cos\left(\pi\overset{\Phi_b \; /\!/}{\frac{\Phi}{\Phi_0}}\right)$$

Bouchiat, Vion, Joyez, Esteve, Devoret, *Physica Scripta* T76, 165 (1998).

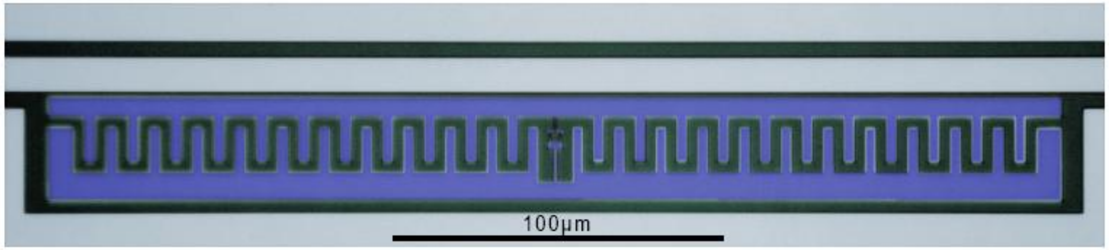
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

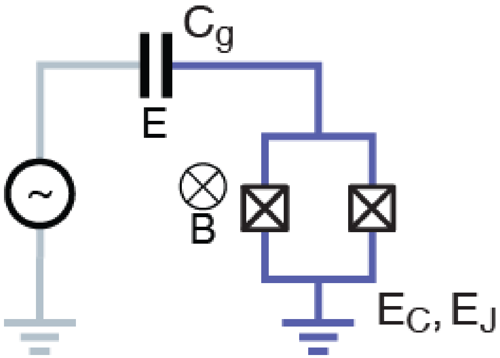
standard CPB:

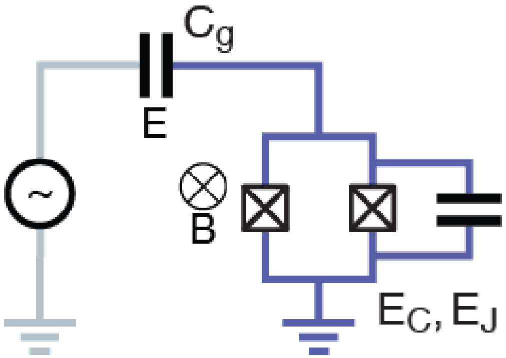


transmon:



circuit diagram:

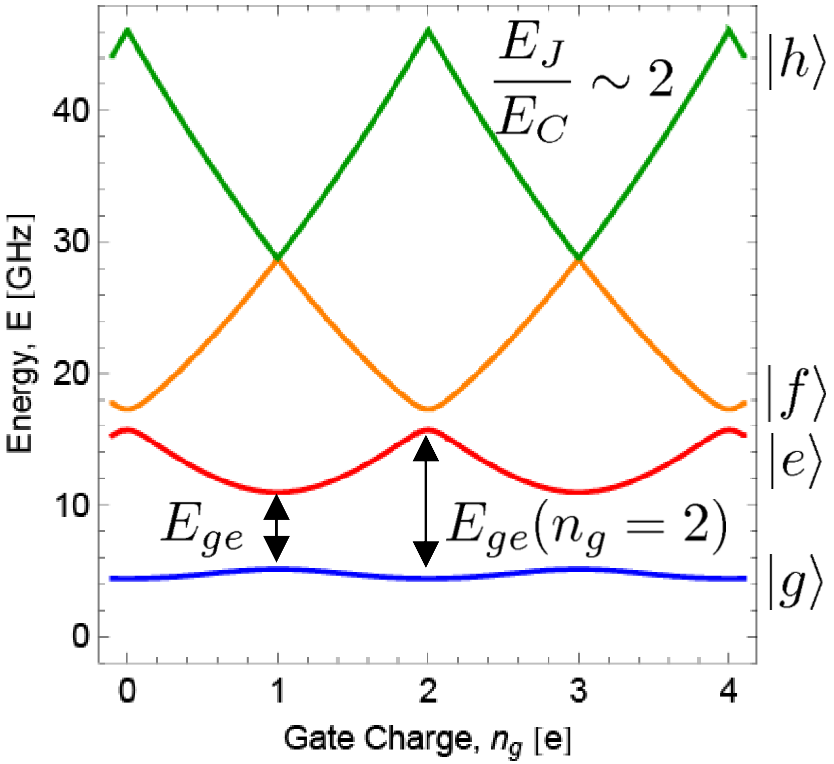




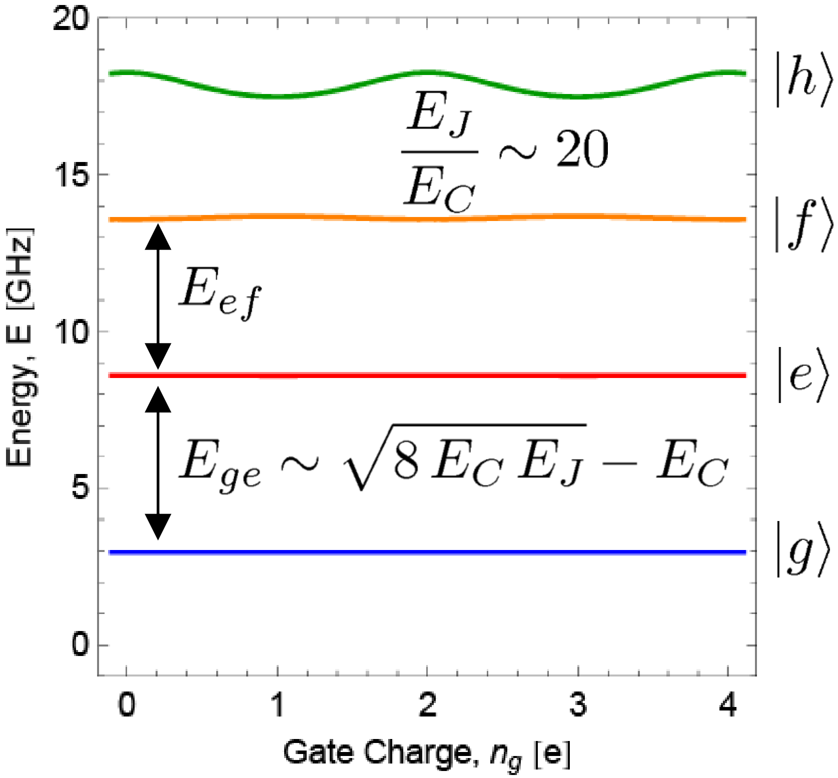
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels



Transmon energy levels



dispersion

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

relative anharmonicity

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

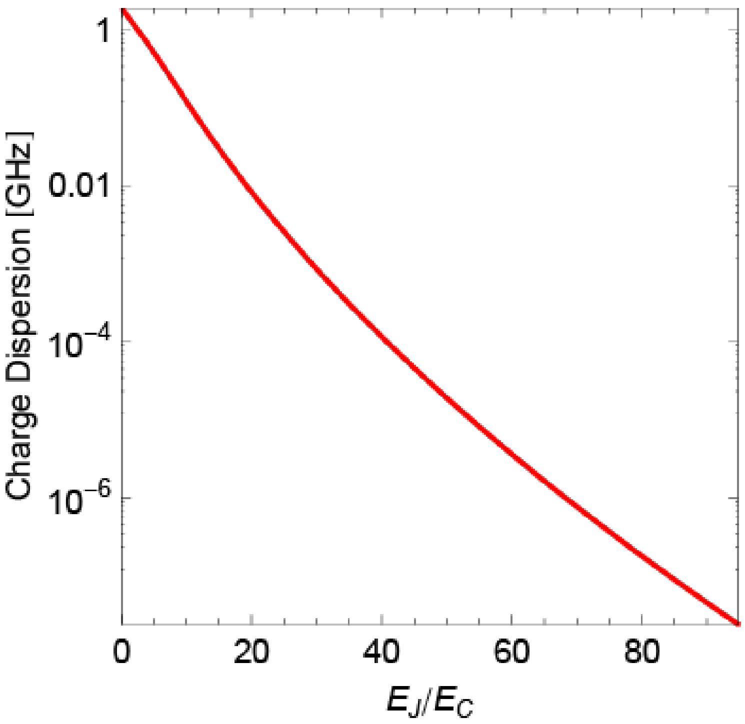
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Dispersion and Anharmonicity

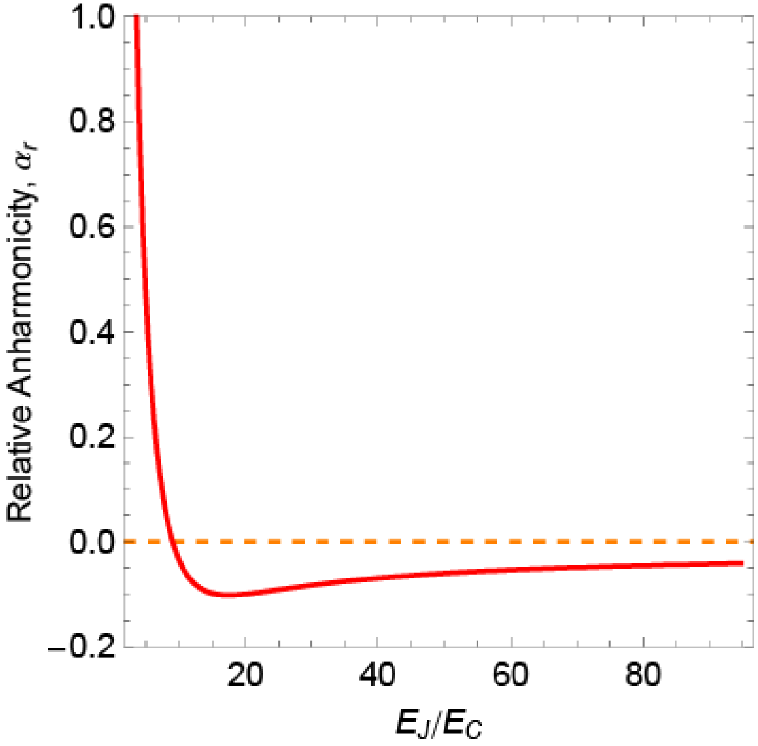
Charge dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$



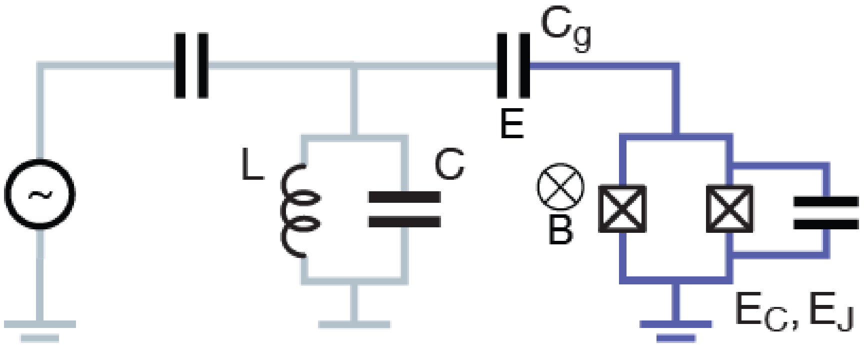
Anharmonicity:

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

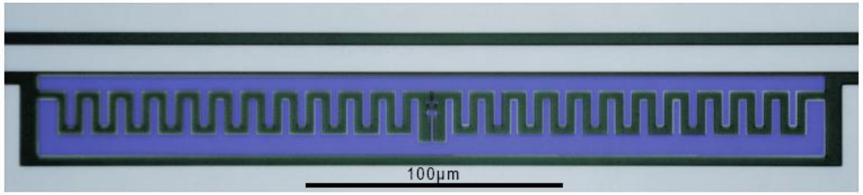


J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Qubit/Photon Coupling and Tunability



qubit coupled to resonator



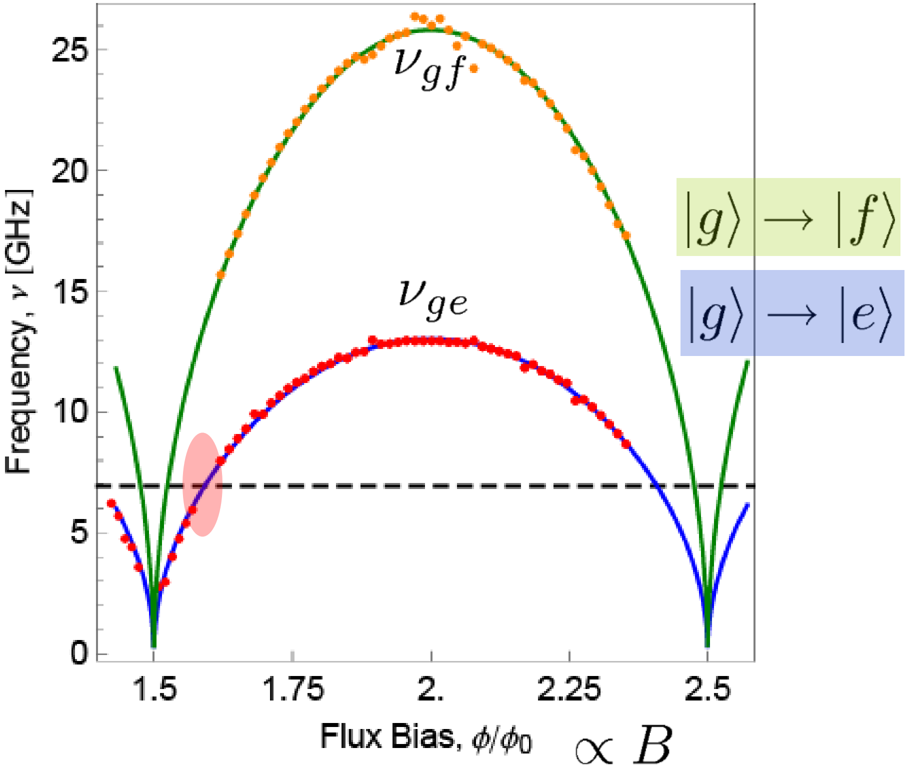
coupling strength:

$$\hbar g = e V_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma}$$

$$\Longrightarrow \nu_{\mathrm{vac}} = \frac{g}{\pi} \approx 1 \ldots 300\,\mathrm{MHz}$$

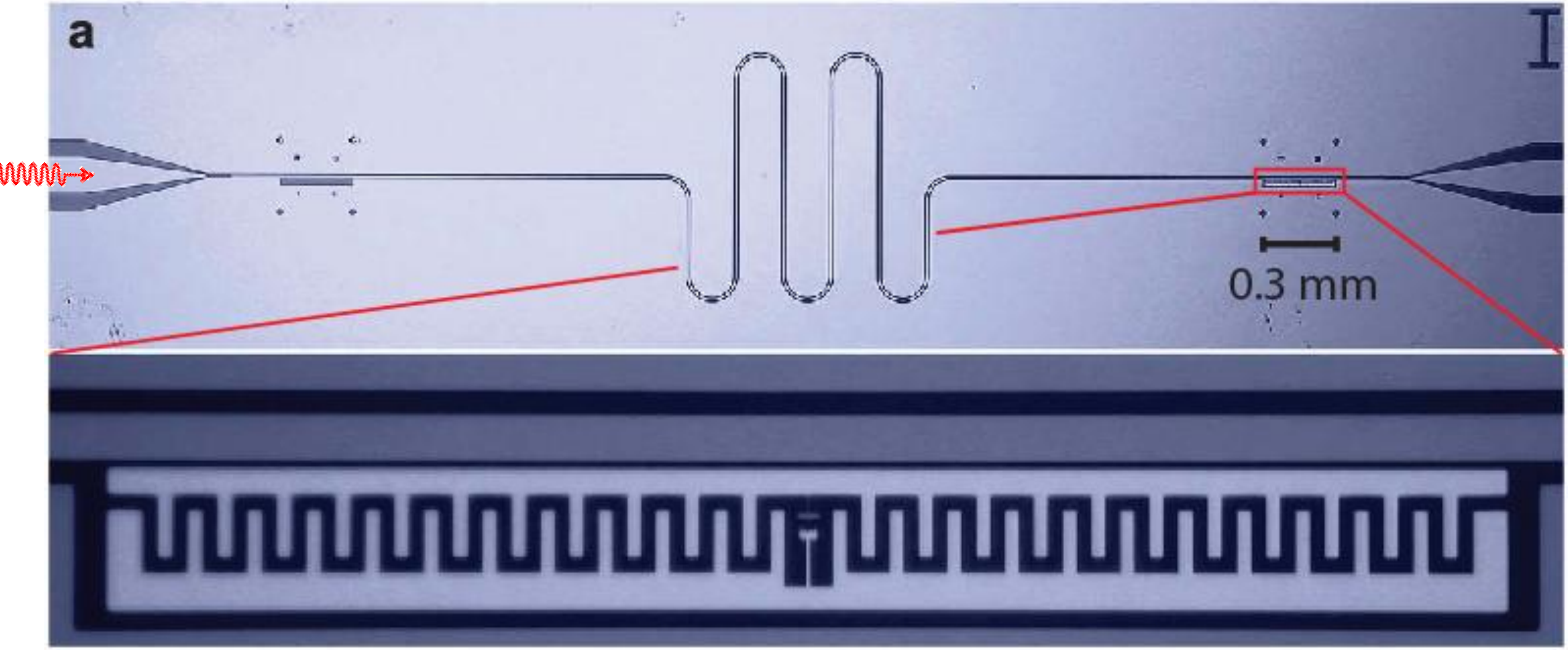
$g \gg [\kappa, \gamma]$ possible!

spectroscopic measurement of transition frequency vs. magnetic field B:

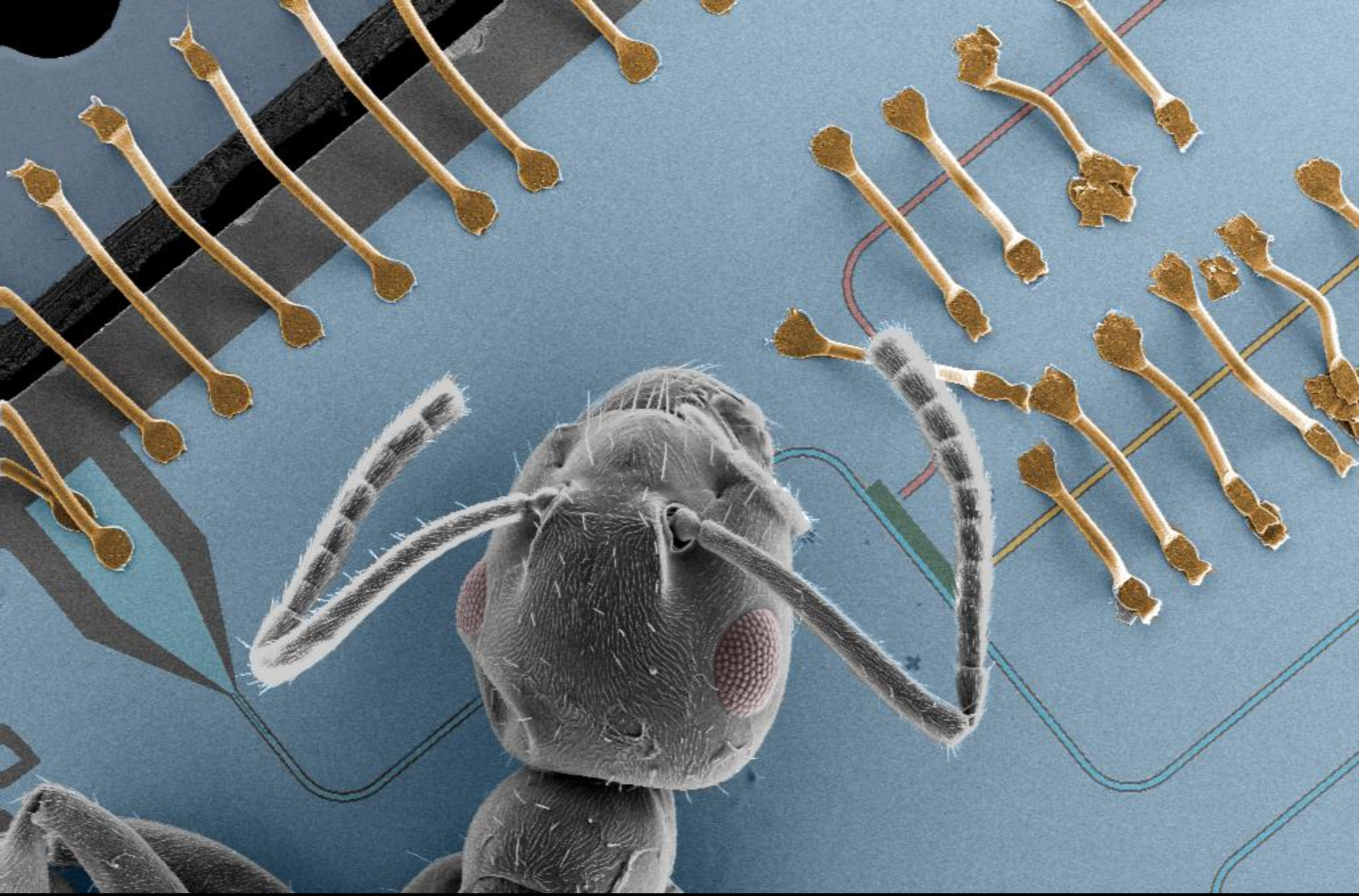


tune qubit into resonance

Realization



device fabrication: L. Frunzio et al., IEEE Trans. on Appl. Supercon. **15**, 860 (2005)



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Swiss Federal Institute of Technology Zurich

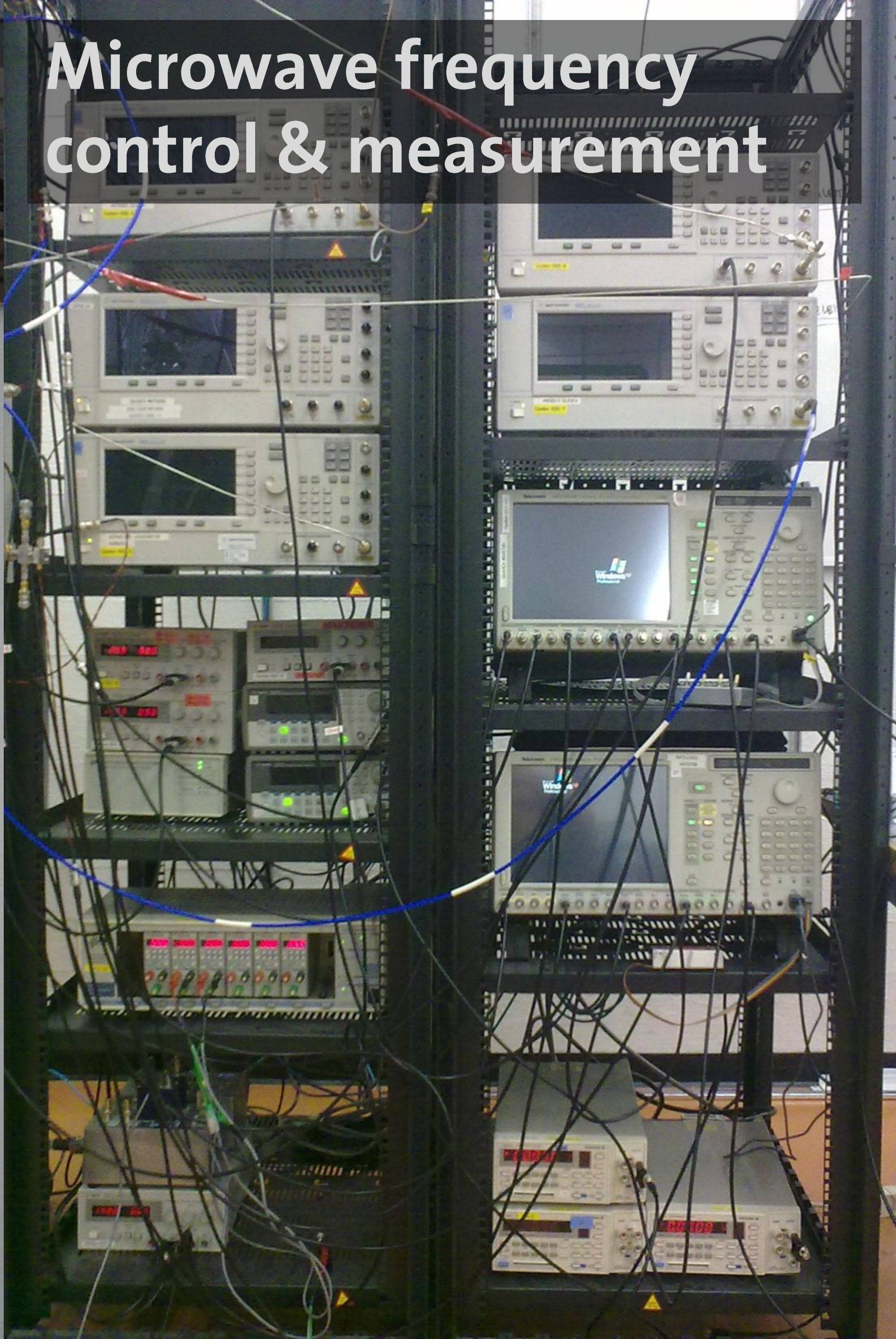
J. Mlynek *et al.*, Quantum Device Lab, ETH Zurich (2012)

Sample Mount

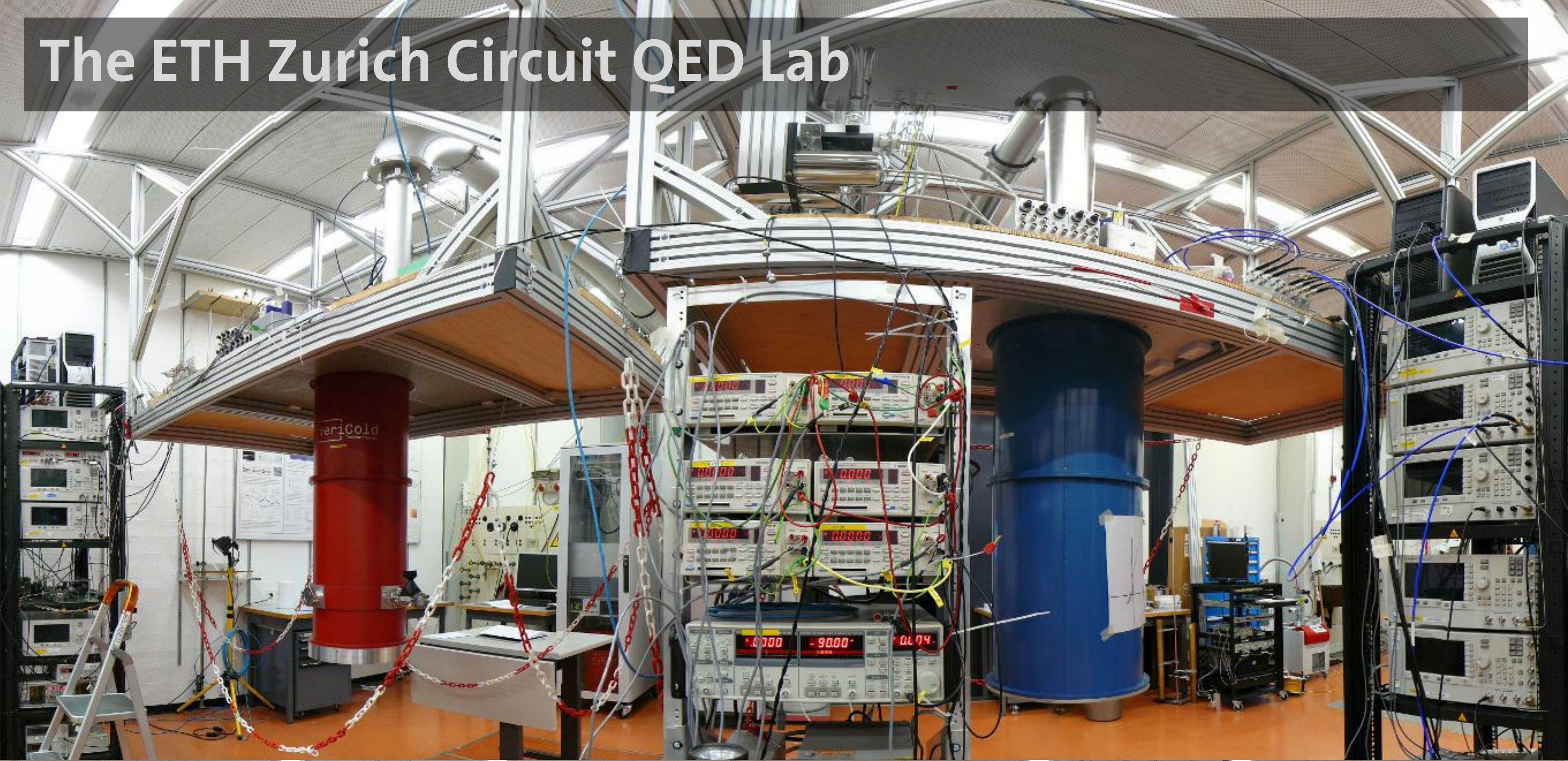
M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)

Cryostate for temperatures down to 0.02 K

Microwave frequency control & measurement

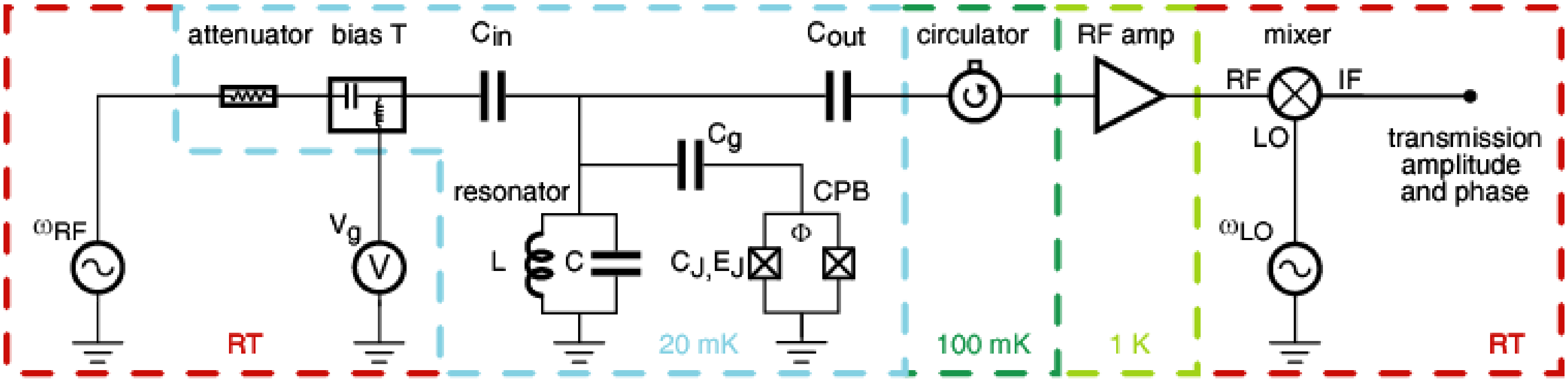


The ETH Zurich Circuit QED Lab



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

How We Do the Measurement



- prevent leakage of thermal photons (cold attenuators and circulators)

- average power to be detected

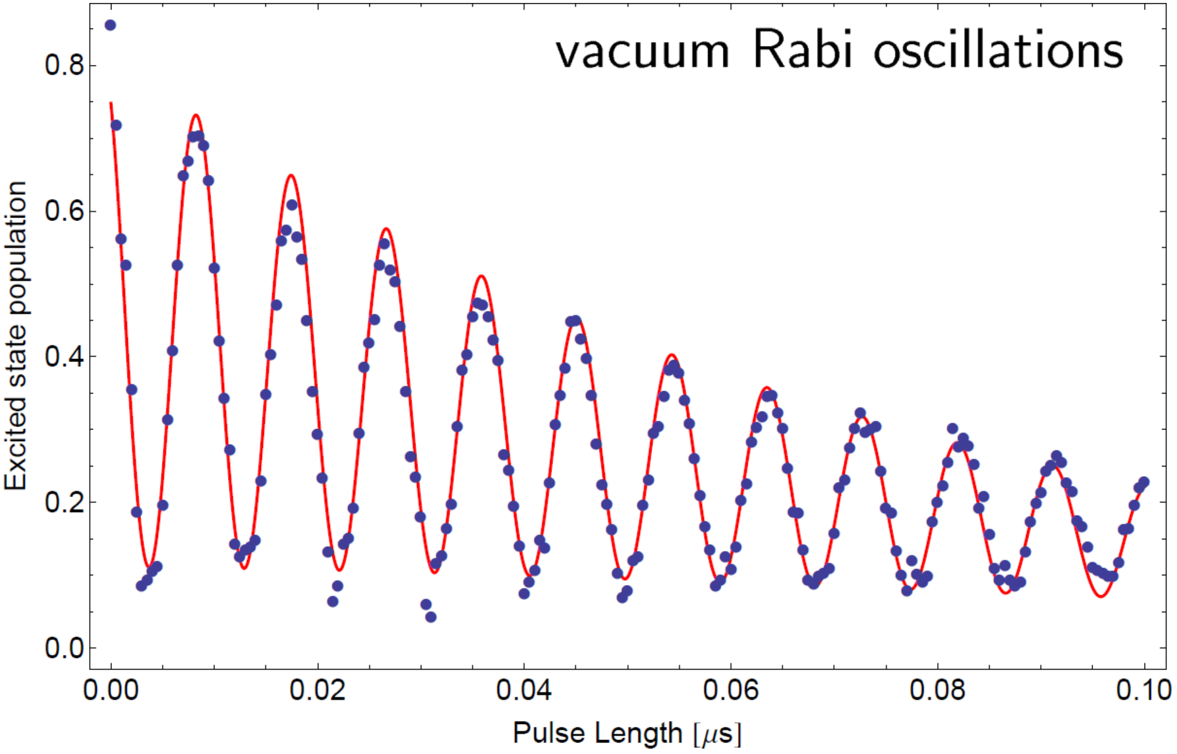
$$\rightarrow \langle n = 1 \rangle \hbar \omega_r \kappa / 2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$

- efficient with cryogenic low noise HEMT amplifier ($T_N = 6\,\mathrm{K}$)

Resonant Vacuum Rabi Mode Splitting ...

