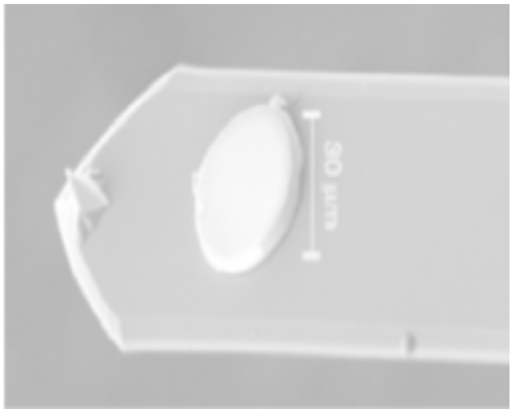
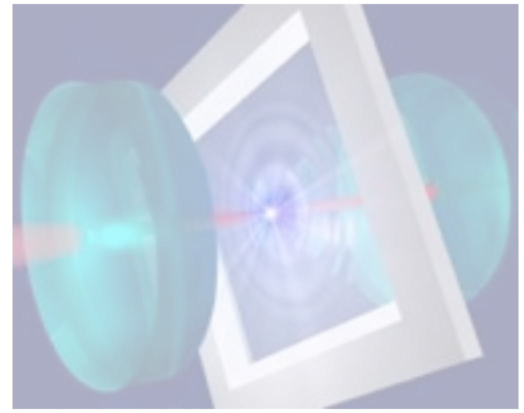
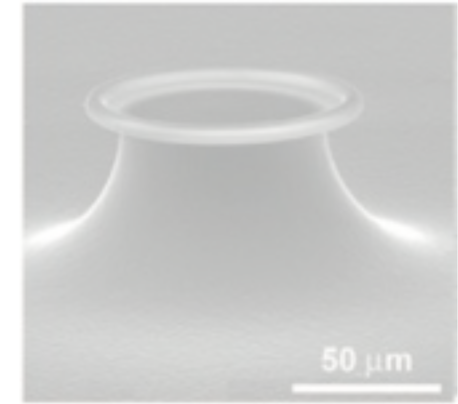






An introduction to cavity optomechanics

Andreas Nunnenkamp
University of Basel

1. What? Why? How?
2. Sideband cooling
3. Strong-coupling regime
4. Dissipative coupling

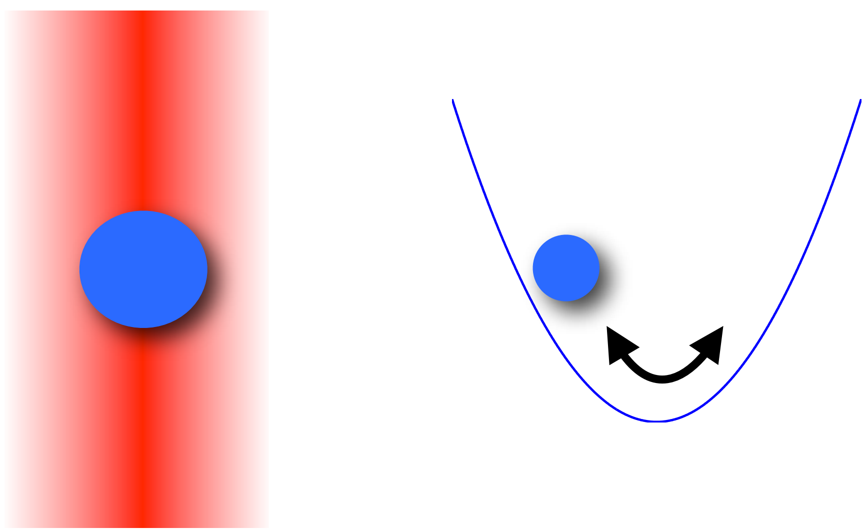
UNI BASEL

QSIT Quantum Science and Technology

Introduction

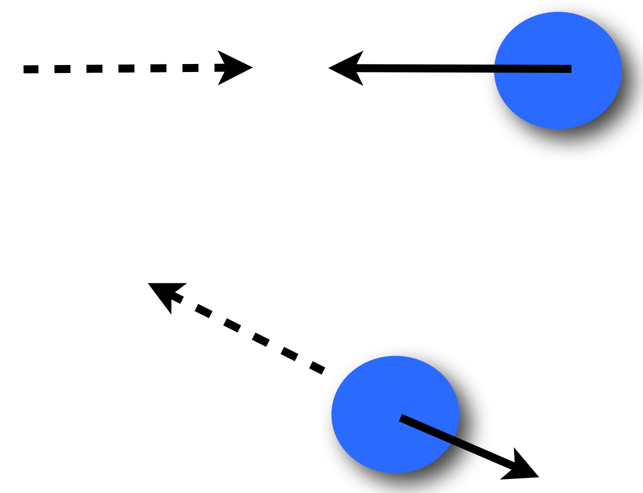
Mechanical effects of light

Dipole or gradient force



- ac-Stark shift
- optical lattices

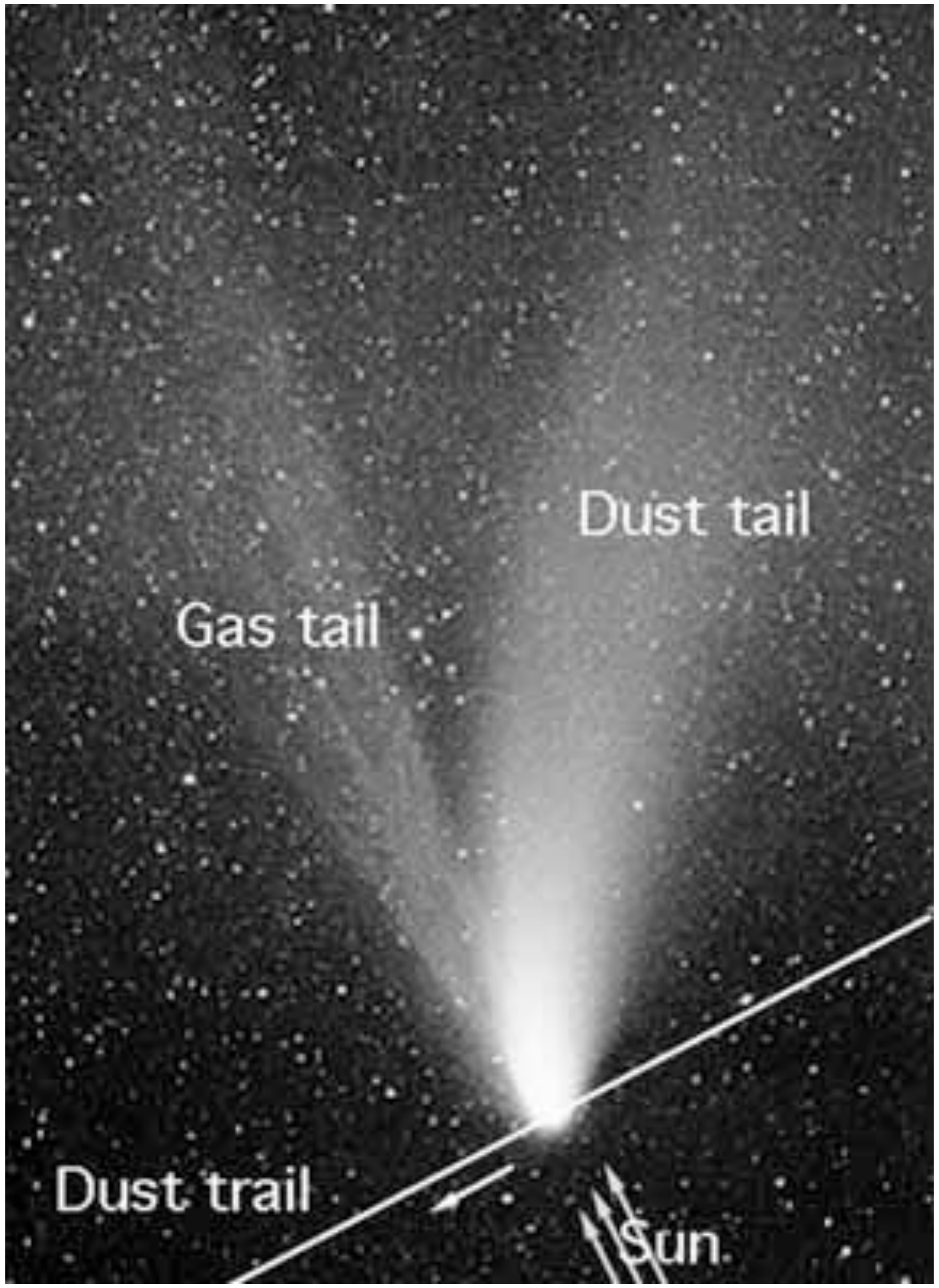
Scattering force

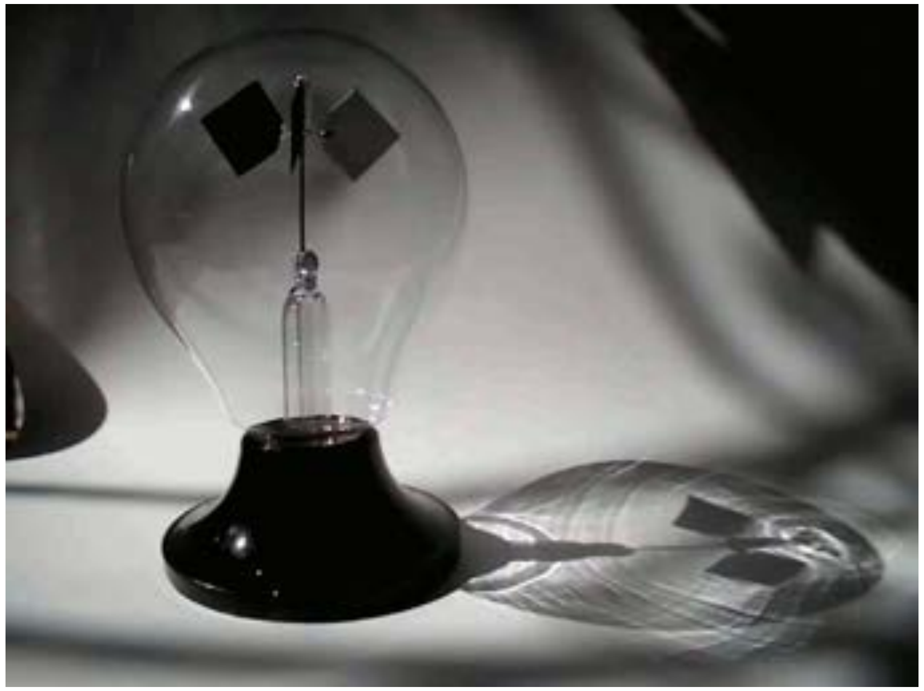


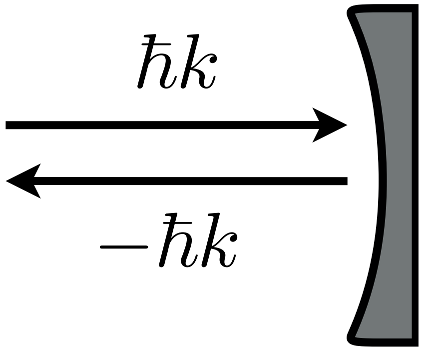
- radiation pressure
- laser cooling

Loudon, *The Quantum Theory of Light* (OSP)
Foot, *Atomic Physics* (OUP)

Radiation-pressure force





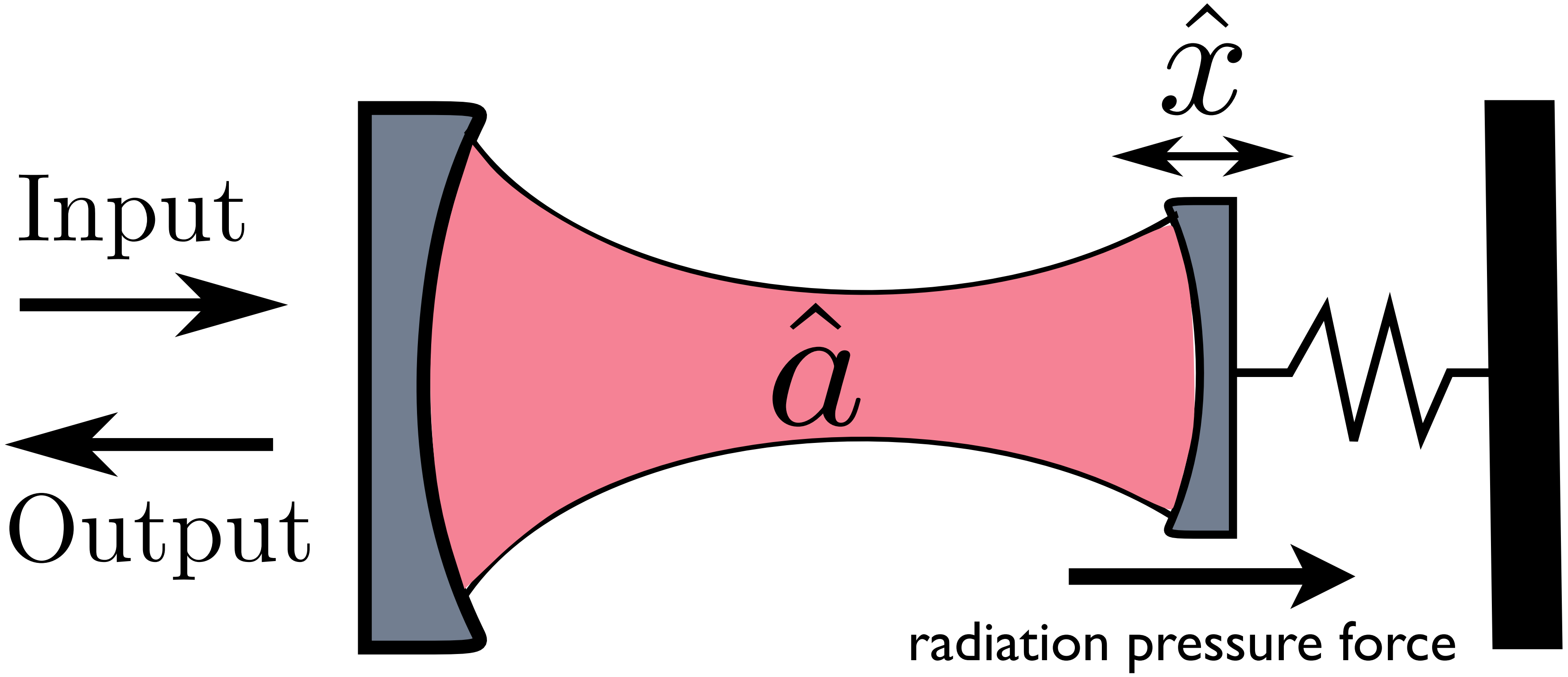


$$\Delta p = 2\hbar k = \frac{2E}{c}$$

$$F = \frac{\Delta p \Delta N}{\Delta t} = \frac{2P_{\text{in}}}{c}$$

Loudon, *The Quantum Theory of Light* (OSP)
Foot, *Atomic Physics* (OUP)