... with one photon ($n = 1$):

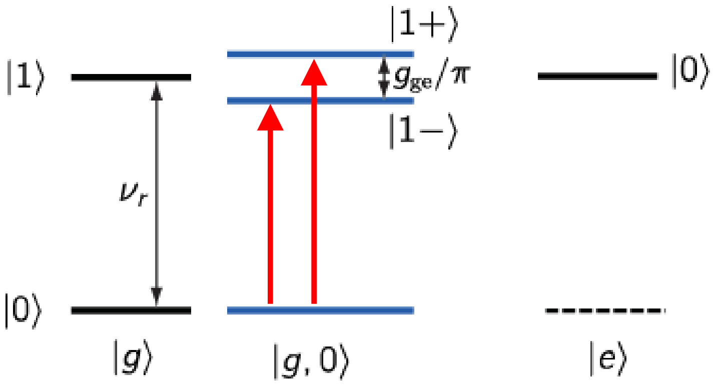
very strong coupling:



$$g_{ge}/\pi = 308\,\text{MHz}$$

$$\kappa, \gamma < 1\ \text{MHz}$$

$$g_{\text{ge}} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Quantum Physics with Circuit QED ... some examples

Vacuum Rabi Mode Splitting
A. Wallraff *et al.*, *Nature* **431**, 162 (2004)

Coherent Flux-Qubit / SQUID Coupling
I. Chiorescu *et al.*, *Nature* **431**, 159 (2004)

Quantum AC-Stark Shift
D. Schuster *et al.*, *Nature* **445**, 515 (2007)

Lamb Shift
A. Fragner *et al.*, *Science* **322**, 1357 (2008)

Fock and Arbitrary Photon States
M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)
M. Hofheinz *et al.*, *Nature* **459**, 546 (2009)

Root n Nonlinearity
J. Fink *et al.*, *Nature* **454**, 315 (2008)

Two Photon Nonlinearities
F. Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)

Parametric Amplification
Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 928 (2008)

Super Splitting and Root n Nonlinearity
L. Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)

Ultrastrong Coupling
T. Niemczyk *et al.*, *Nat. Phys.* **6**, 772 (2010)

Single Photon Source
A. Houck *et al.*, *Nature* **449**, 328 (2007)

Single Qubit MASER
O. Astafiev *et al.*, *Nature* **449**, 588 (2007)

Single Qubit Resonance Fluorescence
O. Astafiev *et al.*, *Science* **327**, 840 (2010)

QND Measurement of Single Photon
B. Johnson *et al.*, *Nat. Phys.* **6**, 663 (2010)

Correlation Function Measurements
D. Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)

Cooling and Amplification
M. Grajcar *et al.*, *Nat. Phys.* **4**, 612 (2008)

Quantum Algorithms & Entangled States
L. DiCarlo *et al.*, *Nature* **460**, 240 (2009)
L. DiCarlo *et al.*, *Nature* **467**, 574 (2010)
A. Fedorov *et al.*, *Nature* **481**, 170 (2012)
M. Reed *et al.*, *Nature* **481**, 382 (2012)

Quantum Bus
M. Sillanpaa *et al.*, *Nature* **449**, 438 (2007)
H. Majer *et al.*, *Nature* **449**, 443 (2007)
M. Mariantoni *et al.*, *Nat.Phys.* **7**, 287 (2011)
M. Mariantoni *et al.*, *Science* **334**, 61 (2011)

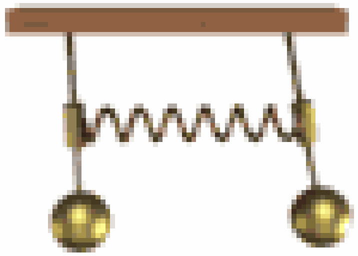
Cavity QED
with one, two, ~~three and many~~ Photons…

… probing quantum nonlinearities on the few photon level

Probing Field Quantization on a Chip ...

... by measuring the quantum nonlinearity of the J-C ladder

What can be learned from a measurement of the vacuum Rabi mode splitting?

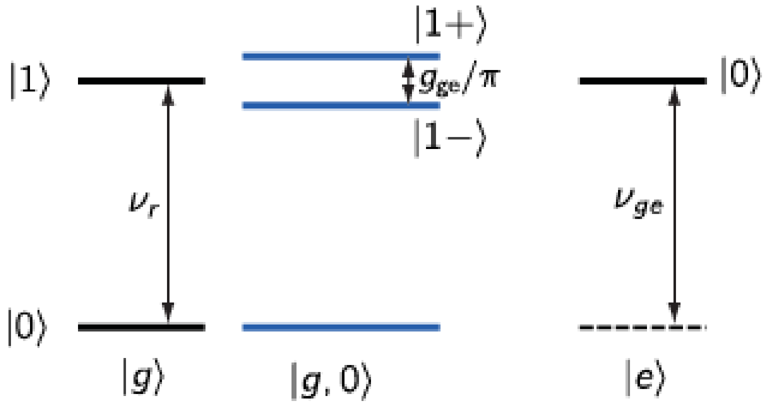


classical interpretation:

- coupled harmonic oscillators
- normal mode coupling

quantum effects:

- scaling of coupling g_{eff} with square root of the photon number n
- direct proof of field quantization



time-resolved data in atomic physics exps. (Haroche, Walther, ...) but no spectroscopic data until recently

$$|n\pm\rangle = \left(|g,n\rangle \pm |e,n-1\rangle\right)/\sqrt{2}$$

Climbing the Jaynes-Cummings Ladder

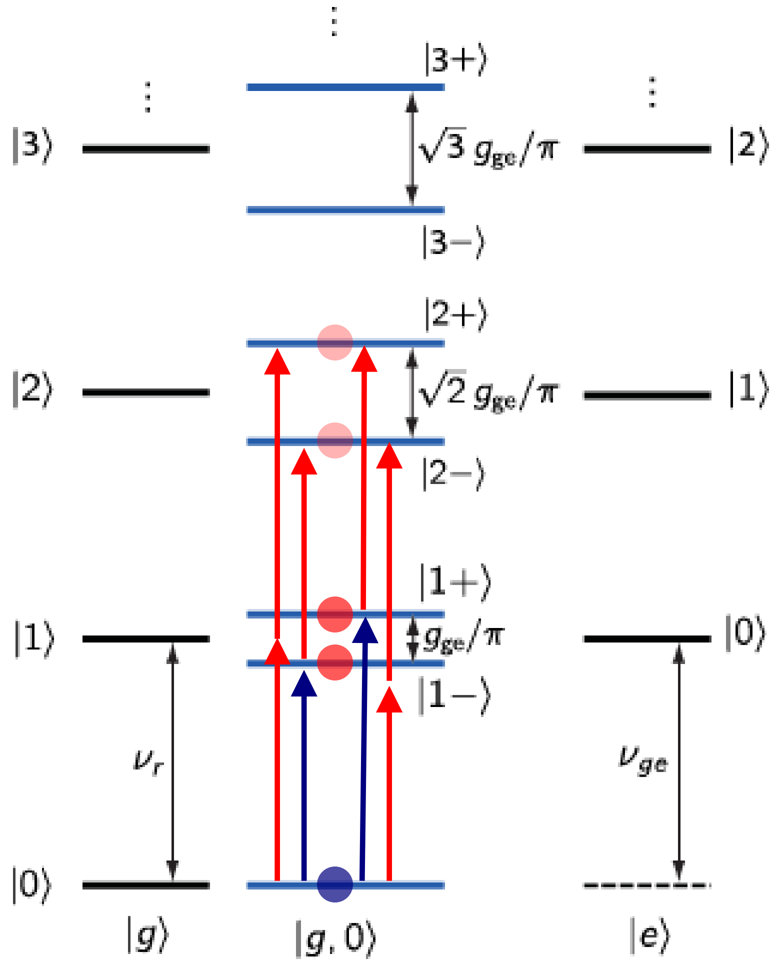
How to climb the ladder?

start on the lowest rung:

- cool to the ground state |g,0>

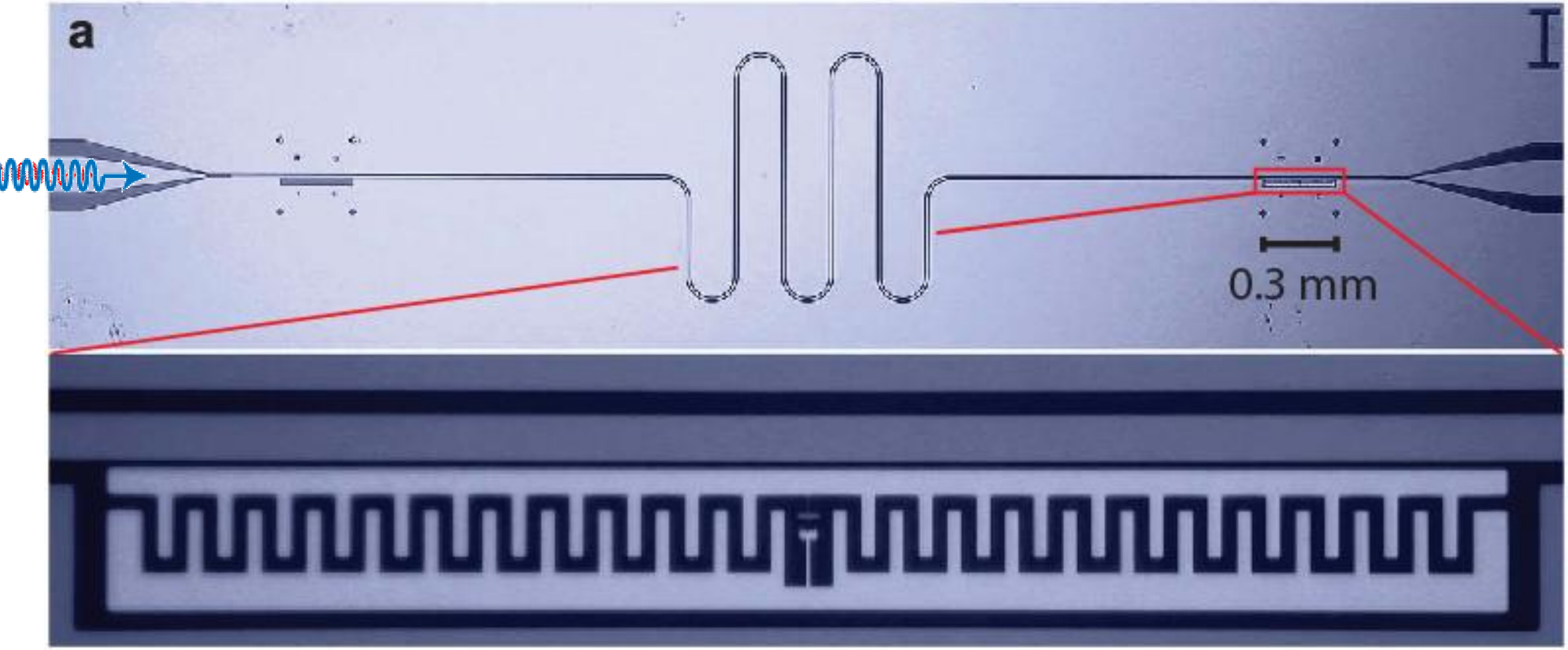
climb towards higher rungs:

- step by step:
  ‚pump & probe‘ excitation
  J. Fink *et al.*, *Nature* **454**, 315 (2008)
  I. Schuster *et al.*, *Nat. Phys.* **4**, 382 (2008)
  M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)
- many rungs at the same time:
  multi-photon excitation
  L. S. Bishop *et al.*, *Nature Phys.* **5**, 105 (2009)
- thermal excitation
  J. Fink *et al.*, *PRL* **105**, 163601 (2010)
  J. Fink *et al.*, *Physica Scripta* **T137**, 014013 (2009)
  with full control over phase:
  M. Hofheinz *et al.*, *Nature* **459**, 546 (2009)



$$|n\pm\rangle = \left(|g,n\rangle \pm |e,n-1\rangle\right)/\sqrt{2}$$

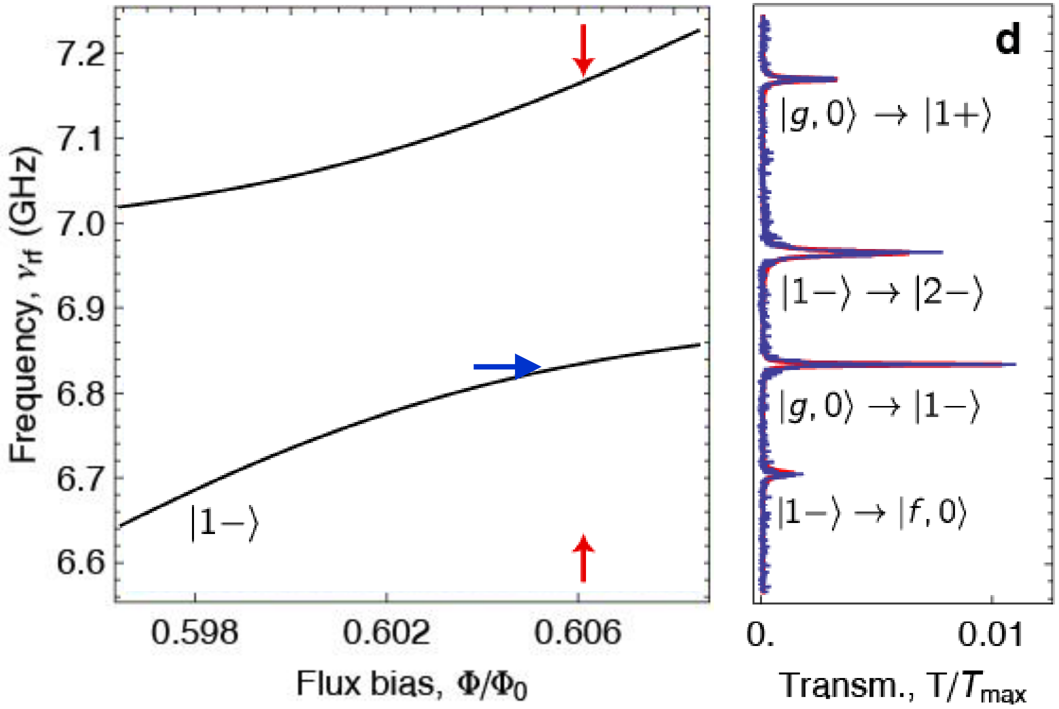
Two-Photon Pump and Probe Spectroscopy



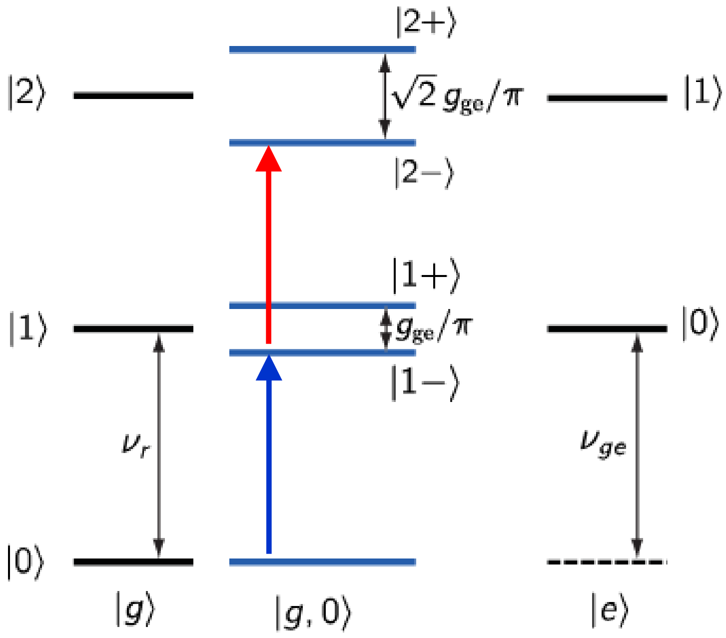
J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff,
Nature (London) **454**, 315 (2008)

Resonant Vacuum Rabi Mode Splitting ...

... with two photons $(n = 2)$:



pump and probe: $|n-\rangle$

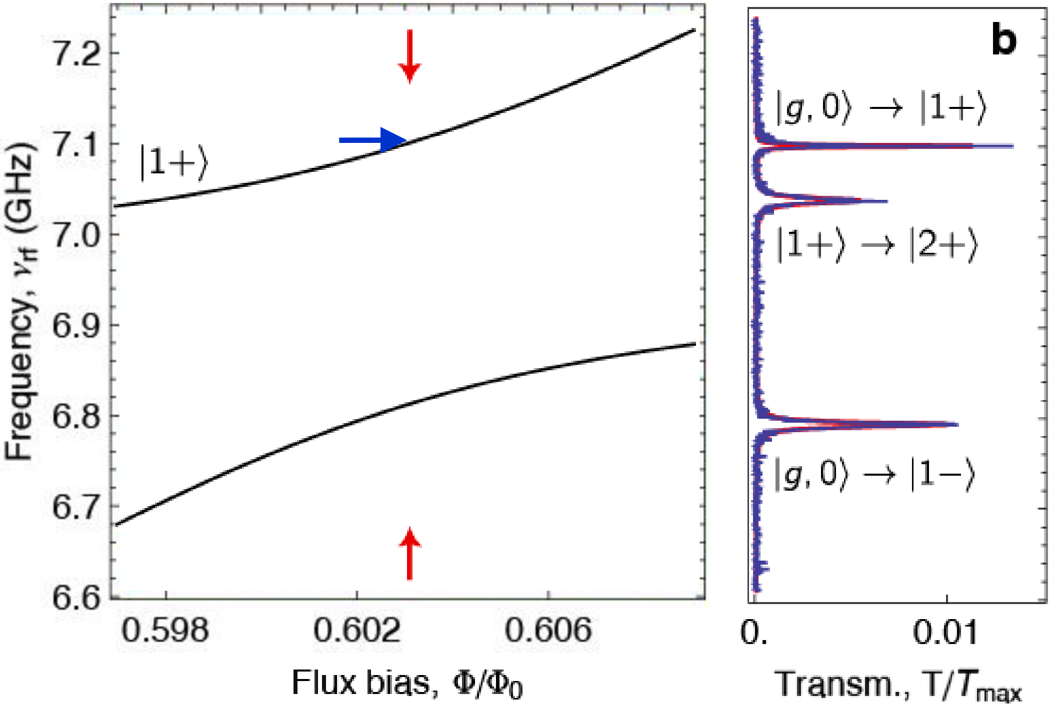


- $|n-\rangle \rightarrow |n+\rangle$ is weak

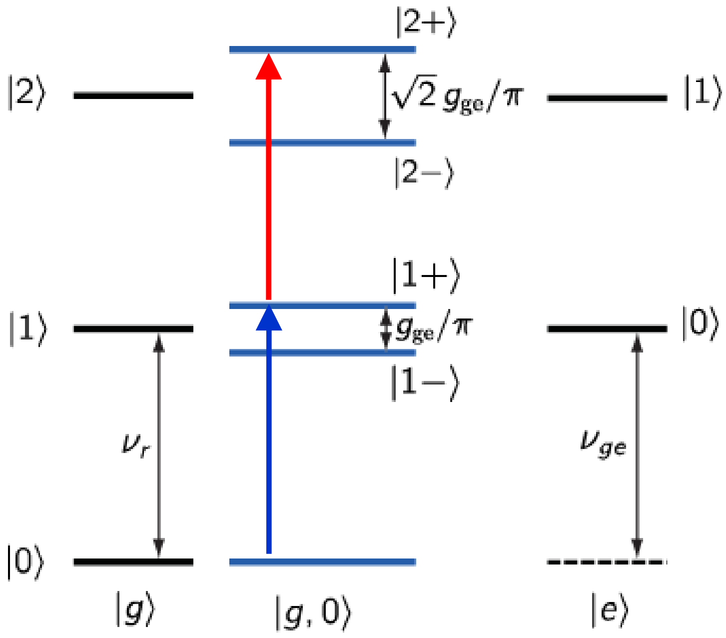
J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff,
Nature (London) **454**, 315 (2008)

Resonant Vacuum Rabi Mode Splitting …

… with two photons ($n = 2$):



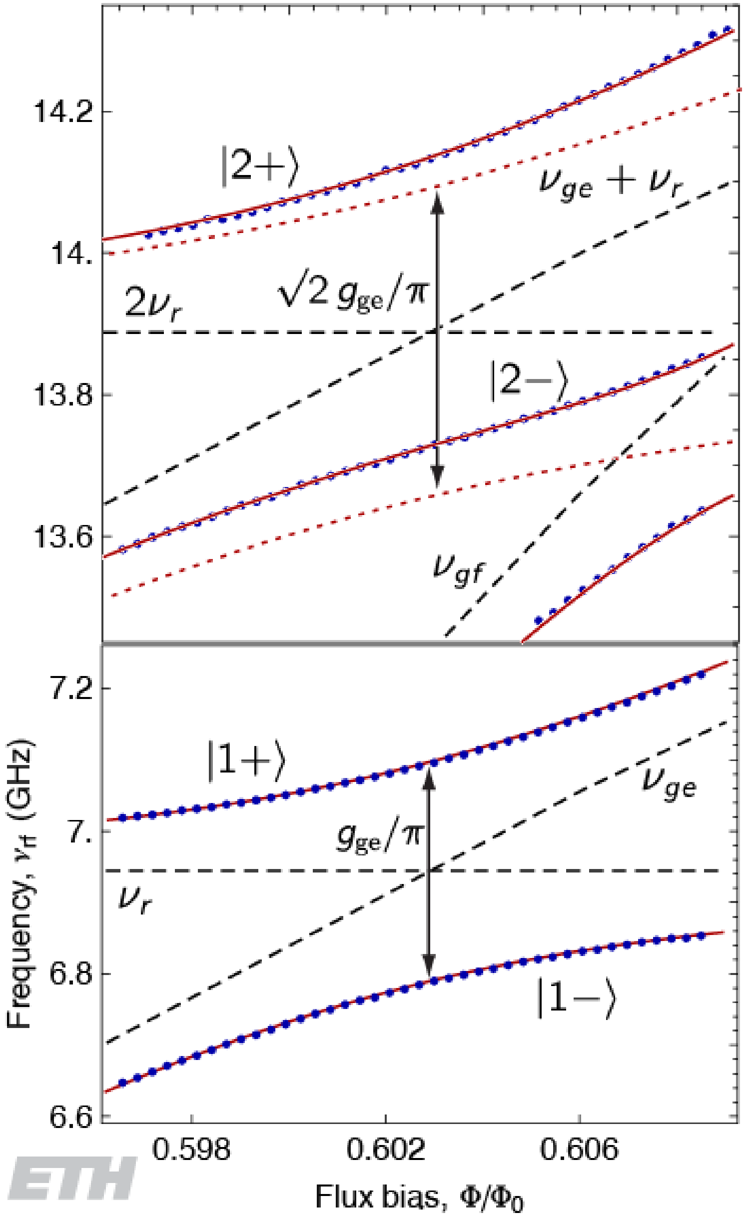
pump and probe: $|n+\rangle$



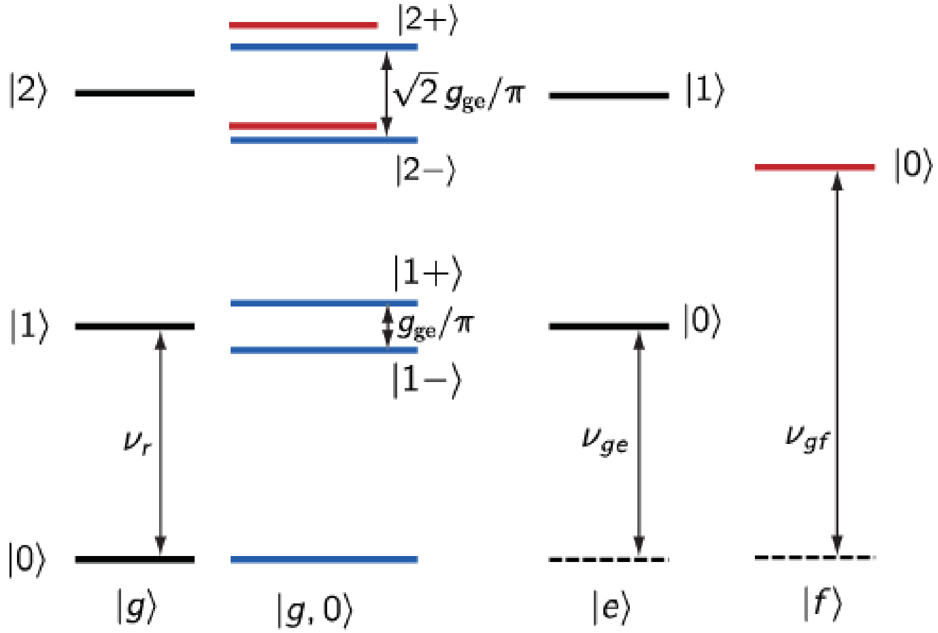
- $|n+\rangle \rightarrow |n-\rangle$ is weak

J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff,
Nature (London) **454**, 315 (2008)

Sqrt(n) Quantum Nonlinearity



- energies reconstructed from pump + probe
- shifts due to 3rd qubit level $|f\rangle$
- full Hamiltonian yields good agreement
- clear spectroscopic demonstration of field quantization in cavity QED



J. Fink, … , A. Wallraff, *Nature (London)* **454**, 315 (2008)

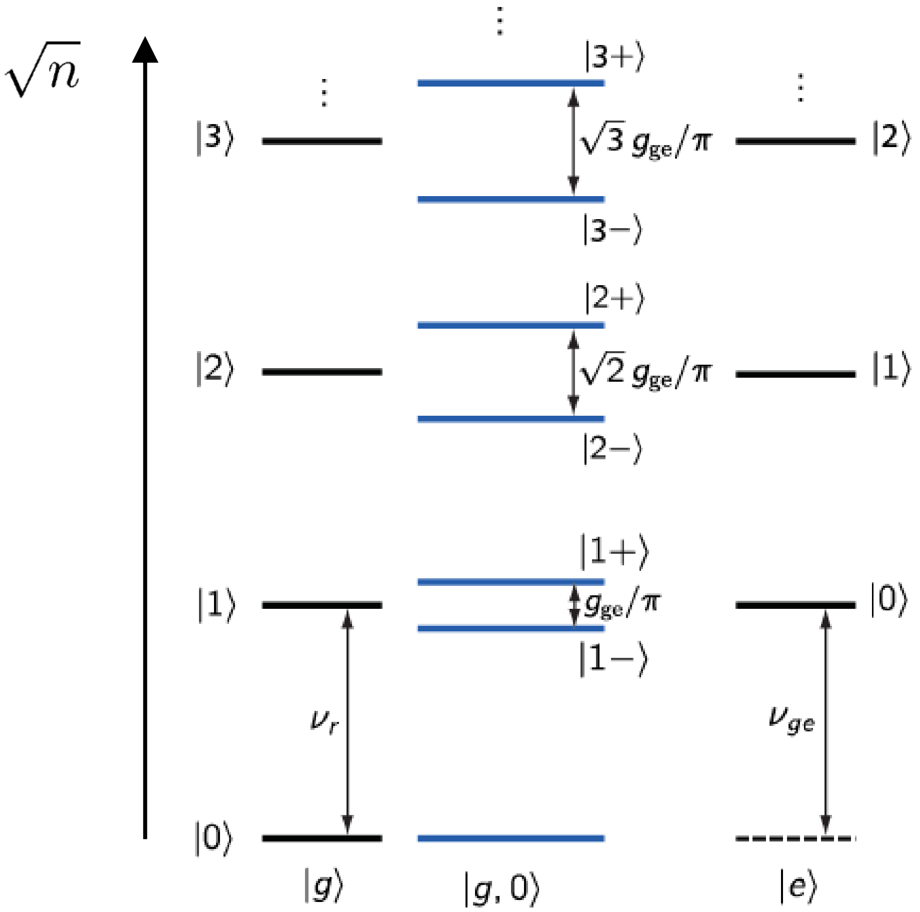
Cavity QED
with one, two and three artificial atoms …

… probing the collective interaction of a number of atoms with a single photon

Cavity QED with Multiple ~~Photons~~ Atoms

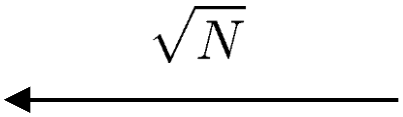
coupling *n* photons to single atom

J. Fink et al., *Nature (London)* **454**, 315 (2008)



coupling *N* atoms to a single photon

J. Fink et al., *Phys. Rev. Lett.* **103**, 083601 (2009)

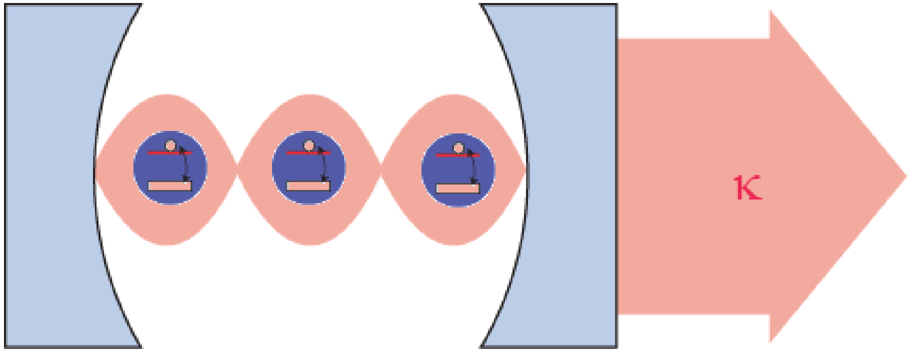


Tavis-Cummings Model

Jaynes-Cummings Model

Multi-Atom Cavity QED

- early on: $\bar{N}$ falling through cavity

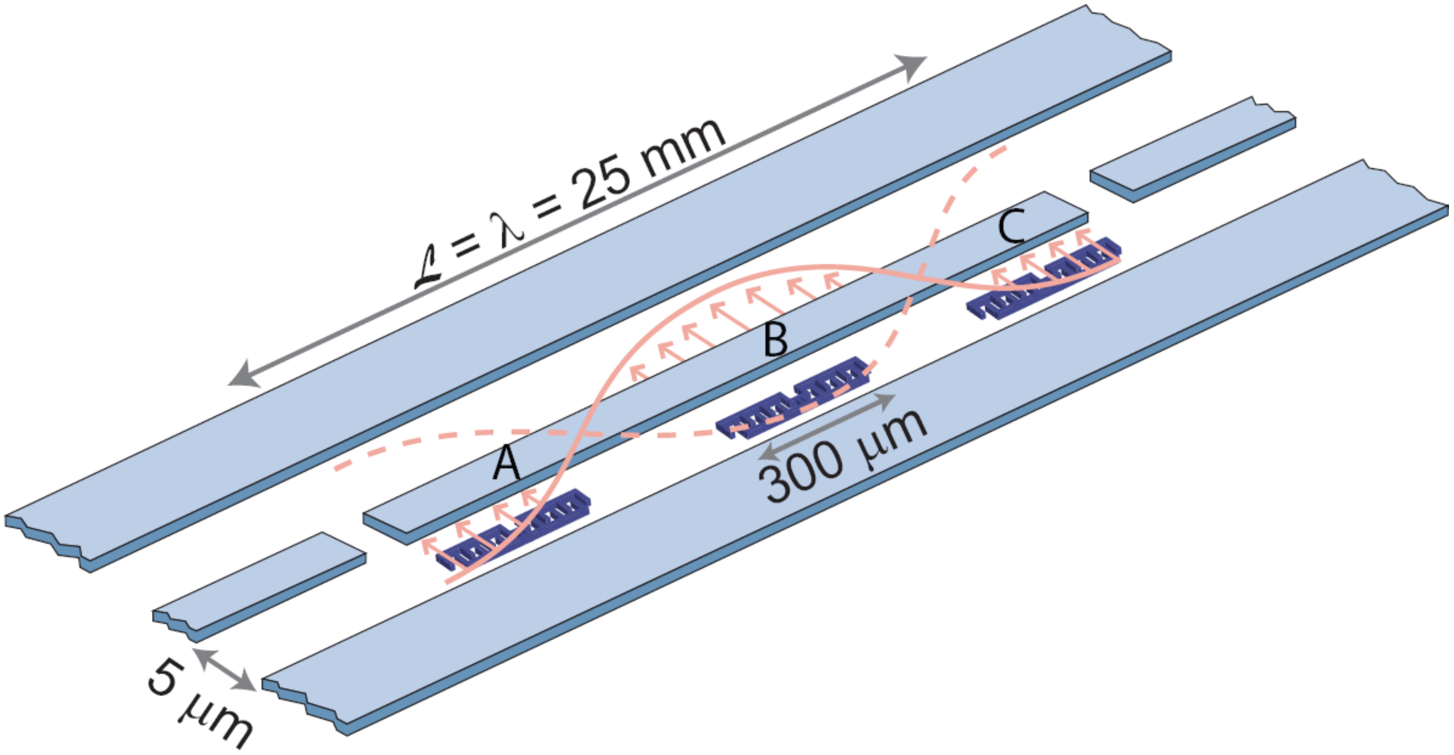


- atom number δN and coupling δg fluctuations
- Tavis-Cummings model

$$\hat{\mathcal{H}}_{\mathrm{TC}} = \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \sum_{j=1}^{N}\left(\frac{\hbar}{2}\omega_{\mathrm{j}}\hat{\sigma}_{\mathrm{j}}^{Z} + \hbar g_{\mathrm{j}}(\hat{a}^{\dagger}\hat{\sigma}_{\mathrm{j}}^{-} + \hat{\sigma}_{\mathrm{j}}^{+}\hat{a})\right)$$