Cavity optomechanics



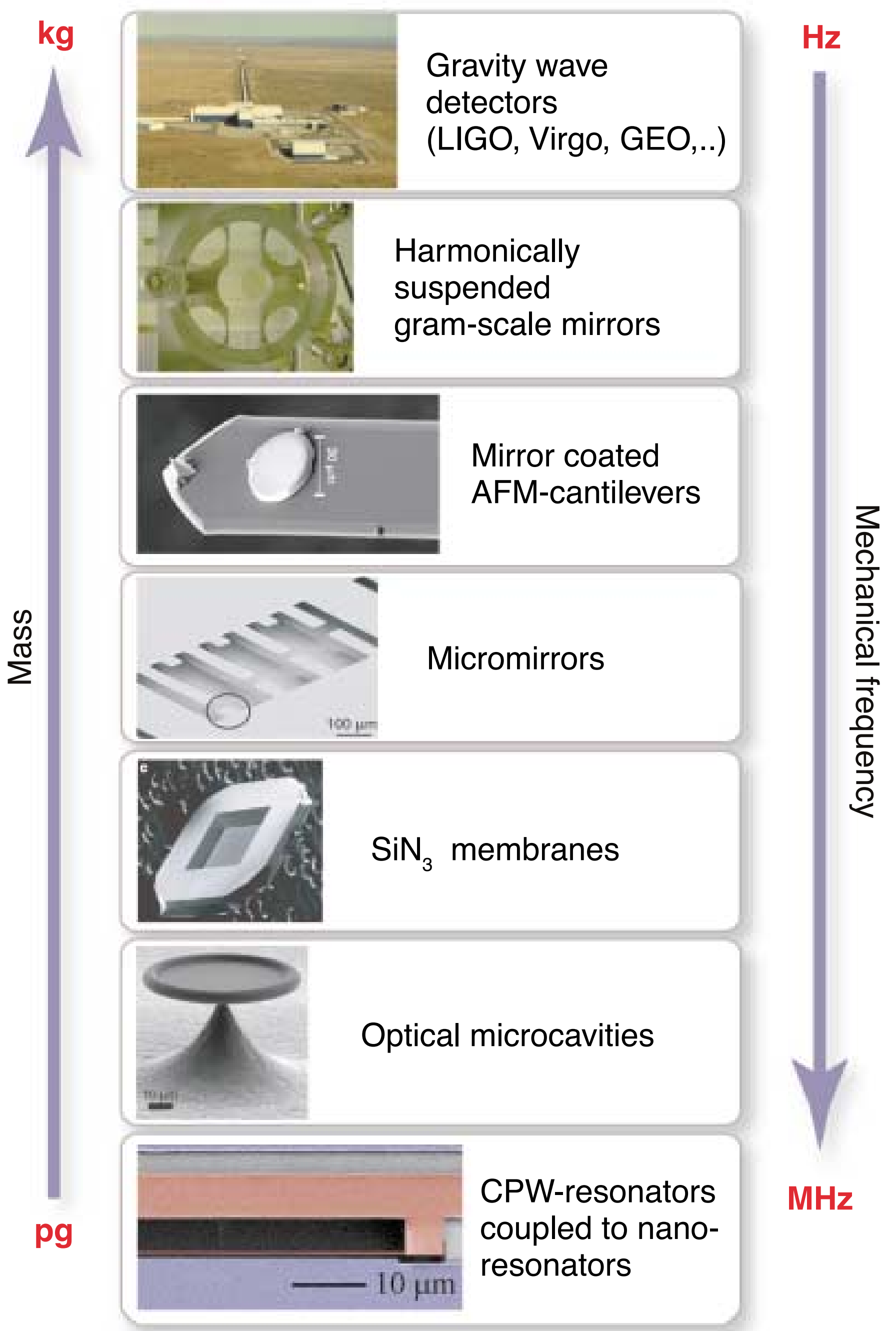
cavity drive

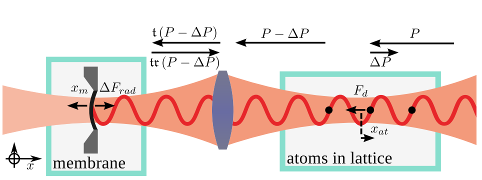
cavity mode

mechanical oscillator

Marquardt & Girvin, Physics **2**, 40 (2009)
Kippenberg & Vahala, Science **321**, 1172 (2008)

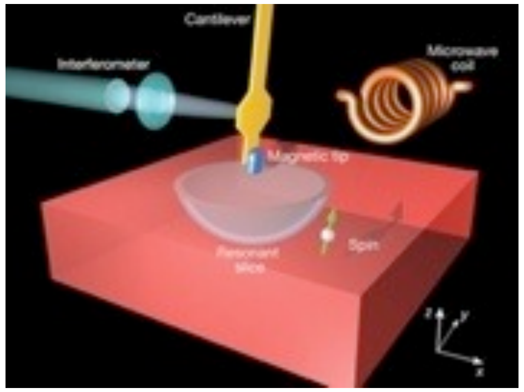
Experiments & Motivation





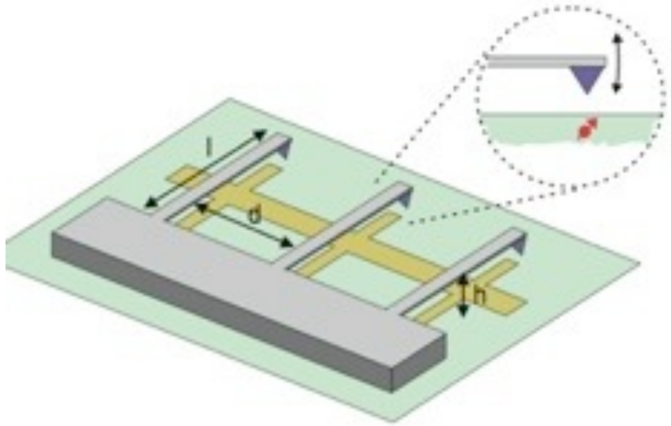
ultracold atoms in optical resonators

1. Mechanical sensing



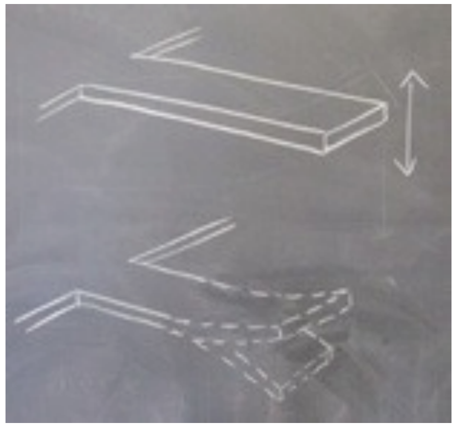
Rugar *et al.*, Nature **430**, 329 (2004)

2. Quantum network



Rabl *et al.*, Nature Physics **6**, 602 (2010)

3. Quantum-classical transition

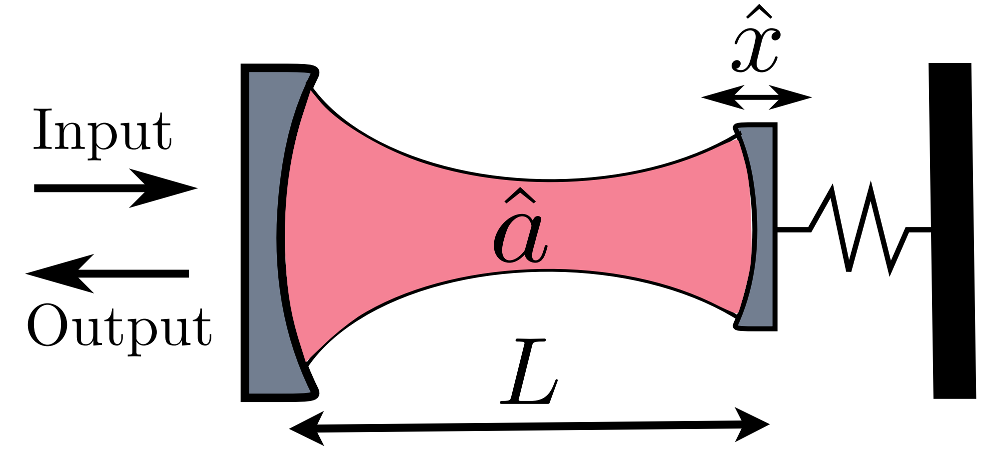


Penrose & Bouwmeester *et al.*, PRL **91**,130401 (2003)

Marquardt & Girvin, Physics **2**, 40 (2009)

Kippenberg & Vahala, Science **321**, 1172 (2008)

Cavity optomechanics



We start with

$$\hat{H} = \hbar\omega_C(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b}$$

where the position of the mechanical oscillator

$$\hat{x} = x_{\mathrm{ZPF}}(\hat{b} + \hat{b}^\dagger) \quad \text{with} \quad x_{\mathrm{ZPF}} = \sqrt{\frac{\hbar}{2m\omega_M}} \sim 10^{-15}\mathrm{m}$$

is parametrically coupled to the cavity mode

$$\omega_C(x) = \frac{L}{L+x}\omega_R \approx \left(1 - \frac{x}{L}\right)\omega_R$$

We obtain the "standard model of optomechanics" $\hbar = 1$

$$\hat{H} = \omega_R\left(1 - \frac{\hat{x}}{L}\right)\hat{a}^\dagger\hat{a} + \omega_M\hat{b}^\dagger\hat{b}$$

"three-wave mixing"

+ optical drive/decay
+ thermal fluctuations

radiation-pressure force

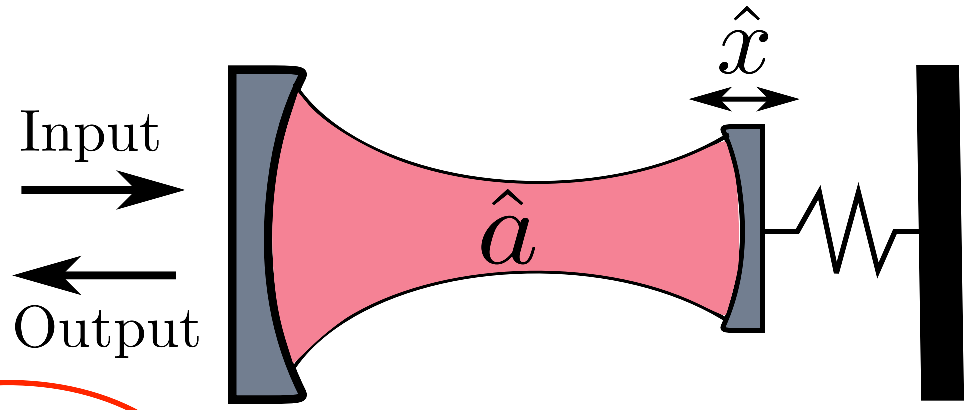
$$\hat{F} = -\frac{\partial\hat{H}_{\mathrm{int}}}{\partial\hat{x}} = \frac{\omega_R}{L}\hat{a}^\dagger\hat{a}$$

N.B. We neglect the dynamical Casimir effect.

Marquardt & Girvin, Physics **2**, 40 (2009)
Kippenberg & Vahala, Science **321**, 1172 (2008)

Cavity optomechanics

The Hamiltonian



$$\hat{H} = \hbar\left(\omega_R + g_0\hat{x}\right)\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b} + \hat{H}_{\mathrm{drive}} + \hat{H}_{\mathrm{env}}$$

$$\hat{x} = x_{\mathrm{zpf}}\left(\hat{b} + \hat{b}^\dagger\right)$$

Position operator

Three-wave mixing
Radiation pressure force

$$x_{\mathrm{zpf}} = \sqrt{\frac{\hbar}{2m\omega_M}}$$

Zero point motion

Far from equilibrium

$$g = g_0 x_{\mathrm{zpf}}$$

Coupling rate

with the dimensionless parameters

Ω/κ Δ/ω_m drive strength & detuning

ω_m/κ good/bad-cavity limit

ω_m/γ mechanical quality factor

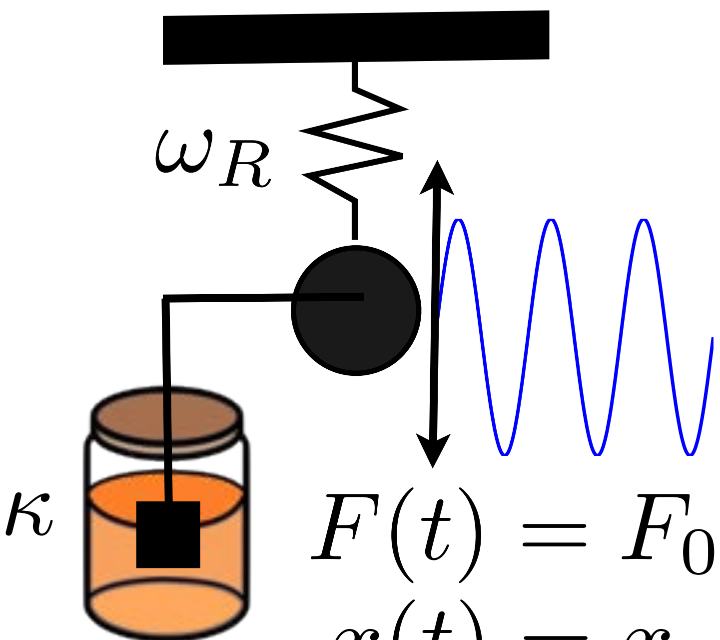
g/κ cavity shift per phonon in units of line width

g/ω_m oscillator displacement per photon in units of its ZPF

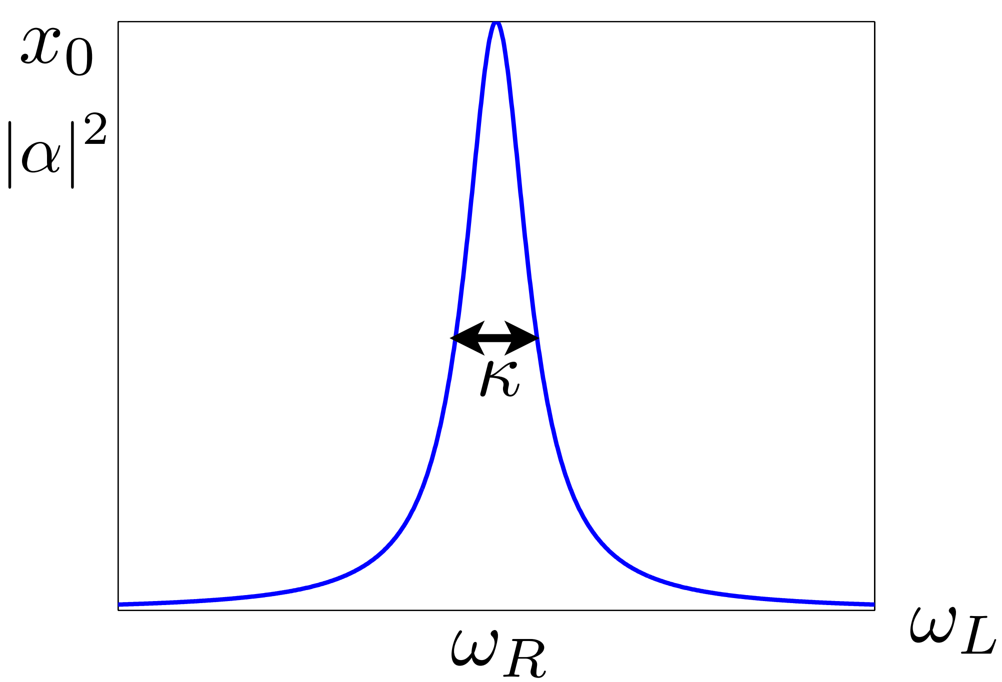
Marquardt & Girvin, Physics **2**, 40 (2009)
Kippenberg & Vahala, Science **321**, 1172 (2008)

Damped & driven optical resonator

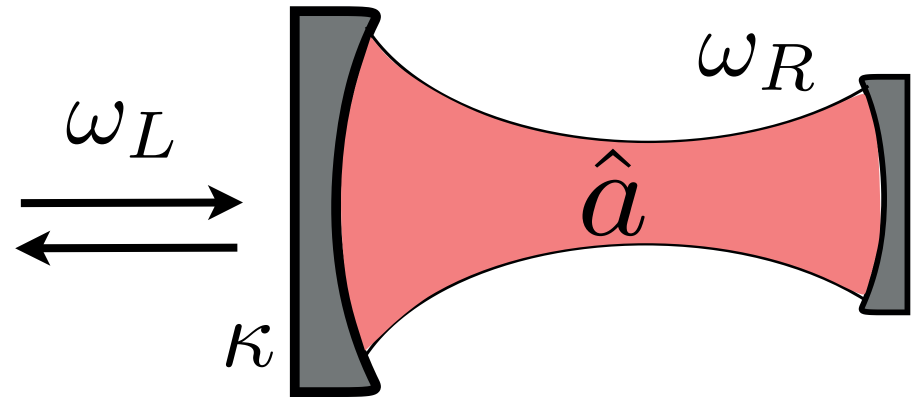
Damped & driven harmonic oscillator



$$F(t) = F_0 \cos \omega_L t$$
$$x(t) = x_0 \cos(\omega_L t - \varphi)$$



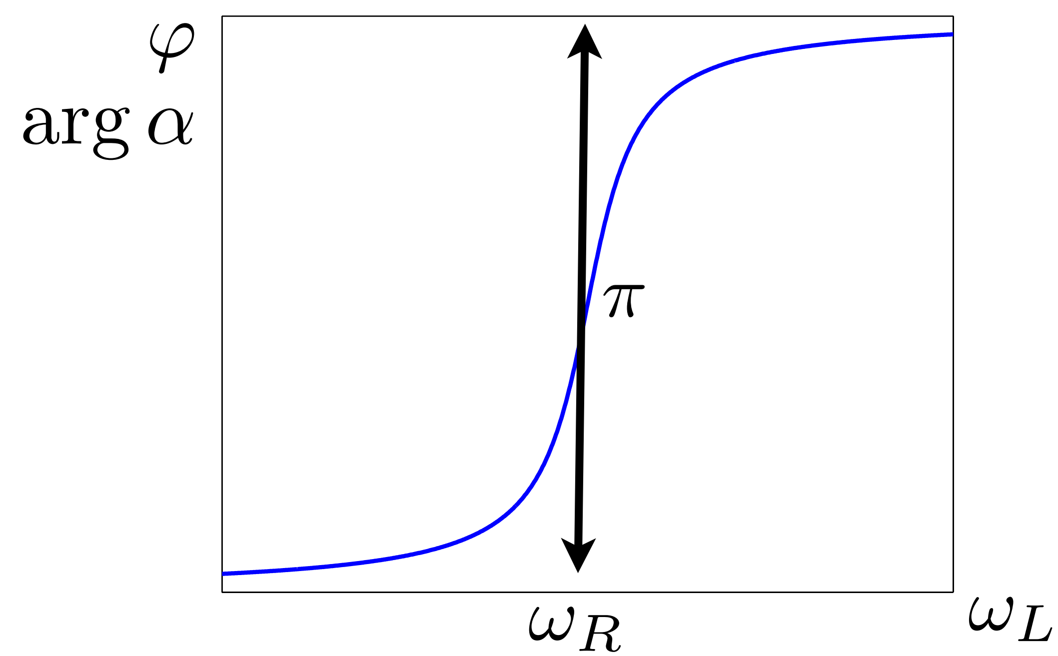
Damped & driven optical resonator



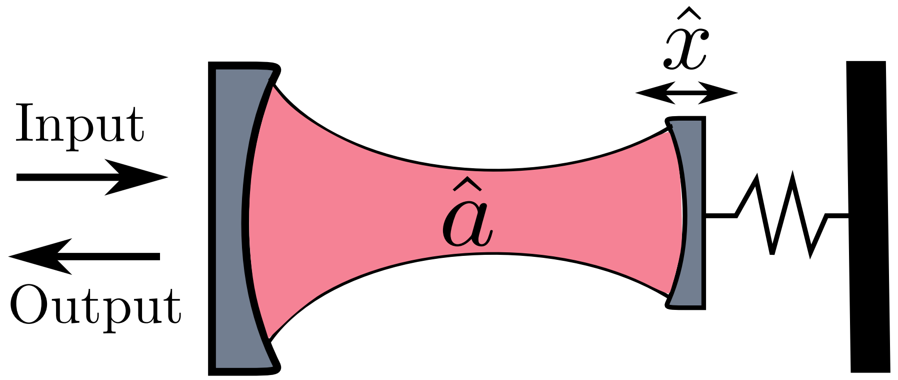
Coherent drive $E(t) = E_0 \cos \omega_L t$

Coherent state $\hat{a}|\alpha\rangle = \alpha e^{-i\omega_L t}|\alpha\rangle$

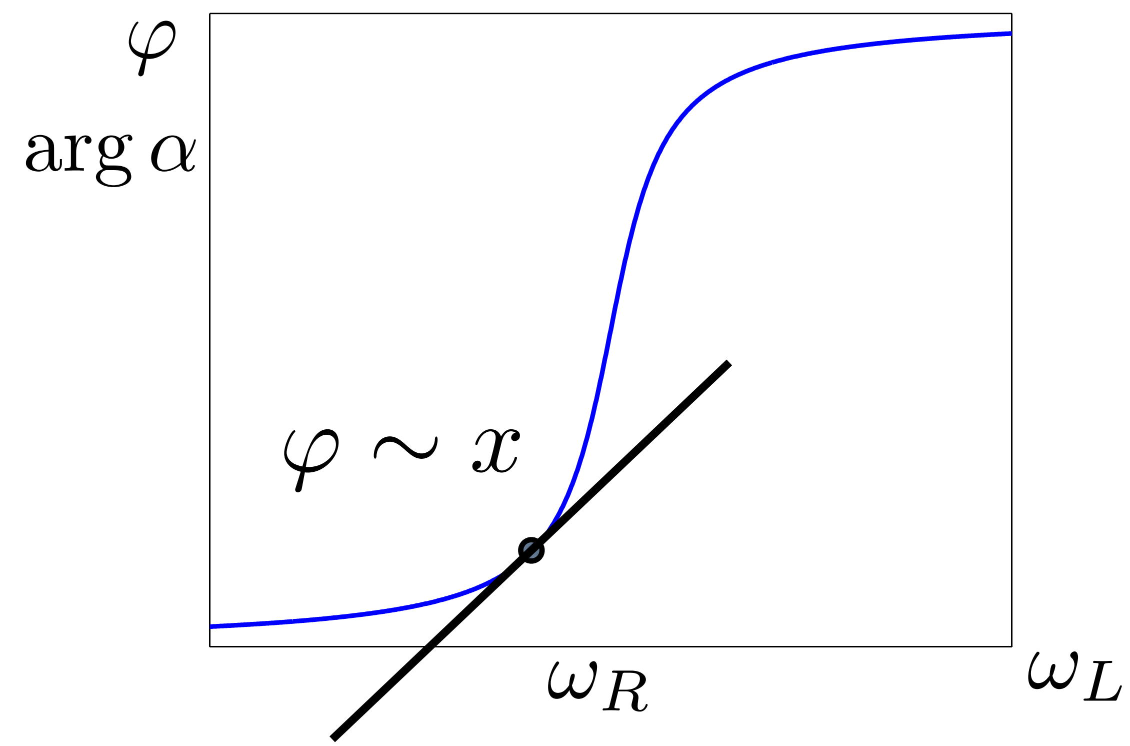
→number fluctuations (shot noise)!



Displacement readout of the mechanical oscillator



$$\hat{H} = \hbar\left(\omega_R + g_0\hat{x}\right)\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b}$$



A single experimental run might look like...

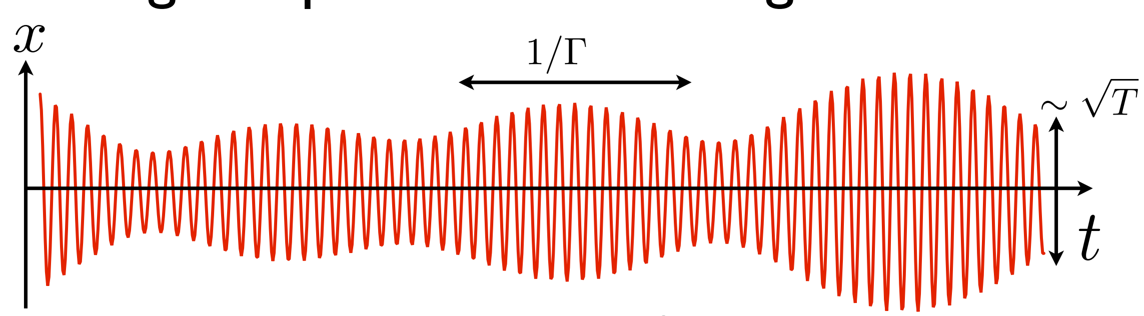


figure by F. Marquardt

we can find the temperature from the area

$$\frac{m\omega_M^2}{2}\left\langle x^2\right\rangle = \frac{k_B T}{2}$$

... and calculating the noise power spectrum

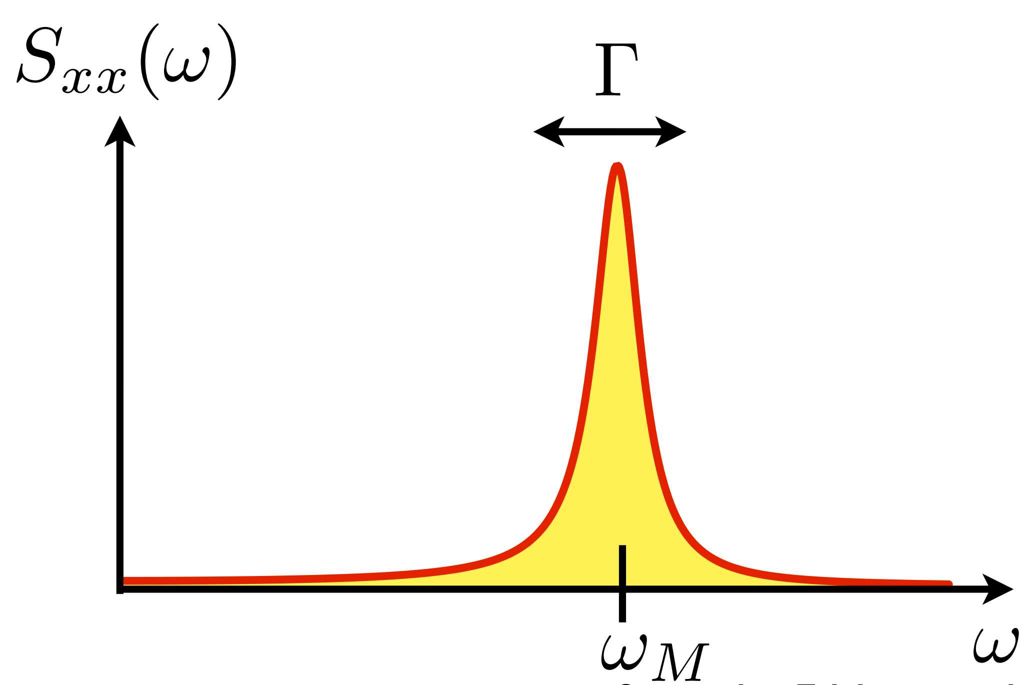


figure by F. Marquardt

Sideband cooling

Sideband cooling: the cavity-enhanced scattering picture

cantilevers, membranes, wires, carbon nanotubes need additional cooling

$$\omega_M = \mathrm{kHz} - \mathrm{GHz}$$

$$k_B T \ll \hbar\omega_M$$

$$1\mathrm{GHz} \sim 50\mathrm{mK}$$

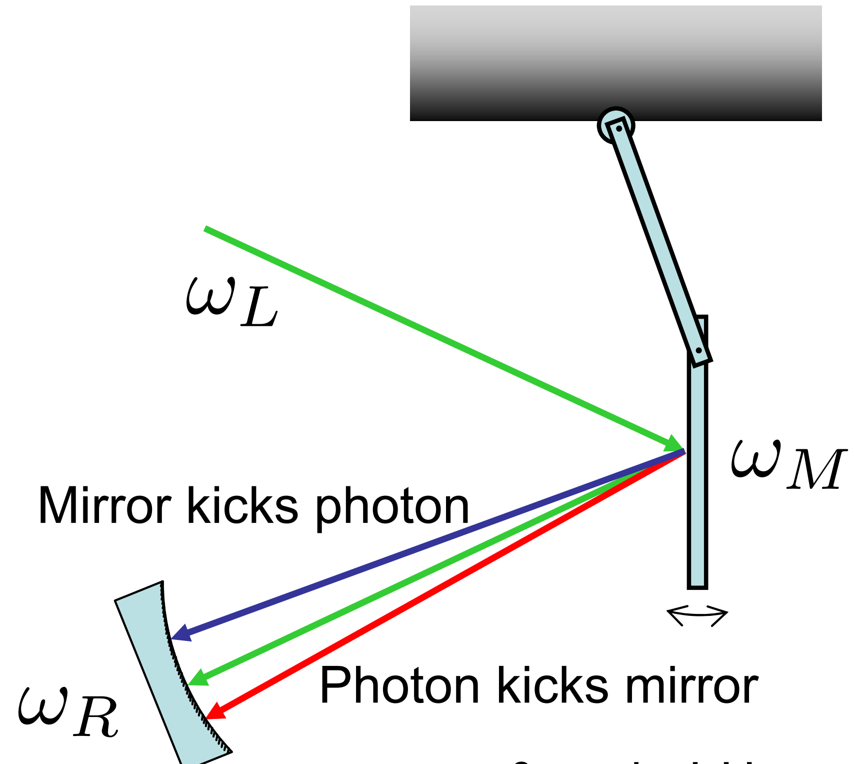
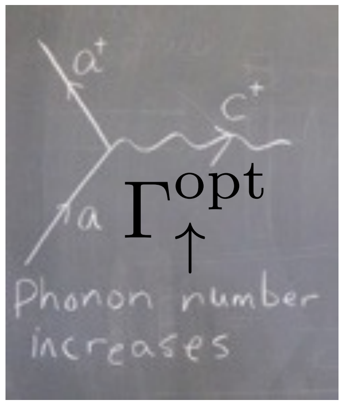
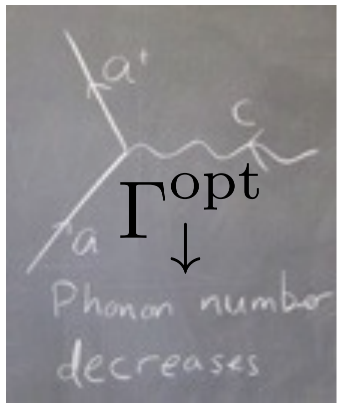
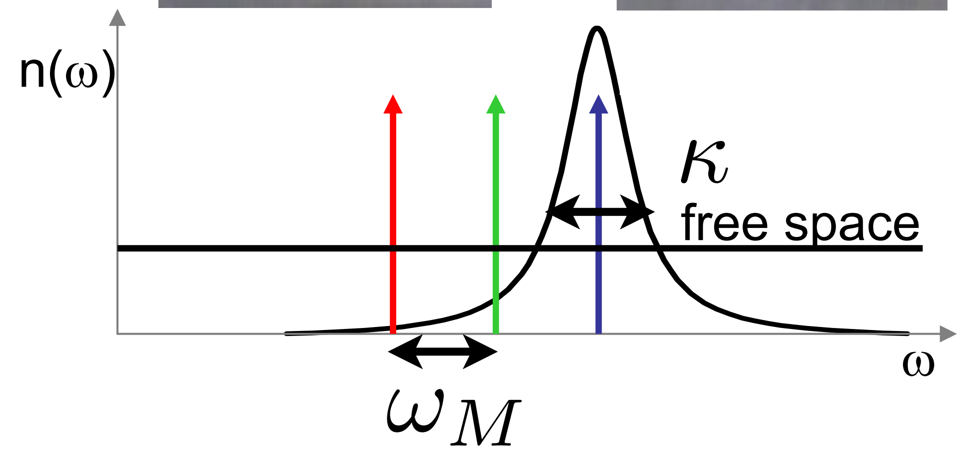