- difficult to trap atoms

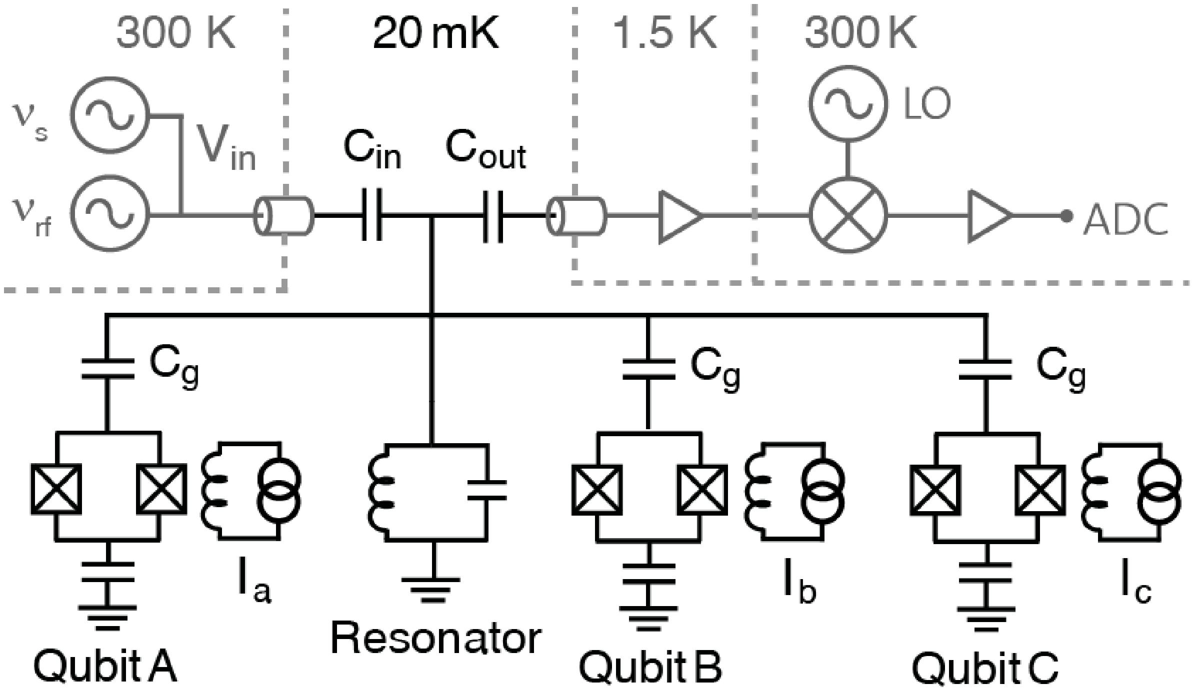
Multi-Qubit Circuit QED Schematic



in circuit QED:

- small well defined number of qubits
- no qubit number fluctuations
- fixed coupling strength
- full single qubit control

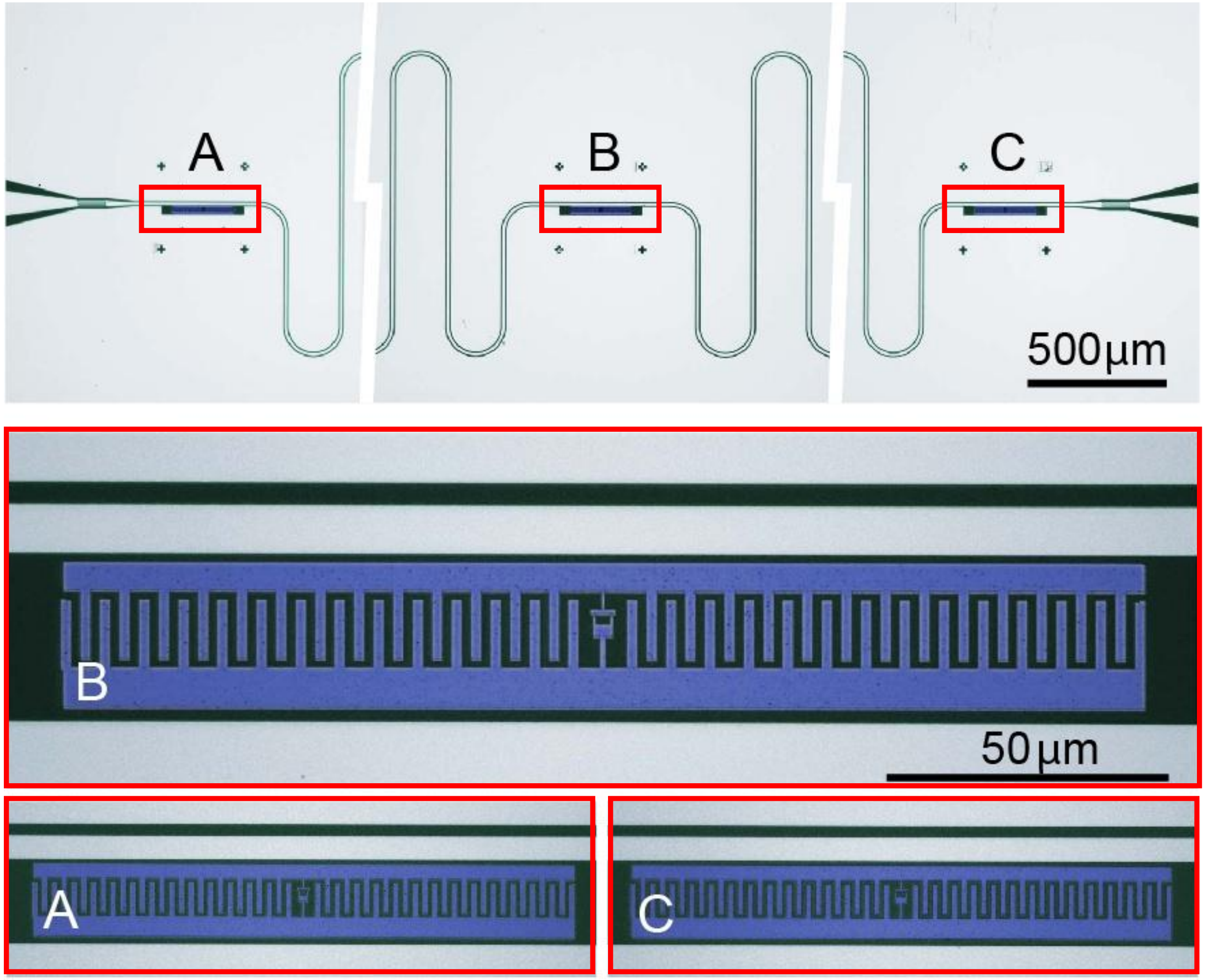
Three Qubit Circuit QED Setup



- one cavity
- three qubits
- local flux control $\Phi_{\mathrm{A,B,C}}$

Three Qubit Circuit QED Sample

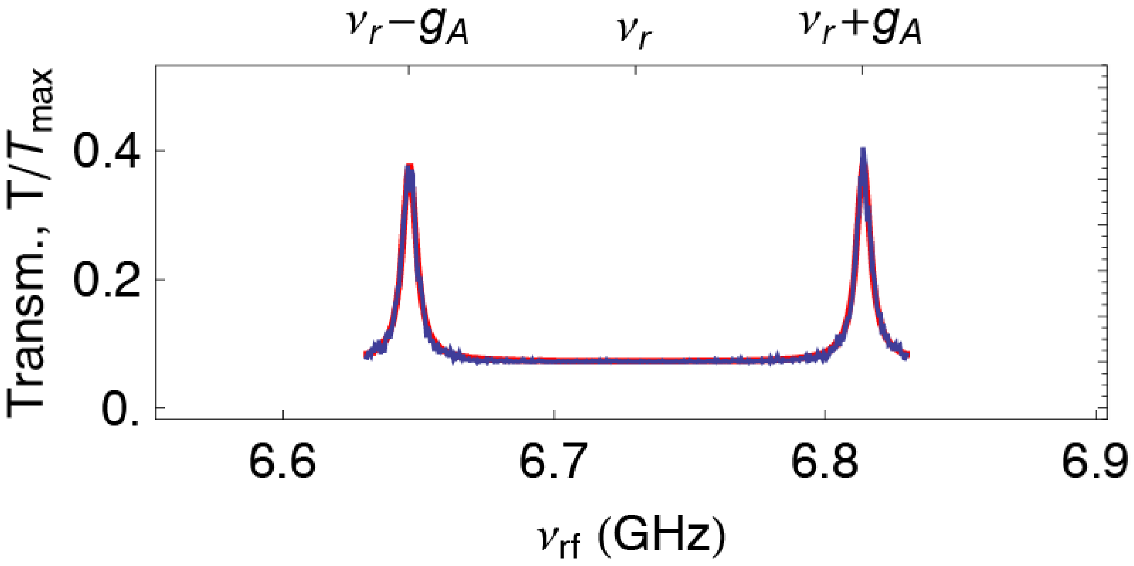
- three qubits integrated into one cavity
- qubits are almost identical
- almost identical coupling constants $g_{\mathrm{A,B,C}}$



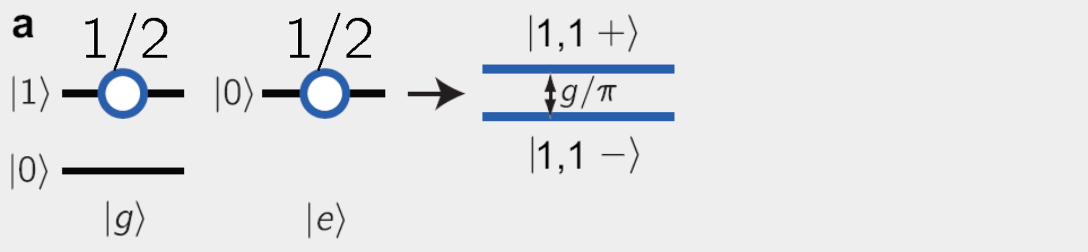
Qubit j	E_{C_j}/h (MHz)	$E_{J\max_j}/h$ (GHz)	$g_j/2\pi$ (MHz)
A	283	224	83.7
B	287	226	−85.7
C	294	214	85.1

Multi-Qubit Vacuum Rabi Mode Splitting

- the spectrum:



- the states:



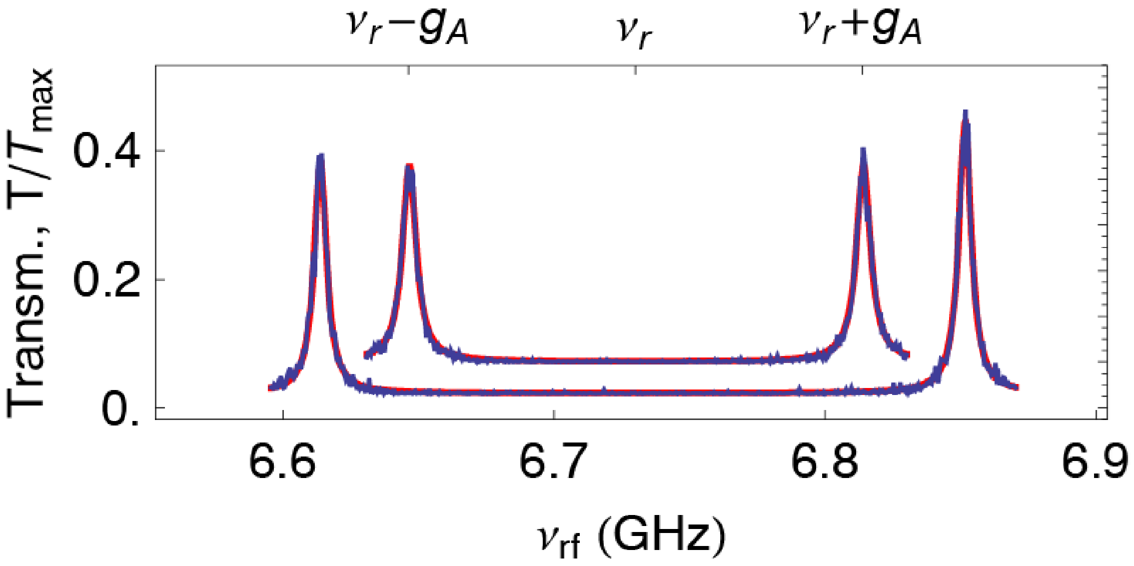
$$|1,1\pm\rangle = 1/\sqrt{2}(|g,1\rangle \pm |e,0\rangle)$$

states equally shared between photon and qubit

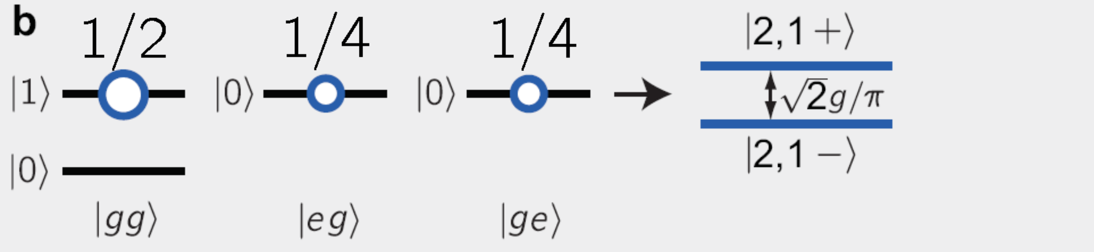
J. Fink *et al.*, Phys. Rev. Lett. **103**, 083601 (2009)

Multi-Qubit Vacuum Rabi Mode Splitting

- the spectrum:



- the states:



$$|2,1\pm\rangle = \ 1/\sqrt{2}\ |g,g\rangle \otimes |1\rangle \pm \\ 1/2\ (|e,g\rangle + |g,e\rangle) \otimes |0\rangle$$

bright states: superposition of a photon and a Bell state

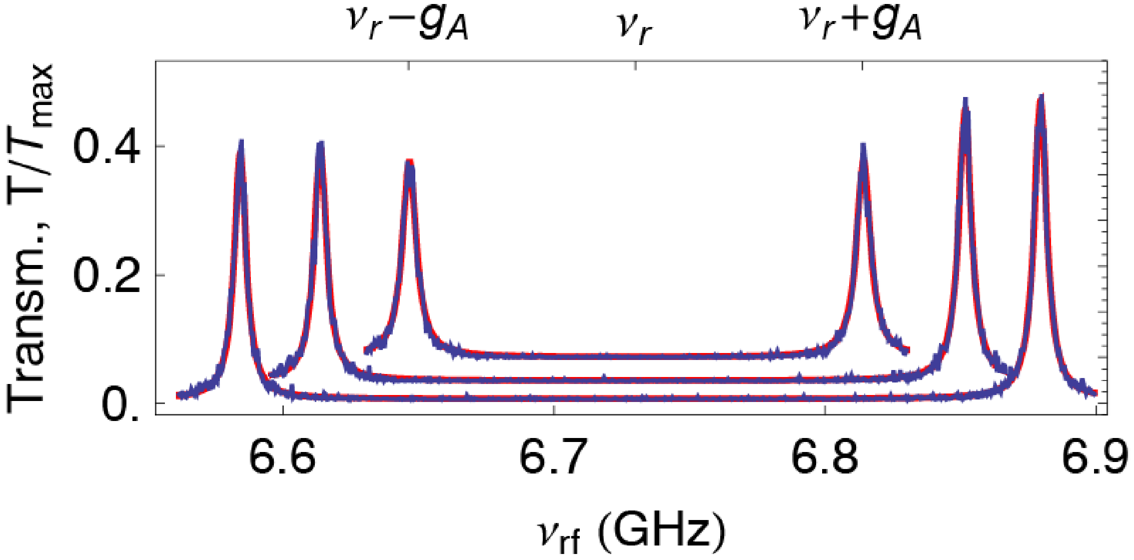
$$|2,1d\rangle = -1/\sqrt{2}(|g,e\rangle + |e,g\rangle) \otimes |0\rangle$$

dark state

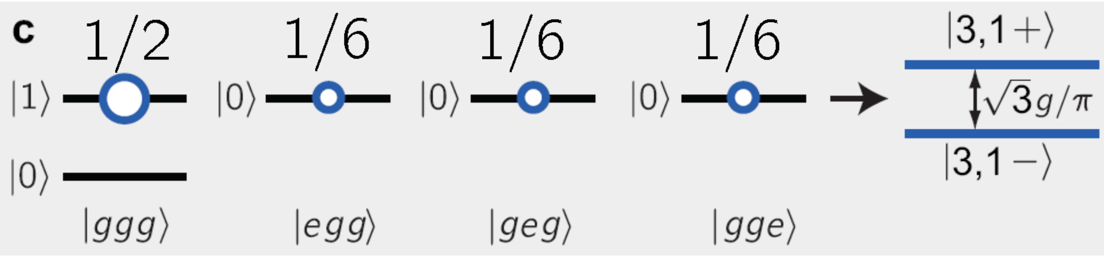
J. Fink *et al.*, Phys. Rev. Lett. **103**, 083601 (2009)

Multi-Qubit Vacuum Rabi Mode Splitting

- the spectrum:



- the states:



$$|3,1\pm\rangle = 1/\sqrt{2}\ |g,g,g\rangle \otimes |1\rangle \\ \pm 1/\sqrt{6}\ (|e,g,g\rangle - |g,e,g\rangle + |g,g,e\rangle) \otimes |0\rangle$$

one photon plus three qubit entagled W-state

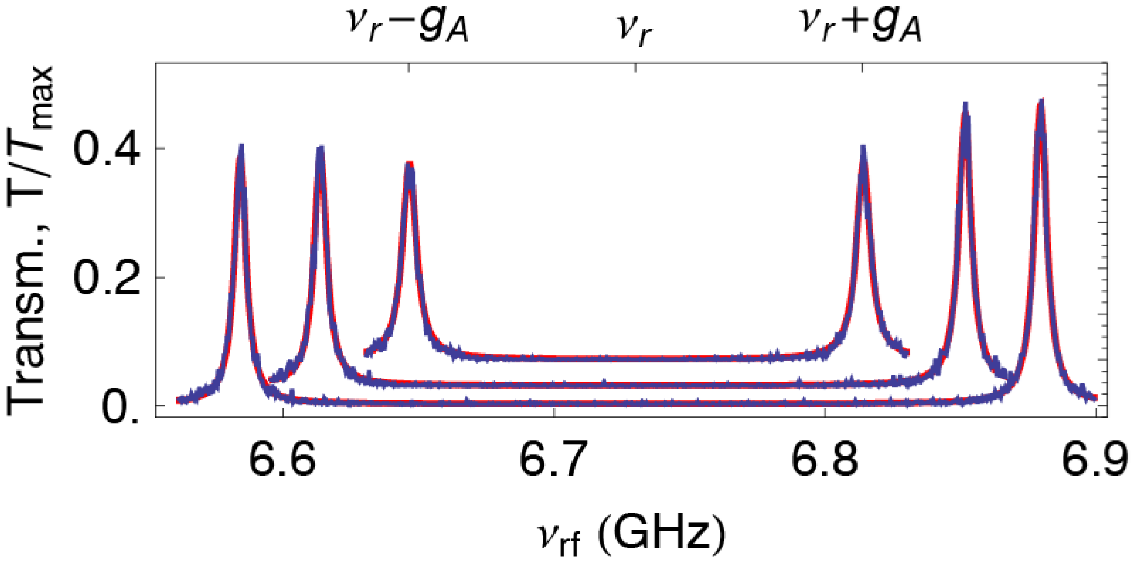
$$|3,1d_1\rangle = 1/\sqrt{2}\ (|e,g,g\rangle - |g,g,e\rangle) \otimes |0\rangle$$
$$|3,1d_2\rangle = 1/\sqrt{2}\ (|g,g,e\rangle + |g,e,g\rangle) \otimes |0\rangle$$

two dark states

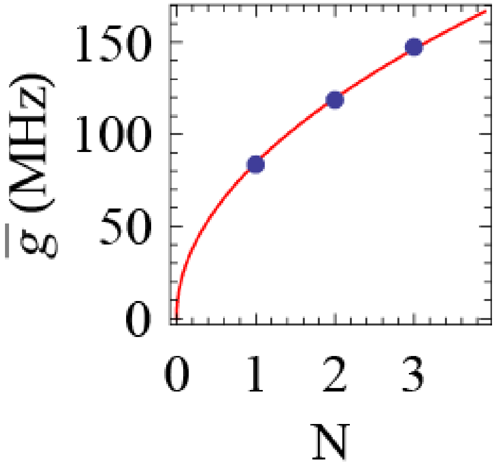
J. Fink *et al.*, Phys. Rev. Lett. **103**, 083601 (2009)

Atom Number Scaling of Coupling Strength

- the spectrum:

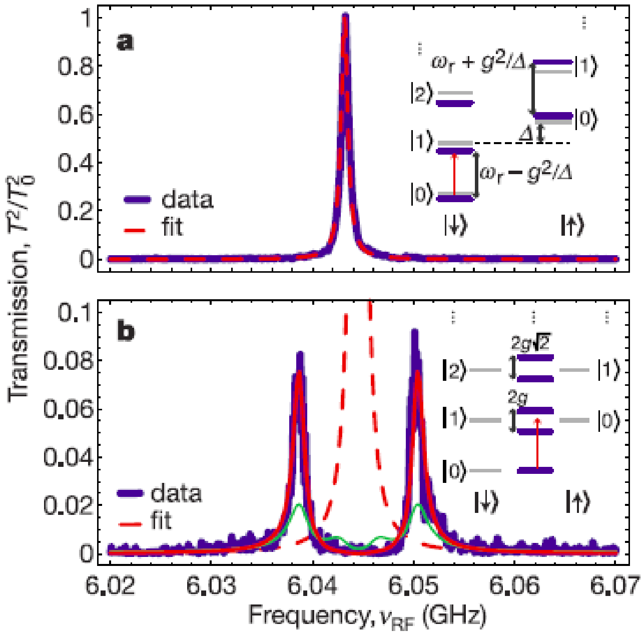


- scaling of collective coupling with $\sqrt{N}$



J. Fink *et al.*, Phys. Rev. Lett. **103**, 083601 (2009)

Quantum Optics using Circuit QED

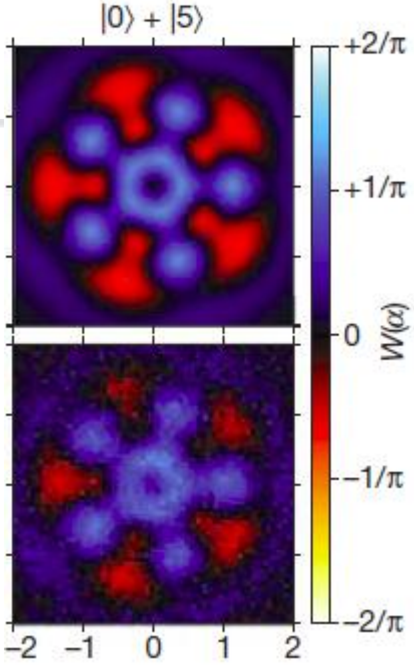


Strong Coherent Coupling

I. Chiorescu *et al.*, *Nature* **431**, 159 (2004)
A. Wallraff *et al.*, *Nature* **431**, 162 (2004)
D. Schuster *et al.*, *Nature* **445**, 515 (2007)

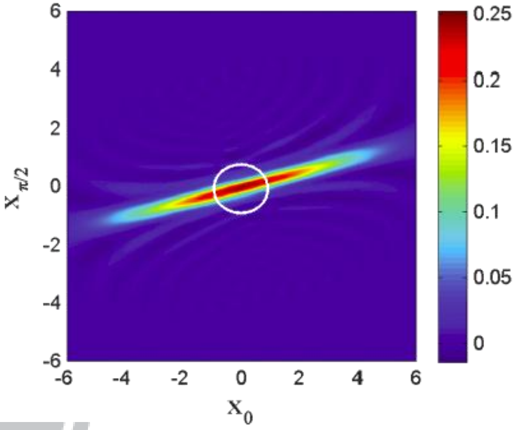
Root n Nonlinearities

J. Fink *et al.*, *Nature* **454**, 315 (2008)
F. Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)
L. Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)



Fock and Arbitrary Photon States

M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)
M. Hofheinz *et al.*, *Nature* **459**, 546 (2009)

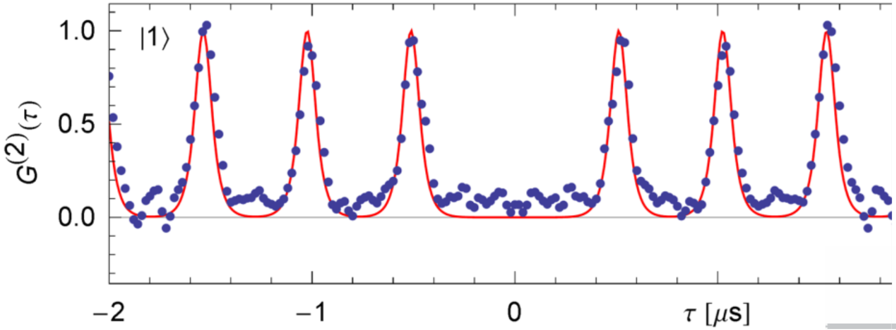


Parametric Amplification & Squeezing

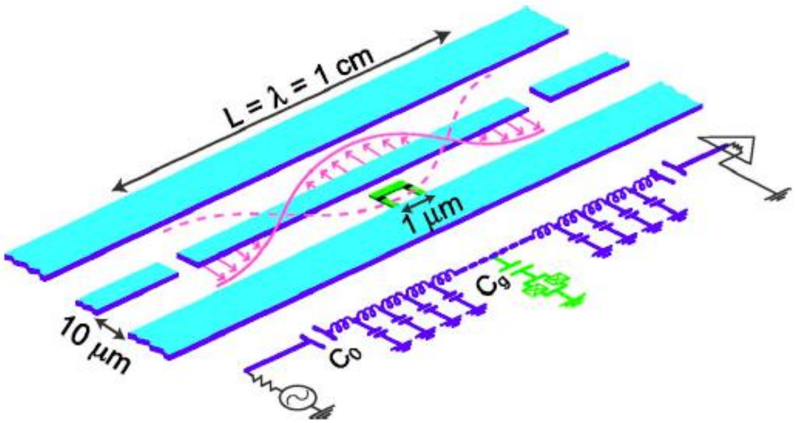
Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 928 (2008)

Single Photons & Correlations

A. Houck *et al.*, *Nature* **449**, 328 (2007)
D. Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)

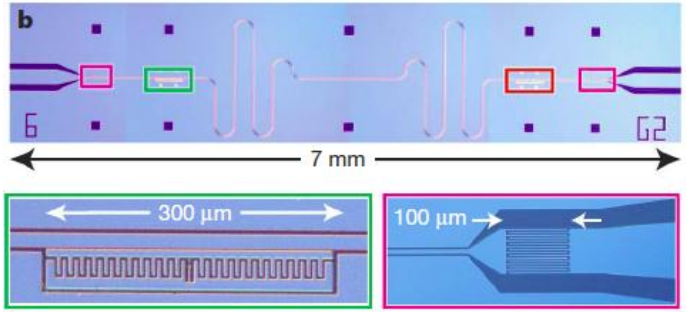


Quantum Computing with Circuit QED



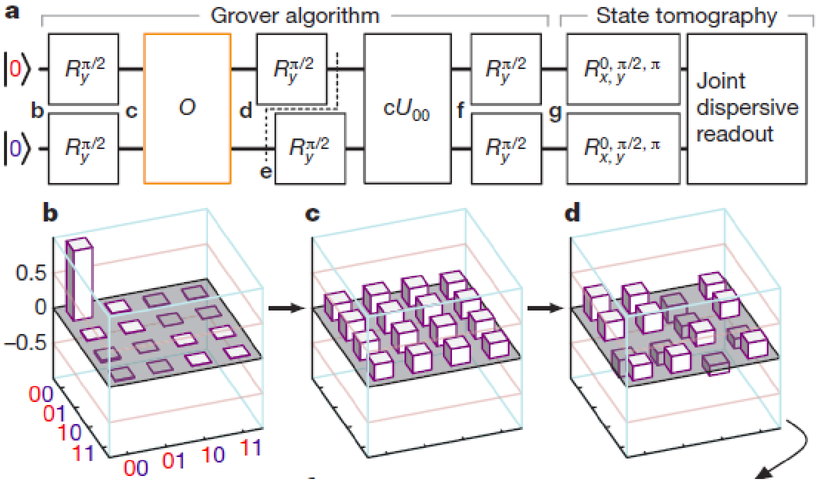
Architecture

A. Blais et al., *PRA* **69**, 062320 (2004)
A. Wallraff *et al.*, *Nature* **431**, 162 (2004)
M. Mariantoni *et al.*, *Science* **334**, 61 (2011)