figure by J. Harris

$$\hat{H} = \hbar\left(\omega_R + g_0\hat{x}\right)\hat{a}^\dagger\hat{a}$$







Red-sideband cooling at

$$\omega_L = \omega_R - \omega_M$$

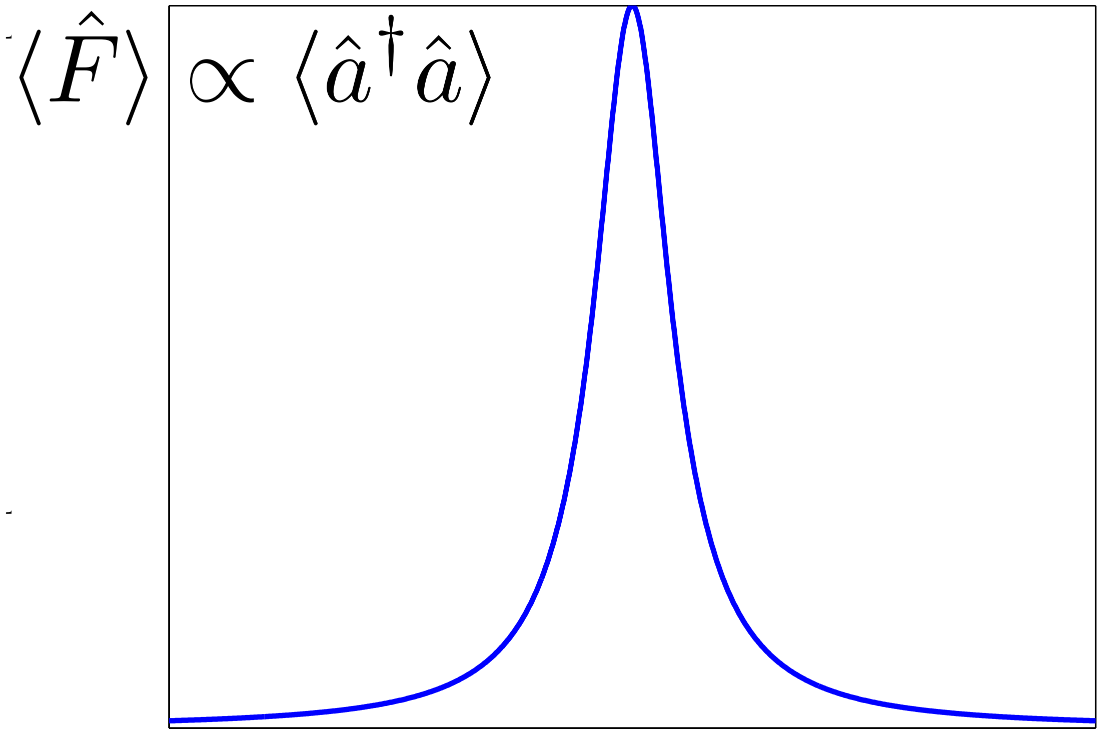
$$\omega_{\mathrm{out}} = \omega_{\mathrm{in}} + \omega_M$$

Ground-state cooling is possible in the sideband-resolved regime $\omega_M \gg \kappa$

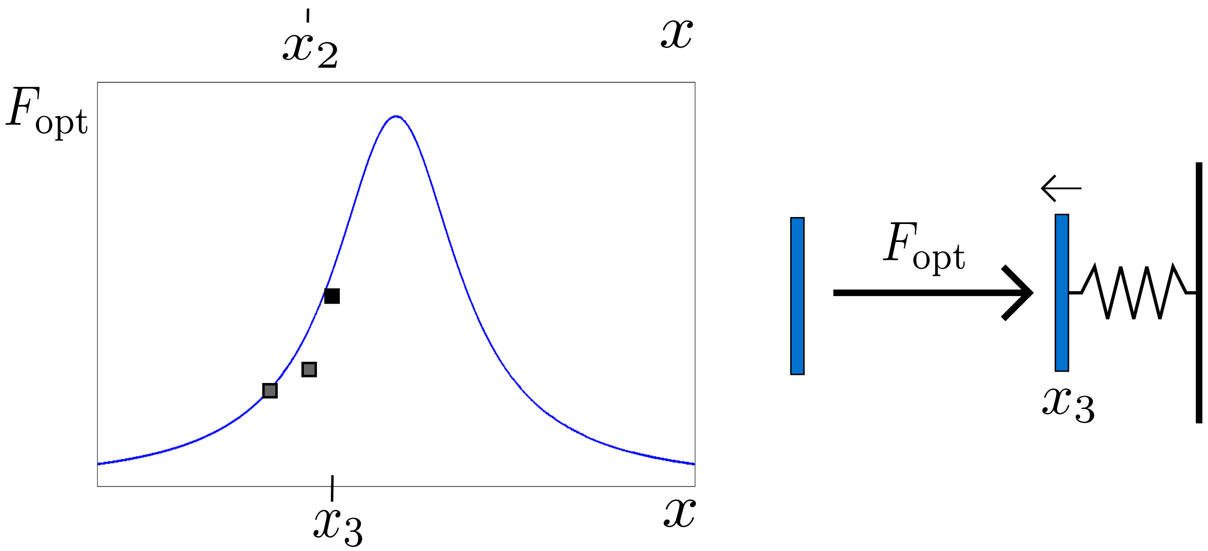
see also: Laser cooling of atoms & ions

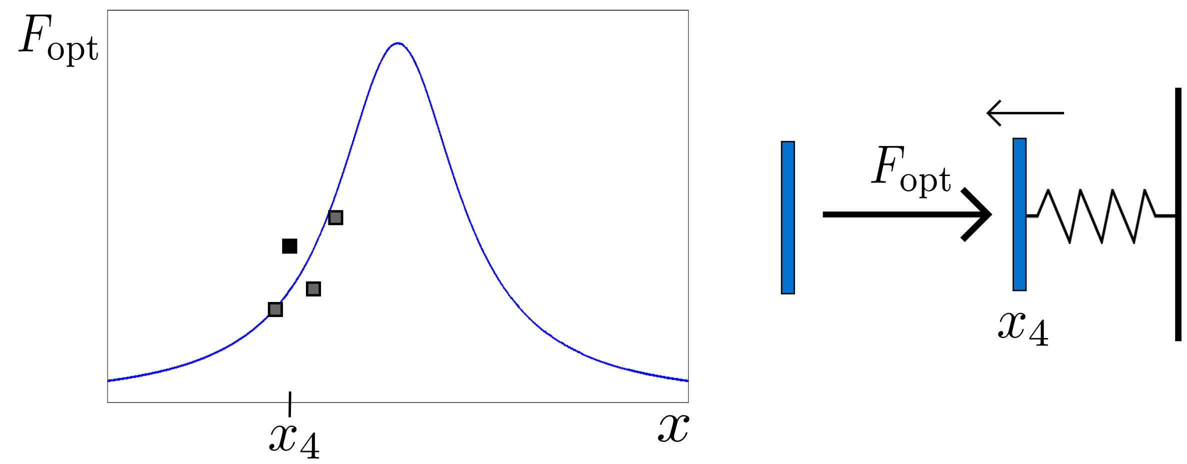
Marquardt *et al.*, PRL **99**, 093902 (2007)
Wilson-Rae *et al.*, PRL **99**, 093901 (2007)

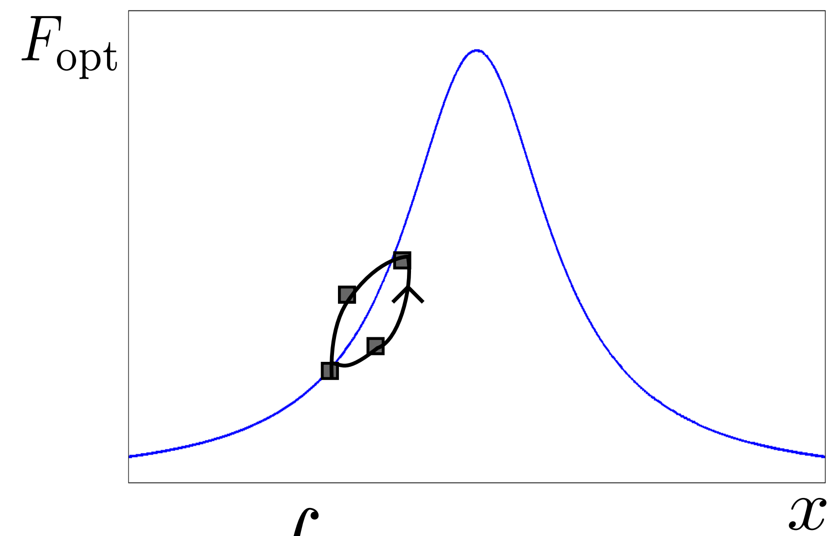
Sideband cooling: the classical picture



$$\Delta = \omega_L - \omega_R \propto x$$





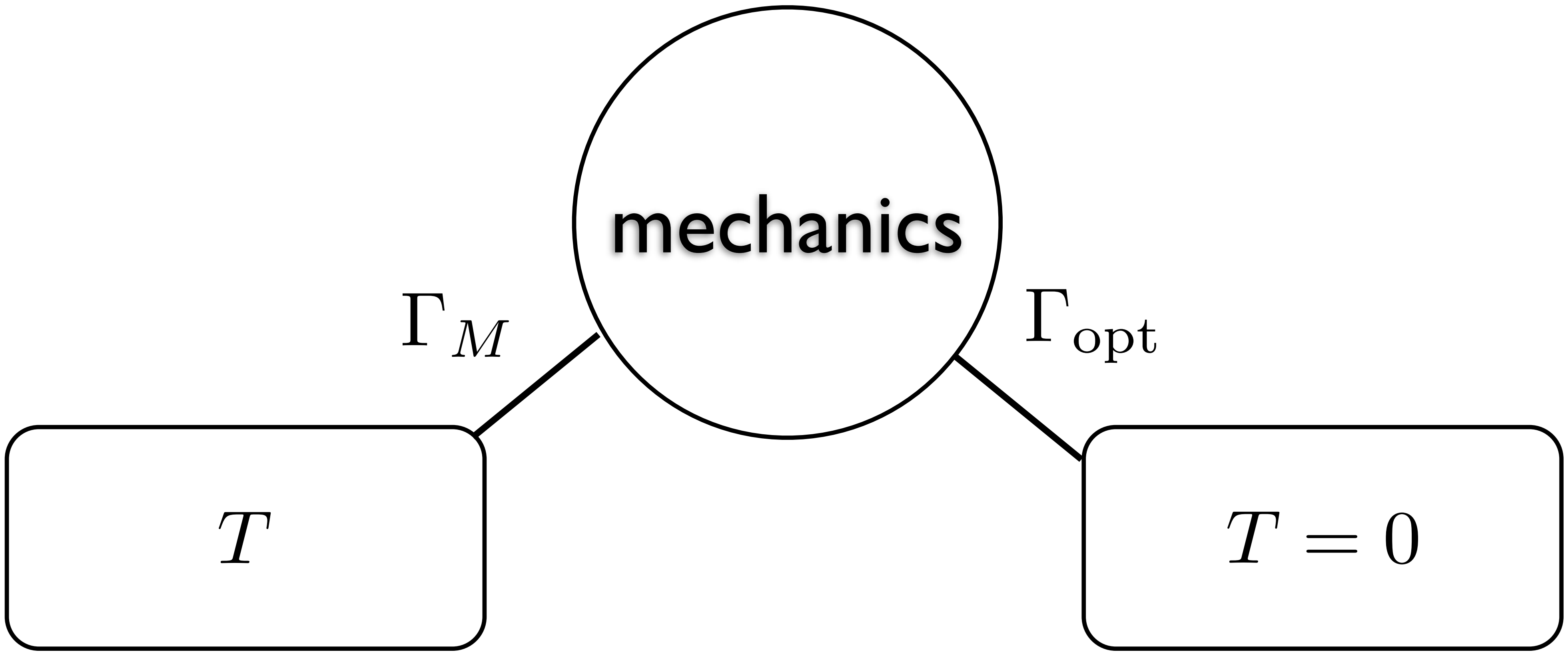


$$\oint dx F_{\mathrm{opt}} < 0$$

Damping is due to the finite time lag between mirror position and radiation-pressure force: $\omega_M \gg \kappa$

Marquardt, Clerk & Girvin, J. Mod. Opt. **55**, 3329 (2008)

Sideband cooling: the classical picture



$$T_{\rm eff} = T \frac{\Gamma_M}{\Gamma_M + \Gamma_{\rm opt}} \xrightarrow{\Gamma_{\rm opt} \to \infty} 0$$

→neglects number fluctuations (shot noise)!

The coupling to the cold optical bath leads to increased damping without additional fluctuations.