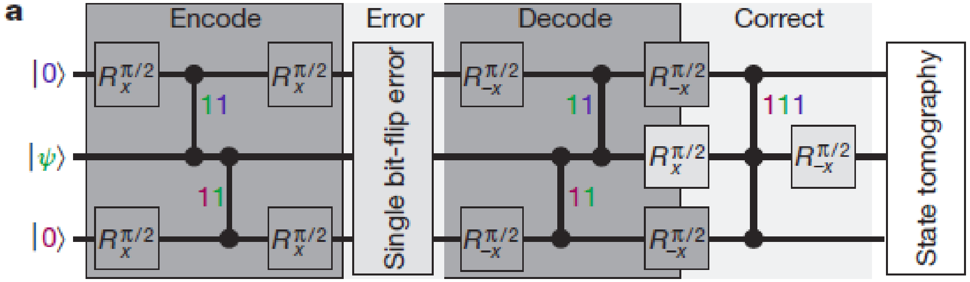
Coupling Bus

M. Sillanpaa *et al.*, *Nature* **449**, 438 (2007)
H. Majer *et al.*, *Nature* **449**, 443 (2007)



Deutsch, Grover Algorithms

L. DiCarlo *et al.*, *Nature* **460**, 240 (2009)
L. DiCarlo *et al.*, *Nature* **467**, 574 (2010)



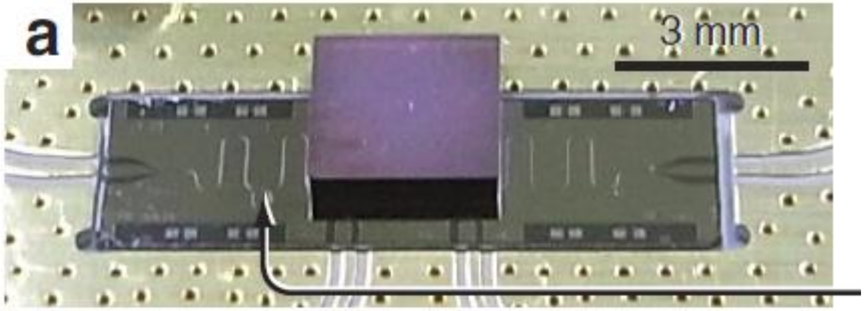
Toffoli Gates & Error Correction

A. Fedorov *et al.*, *Nature* **481**, 170 (2012)
M. Reed *et al.*, *Nature* **481**, 382 (2012)

Hybrid Systems using Circuit QED

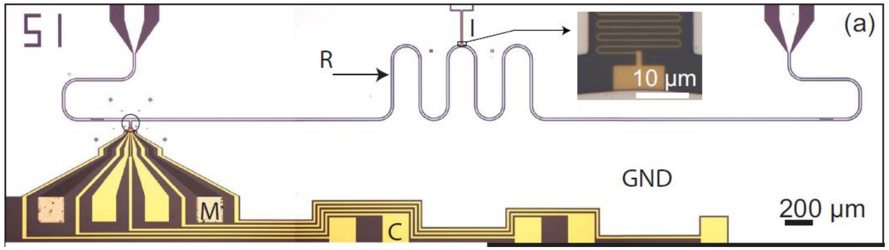
Spin Ensembles: e.g. NV centers

D. Schuster *et al.*, *PRL* **105**, 140501 (2010)
Y. Kubo *et al.*, *PRL* **105**, 140502 (2010)



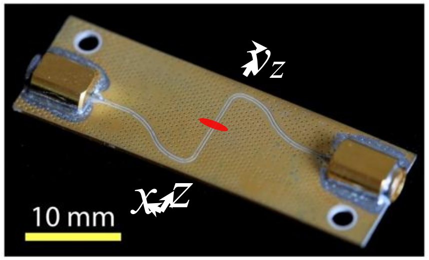
CNT, Gate Defined 2DEG, or nanowire Quantum Dots

M. Delbecq *et al.*, *PRL* **107**, 256804 (2011)
T. Frey *et al.*, *PRL* **108**, 046807 (2012)
K. Petersson *et al.*, *arXiv*:1205.6767 (2012)



Rydberg Atoms

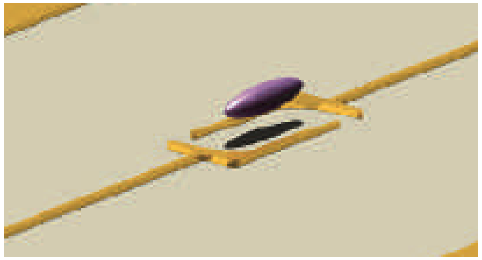
S. Hogan *et al.*, *PRL* **108**, 063004 (2012)



Proposals:

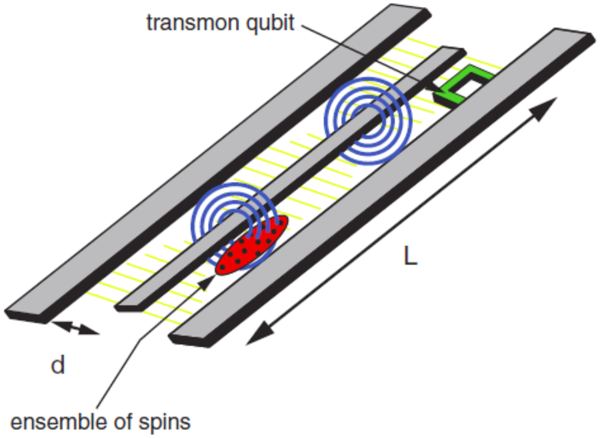
Polar Molecules, Rydberg, BEC

P. Rabl *et al*, *PRL* **97**, 033003 (2006)
A. Andre *et al*, *Nat. Phys.* **2**, 636 (2006)
D. Petrosyan *et al*, *PRL* **100**, 170501 (2008)
J. Verdu *et al*, *PRL* **103**, 043603 (2009)



Spin Ensembles

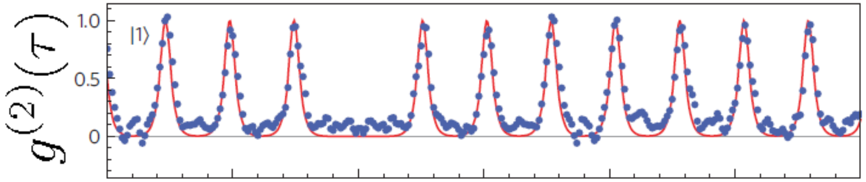
A. Imamoglu *et al.*, *PRL* **102**, 083602 (2009)
J. Wesenberg *et al.*, *PRL* **103**, 070502 (2009)



... and many more

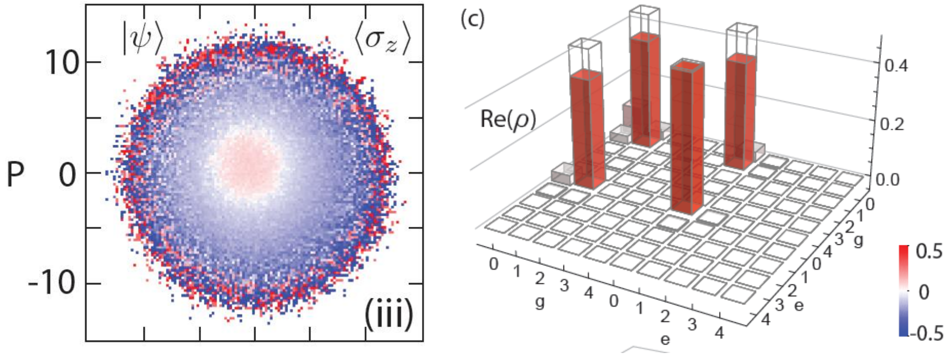
Experiments with Propagating Quantum Microwaves

Single photon sources and their anti-bunching



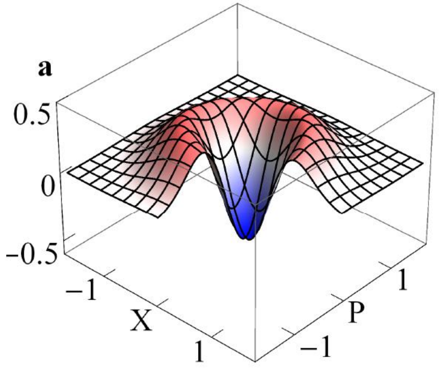
D. Bozyigit et al., Nat. Phys **7**, 154 (2011)
Lang *et al.*, *PRL* **107**, 073601 (2011)

Preparation and characterization of qubit-**propagating** photon entanglement



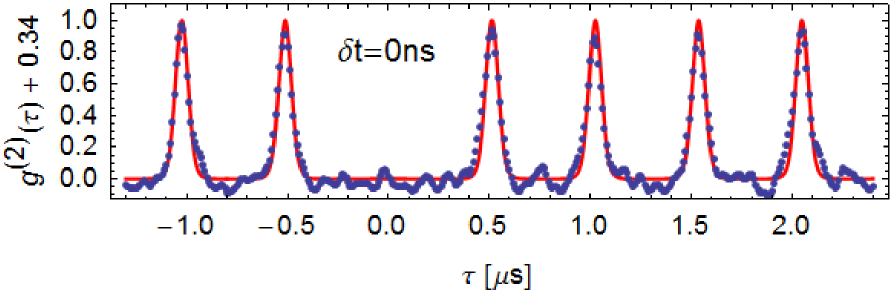
Eichler *et al.*, *ETH Zurich* (2012)

Wigner functions and full state tomography of propagating photons:



Eichler *et al.*, *PRL* **106**, 220503 (2011)

Hong-Ou-Mandel: Two-photon interference with mirowaves



Lang *et al.*, *ETH Zurich* (2012)

Exploring Quantum Properties of Propagating Microwave Photons

Andreas Wallraff *(ETH Zurich)*
www.qudev.ethz.ch

Team: A. Abdumalikov, M. Baur, S. Berger, C. Eichler, A. Fedorov,
S. Filipp, J. Fink, T. Frey, C. Lang, J. Mlynek, M. Oppliger,
M. Pechal, G. Puebla-Hellmann, K. Reim, Y. Salathe,
M. Stammeier, L. Steffen, T. Thiele, A. van Loo
(ETH Zurich)

Collaborations:
A. Blais *(Sherbrooke, Canada)*
M. da Silva *(Raytheon, USA)*
K. Ensslin, T. Ihn, F. Merkt,
V. Wood *(ETH Zurich)*

SEVENTH FRAMEWORK PROGRAMME
(·)QED
CIRCUIT AND CAVITY QUANTUM ELECTRODYNAMICS
SOLID
GEOMDISS
aqute
erc
QSIT Quantum Science and Technology
National Centre of Competence in Research
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

Itinerant Single Photons in the Microwave Domain

quantum optics in the **visible**:

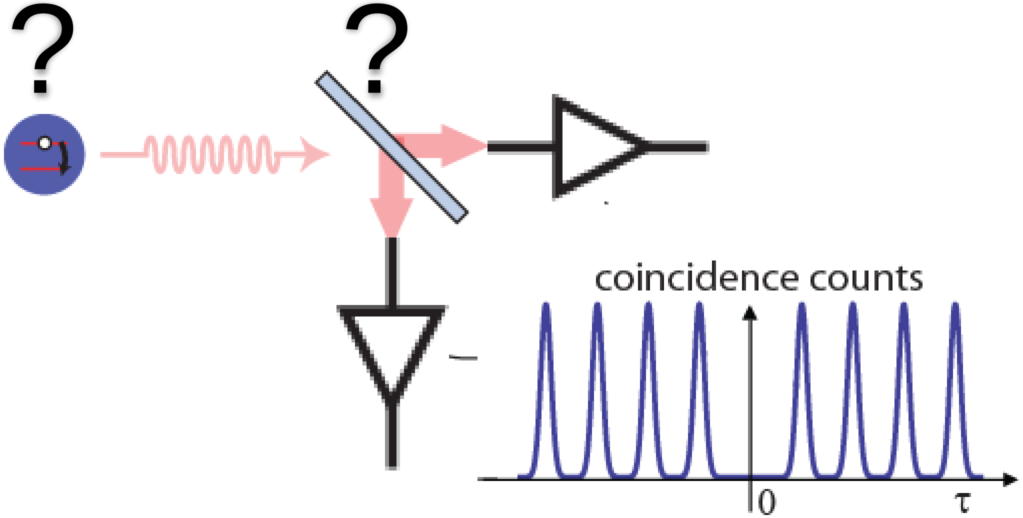
- single photon sources
- beam splitters
- photon counters

o.k. at **optical frequencies**

But in the **microwave domain**?

- smaller photon energy ...

$$\frac{\nu_{\text{opt}}}{\nu_{\mu\text{w}}} = \frac{500\,\text{THz}}{5\,\text{GHz}} = 10^5$$



instead:

- linear amplifiers
- signal processing

J. Gabelli et al., *Phys. Rev. Lett.* **93**, 056801 (2004)
E. P. Menzel et al., *Phys. Rev. Lett.* **105**, 100401 (2010)
M. P. da Silva et al.. Phys. *Rev. A* **82**, 043804 (2010)

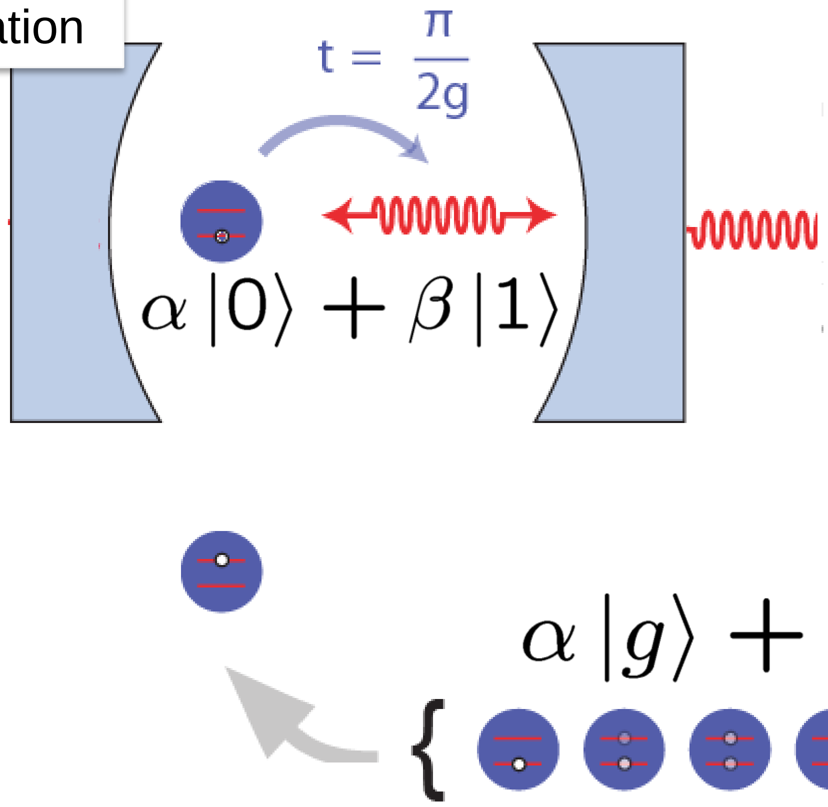
On-Demand Pulsed Single Photon Source

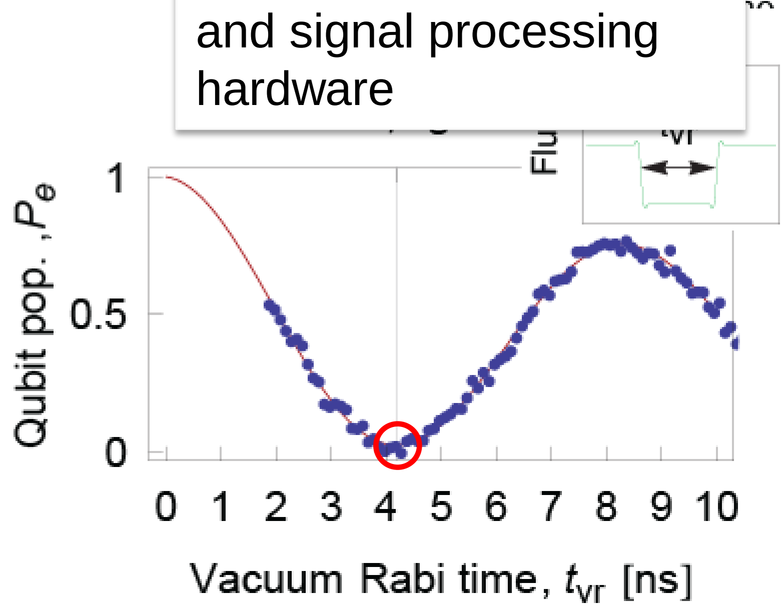
Step 2:

Map qubit state to resonator by 1/2 vacuum Rabi oscillation

Step 3:

Measure at the output using linear amplifier and signal processing hardware





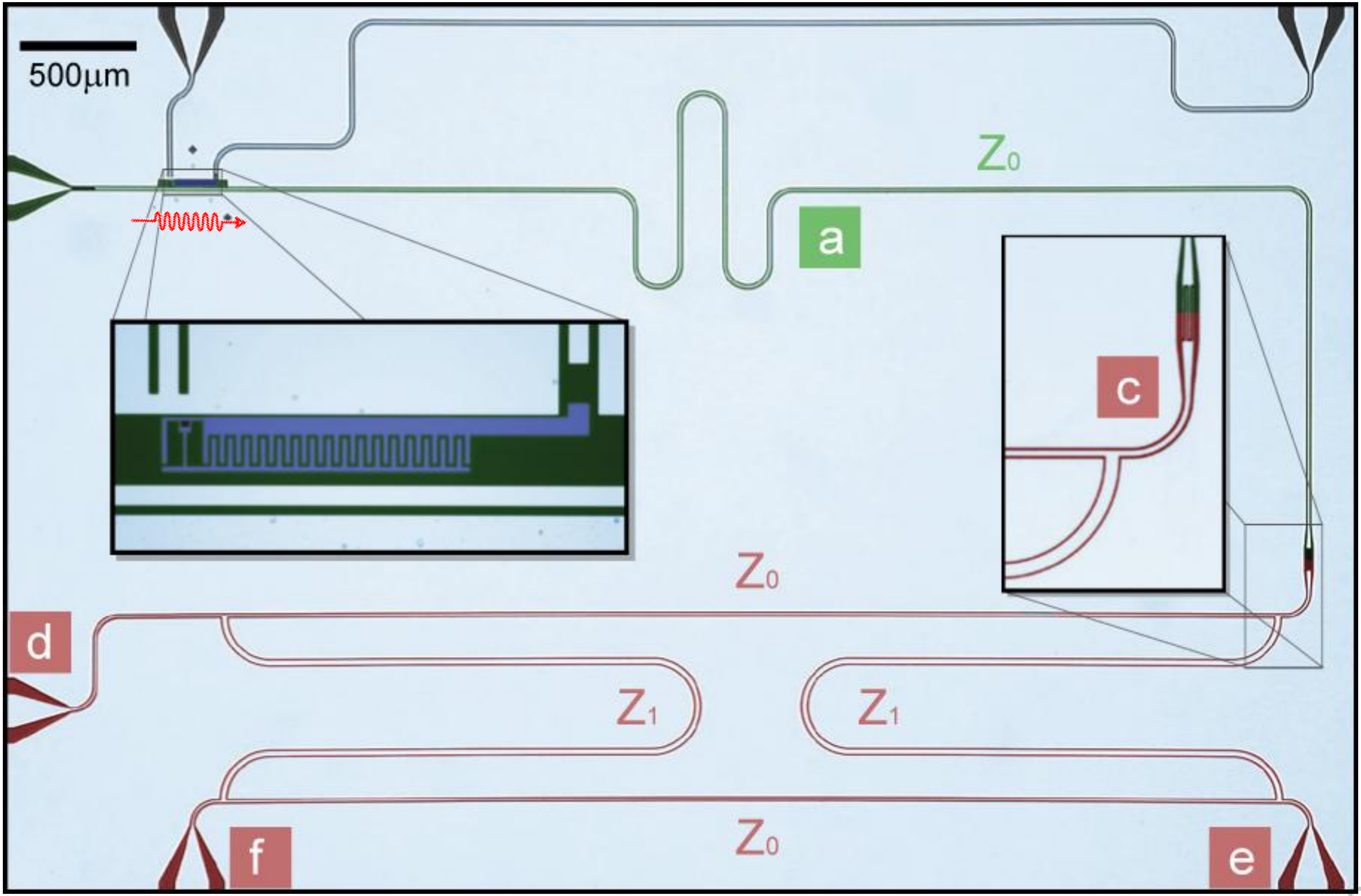
Step 1:

Prepare qubit state by Rabi oscillation

D. Bozyigit et al., *Nat. Phys.* **7**, 154 (2011)

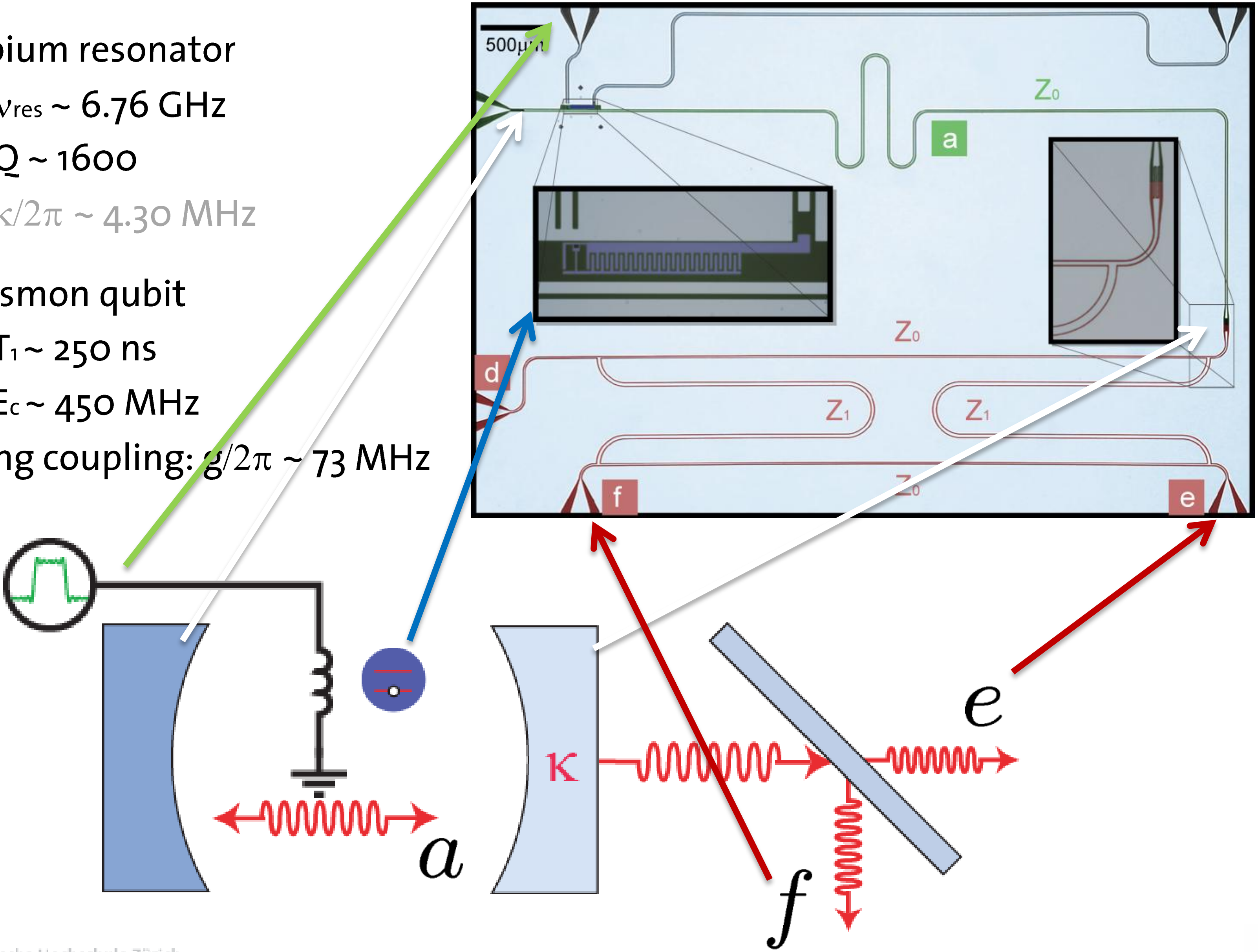
S. Deleglise et al. Nature **455**, 510514 (2008); M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)

Single Sided Cavity and Beam Splitter

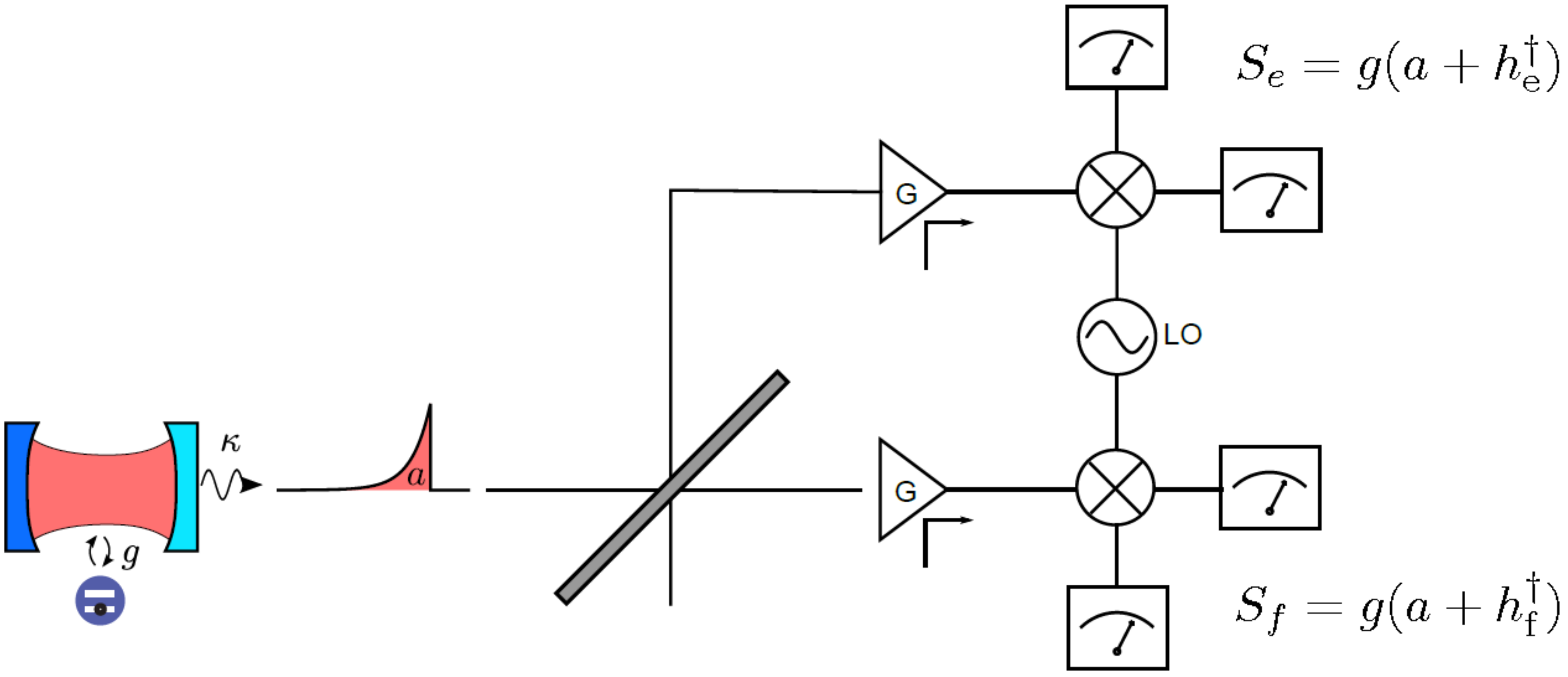


Single Sided Cavity and Beam Splitter

- Niobium resonator
  - ν_{res} ~ 6.76 GHz
  - Q ~ 1600
  - $\kappa/2\pi$ ~ 4.30 MHz
- Transmon qubit
  - T_1 ~ 250 ns
  - E_c ~ 450 MHz
- Strong coupling: $g/2\pi$ ~ 73 MHz



Schematic of Measurement Setup



$$S_e = g(a + h_\mathrm{e}^\dagger)$$

$$S_f = g(a + h_\mathrm{f}^\dagger)$$

$$g \equiv \sqrt{G/2}$$

$h_\mathrm{e}, h_\mathrm{f}$ effective noise modes

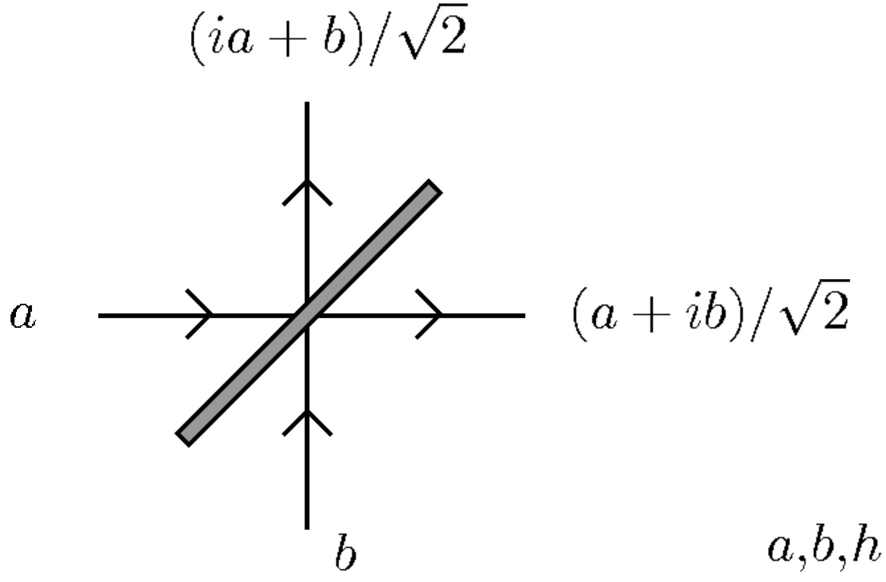
generalization of accessible expectation values:

$$\langle (S^\dagger)_\mathrm{e}^n S_\mathrm{f}^m \rangle = g^{n+m} \langle (a^\dagger)^n a^m \rangle$$

M. P. da Silva et al., *PRA* **82,** 043804 (2010)
D. Bozyigit et al., *Nat. Phys.* **7,** 154 (2011)
S. L. Braunstein et al., *PRA* **43,** 1153 (1991)
G. S. Argawal et al., *PRA* **49,** 2 (1994)

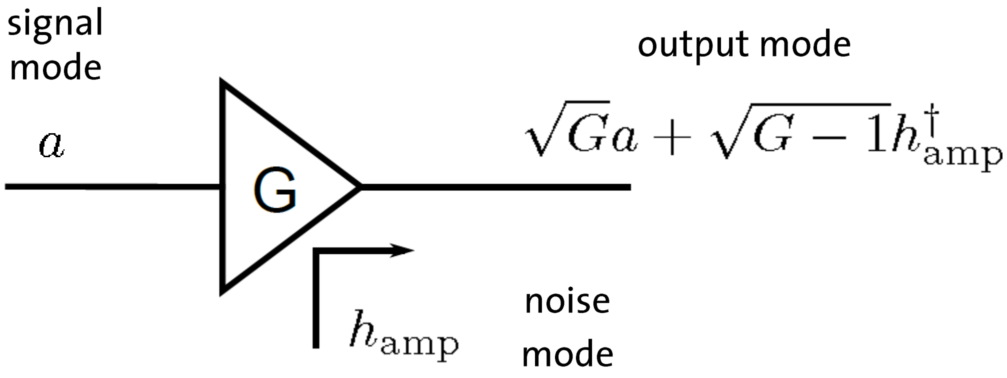
Detection Scheme using Beam Splitters, Amplifiers ...

beamsplitter:



$a, b, h, \dots$ bosonic field operators

linear amplifier:



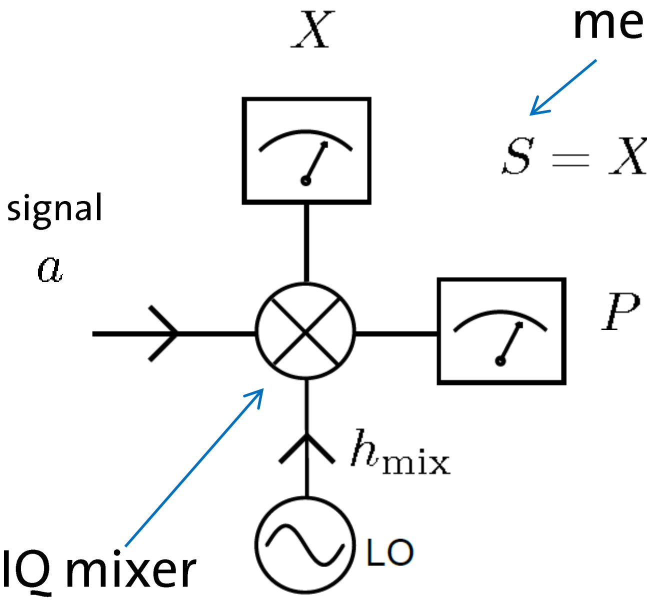
C. Caves, *PRD* **26**, 1817 (1982).

… and Quadrature Detectors

measuring field amplitude instead of photon number:

measured observable

$$S = X + iP = a + h^{\dagger}_{\text{mix}}$$

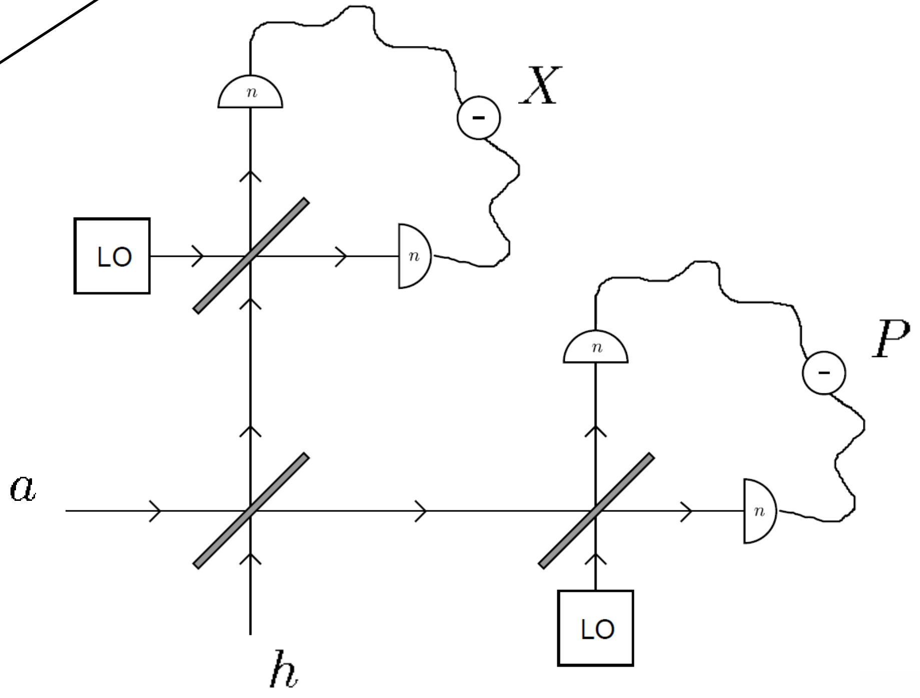


IQ mixer

M. P. da Silva et al., *PRA* **82**, 043804 (2010).

optical analogue

homodyne detection scheme



The Signal in One Channel of the Setup

signal mode a , vacuum mode v

$$\longrightarrow (a+iv)/\sqrt{2}$$

Beam splitter

$$\longrightarrow \sqrt{G/2}(a+iv)+\sqrt{G-1}h^\dagger_{\text{amp}}$$

Linear amplifier

$$\longrightarrow \sqrt{G/2}(a+\underbrace{iv)+\sqrt{G-1}h^\dagger_{\text{amp}}+h^\dagger_{\text{mix}}}_{\propto h^\dagger}$$

Mixer

$$\equiv \sqrt{G/2}(a+h^\dagger)$$

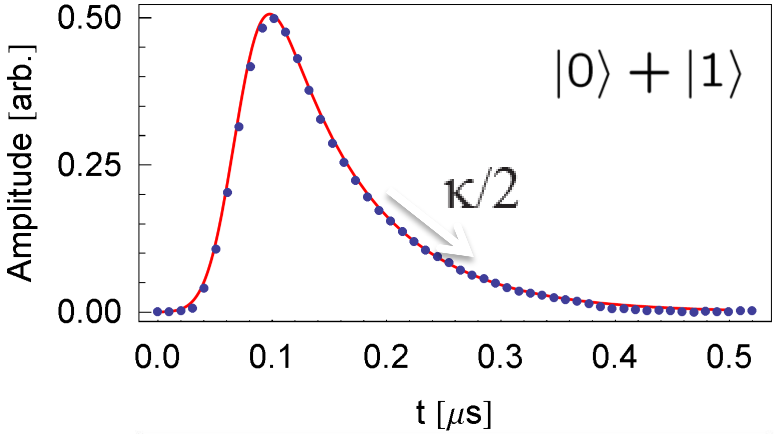
$$\equiv S = X + iP$$

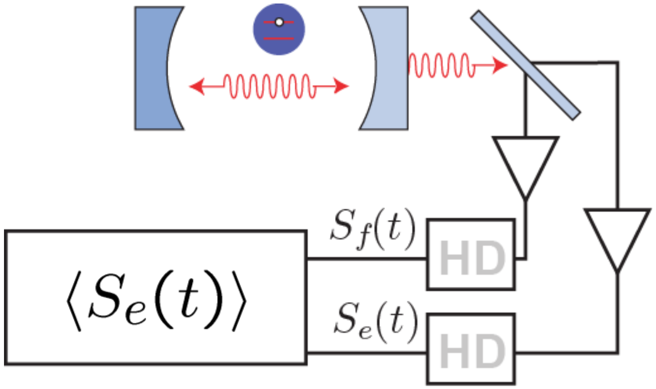
effective noise mode

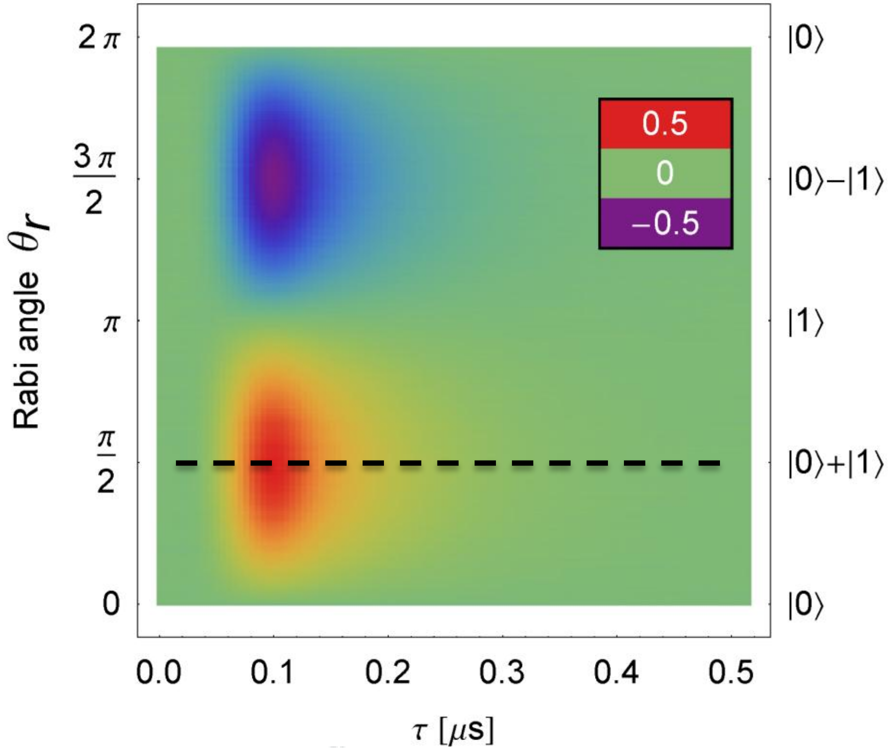
analogous for second channel!

Cavity Field Quadrature Measurement

Measure quadratures at channel b:







$$\langle S_e(t) \rangle \propto \langle a(t) \rangle$$

$$\left\langle S_e^*(t) S_f(t) \right\rangle \propto \left\langle a^\dagger(t) a(t) \right\rangle + N_{ef}$$

Time-dependence:

- Falling edge: cavity decay
- Rising edge: detection bandwidth

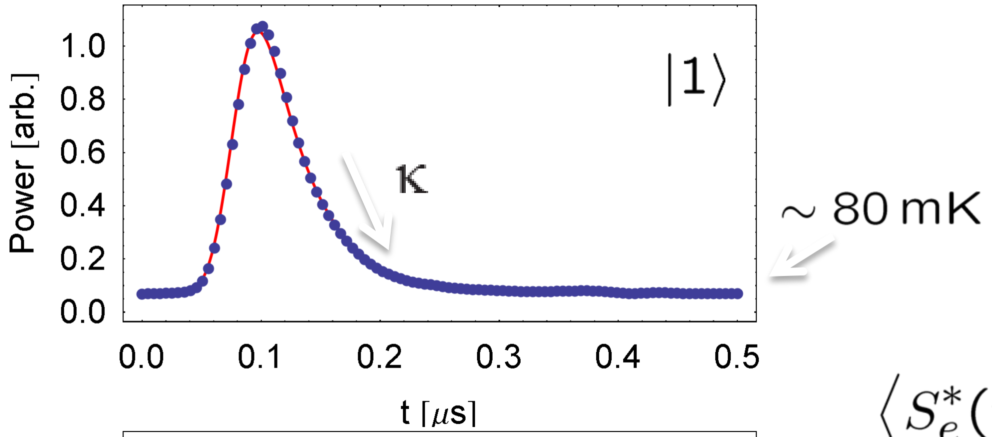
Rabi angle dependence:

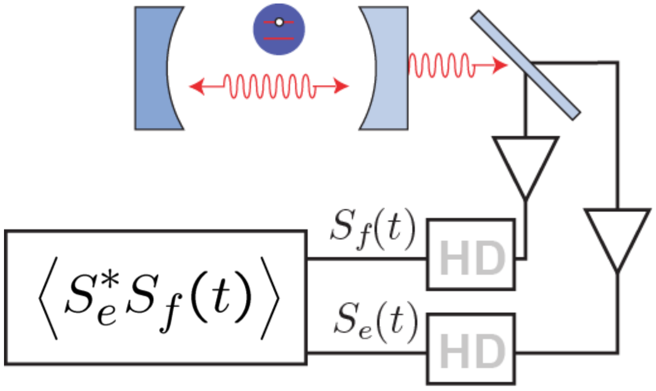
- Maximum signal for $|0\rangle + |1\rangle$
- No signal for $|1\rangle$

D. Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)
A. Houck *et al.*, *Nature* **449**, 328 (2007)

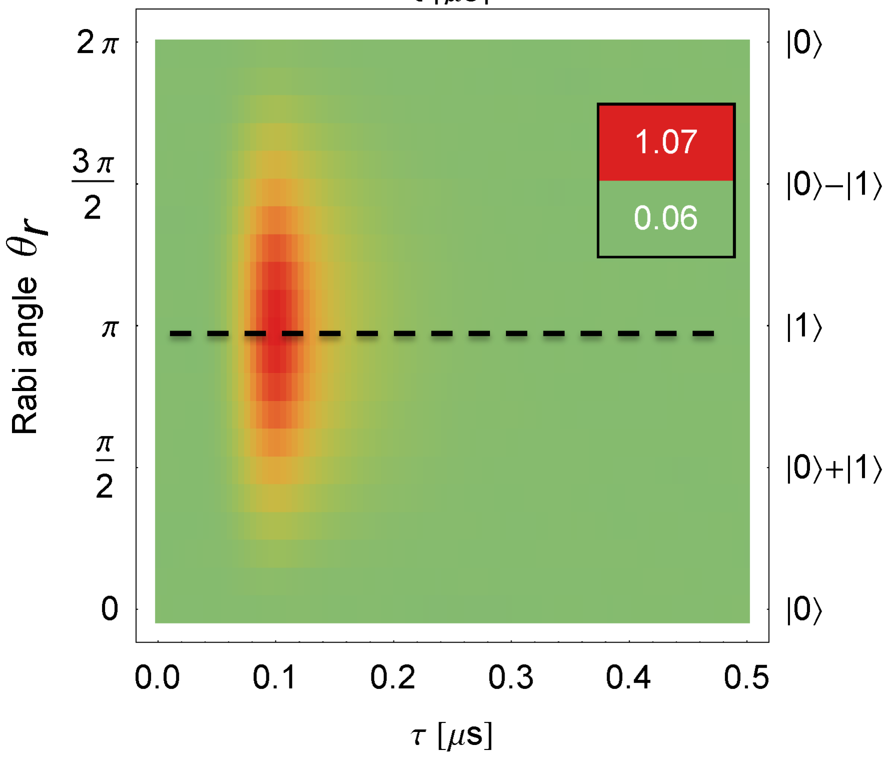
Cavity Photon Number Measurement

Measure crosspower between channel e&f:





$$\left\langle S_e^*(t) S_f(t)\right\rangle \propto \left\langle a^\dagger(t) a(t)\right\rangle + N_{ef}$$
$$\propto \sin^2(\theta_r/2)$$



Time-dependence:

- Falling edge: cavity decay
- Rising edge: detection bandwidth

Rabi angle dependence:

- **Maximum signal for** $|1\rangle$
- Excellent agreement with theory

D. Bozyigit et al., *Nat. Phys.* **7**, 154 (2011)

Single-Channel Power vs. Two-Channel Cross Power

Single channel power:

system noise uncorrelated from signal

$$\langle a h_e^\dagger\rangle = \langle a\rangle\langle h_e^\dagger\rangle$$

$$\langle S_e^\dagger S_e\rangle/g^2 = \langle a^\dagger a\rangle + \langle h_e h_e^\dagger\rangle + \underbrace{\cancel{\langle a\rangle\langle h_e^\dagger\rangle + \langle a^\dagger\rangle\langle h_e\rangle}}_{=0}$$

signal photons

added noise photons

system noise is Gaussian with vanishing mean

... vs. cross power:

$$\langle S_e^\dagger S_f\rangle/g^2 = \langle a^\dagger a\rangle + \underbrace{\langle h_e h_f^\dagger\rangle}_{=0} + \underbrace{\langle a\rangle\langle h_e^\dagger\rangle + \langle a^\dagger\rangle\langle h_e\rangle}_{=0} = \langle a^\dagger a\rangle$$

v in vacuum

Noise in 2 detection channels uncorrelated

... similar for higher order moments:

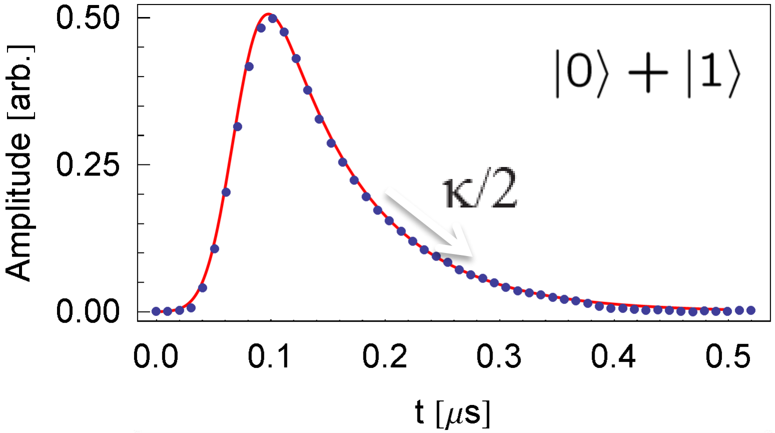
$$\langle (S_e^\dagger)^2 S_f^2\rangle = \langle (a^\dagger)^2 a^2\rangle$$

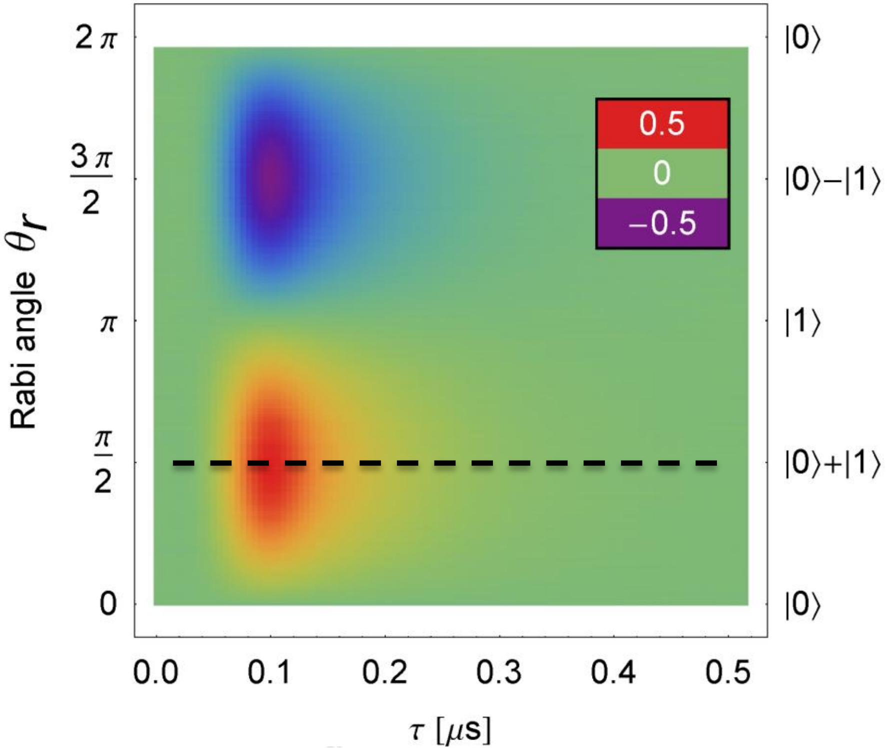
whereas

$$\langle S_e^\dagger S_e S_f^\dagger S_f\rangle = \langle (a^\dagger)^2 a^2\rangle + \langle a^\dagger a\rangle\langle h_e h_e^\dagger\rangle + \ldots$$

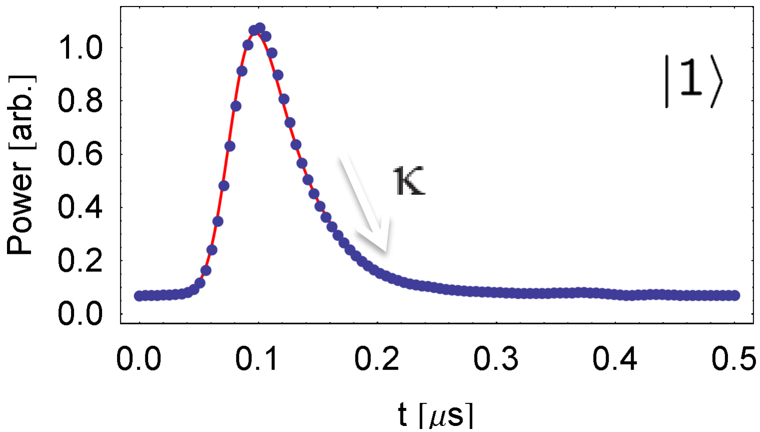
Field Quadrature and Photon Number Measurements

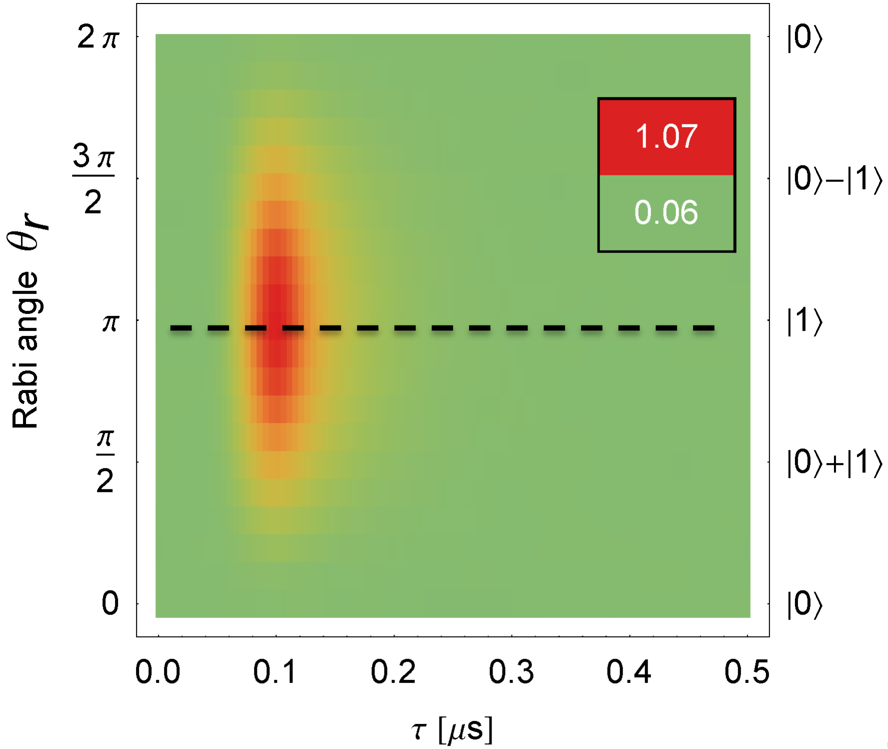
Measure quadratures at channel b:





Measure crosspower between channel e&f:





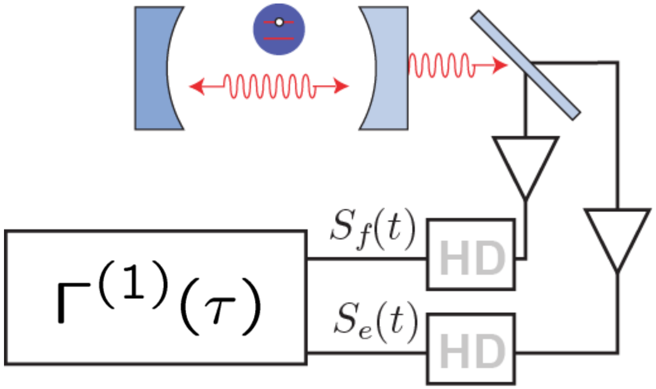
1^{st}-Order Correlation Measurement

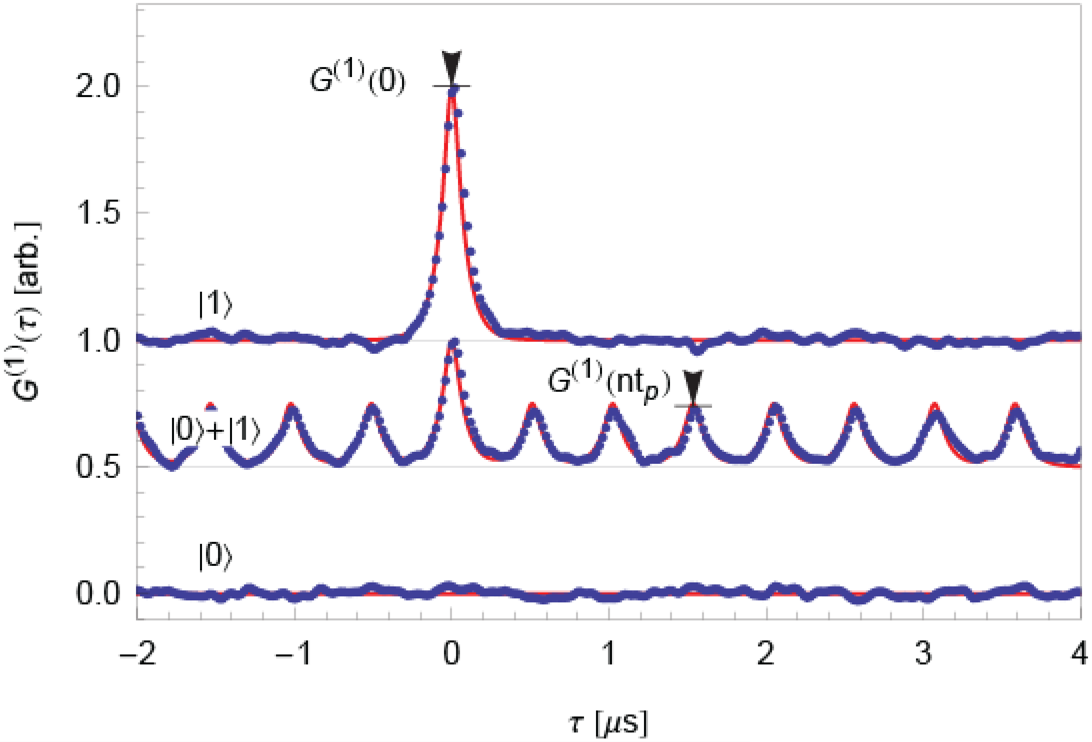
Measure 1st-order cross correlation:

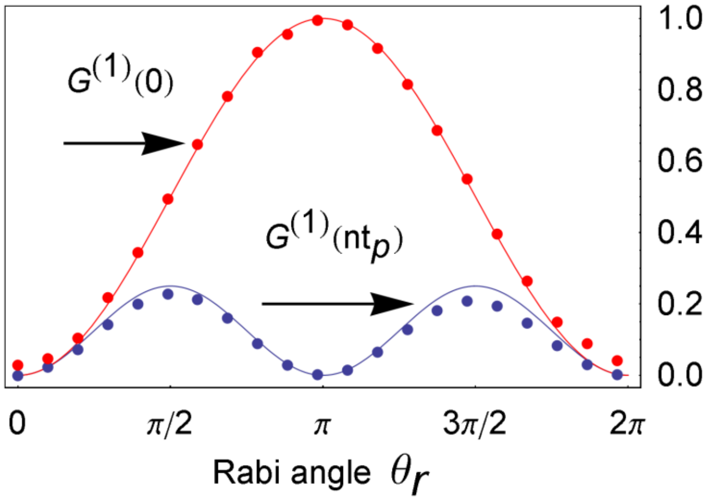
$$\Gamma^{(1)}(\tau) = \int \left\langle S_e^*(t) S_f(t+\tau) \right\rangle dt$$

$$= G^{(1)}(\tau) + N_{ef}(\tau)$$

$$G^{(1)}(\tau) = \Gamma^{(1)}(\tau) - \Gamma^{(1)}_{ss}(\tau)$$







64 M averages ~ 0.5 h ~ 0.5 TByte

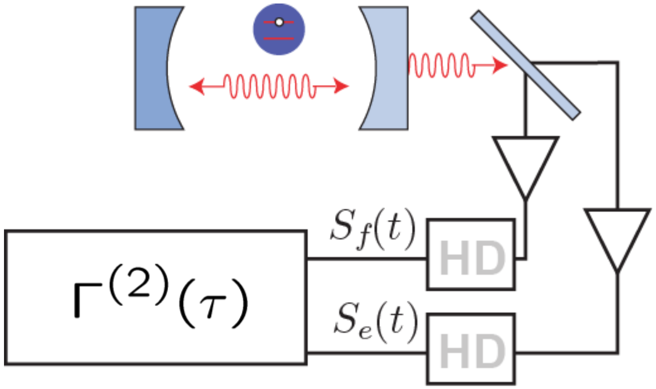
D. Bozyigit et al., *Nat. Phys.* **7**, 154 (2011)

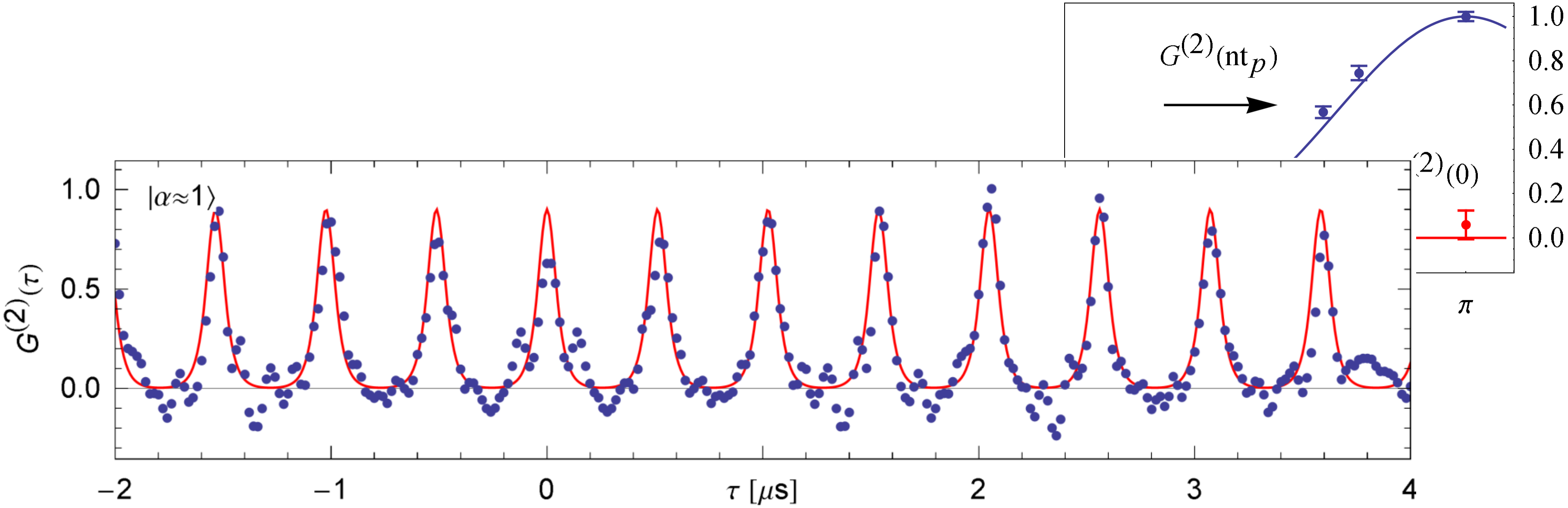
$G^{(2)}$ Measurement

Measure power correlation between channel e & f:

$$\Gamma^{(2)}(\tau) = \int \left\langle S_e^*(t) S_e^* S_f(t+\tau) S_f(t) \right\rangle dt$$

$$G^{(2)}(\tau) = \Gamma^{(2)}_{prep}(\tau) - \Gamma^{(2)}_{ss}(\tau)$$

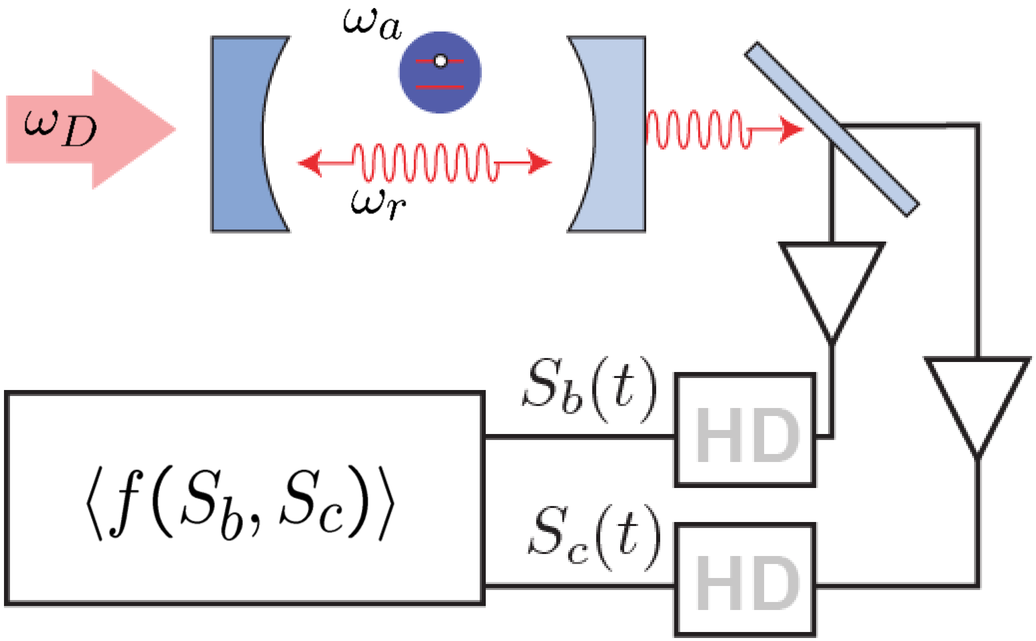




$G^{(2)}$ measurement for a microwave frequency single photon source

D. Bozyigit et al., *Nat. Phys.* 7, 154 (2011)

A Continuously Pumped Single Photon Source

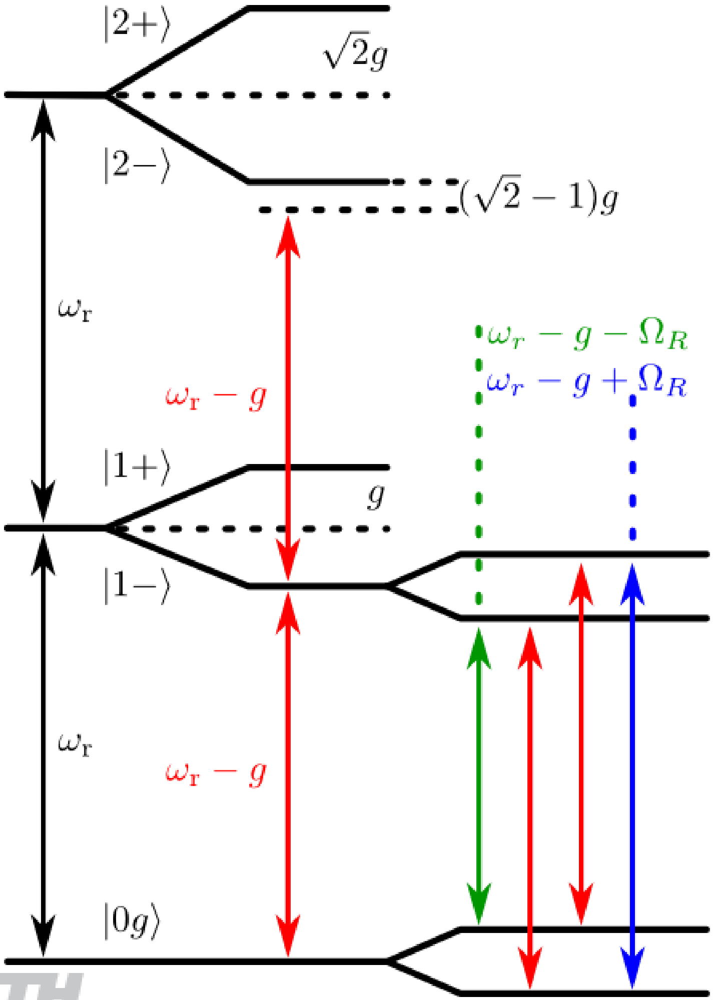


- atom and cavity in resonance $\omega_a = \omega_r$
- driving the lower Jaynes-Cummings doublet $\omega_D = \omega_r - g$