Marquardt, Clerk & Girvin, J. Mod. Opt. **55**, 3329 (2008)

Sideband cooling: the quantum noise approach

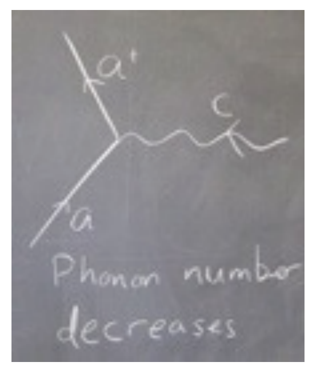
At weak coupling all you need to know is the force spectrum

$$\hat{H}_{\text{int}} = -\hat{F}\hat{x} \qquad S_{FF}(\omega) = \int dt\, e^{i\omega t} \left\langle \hat{F}(t)\hat{F}(0) \right\rangle$$

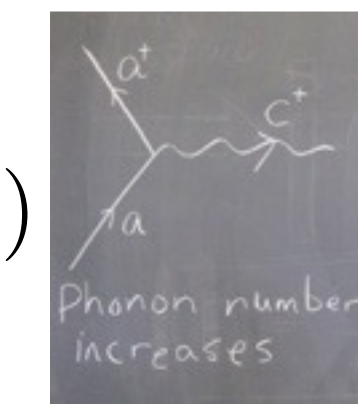
with $\hat{F} = \frac{\omega_R}{L}\hat{a}^\dagger\hat{a}$

Obtain the rates with Fermi's Golden Rule

$$\Gamma_{\downarrow}^{\text{opt}} = \frac{x_{\text{ZPF}}^2}{\hbar^2} S_{FF}(\omega_M)$$



$$\Gamma_{\uparrow}^{\text{opt}} = \frac{x_{\text{ZPF}}^2}{\hbar^2} S_{FF}(-\omega_M)$$



And calculate the steady-state phonon number

$$\bar{n}_M = \frac{\Gamma_M \bar{n}_{\text{th}} + \Gamma_{\text{opt}} \bar{n}_M^O}{\Gamma_M + \Gamma_{\text{opt}}}$$

$\Gamma_{\text{opt}} = \Gamma_{\downarrow}^{\text{opt}} - \Gamma_{\uparrow}^{\text{opt}}$ optical damping

$\bar{n}_M^O = \frac{\Gamma_{\uparrow}^{\text{opt}}}{\Gamma_{\text{opt}}}$ minimal phonon number

Marquardt *et al.*, PRL **99**, 093902 (2007)
Wilson-Rae *et al.*, PRL **99**, 093901 (2007)

Clerk *et al.*, RMP **82**, 1155 (2010)

Sideband cooling: the quantum noise approach

Calculate the force (i.e. shot-noise) spectrum

$$S_{FF}(\omega) = \int dt\, e^{i\omega t} \left\langle \hat{F}(t)\hat{F}(0) \right\rangle \quad \text{with} \quad \hat{F} = \frac{\omega_R}{L}\hat{a}^\dagger \hat{a}$$

$$= \left(\frac{\hbar\omega_R}{L}\right)^2 \bar{n}_{\mathrm{p}} \frac{\kappa}{(\omega+\Delta)^2 + (\kappa/2)^2}$$

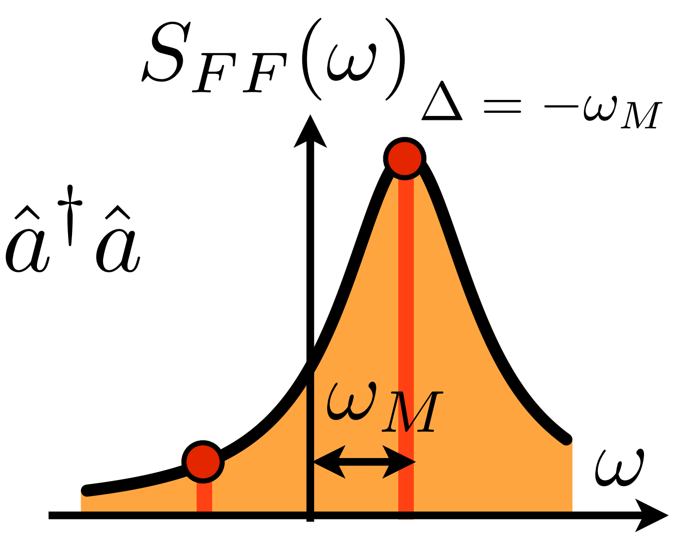


figure by F. Marquardt

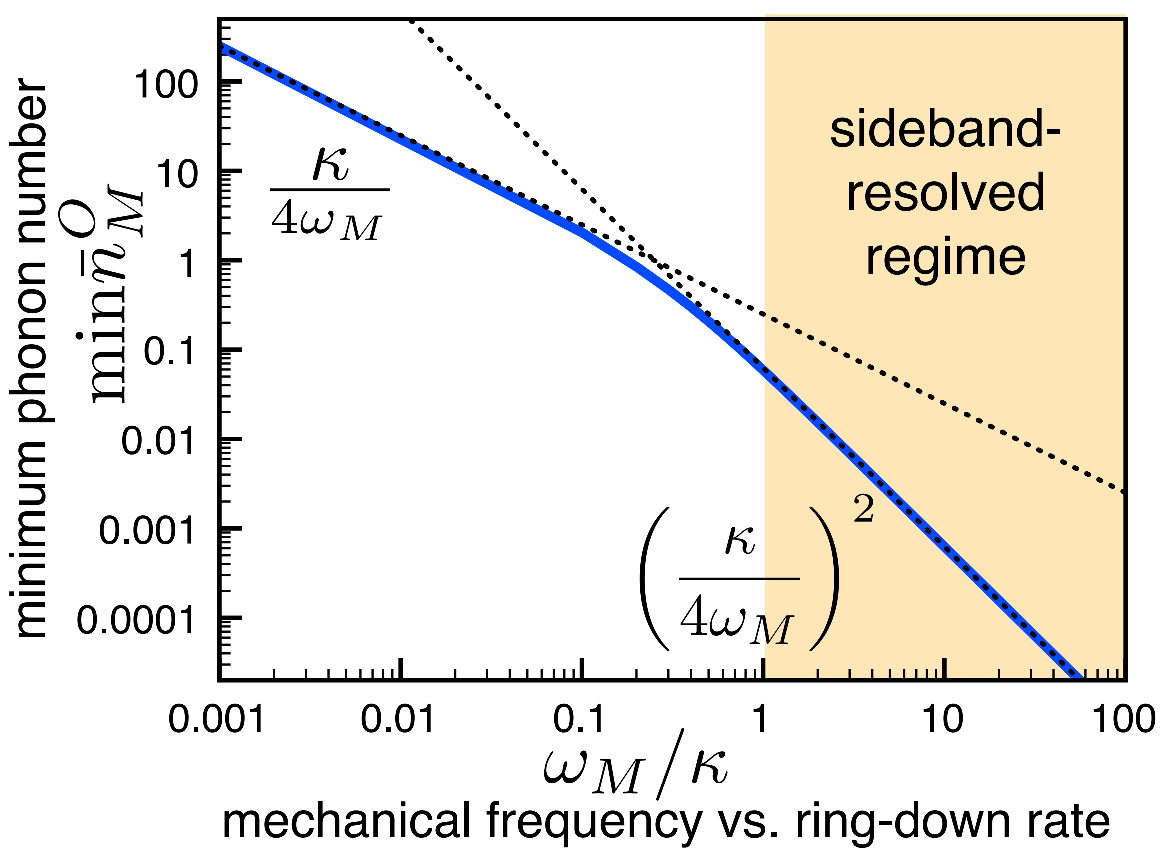


figure by F. Marquardt

Clerk *et al.*, RMP **82**, 1155 (2010)

Red-sideband cooling at

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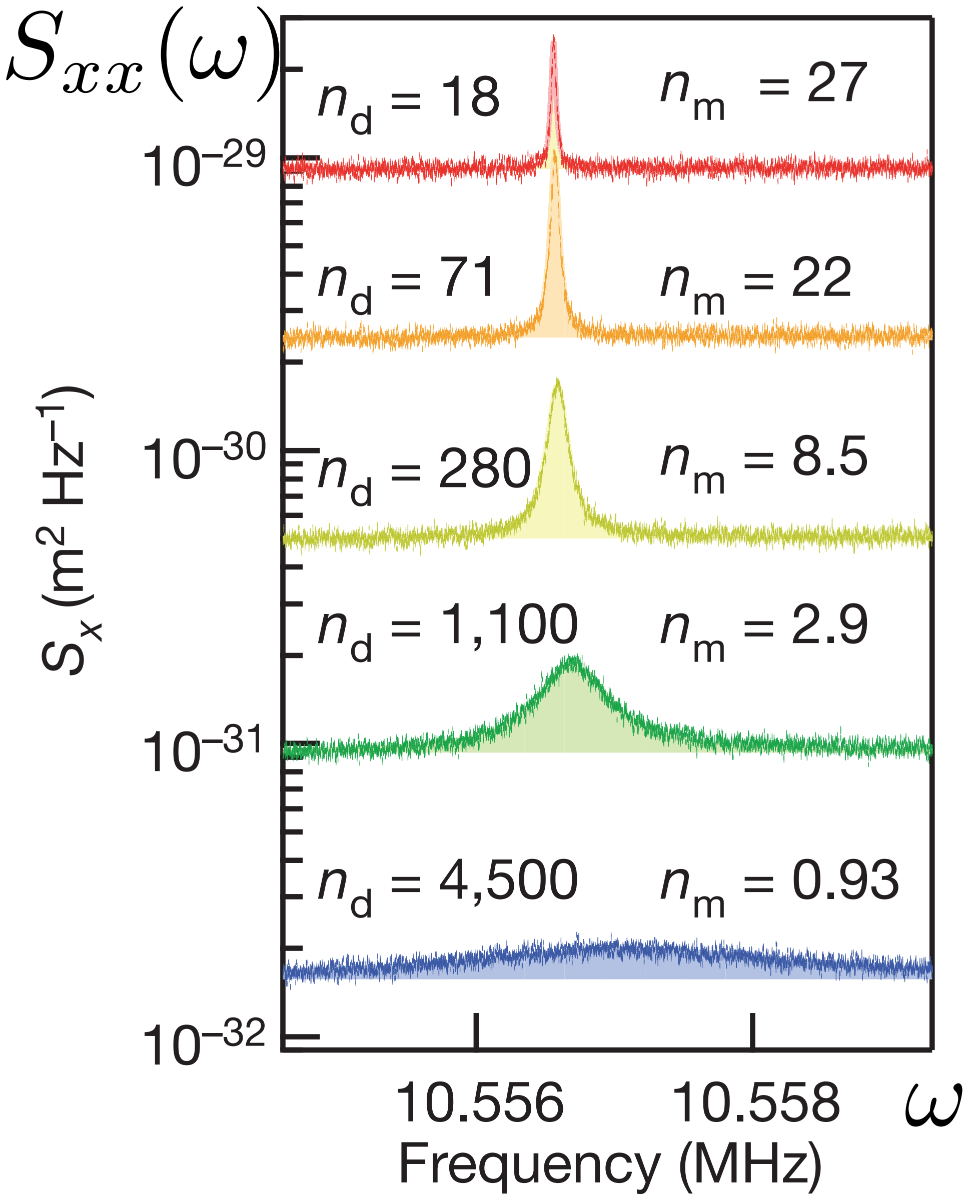
$$\omega_{\mathrm{out}} = \omega_{\mathrm{in}} + \omega_M$$

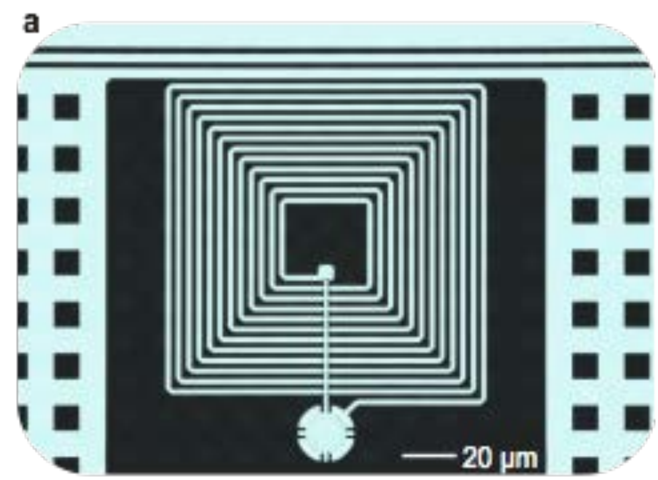
Ground-state cooling is possible in the sideband-resolved regime $\omega_M \gg \kappa$

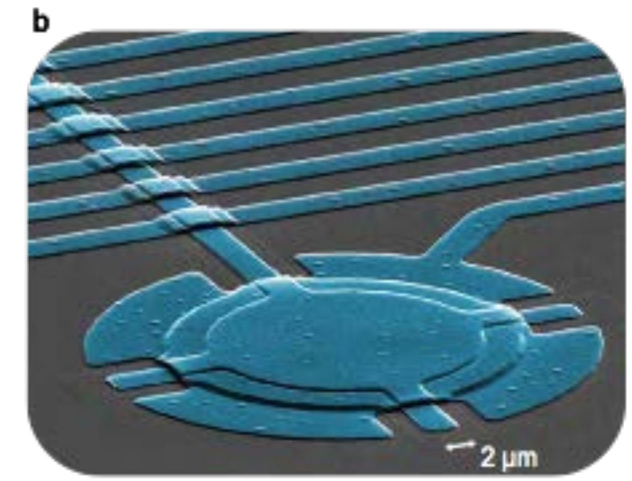
Marquardt *et al.*, PRL **99**, 093902 (2007)
Wilson-Rae *et al.*, PRL **99**, 093901 (2007)

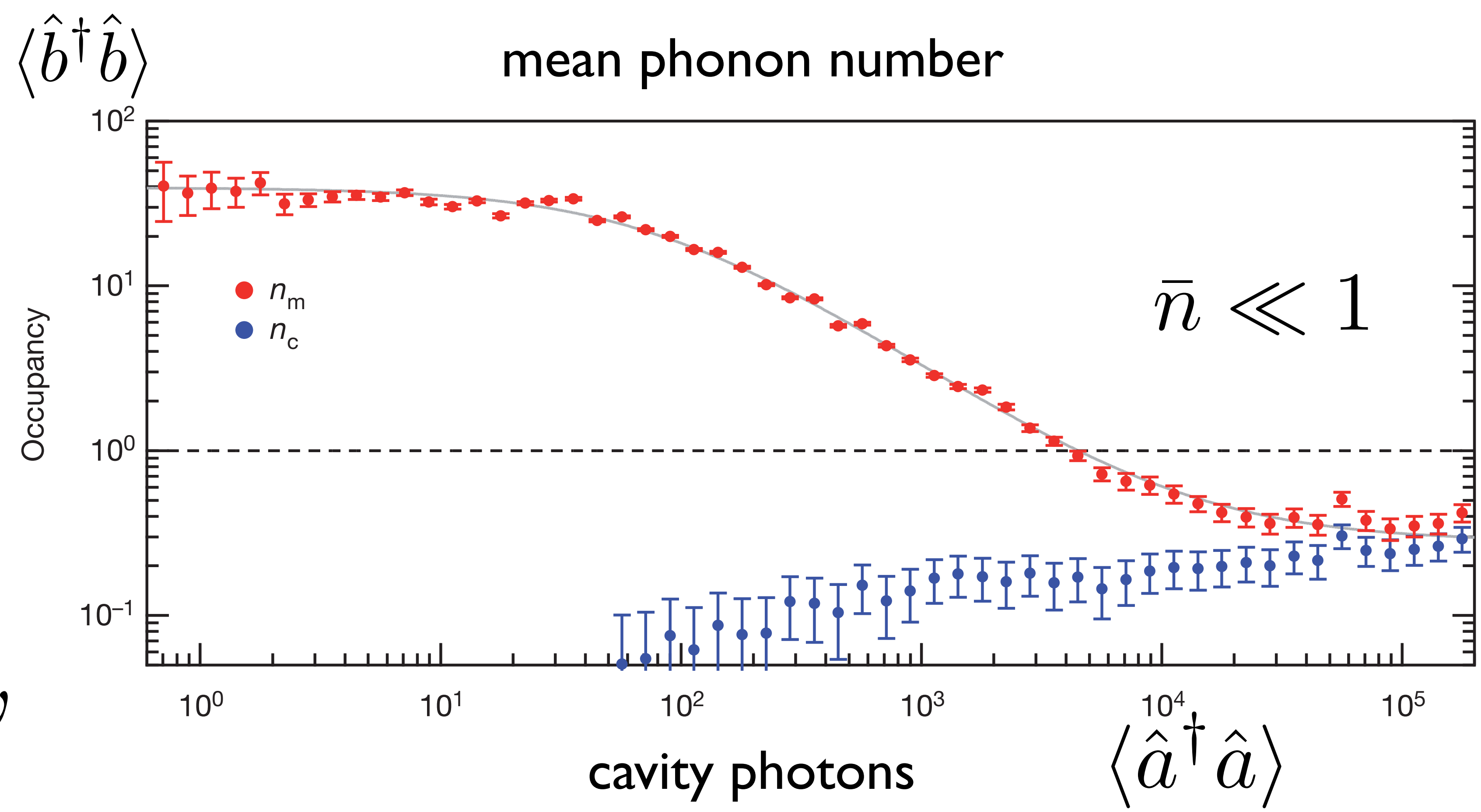
Sideband cooling: the experimental results

displacement spectrum









Mechanical systems are now firmly in the quantum regime.