C. Lang *et al.*, *PRL* **106**, 243601 (2011)

Photon Blockade: A Single Photon Turnstile

Level diagram:



- Vacuum Rabi mode splitting:

$$|n, \mp\rangle = 1/\sqrt{2} \cdot (|n, g\rangle \mp |n-1, e\rangle)$$

- Drive:

$$\omega_\mathrm{p} = \omega_\mathrm{r} - g$$

- Photon blockade: first photon enters cavity second is blocked

- mediated photon/photon interactions

- Effective two-level system (polariton)

$$|\downarrow\rangle = |0, g\rangle \qquad |\uparrow\rangle = |1, -\rangle$$

  - Mollow-type triplet:

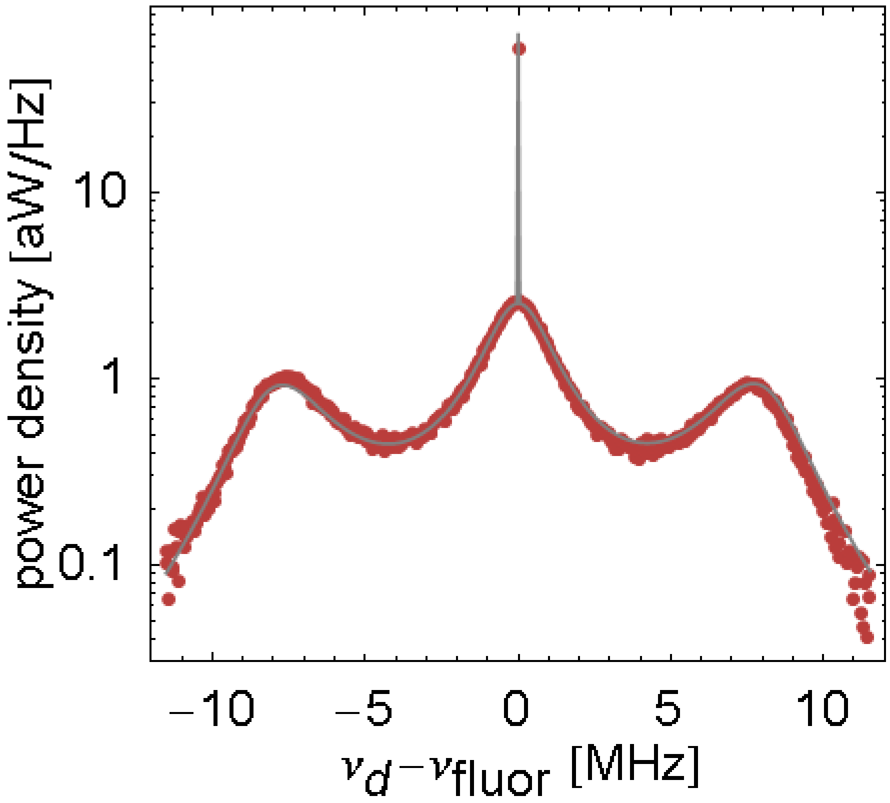
$$\omega_{1,2,3} = \omega_\mathrm{p} \begin{cases} +0 \\ \pm\Omega_R \end{cases}$$

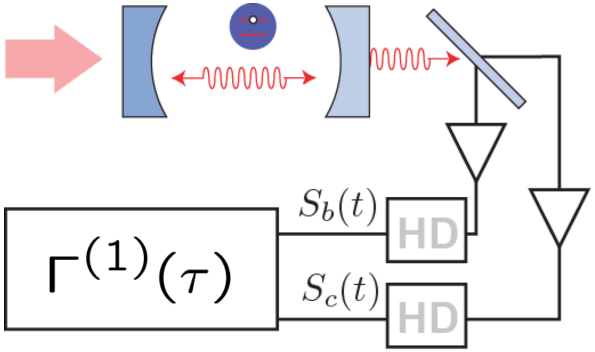
C. Lang *et al.*, *PRL* **107**, 243601 (2011)

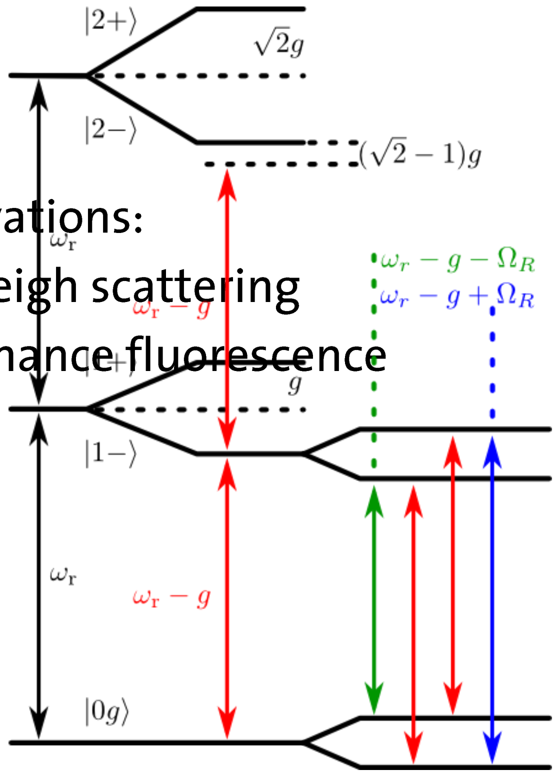
Polariton Mollow Triplet Measurement

(cross-)power spectrum:

$$\mathcal{F}\{\Gamma^{(1)}(\tau)\} = \langle \mathcal{F}\{S_b(t)\} \cdot \mathcal{F}\{S_c(t)\}^* \rangle$$





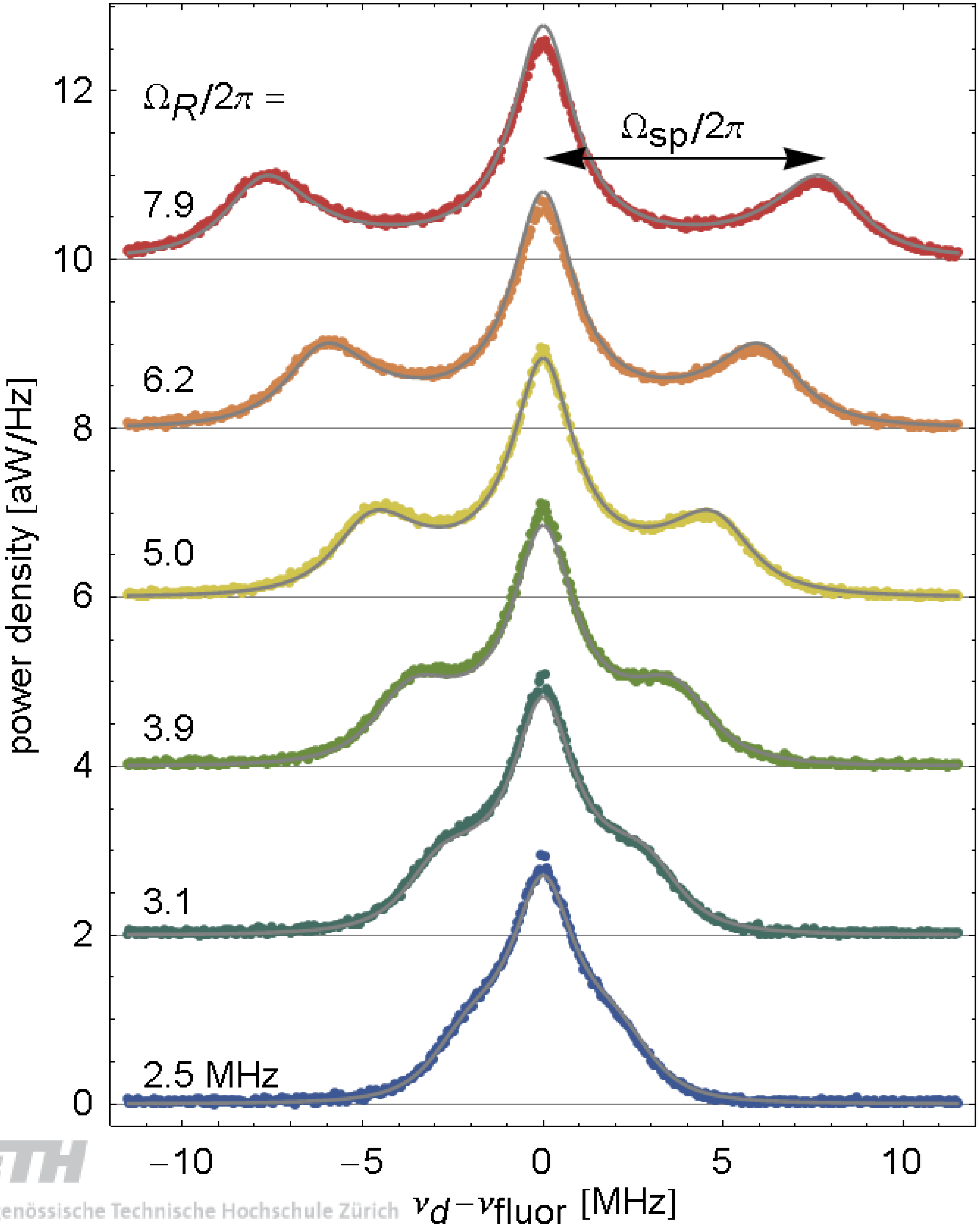


observations:

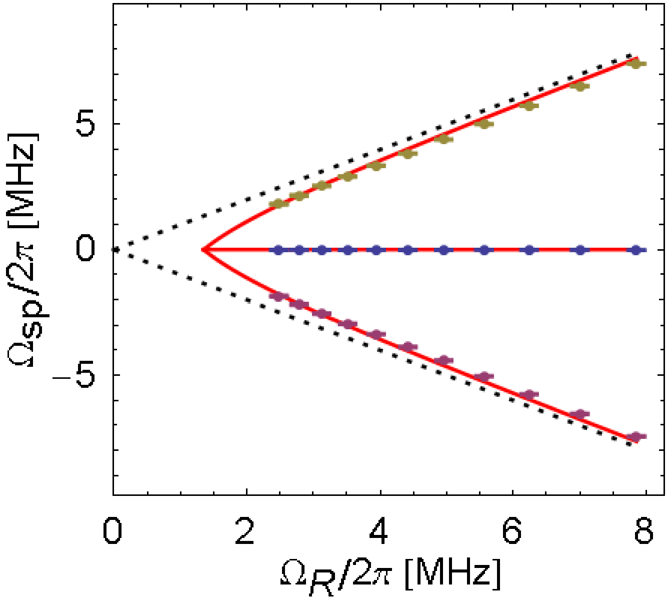
- Rayleigh scattering
- resonance fluorescence

C. Lang *et al.*, *PRL* **106**, 243601 (2011)

Dependence on Drive Amplitude



- ‘Mollow’ fluorescence sidebands at Rabi frequency Ω_{sp}



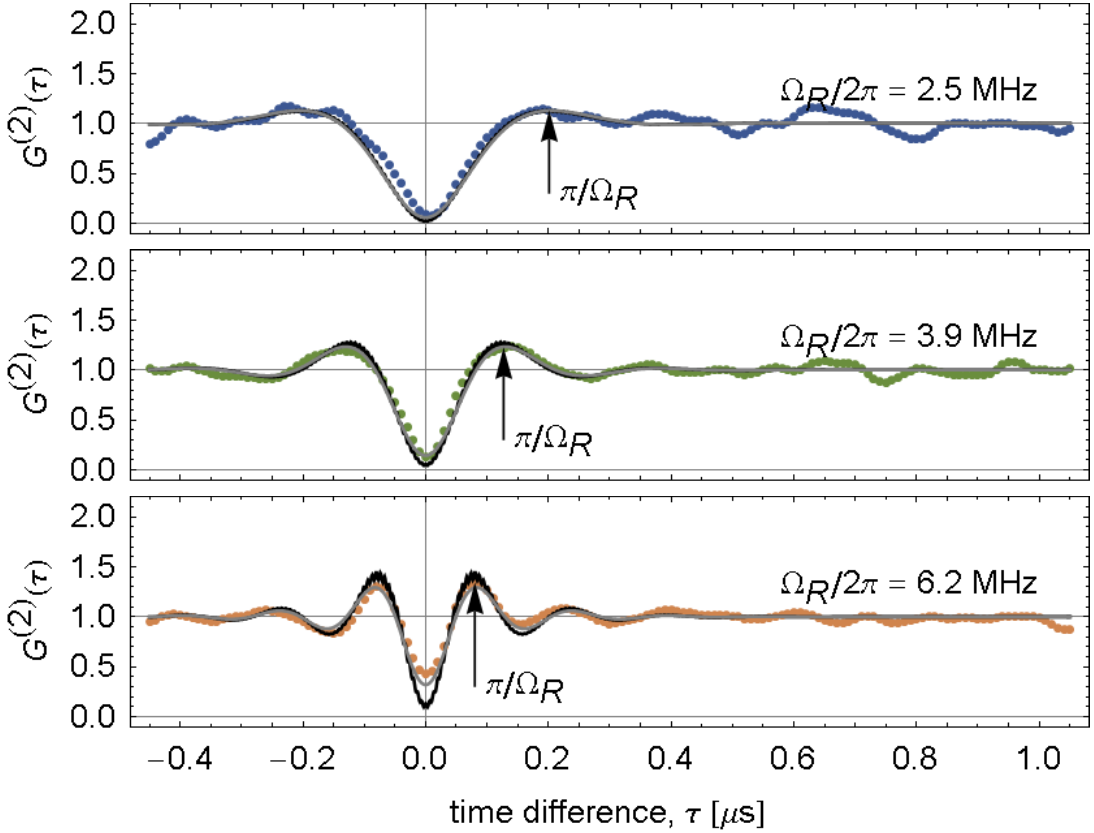
- analytical expression explains nonlinear drive scaling

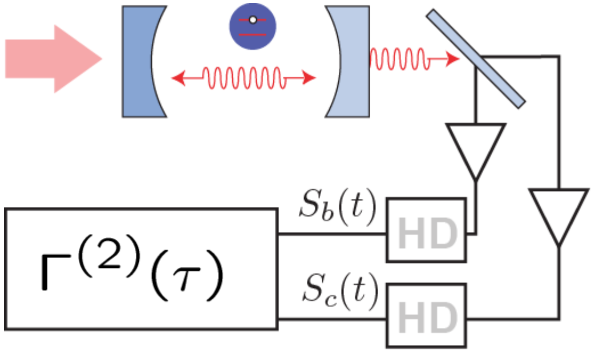
C. Lang *et al.*, *PRL* **106**, 243601 (2011)

Antibunching and Subpoissonian Statistics

- intensity/intensity correlation function (dots)

$$\Gamma^{(2)}(\tau) = \int \langle S_b^* S_b(t) S_c^* S_c(t+\tau) \rangle \, dt$$





observations:

- sub-Possionian statistics
- anti-correlation at τ = 0
- Rabi oscillations visible

solid lines are master equation simulations

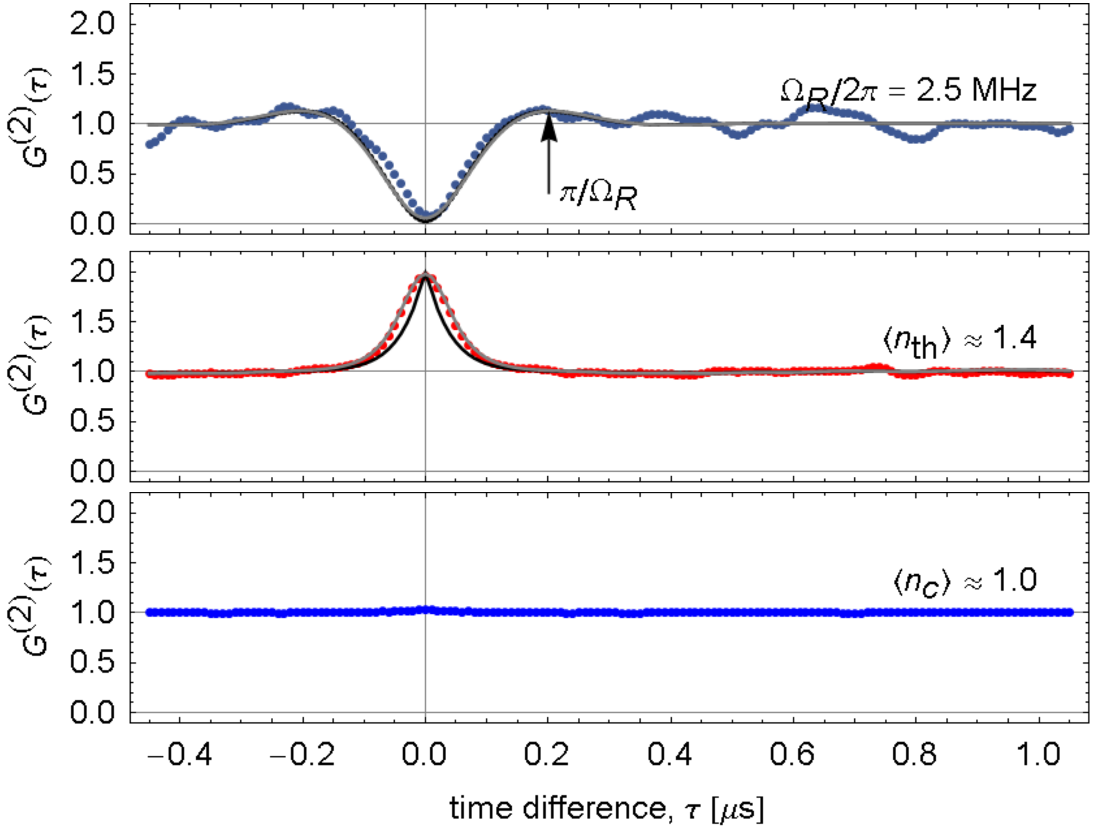
C. Lang *et al.*, *PRL* **106**, 243601 (2011)

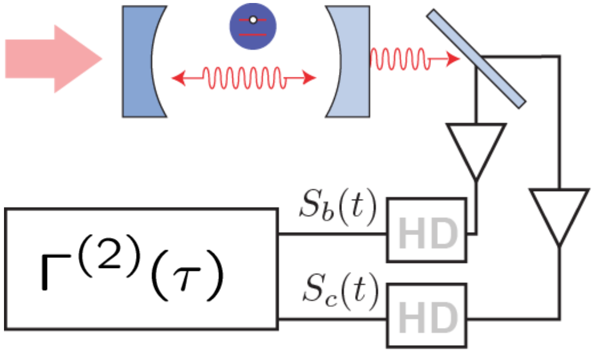
also dispersive photon blockade: A. J. Hoffman *et al.*, *PRL* **107**, 053602 (2011)

Compare to Thermal and Coherent Fields

- intensity/intensity correlation function (dots)

$$\Gamma^{(2)}(\tau) = \int \langle S_b^* S_b(t) S_c^* S_c(t+\tau)\rangle \, dt$$





thermal field:

- $G^{(2)}(0) = 2$
- $G^{(2)}(\tau) = 1$ for large τ

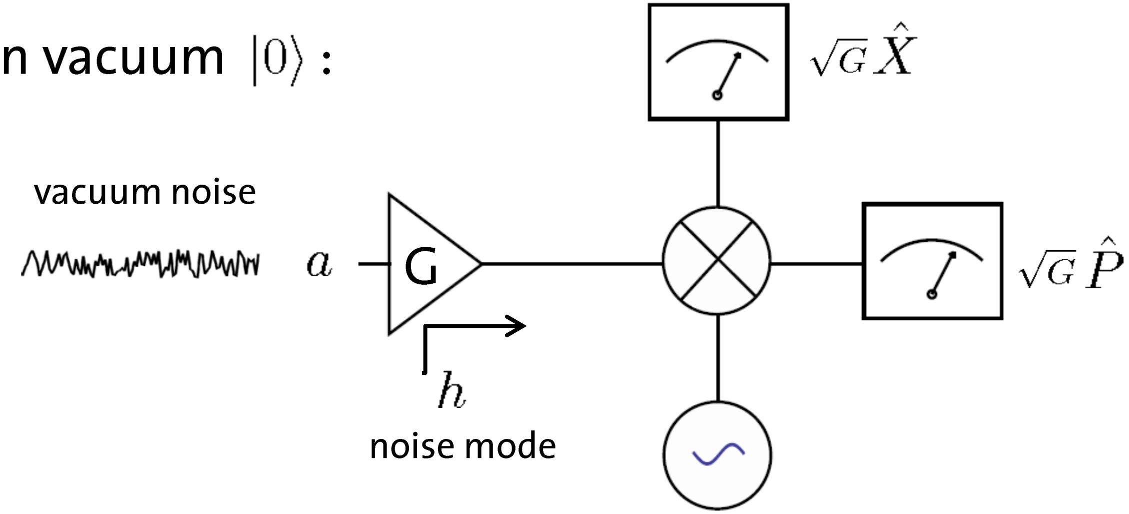
coherent field:

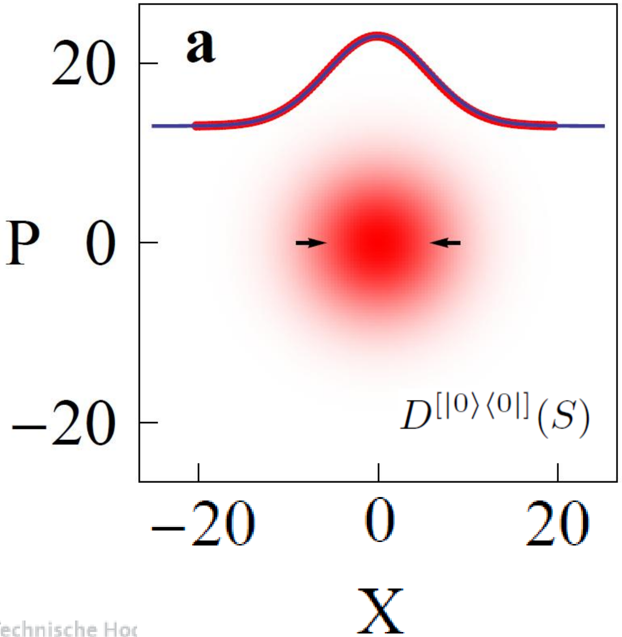
- $G^{(2)} = 1$

C. Lang *et al.*, *PRL* **106**, 243601 (2011)

Full Tomography of a Single Propagating Mode

1) prepare a in vacuum $|0\rangle$:





record histogram $D^{[|0\rangle\langle 0|]}(S)$

of measurement results $S/\sqrt{G} = X + iP$

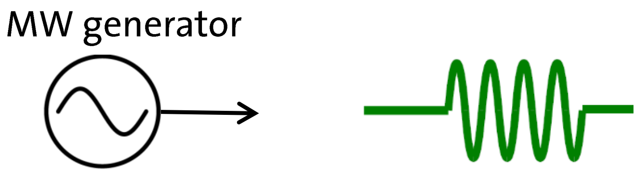
normal distribution with variance

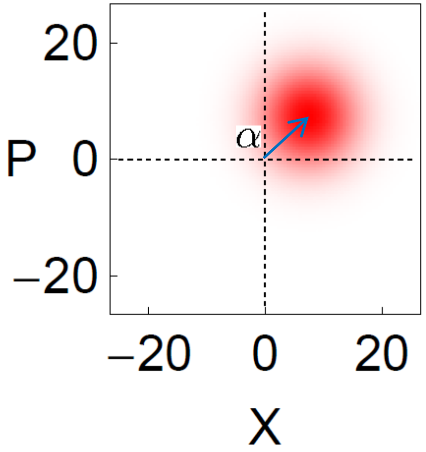
$$2\sigma^2 = \langle \hat{S}^\dagger \hat{S} \rangle / G = \frac{1}{G} \int \mathrm{d}^2 S \, D^{[|0\rangle\langle 0|]}(S) \, S^* S = 67$$

h introduces thermal noise with mean photon number N_{noise}

Coherent State Histograms

2) prepare a in coherent state $|\alpha\rangle$:



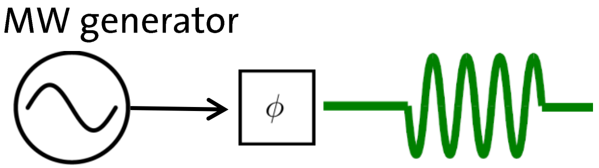


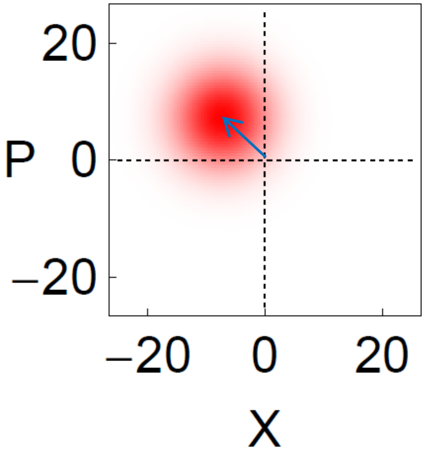
$$|\alpha| \approx 6.3$$

$$\Leftrightarrow$$

$$\langle a^{\dagger} a \rangle \approx 41 \sim N_{\text{noise}}$$

3) rotate phase $|e^{i\phi}\alpha\rangle$:

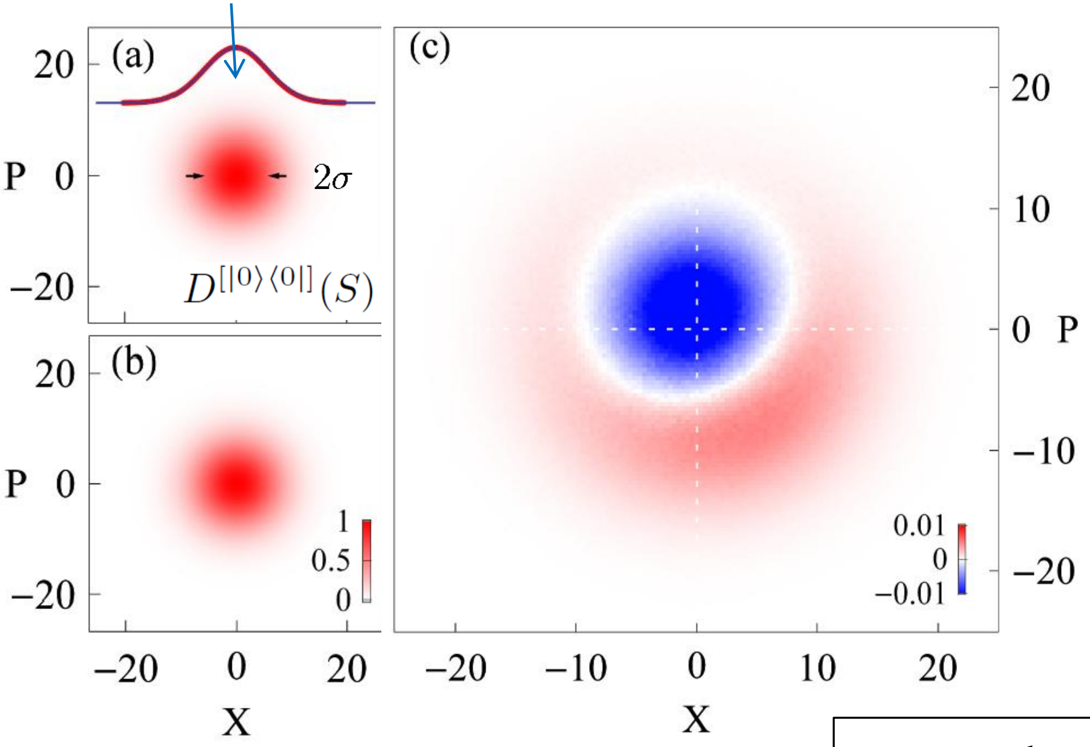




Question: What can we learn about state when $\langle a^{\dagger} a \rangle \leq 1$?

Single Photon Source Histogram

store 2D histogram $D^{[\rho]}(S)$ from $S/\sqrt{G} = X + iP$ measurement results:



corresponding
phase space
distribution

signal mode a
in **vacuum**

Q - function
of noise mode :

Q_h

convolution
with **P** – function
of signal

$Q_h * P_a$

subtracted
histograms
to visualize
difference

signal mode a
in **single photon**
Fock state

separate noise h from
signal a systematically!