→ atoms, superconducting circuits, and mechanics

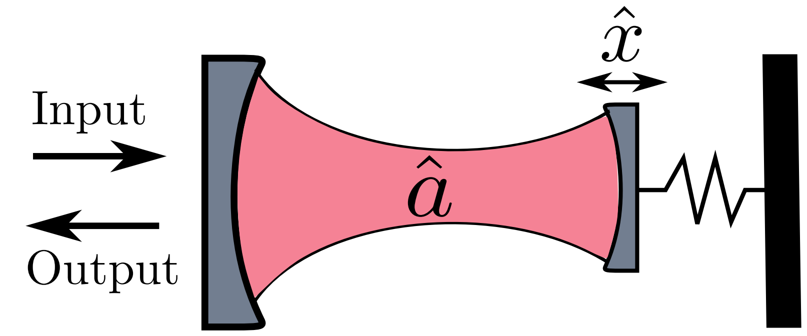
Teufel *et al.*, Nature **475**, 359 (2011)

Strong-coupling regime

Linear optomechanics

The Hamiltonian

$$\hat{H} = \hbar\omega_R \hat{a}^\dagger \hat{a} + \hbar\omega_M \hat{b}^\dagger \hat{b} + \hbar g(\hat{b} + \hat{b}^\dagger)\hat{a}^\dagger \hat{a}$$



For drive and decay we write $\hat{a} = e^{-i\omega_L t}\left(\bar{a} + \hat{d}\right)$ and $\hat{b} = \bar{b} + \hat{c}$

→ bistability

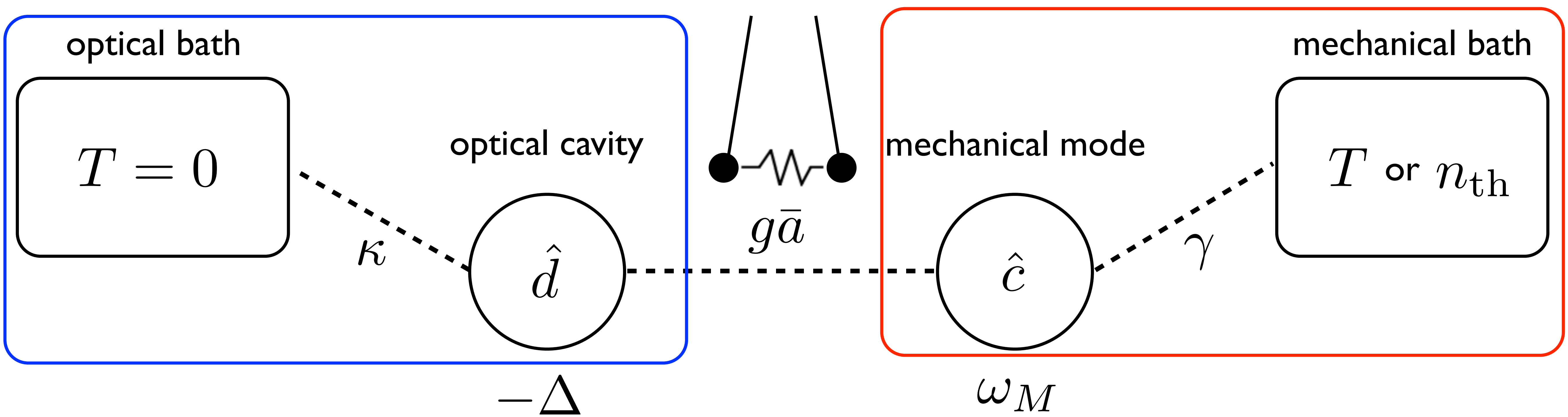
$$\hat{H} = -\Delta \hat{d}^\dagger \hat{d} + \omega_M \hat{c}^\dagger \hat{c} + g\bar{a}(\hat{c} + \hat{c}^\dagger)(\hat{d} + \hat{d}^\dagger) + g\hat{d}^\dagger \hat{d}(\hat{c} + \hat{c}^\dagger)$$

detuning $\Delta = \omega_L - \omega_R$

# of photons $|\bar{a}|^2$

Effective coupling rate

Bilinear coupling



Static part of radiation-pressure force

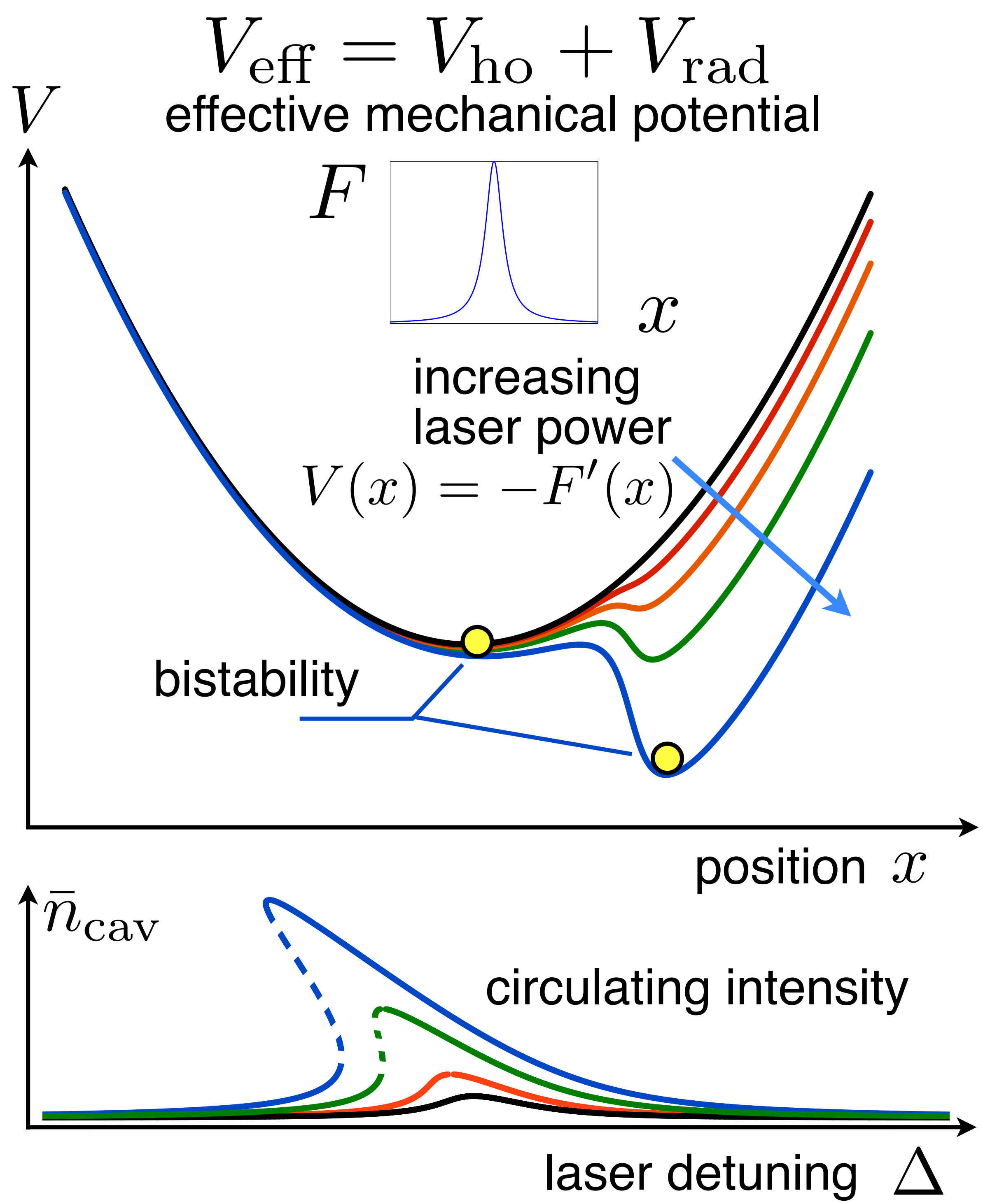


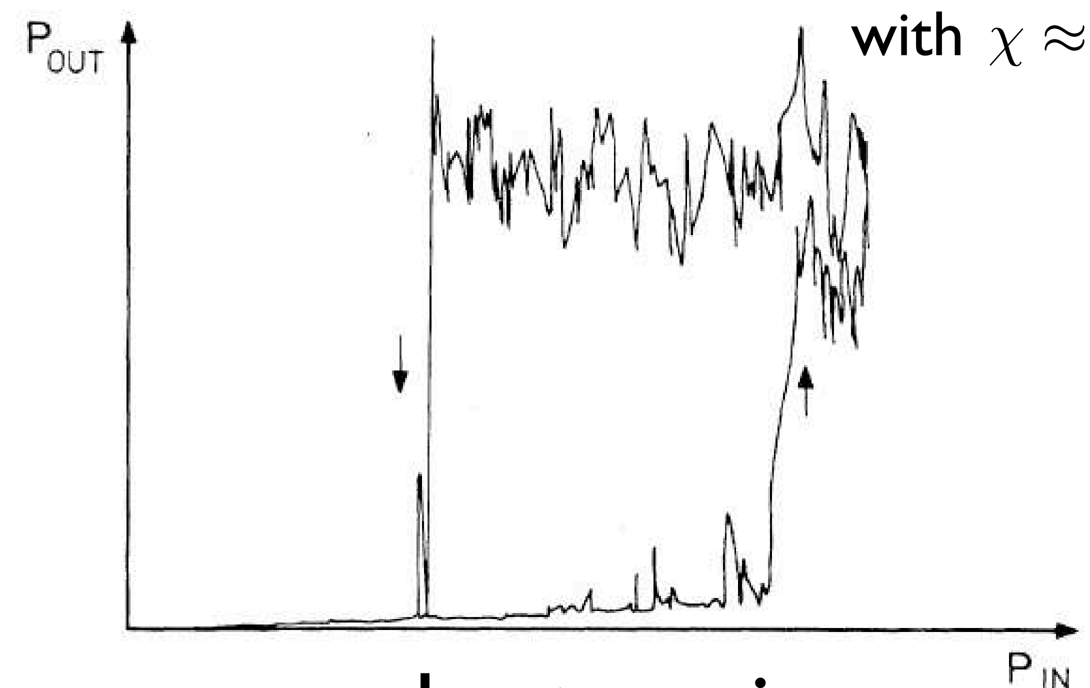
figure adapted from F. Marquardt

$$0 = i\Delta\bar{a} - \frac{\kappa}{2}\bar{a} - i\Omega - ig(\bar{b}+\bar{b}^*)\bar{a}$$

$$0 = -i\omega_M\bar{b} - \frac{\gamma}{2}\bar{b} - ig|\bar{a}|^2$$

$$\bar{n}_{\mathrm{cav}} = \frac{\Omega^2}{\left(\frac{\kappa}{2}\right)^2 + \left(\Delta + \chi\bar{n}_{\mathrm{cav}}\right)^2}$$

with $\chi \approx \frac{2g^2}{\omega_M}$



hysteresis

Experiment: Dorsel *et al.*, PRL **51**, 1550 (1983)

Linear optomechanics

The Hamiltonian

$$\hat{H} = -\Delta \hat{d}^\dagger \hat{d} + \omega_M \hat{c}^\dagger \hat{c} + g\bar{a}(\hat{c} + \hat{c}^\dagger)(\hat{d} + \hat{d}^\dagger) + g\hat{d}^\dagger \hat{d}(\hat{c} + \hat{c}^\dagger)$$

detuning $\Delta = \omega_L - \omega_R$

Effective coupling rate

Bilinear coupling

# of photons $|\bar{a}|^2$

The equations of motion

$$\dot{\hat{d}} = i\Delta \hat{d} - \frac{\kappa}{2}\hat{d} - \sqrt{\kappa}\hat{d}_{\text{in}} - ig\bar{a}(\hat{c} + \hat{c}^\dagger)$$

$$\dot{\hat{c}} = -i\omega_M \hat{c} - \frac{\gamma}{2}\hat{c} - \sqrt{\gamma}\hat{c}_{\text{in}} - ig\bar{a}(\hat{d} + \hat{d}^\dagger)$$

Exact solution

$$\hat{c}(\omega) = \ldots \hat{c}_{\text{in}} + \ldots \hat{c}_{\text{in}}^\dagger + \ldots \hat{d}_{\text{in}} + \ldots \hat{d}_{\text{in}}^\dagger$$

$$\hat{d}(\omega) = \ldots \hat{c}_{\text{in}} + \ldots \hat{c}_{\text{in}}^\dagger + \ldots \hat{d}_{\text{in}} + \ldots \hat{d}_{\text{in}}^\dagger$$

$$\langle \hat{c}_{\text{in}}^\dagger(\omega)\hat{c}_{\text{in}}(\omega')\rangle = n_{\text{th}}\delta(\omega + \omega')$$

$$\langle \hat{c}_{\text{in}}(\omega)\hat{c}_{\text{in}}^\dagger(\omega')\rangle = (n_{\text{th}} + 1)\delta(\omega + \omega')$$

$$\langle \hat{c}_{\text{in}}^\dagger(\omega)\hat{c}_{\text{in}}^\dagger(\omega')\rangle = \langle \hat{c}_{\text{in}}(\omega)\hat{c}_{\text{in}}(\omega')\rangle = 0$$

We can calculate all observables!