C. Eichler, et al., *PRL* **106**, 220503 (2011)

Statistical Analysis of Histograms

systematic mode separation:

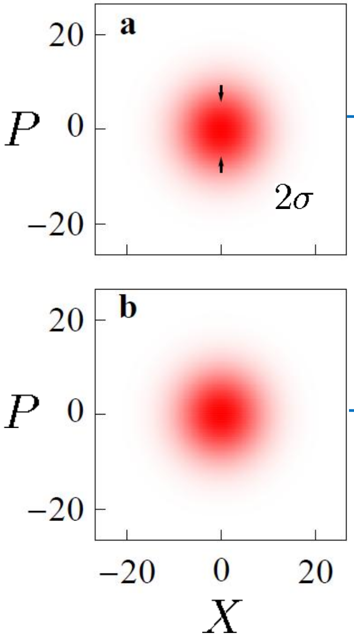
histogram moments: $\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_\rho = \int \mathrm{d}^2 S \; (S^*)^n S^m \; D^{[\rho]}(S)$

1. calculate histogram moments

2. algebraic inversion

vacuum $|0\rangle$

Fock state $|1\rangle$



$$\left.\begin{array}{l} \dfrac{\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_{|0\rangle\langle 0|}}{G^{(n+m)/2}} = \langle h^n (h^\dagger)^m \rangle \\[3em] \dfrac{\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_{|1\rangle\langle 1|}}{G^{(n+m)/2}} = \langle (h + a^\dagger)^n (h^\dagger + a)^m \rangle \end{array}\right\} \langle (a^\dagger)^n a^m \rangle$$

reminder: $X + iP = S/\sqrt{G} = (a + h^\dagger)$

State Dependent Moments of Probability Distribution

moments $|\langle (a^\dagger)^n a^m \rangle|$ for different prepared states:

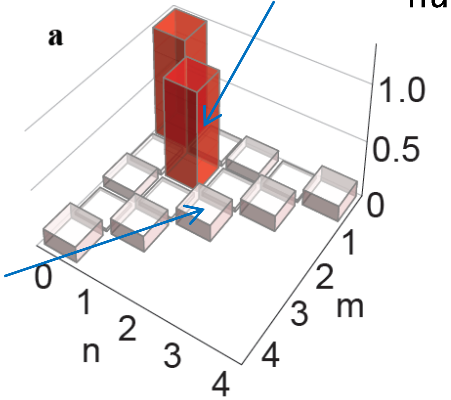
Fock state

$|1\rangle$

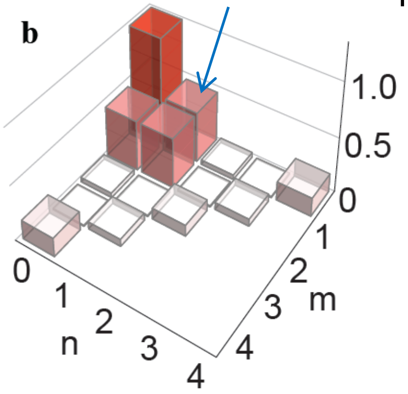
$\langle a^\dagger a \rangle =$ “mean photon number”

$\langle (a^\dagger)^2 a^2 \rangle \approx 0$

“anti bunching”

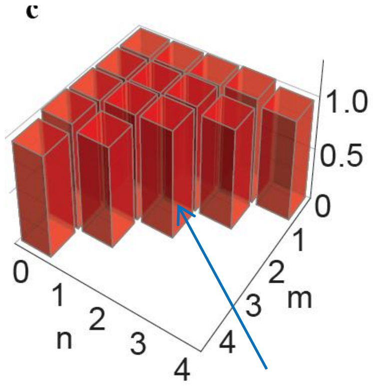


$\langle a \rangle =$ “mean amplitude”



superposition

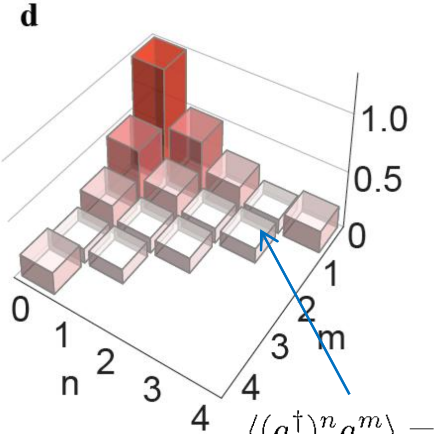
$\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$



coherent state

$|\alpha = 1\rangle$

$\langle (a^\dagger)^n a^m \rangle = (\alpha^*)^n \alpha^m = 1$



coherent state

$|\alpha = 0.5\rangle$

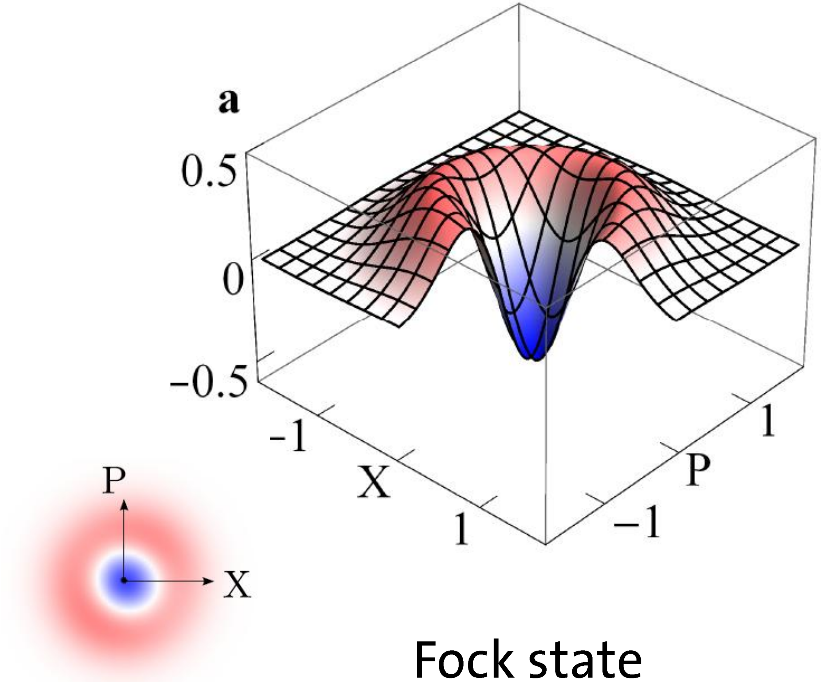
$\langle (a^\dagger)^n a^m \rangle = (1/2)^{n+m}$

C. Eichler, et al., *PRL* **106**, 220503 (2011)

Reconstructed Wigner Function of Itinerant Photon

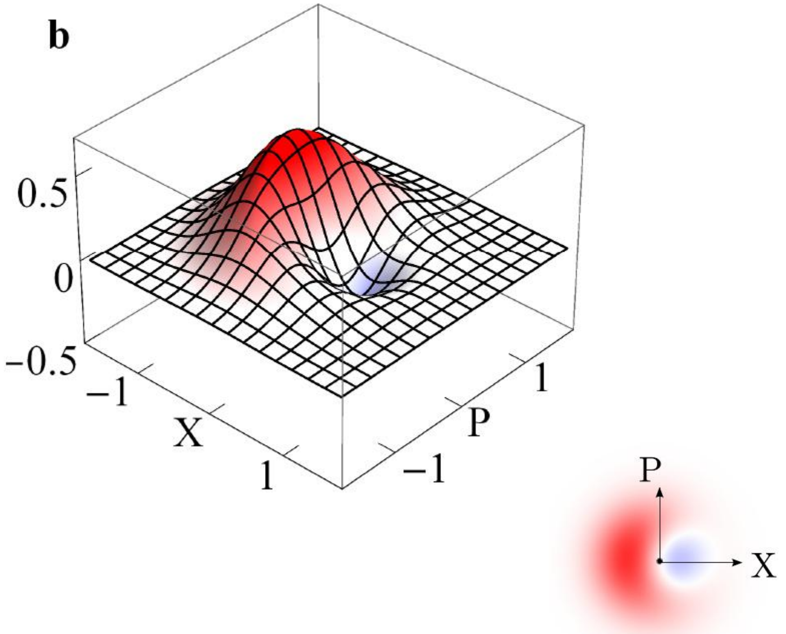
Wigner function reconstructed from measured moments:

$$W(\alpha) = \sum_{n,m} \int d^2\lambda \, \frac{\langle (a^\dagger)^n a^m \rangle (-\lambda^*)^m \lambda^n}{\pi^2 n! m!} e^{(-1/2)|\lambda|^2 + \alpha\lambda^* - \alpha^*\lambda} \qquad \text{with} \qquad n + m < 4$$



Fock state

$|1\rangle$



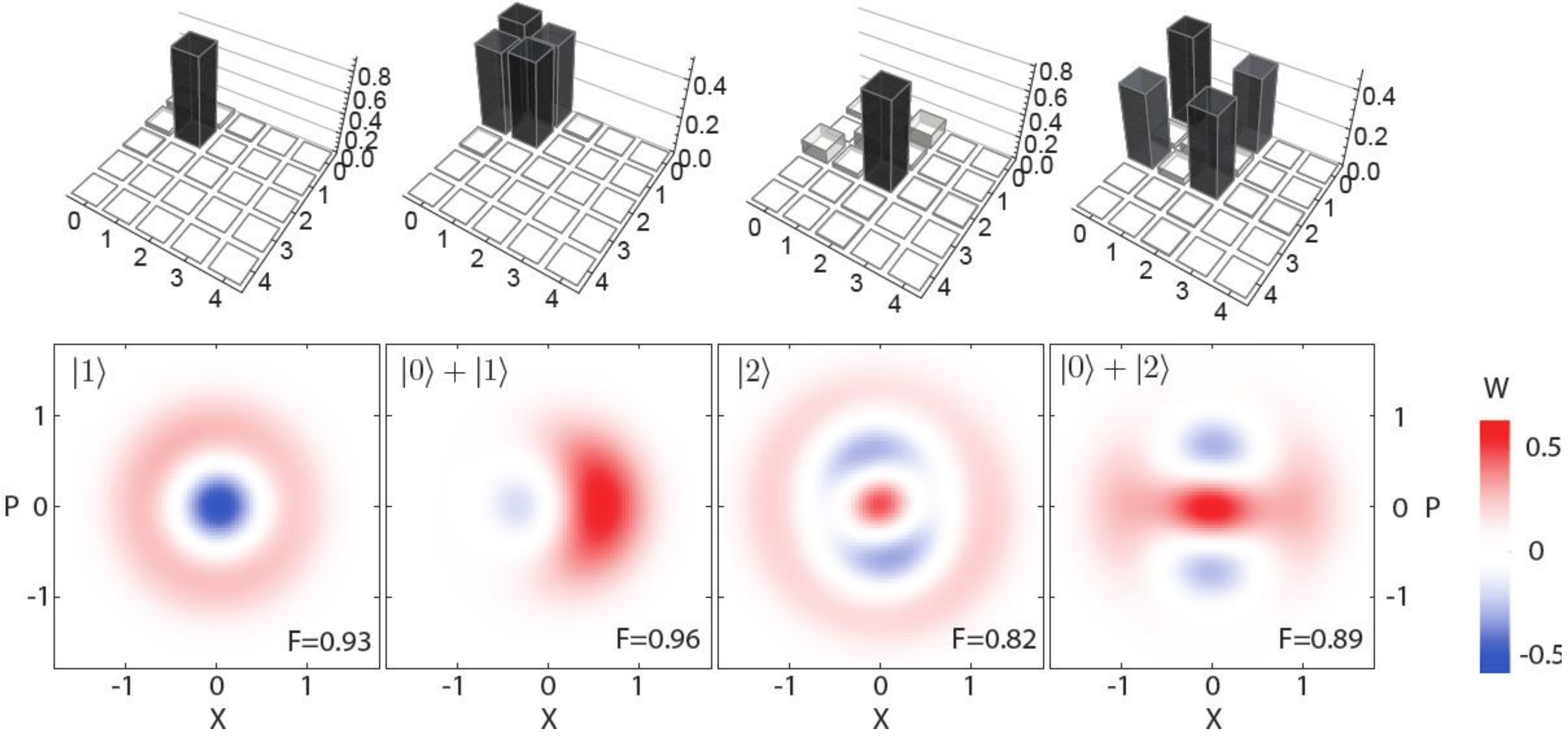
superposition

$\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$

C. Eichler, et al., *PRL* **106**, 220503 (2011)

Wigner Function and Density Matrices ...

... for **propagating** multi-photon Fock states and their superpositions:

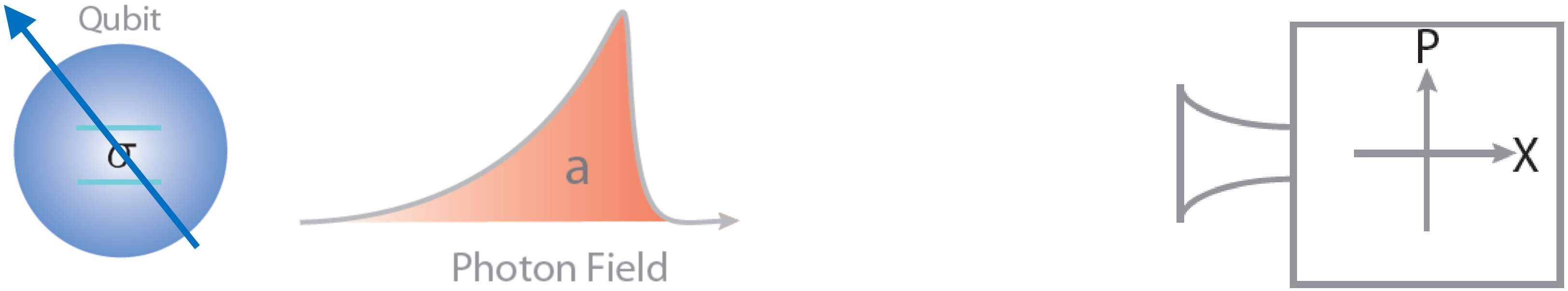


measured using near-quantum-limited parametric amplifier

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

C. Eichler, et al., *ETH Zurich* (2012)

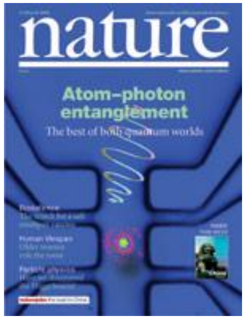
Entanglement of Localized and Propagating Modes



- test of correlations between propagating photon and qubit
- probe non-local aspects of quantum mechanics in circuits

- interfacing stationary and flying qubits
- entanglement distribution in a quantum network

Photon/Qubit Entanglement at Optical Frequencies



Atom–Photon Entanglement

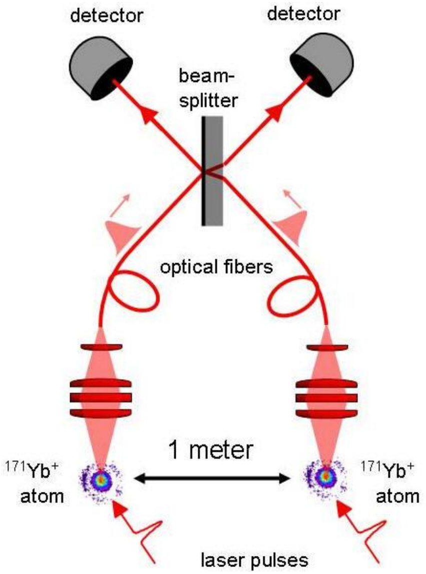
Blinov *et al., Nature* **428**, 153 (2004)

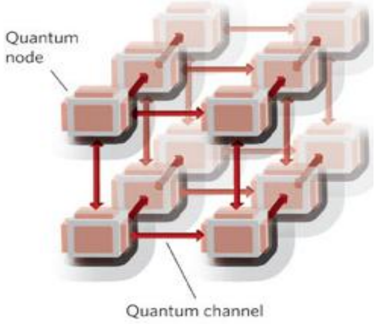
Volz *et al., PRL* **96**, 030404 (2006)

Atom–Atom Entanglement

Moehring *et al., Nature* **449**, 68 (2007)

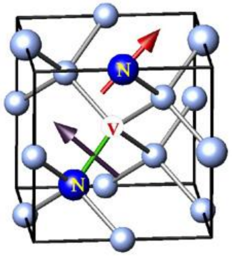
Ritter *et al., Nature* **484**, 195 (2012)





The quantum internet

Kimble, *Nature* **453**, 1023 (2008)

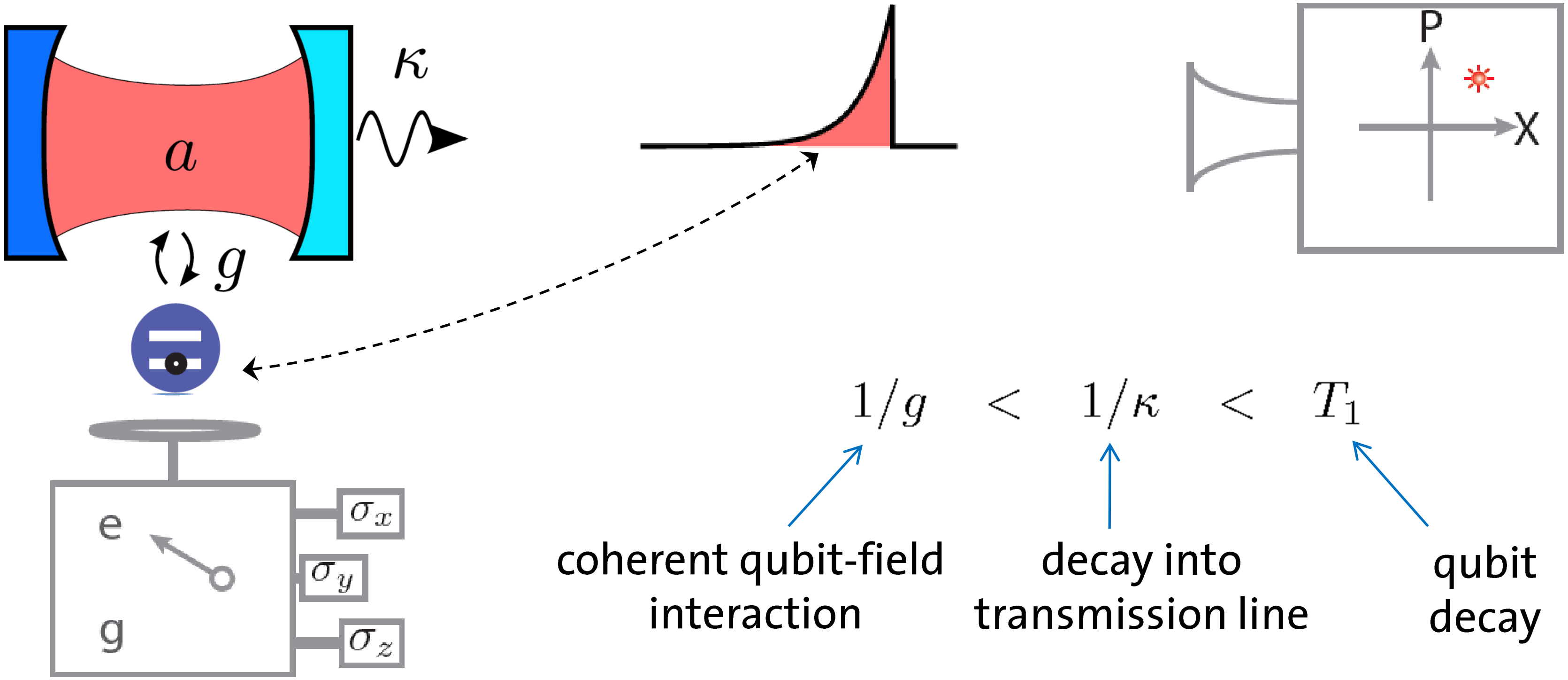


Spin–Photon Entanglement

Togan *et al., Nature* **466**, 730 (2010)

What about superconducting circuits?

Concept of Photon/Qubit Entanglement Experiment



$$1/g \quad < \quad 1/\kappa \quad < \quad T_1$$

coherent qubit-field interaction

decay into transmission line

qubit decay

Conditions for generation and detection of qubit/photon entanglement

Experimental Setup

- Transmon qubit

  $T_1 = 1.1\,\mu s$

  $T_2 = 550\,ns$

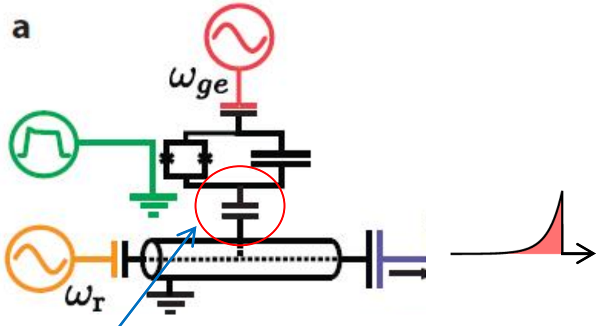
  $T_2^* = 220\,ns$

- Single sided resonator

  $1/\kappa = 25\,ns$

- Coupling strength

  $\pi/g = 7.7\,ns$



ctor = ?

Strong coupling limit!

Experimental Setup

- Transmon qubit

$$T_1 = 1.1\,\mu s$$
$$T_2 = 550\,ns$$
$$T_2^* = 220\,ns$$

- Single sided resonator

$$1/\kappa = 25\,ns$$

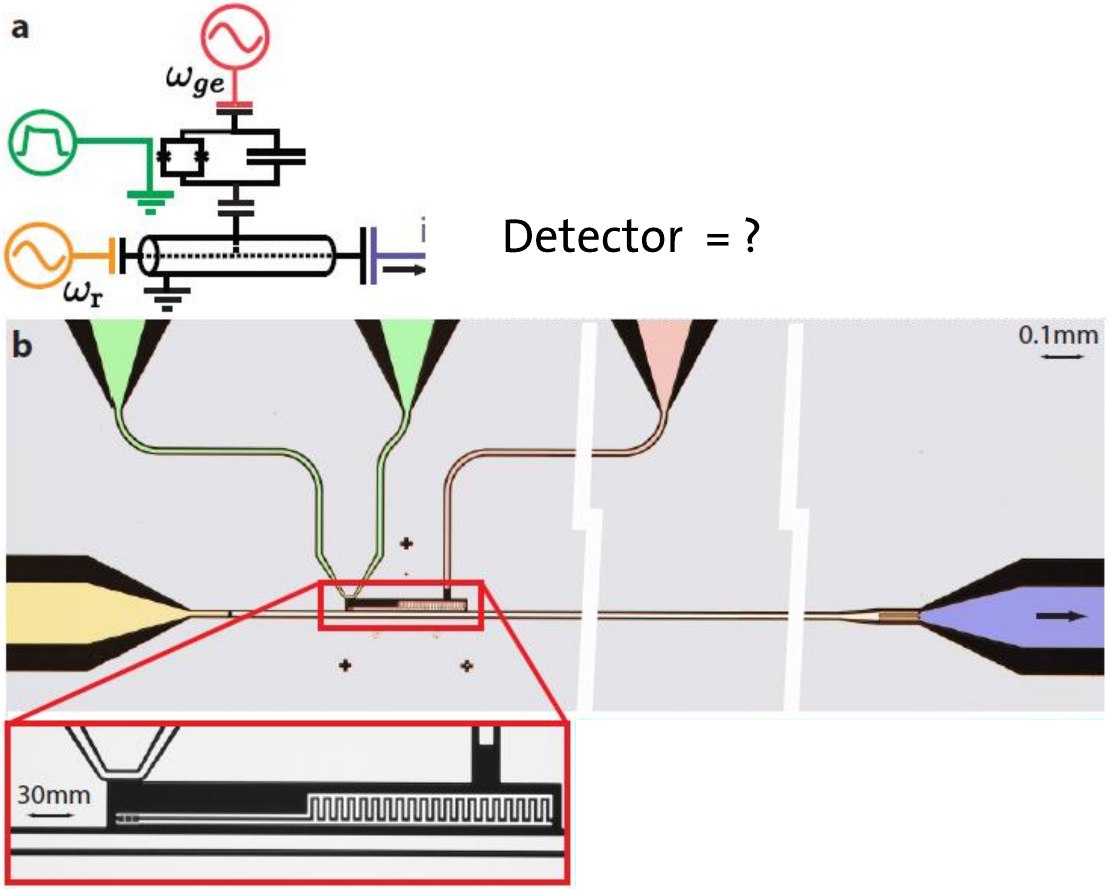
- Coupling strength

$$\pi/g = 7.7\,ns$$

- Parametric amplifier

$$\sqrt{G}B = 178\,\mathrm{MHz}$$
$$P_{1dB}\,@\sim 16\,\mathrm{photons}$$

Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 929 (2008)



Detector = ?

C. Eichler *et al.*, ETH Zurich *unpublished* (2012)

Prepare and Measure Qubit/Photon Entanglement

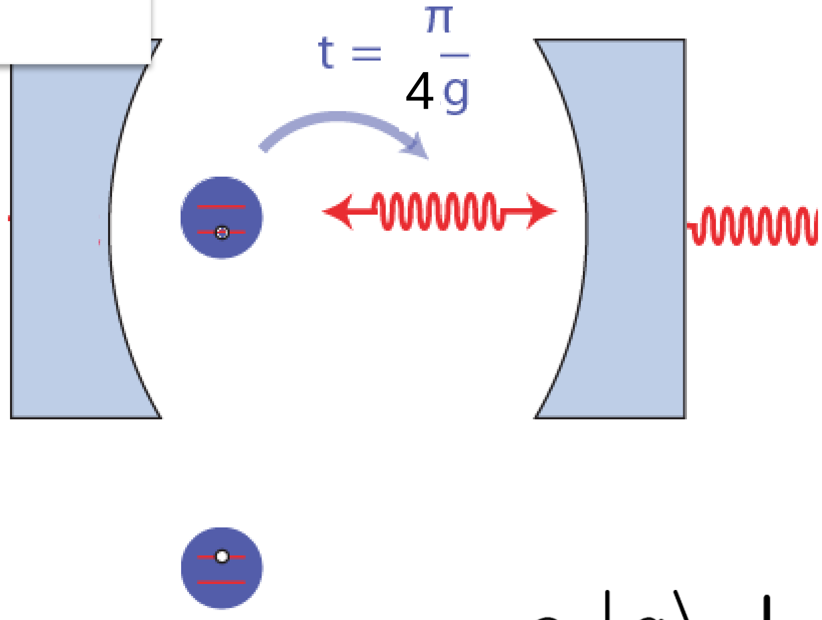
Step 2:

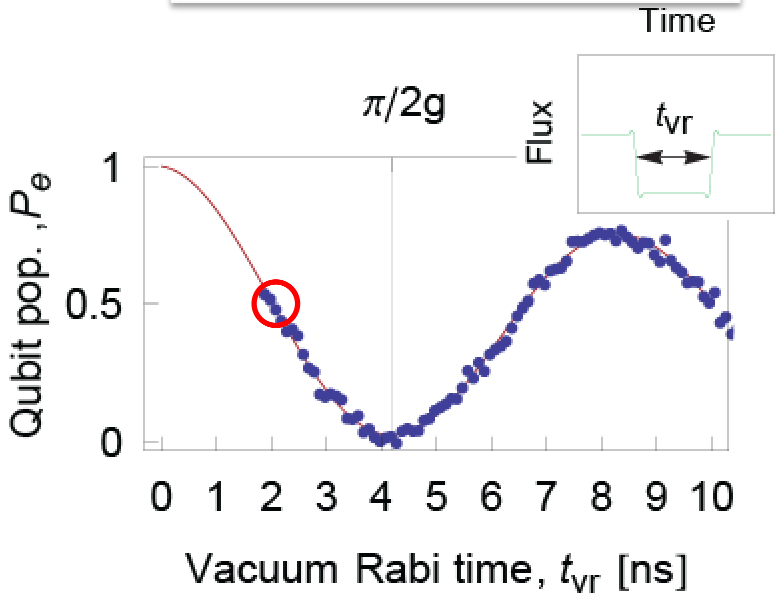
Entangle qubit with resonator by 1/4 vacuum Rabi oscillation

$$\frac{1}{\sqrt{2}}(|0e\rangle + |1g\rangle)$$

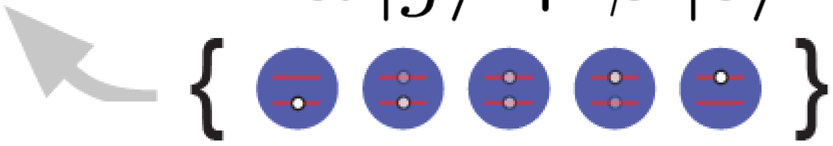
Step 3:

Measure qubit and photon state.





$$\alpha\,|g\rangle + \beta\,|e\rangle$$

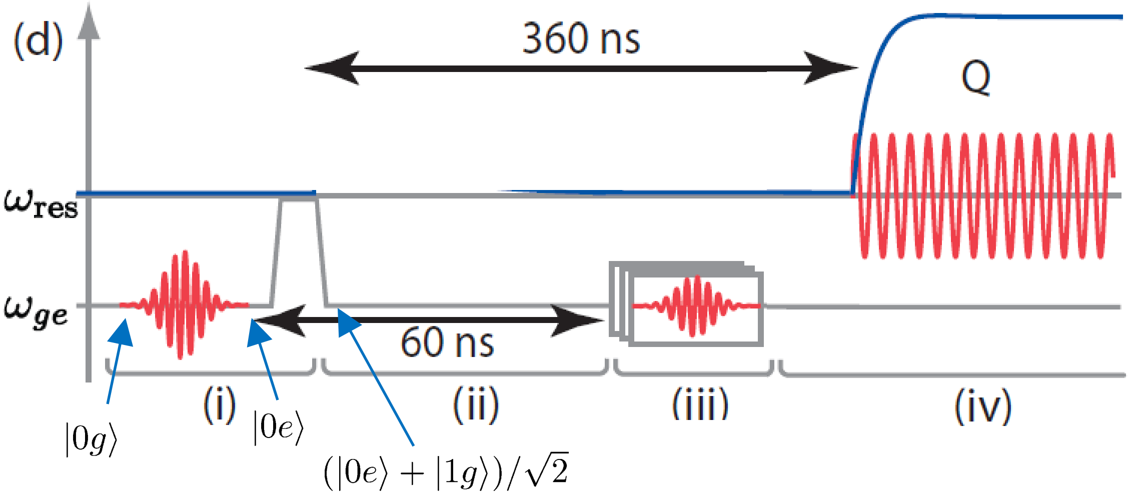


Step 1:

Prepare qubit state by Rabi oscillation

D. Bozyigit et al., *Nat. Phys.* **7**, 154 (2011), S. Deleglise et al. Nature **455**, 510514 (2008) M. Hofheinz *et al.*, *Nature* **454**, 310 (2008), A. Houck *et al.*, *Nature* **449**, 328 (2007)

Exp. Entanglement Generation and Detection Sequence



- (i) preparation of Bell state

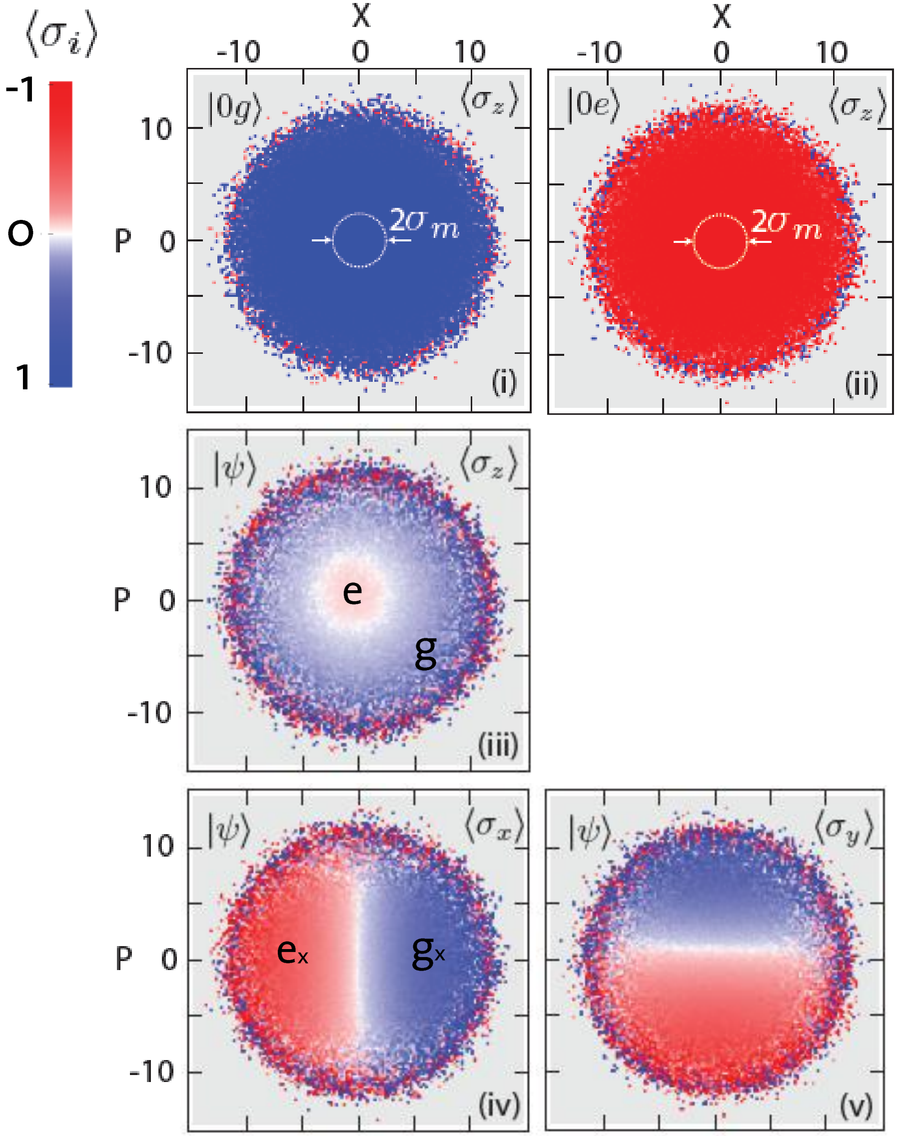
$$\frac{1}{\sqrt{2}}(|0e\rangle + |1g\rangle)$$

- (ii) field decay into transmission line and measurement of X and P
- (iii) qubit tomography pulses
- (iv) dispersive qubit read-out

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

C. Eichler *et al.*, ETH Zurich *unpublished* (2012)

Measurement Results



und state

'e

ation vs.

$+ iP$

→

as expected $\langle\sigma_z\rangle_\alpha$
independent
of α

Analyzing the Bell state

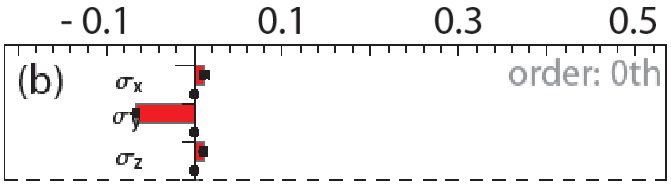
$$|\psi\rangle = |0e\rangle + |1g\rangle$$

Probing coherences:

$$= |e_x\rangle(\underbrace{|1\rangle - |0\rangle}_{\langle\hat{X}\rangle < 0}) + |g_x\rangle(\underbrace{|1\rangle + |0\rangle}_{\langle\hat{X}\rangle > 0})$$

exp: C. Eichler *et al.*, ETH Zurich *unpublished* (2012)
theo: C. Eichler *et al.*, *arXiv:1206.3405* (2012)

Extract Expectation Values of Moments of Distribution



$$\langle (a^\dagger)^n a^m \sigma_i \rangle$$

🟥 Re 🟦 Im

0th :	qubit state with **photon** traced out
1st :	**phase correlations** between qubit and photon field
2nd :	**number** correlations! e <-> no photon g <-> one photon
3rd , 4th :	no higher photon number states!

exp: C. Eichler *et al.*, ETH Zurich *unpublished* (2012)

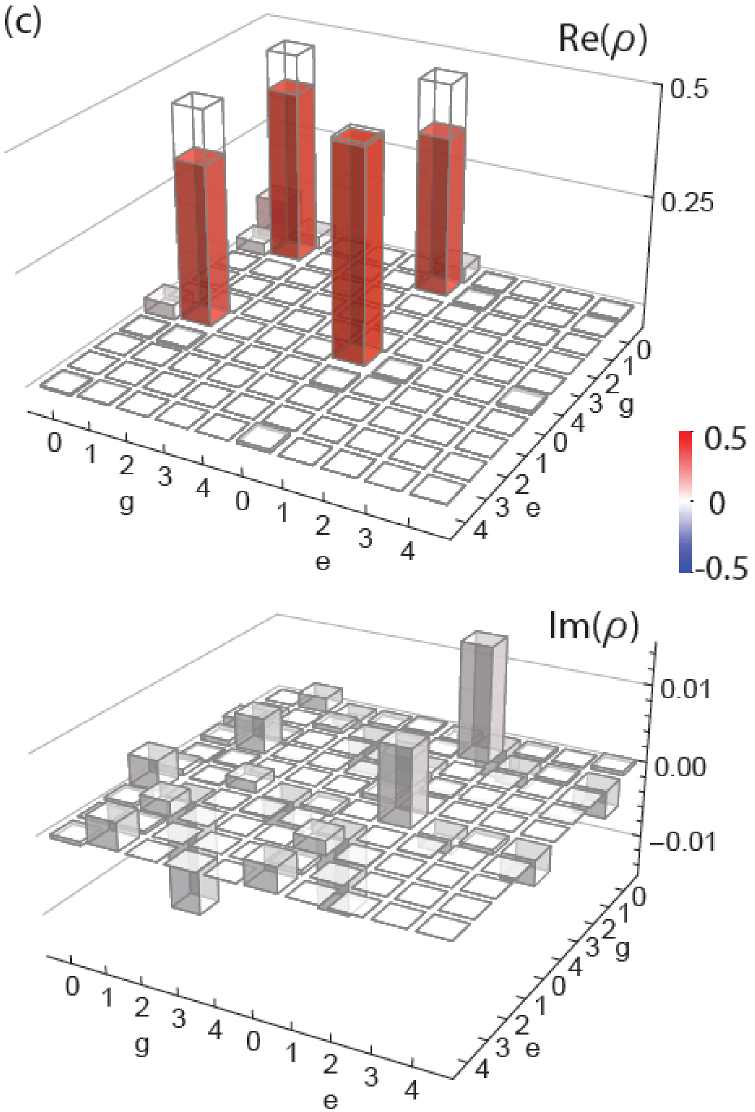
theo: C. Eichler *et al.*, *arXiv:1206.3405* (2012)

Photon/Qubit Joint State Density Matrix

Reconstruction from measured moments

Fidelity: $\langle\psi|\rho|\psi\rangle = 0.83$

Limited by qubit decay during time required for photon detection in same mode.