Walls and Milburn, *Quantum Optics* (OUP)

Normal-mode splitting in the strong-coupling limit

$$S_{cc}(\omega) = \int dt\, e^{i\omega t} \langle \hat{c}^\dagger(t)\hat{c}(0)\rangle$$

$$S_{\text{out}}(\omega) = \int dt\, e^{i\omega t} \langle \hat{d}^\dagger_{\text{out}}(t)\hat{d}_{\text{out}}(0)\rangle$$

with $\hat{d}_{\text{out}} = \hat{d} + \sqrt{\kappa}\hat{d}_{\text{in}}$

$$S_{\text{out}}(\omega) = S_{FF}(-\omega) S_{xx}(\omega)$$

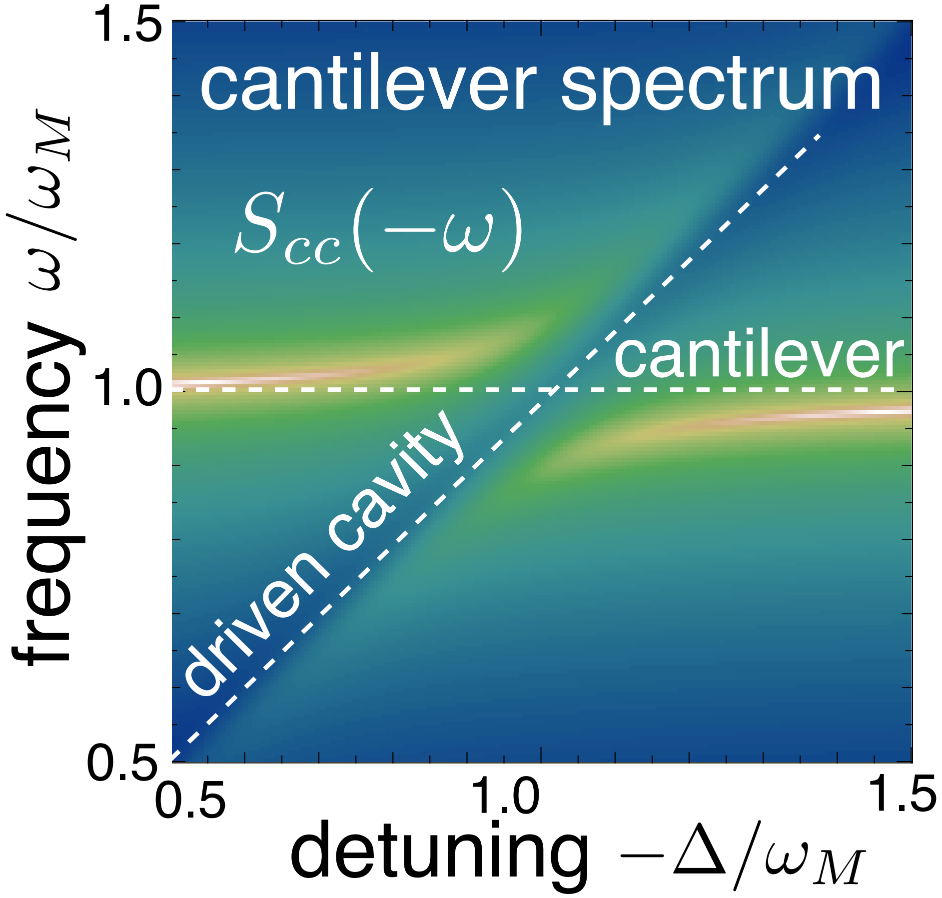
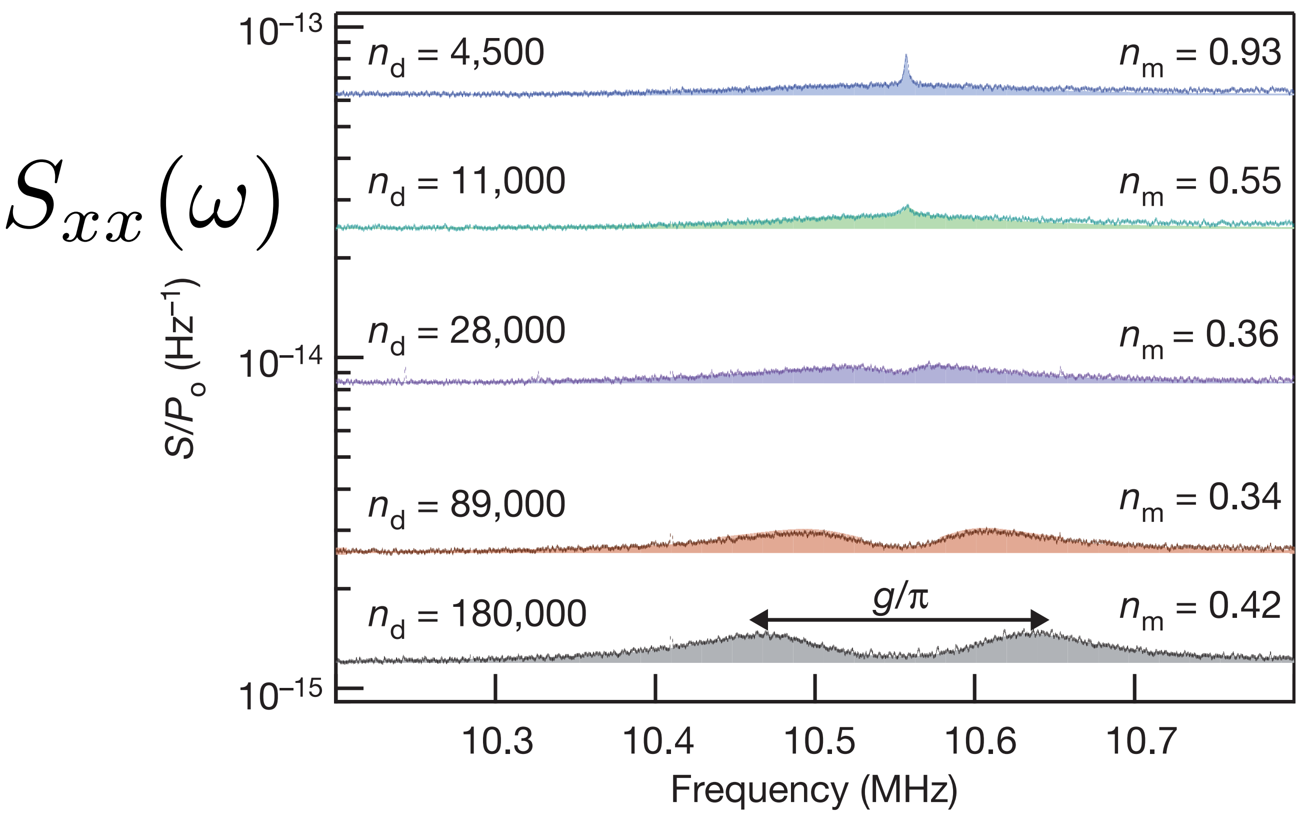


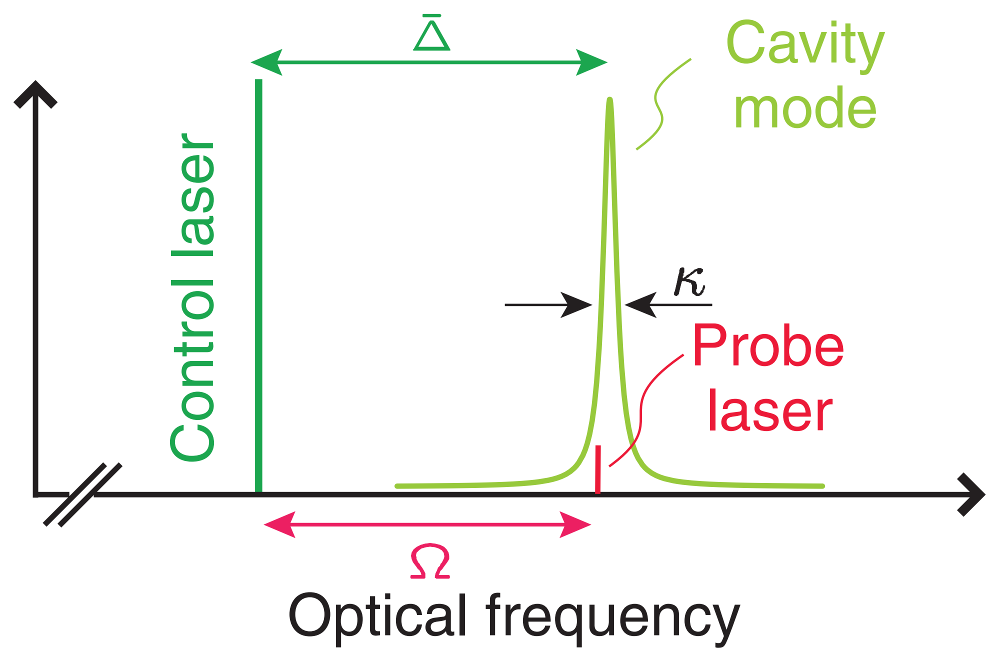
figure by F. Marquardt

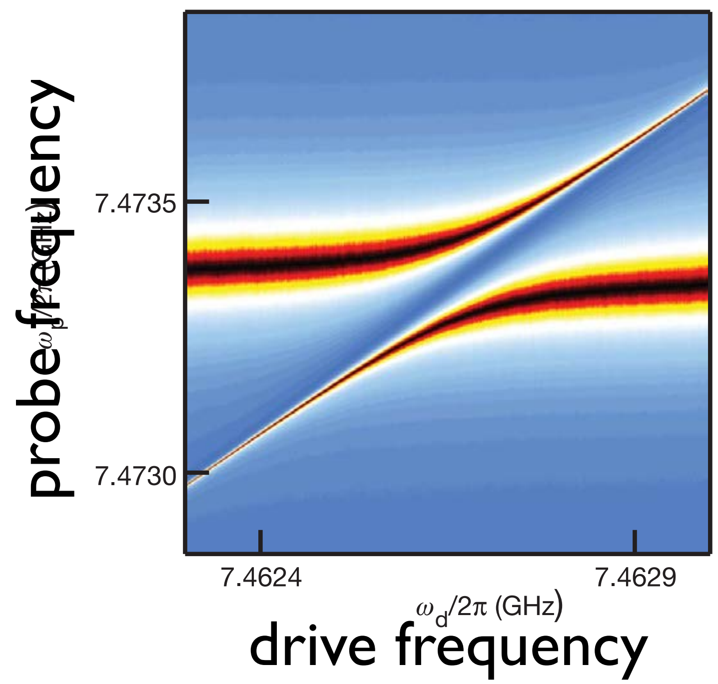


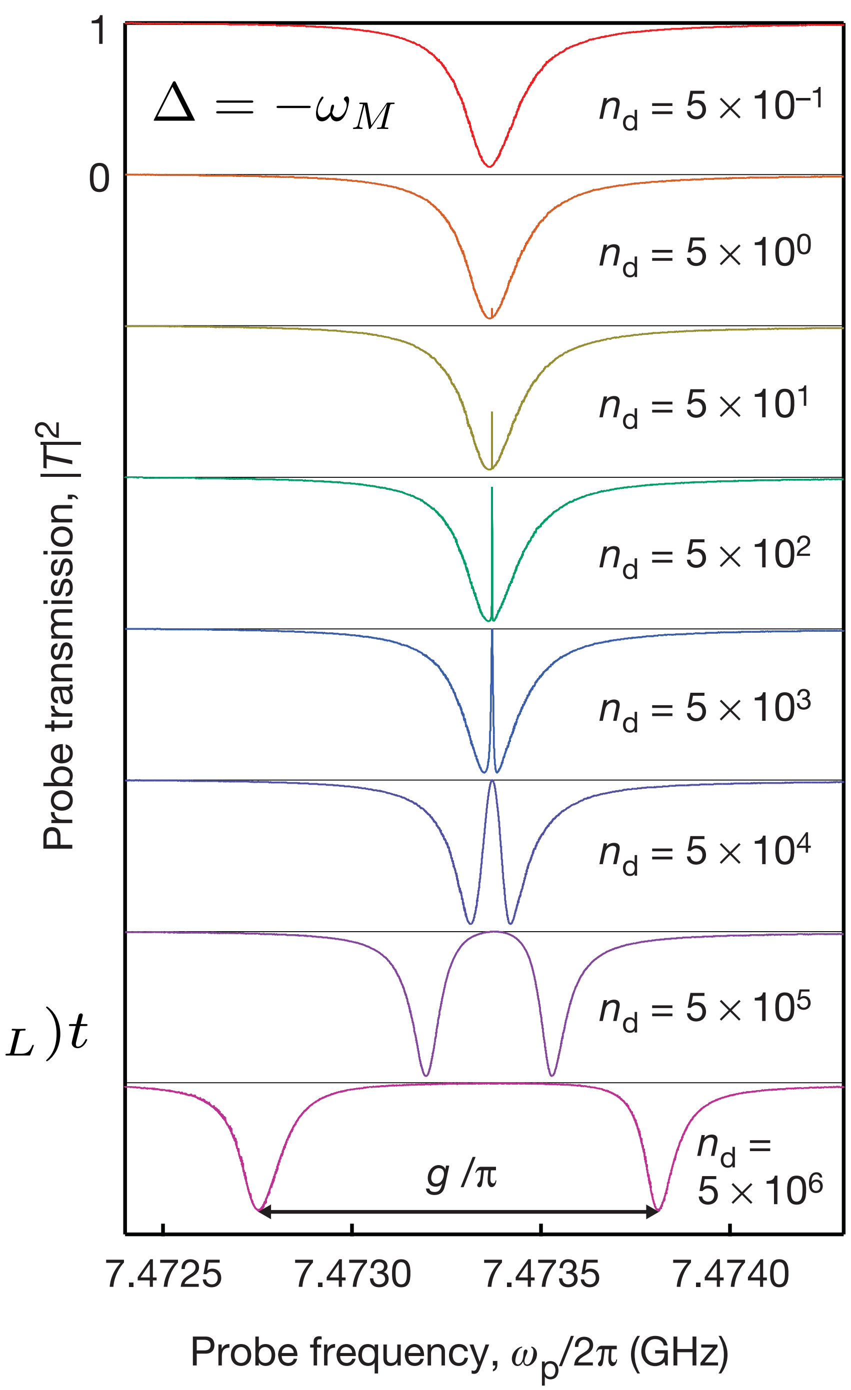
In the strong-coupling limit cavity and mechanics hybridize.

Gröblacher *el al.*, Nature **460**, 724 (2009)
Teufel *et al.*, Nature **471**, 204 (2011)
Theory: Dobrindt *et al.*, PRL **101**, 263602 (2008)

Optomechanically-induced transparency (OMIT)







coherent pump $\langle \hat{d}_{\rm in}(t) \rangle \propto e^{-i(\omega_P - \omega_L)t}$

coherent signal $\langle \hat{d}(t) \rangle = A^- e^{-i(\omega_P - \omega_L)t} + A^+ e^{+i(\omega_P - \omega_L)t}$

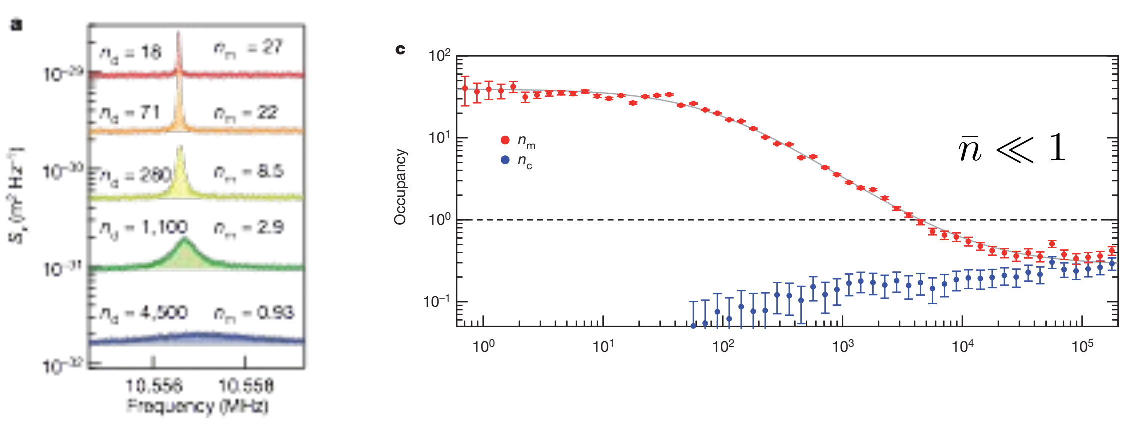
$$A^- \propto \frac{1}{\frac{\kappa}{2} - i(\Omega + \Delta) + \frac{g^2 \bar{a}^2}{\frac{\gamma}{2} - i(\Omega - \omega_M)}}$$

with $\Omega = \omega_P - \omega_L$

Weis *et al.*, Science **330**, 1520 (2010)
Teufel *et al.*, Nature **471**, 204 (2011)
Theory: Agarwal and Huang, PRA **81**, 041803 (2010)