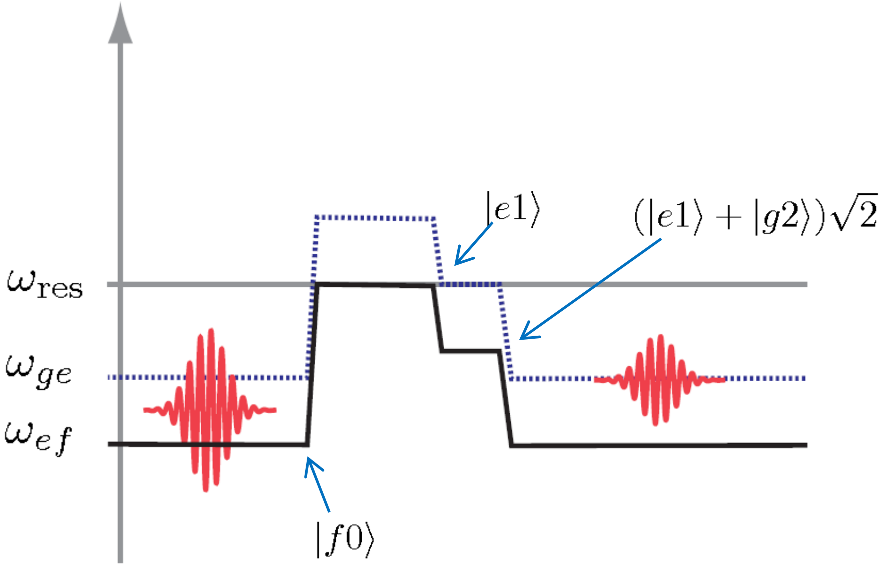


Extension to states with more than a single photon possible!

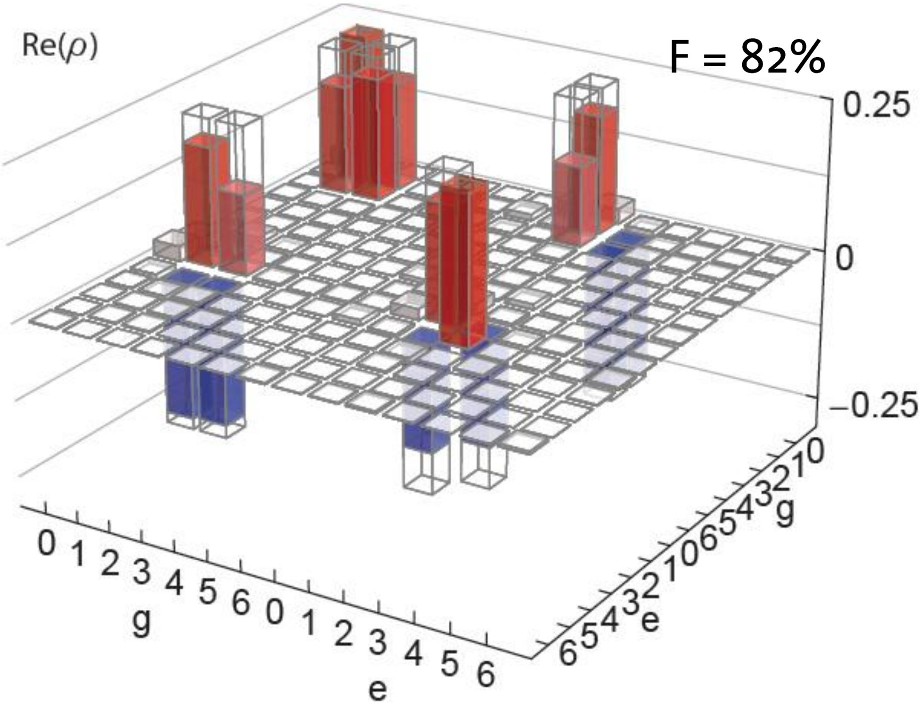
exp: C. Eichler *et al.*, ETH Zurich *unpublished* (2012)
theo: C. Eichler *et al.*, *arXiv:1206.3405* (2012)

Qubit Entangled with Two Propagating Photons

state: $\frac{1}{2}[|g\rangle(|1\rangle + |2\rangle) + |e\rangle(|1\rangle - |2\rangle)]$ → use second excited state of qubit for preparation



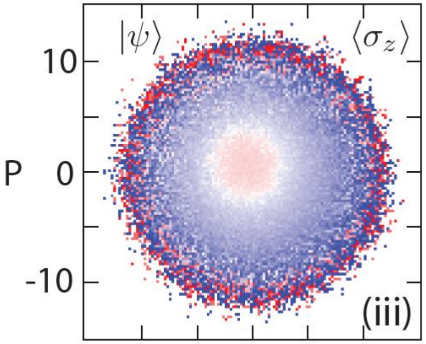
joint state tomography

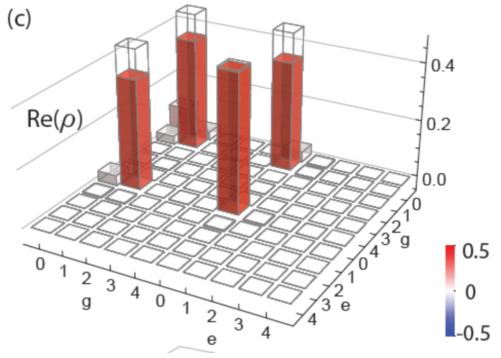


exp: C. Eichler *et al.*, ETH Zurich *unpublished* (2012)
theo: C. Eichler *et al.*, *arXiv:1206.3405* (2012)

Summary Qubit/Photon Entanglement

- First observation of entanglement between superconducting qubit and itinerant microwave photon field
- Characterization in full tomography.



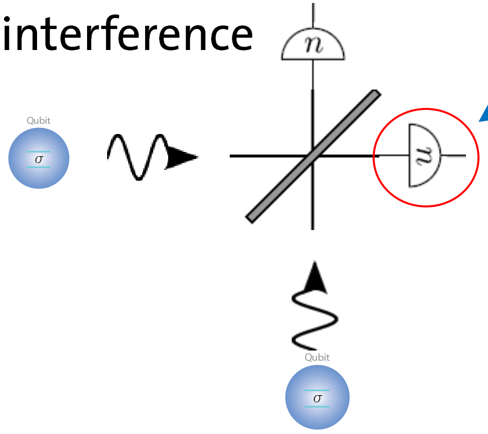


C. Eichler *et al.*, ETH Zurich *unpublished* (2012)
C. Eichler *et al.*, *arXiv:1206.3405* (2012)

Recent experiment:

Hong-Ou-Mandel two-photon interference

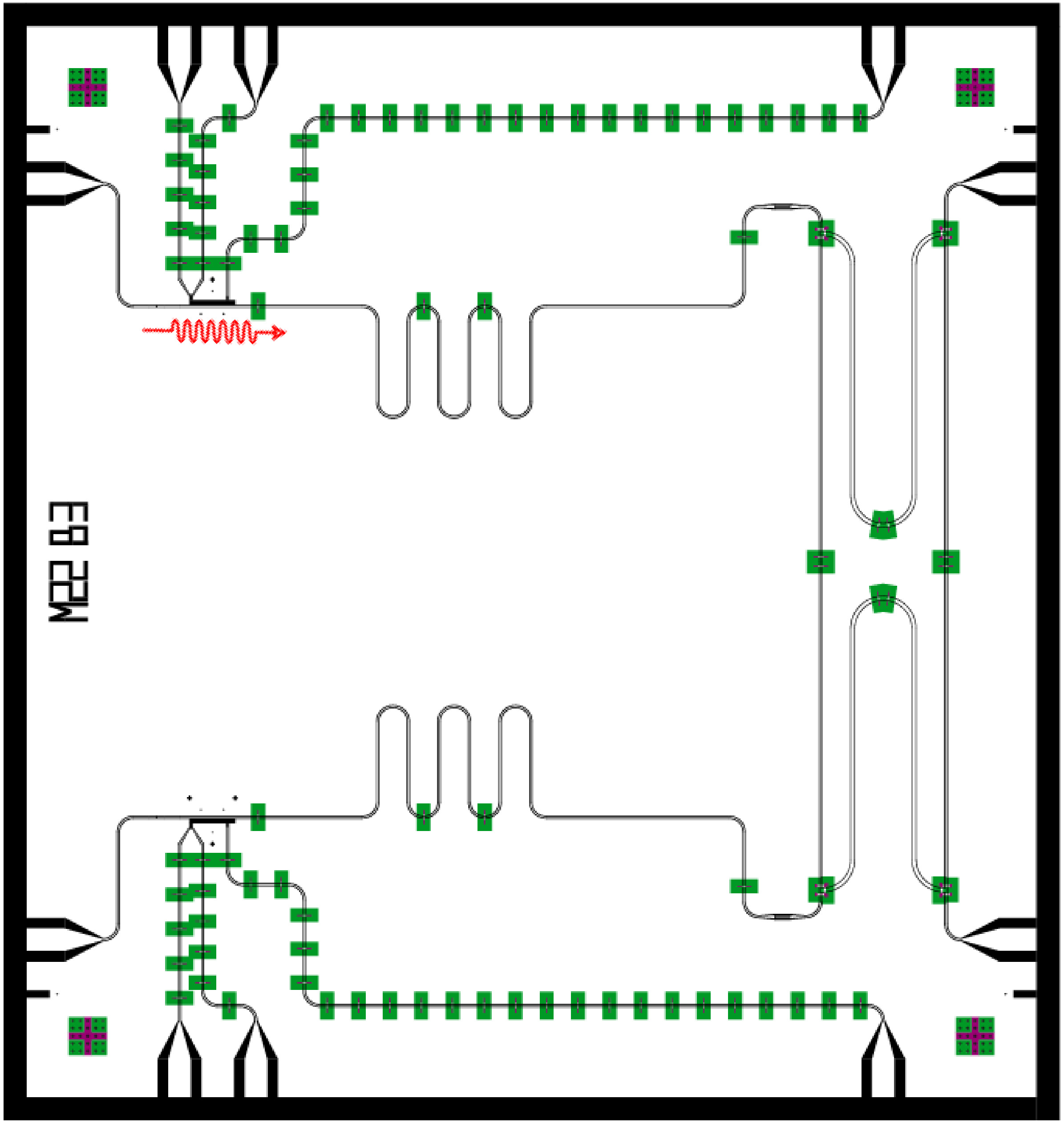
C. Lang et al. , ETH Zurich (2012)



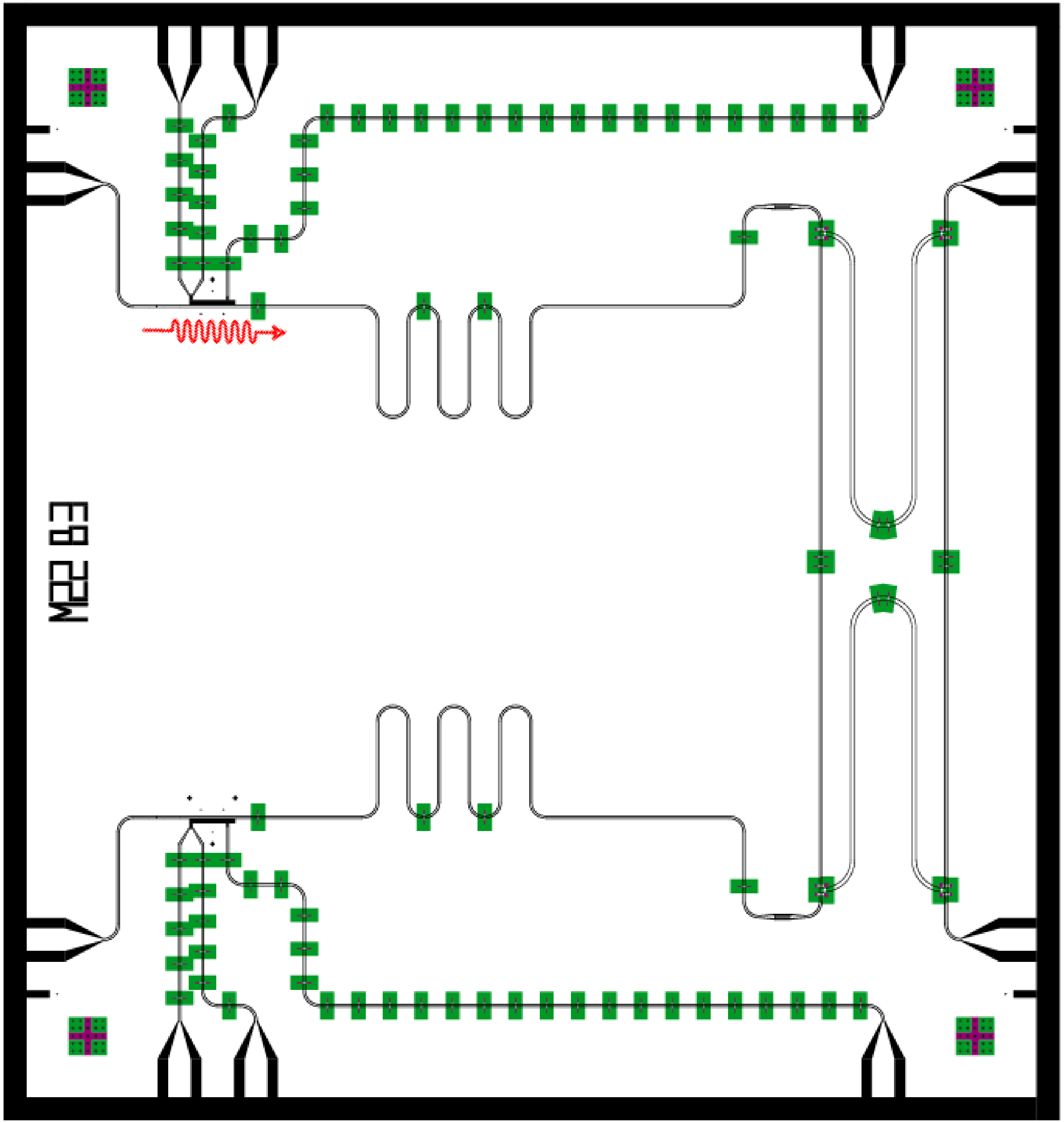
microwave photon counter

Chen *et al.*, *PRL* **107**, 217401 (2011)
Romero et al., *PRL* **102**, 173602 (2009)

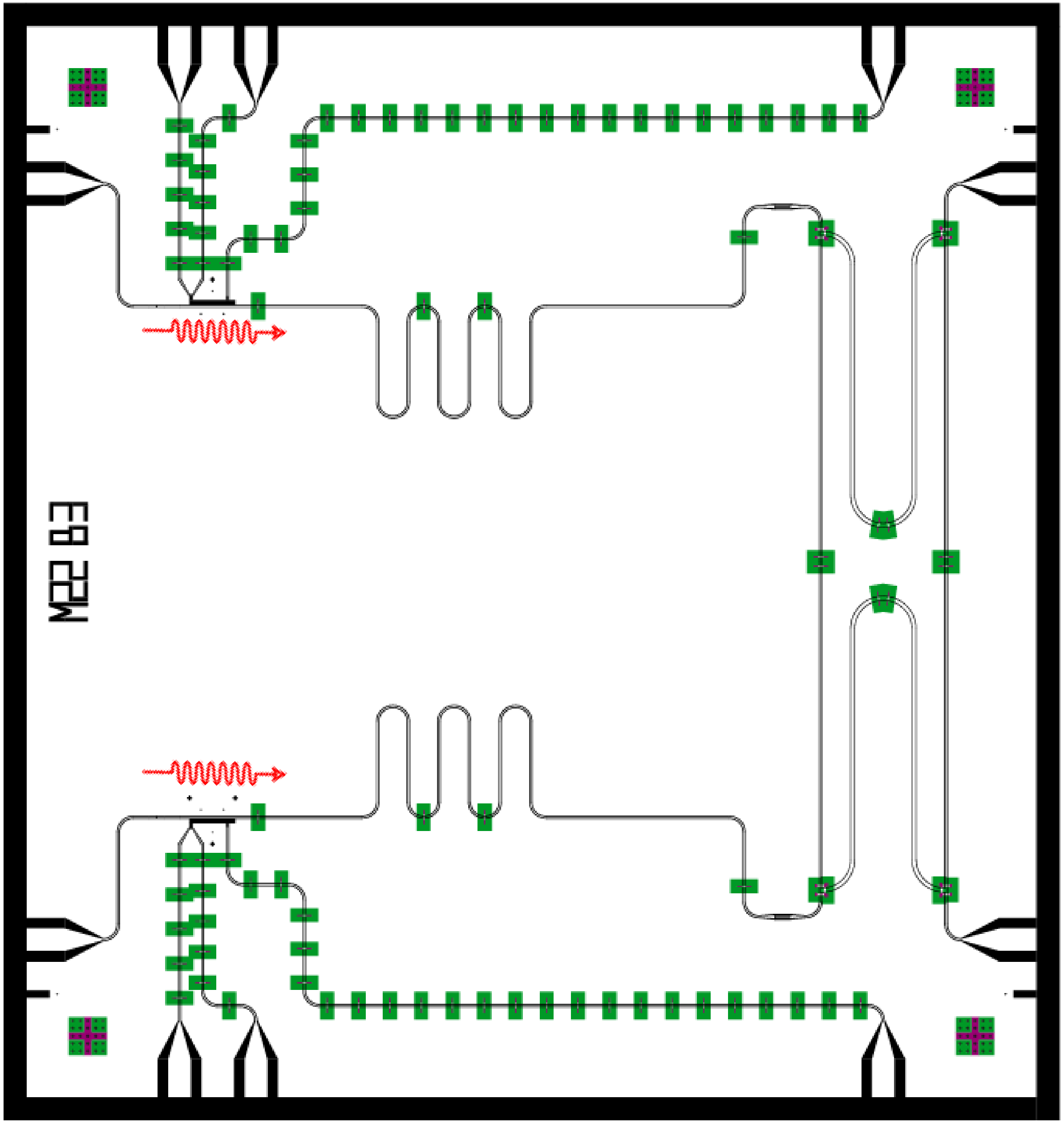
Design: Two Single Photon Sources and Beam Splitter



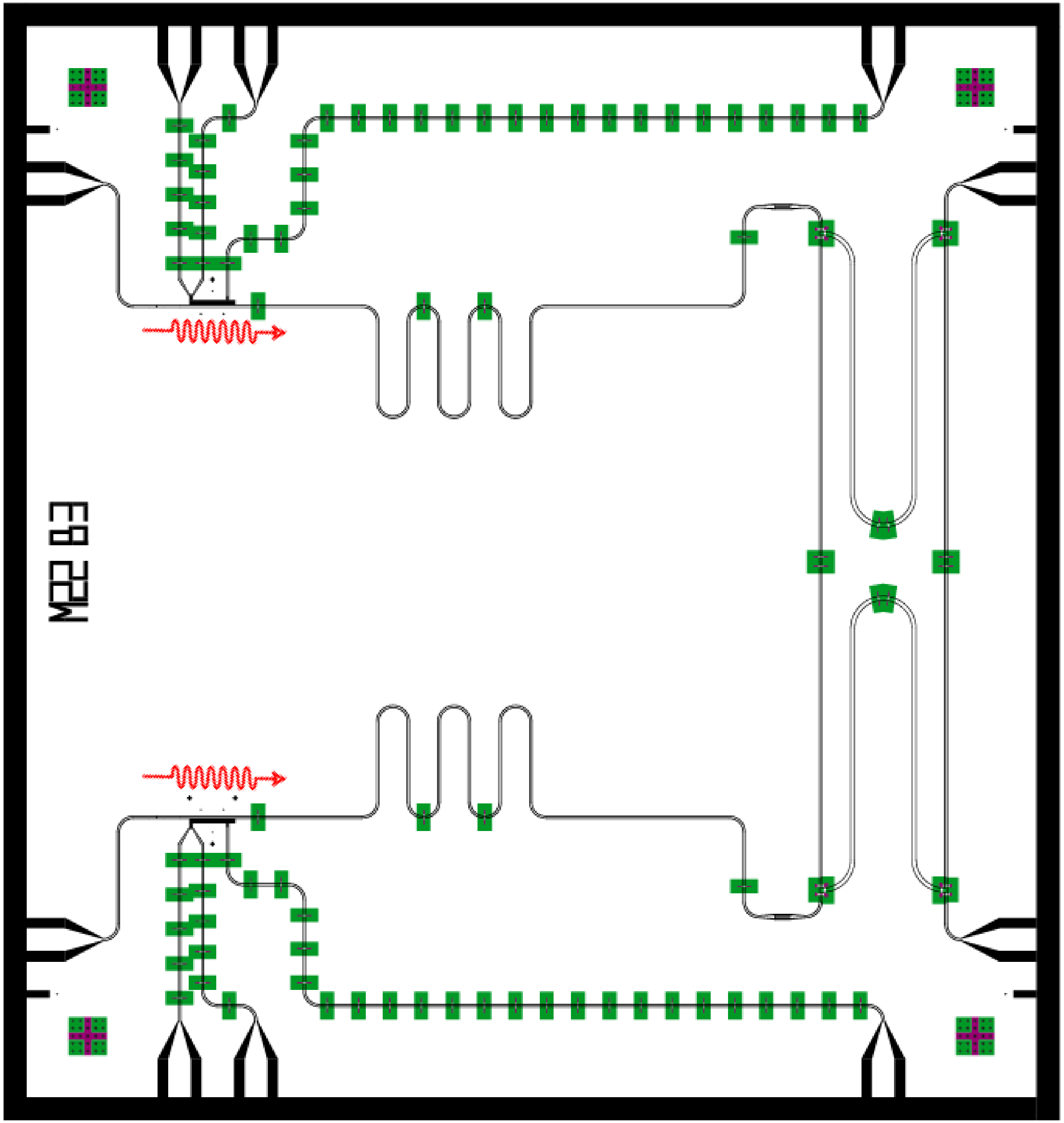
Design: Two Single Photon Sources and Beam Splitter



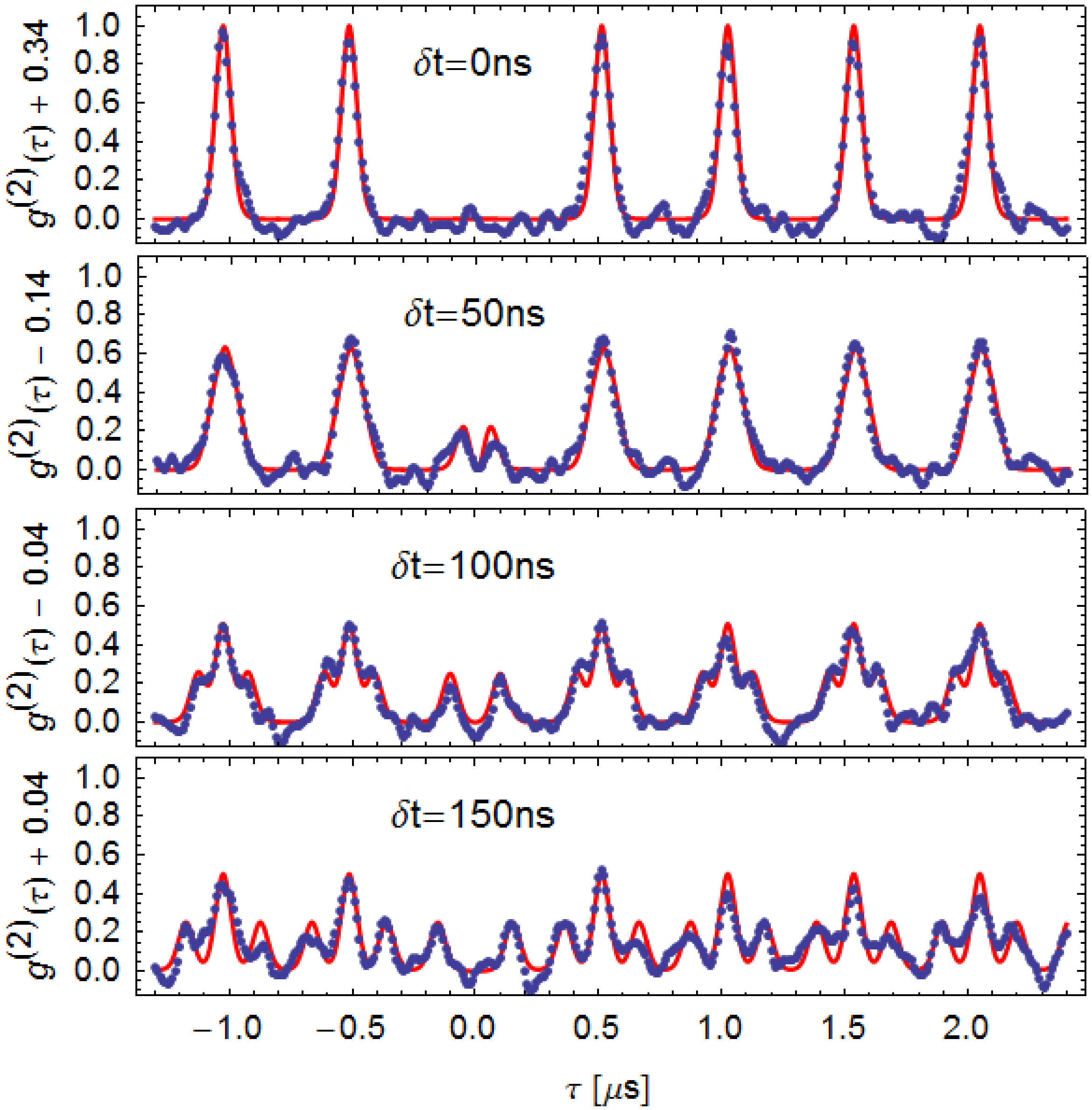
Design: Two Single Photon Sources and Beam Splitter



Design: Two Single Photon Sources and Beam Splitter



Hong-Ou-Mandel $g^{(2)}(\tau)$ for Microwave Photons



Observations:

- Photon-Pair anti-bunching

For $\tau > 0$:

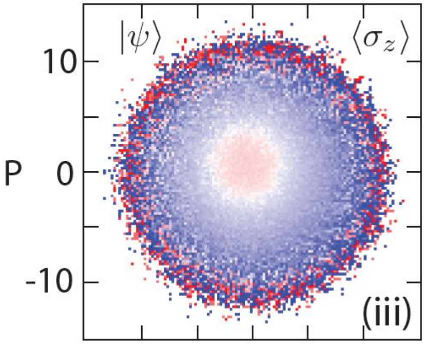
- Broadening of satellite peaks
- Triple-peak structure of satellite peaks
- Full recovery of double-peak at $\tau \approx 0$

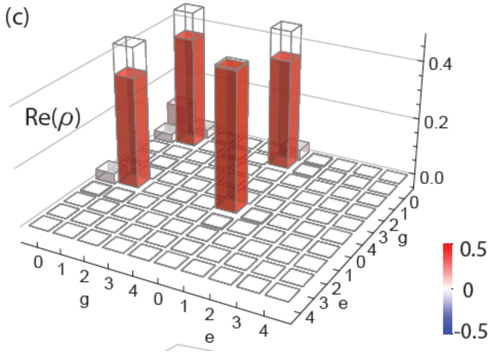
C. Lang et al. , ETH Zurich (2012)

Summary Qubit/Photon Entanglement

- First observation of entanglement between superconducting qubit and itinerant microwave photon field

- Characterization in full tomography.





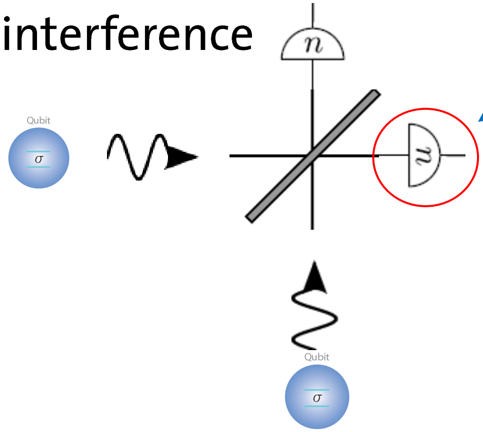
C. Eichler *et al.*, ETH Zurich *unpublished* (2012)
C. Eichler *et al.*, *arXiv:1206.3405* (2012)

First results:

microwave photon counter

Hong-Ou-Mandel two-photon interference

C. Lang et al. , ETH Zurich (2012)



Chen *et al.*, *PRL* **107**, 217401 (2011)
Romero et al., *PRL* **102**, 173602 (2009)

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Selected Circuit QED Publications

Circuit QED Proposal:

- Blais et al., *PRA* **69,** 062320 (2004)

Strong Coupling & Vacuum Rabi Mode Splitting:

- Wallraff et al., *Nature* **431**, 162 (2004)
- Fink et al., *Nature* **454**, 315 (2008)
- Fink et al., *PRL* **105**, 163601 (2010)

Tavis-Cummings Multi-Atom QED:

- Fink et al., *PRL* **103**, 083601 (2009)

AC-Stark & Lamb Shift, Autler-Townes and Mollow Transitions

- Schuster et al., *PRL* **94,** 123062 (2005)
- Gambetta et al., *PRA* **74**, 042318 (2006)
- Schuster et al., *Nature* **445**, 515 (2007)
- Fragner et al., *Science* **322**, 1357 (2008)
- Baur et al., *PRL* **102**, 243602 (2009)

Itinerant Photons, Tomography, Photon Blockade

- da Silva et al., *PRA* **82**, 043804 (2010)
- Bozyigit et al., *Nat. Phys.* **7**, 154(2011)
- Eichler et al., *PRL* **106**, 220503 (2011)
- Lang et al., *PRL* **106**, 243601 (2011)
- Eichler et al., *PRL* **107**, 113601 (2011)

One-, Two-, Three-Qubit Gates and Algorithms:

- Wallraff et al., *PRL* **95**, 060501 (2005)
- Blais et al., *PRA* **75**, 032329 (2007)
- Wallraff et al., *PRL* **99**, 050501 (2007)
- Majer et al., *Nature* **449**, 443 (2007)
- Leek et al., *Science* **318**, 1889 (2007)
- Leek et al., *PRB* **79**, 180511(R) (2009)
- Filipp et al., *PRL* **102**, 200402 (2009)
- Leek et al., *PRL* **104**, 100504 (2010)
- Bianchetti et al., *PRL* **105**, 223601 (2010)
- Fedorov et al., *Nature* **481**, 170 (2012)
- Baur et al., *PRL* **108**, 040502 (2012)

Hybrid Systems:

- Frey et al., *PRL* **108**, 046807 (2012)
- Hogan et al., *PRL* **108**, 063004 (2012)

Device Fabrication:

- Frunzio et al., *IEEE Trans. Appl. Sup.*. **15**, 860 (2005)
- Goeppl et al., *J. Appl. Phys.* **104**, 113904 (2008)

Review (gr.):

- Wallraff, *Physik Journal* **7 (12)**, 39 (Dez. 2008)

Additional Information: www.qudev.ethz.ch