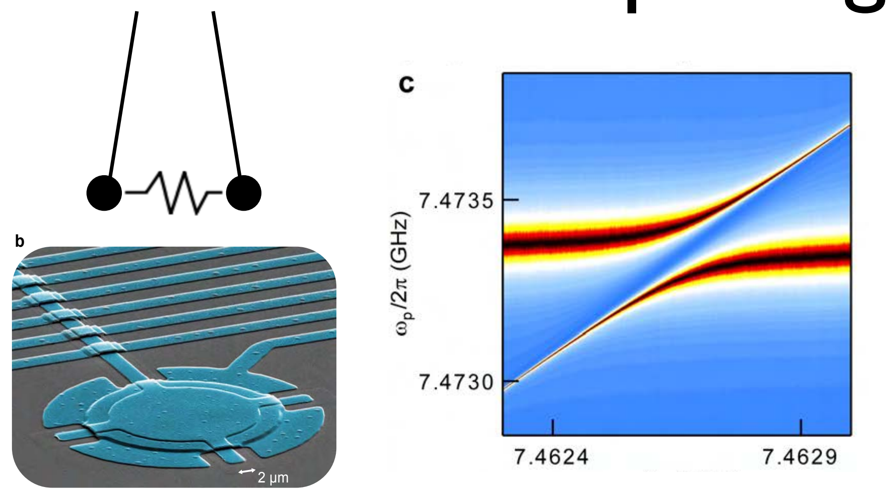
The state of the art in optomechanics

Ground-state cooling



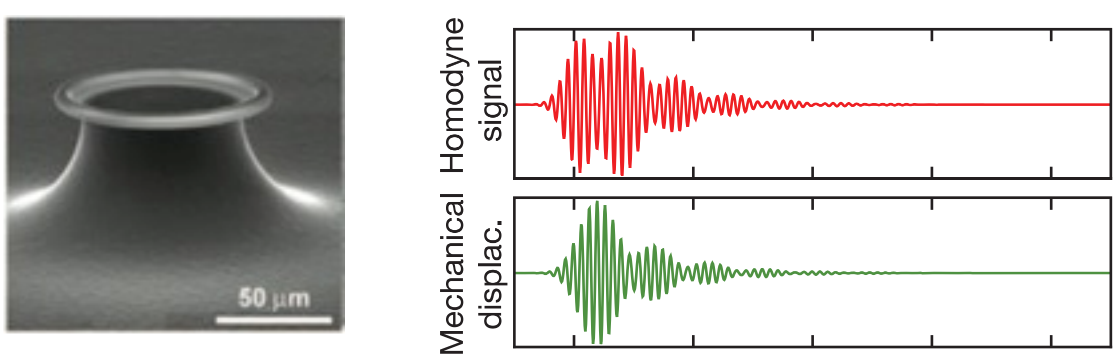
Nature **475**, 359 (2011)

Normal-mode splitting



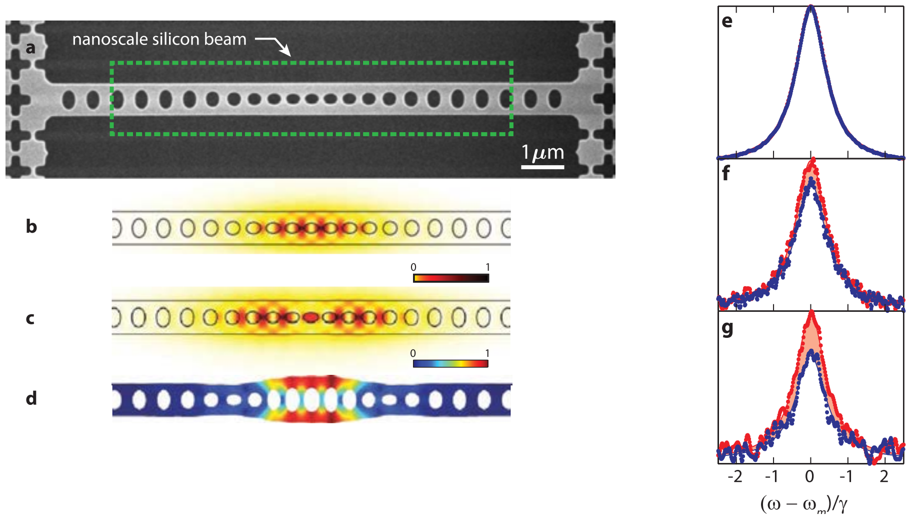
Nature **471**, 204 (2011)

Coherent coupling



Nature **482**, 63 (2012)

Sideband thermometry

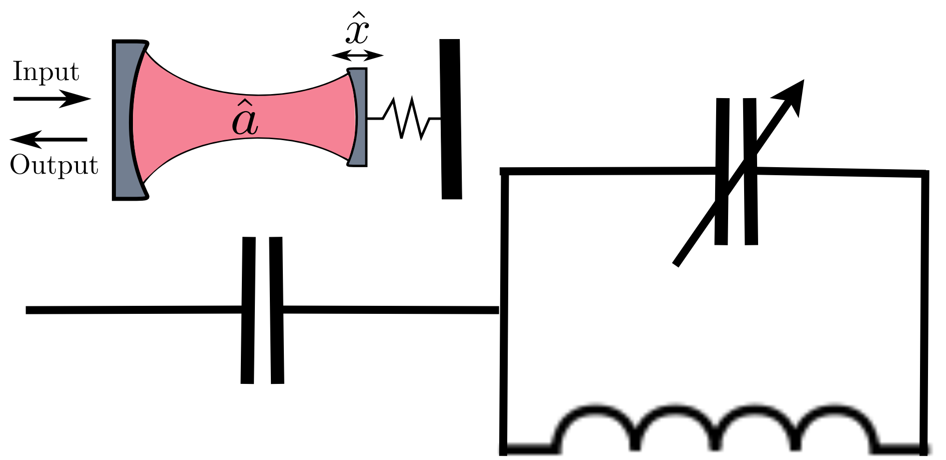


PRL **108**, 033602 (2012)

Dissipative coupling

Dispersive vs. dissipative coupling

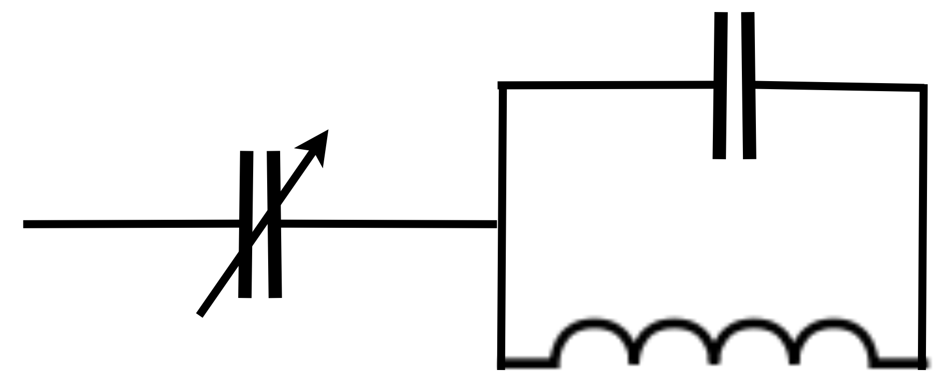
Dispersive coupling



$$\omega_R(\hat{x}) = \omega_R + \frac{\partial \omega_R}{\partial x}\hat{x}$$

$$\hat{H}_{\mathrm{int}} \propto \hat{a}^\dagger \hat{a} \hat{x}$$

Dissipative coupling



$$\kappa(\hat{x}) = \kappa + \frac{\partial \kappa}{\partial x}\hat{x}$$

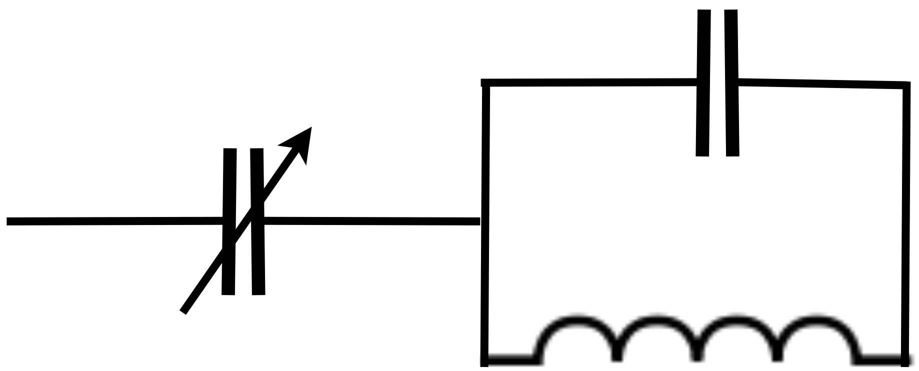
$$\hat{H}_{\mathrm{int}} \propto \hat{H}_{\mathrm{damp}} \hat{x}$$

with

$$\hat{H}_{\mathrm{damp}} \propto \sqrt{\kappa} \sum_q (\hat{a}^\dagger \hat{a}_q + \hat{a}_q^\dagger \hat{a})$$

Elste *et al.*, PRL **102**, 207209 (2009)

Cooling with dissipative coupling



$$\hat{H}_{\text{int}} \propto \hat{H}_{\text{damp}} \hat{x}$$

with

$$\hat{H}_{\text{damp}} \propto \sqrt{\kappa} \sum_q (\hat{a}^\dagger \hat{a}_q + \hat{a}_q^\dagger \hat{a})$$

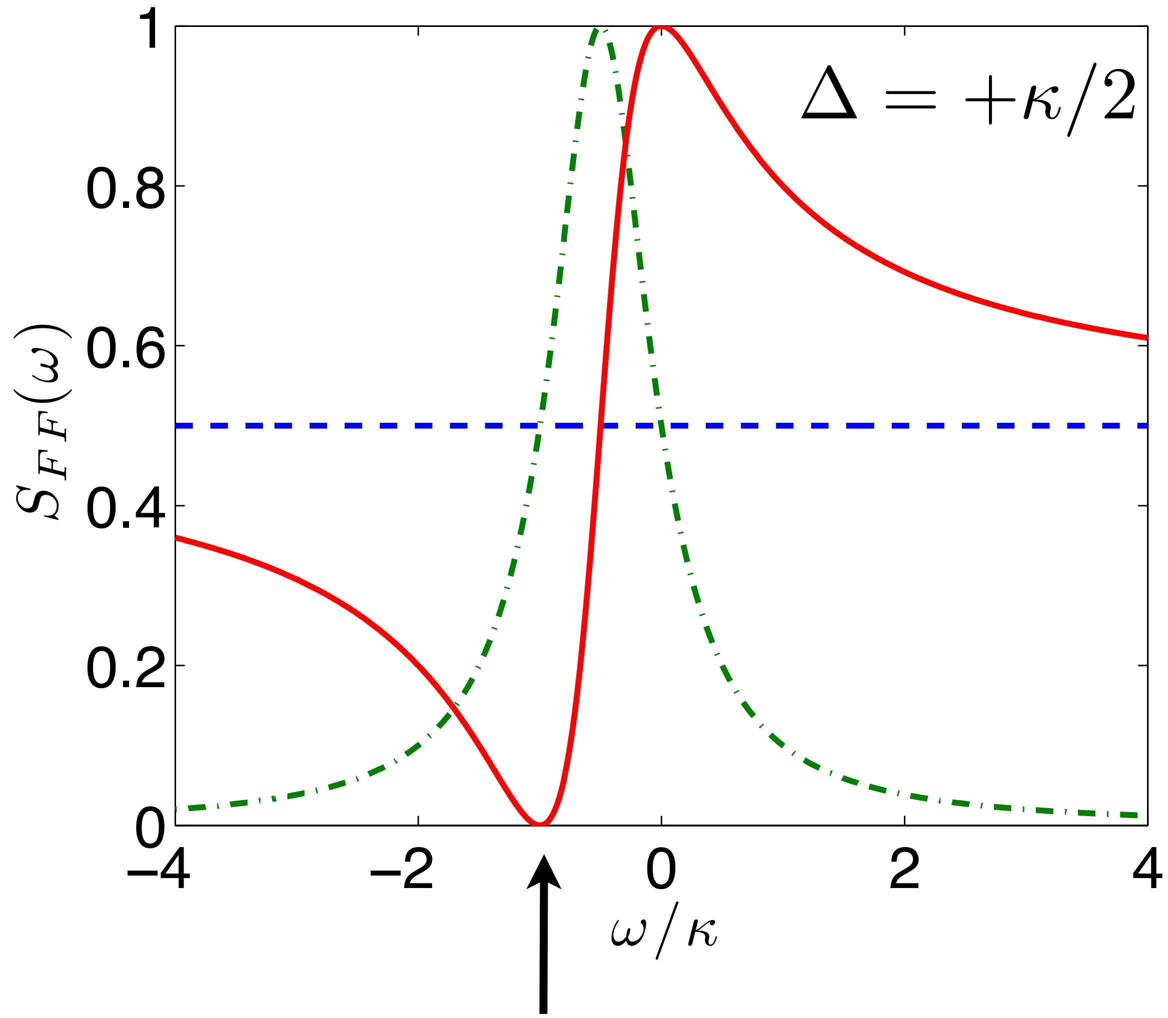
Two noise sources! $\hat{F} = \hat{F}_1 + \hat{F}_2$

$$\hat{F}_1 \propto \hat{d}_{\text{in}} + \hat{d}_{\text{in}}^\dagger \qquad \hat{F}_2 \propto \hat{d} + \hat{d}^\dagger$$

They are random, but not independent! $\hat{d}(\omega) \propto \chi_R(\omega) \hat{d}_{\text{in}}$

$$S_{FF}(\omega) \propto \left| 1 - \left(\frac{\kappa}{2} + i\Delta \right) \chi_R(\omega) \right|^2$$

with $\chi_R(\omega) = [\kappa/2 - i(\omega + \Delta)]^{-1}$ →Fano line shape

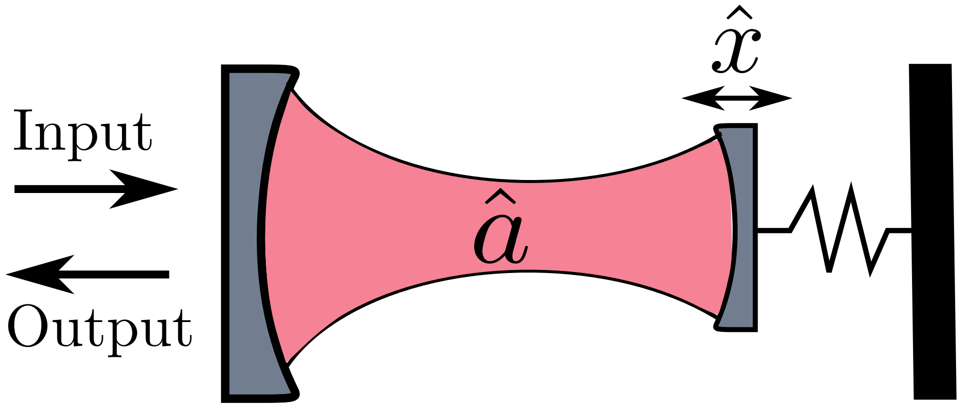


$$\Gamma_{\text{opt}}^\uparrow \propto S_{FF}(-\omega_M)$$

This enables ground state cooling outside the resolved-sideband limit!

Elste *et al.*, PRL **102**, 207209 (2009)

Conclusions



$$\hat{H} = \omega_R \left(1 - \frac{\hat{x}}{L}\right) \hat{a}^\dagger \hat{a} + \omega_M \hat{b}^\dagger \hat{b}$$

radiation-pressure force

- Applications: sensors, transducers, decoherence
- Damping is due to the finite time lag between mirror and radiation-pressure force: $\omega_M \gg \kappa$
- With red-sideband cooling $\omega_L = \omega_R - \omega_M$ the ground state was reached in experiments: $\bar{n} \ll 1$
- At strong coupling optics and mechanics hybridize.

http://www.physinfo.fr/houches/pdf/Marquardt.pdf