Bouncing Droplets, Pilot-Waves, and Quantum Mechanics

Tomas Bohr, Anders Andersen and Benny Lautrup

Abstract Bouncing droplets on a fluid surface have recently been shown to provide a surprising analogy to quantum behaviour. Here we discuss the limitation of this analogy in the context of the double-slit experiment, which our colleagues and we have analysed in a recent paper [Phys. Rev. E **92**, 013006 (2015)]. The present paper is based on the talk given by Tomas Bohr at the XX Congreso de la División de Dinámica de Fluidos, Sociedad Mexicana de Física, Centro Mesoamericana de Física Teórica, Tuxtla Gutiérrez, November 2014.

1 Introduction

Recently, it has been suggested that it is possible to "simulate" quantum mechanics by wave-driven particles (droplets) bouncing on a fluid surface (Eddi et al. 2009, 2012; Fort et al. 2010; Bush 2010, 2015a, b; Couder and Fort 2012; Harris and Bush 2014; Oza et al. 2014; Perrard et al. 2014a, b). Indeed it has been possible to demonstrate typical quantum effects such as discrete orbits and tunnelling. Furthermore, experimental results have provided evidence that the famous double-slit experiment could be reproduced in this way (Couder and Fort 2006). The fact that quantum particles can show interference just like light waves, lies at the heart of quantum mechanics, since it shows that quantum particles, although retaining their integrity and remaining localized, can interfere like waves. Thus the realization of the double-slit experiment with bouncing droplets would be a strong indication of a deep correspondence between the two systems. In a recent paper (Andersen et al.

T. Bohr (✉) · A. Andersen
Department of Physics, Technical University of Denmark, Kgs. Lyngby, Denmark
e-mail: tbohr@fysik.dtu.dk

A. Andersen
e-mail: aanders@fysik.dtu.dk

B. Lautrup
Niels Bohr International Academy, The Niels Bohr Institute, Copenhagen, Denmark
e-mail: lautrup@nbi.dk

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2015), our colleagues and we have, however, argued that the double-slit experiment will be qualitatively different for quantum particles and bouncing droplets. The very fact that we can determine the precise paths followed by the droplets makes it impossible to reproduce the quantum results with such "wave-driven" particles.

In 1913, when Niels Bohr published his trilogy *On the Constitution of Atoms and Molecules*, where the structure of atoms and their characteristic spectra are derived on the basis of two novel postulates that clash violently with classical mechanics and electromagnetism, he knew that the incorporation of such new phenomena would require a radical rethinking of the basic laws of physics. Decoupling the orbital period of the electron around the nucleus from the frequency of the radiation it emits or absorbs, actually implies that the very notion of an *orbit* is ill-defined. To him it was therefore no great surprise that the new "Quantum Mechanics", when it finally emerged in 1925–26 turned out to be a *statistical* theory, only able to predict the *probability* for the time and place of certain events, and not the detailed outcome of a single event. Even though Schrödinger's wave mechanics has the appearance of a classical wave equation, these waves are not "real" waves, but probability amplitudes, complex fields, whose square modulus is a probability density.

This created a rift through the physics community between the majority, led by Heisenberg and Bohr, who believed that it was a necessity—or at least, a pragmatic approach allowing very successful calculations and predictions—and those, like Einstein, de Broglie, and Schrödinger—who felt that denouncing "realism" in such a radical fashion was not the right way forward.

In 1927, de Broglie published his reaction to the work of Schrödinger and the statistical interpretation of Born, claiming that a deterministic interpretation of the wave equation could be maintained if particles were introduced as singularites in the wave field (Broglie 1927). On p. 226, he writes: *Pour M. Born il n'y a que des probabilités ; la déterminisme des phénomènes individuels devrai être abandonné, la probabilité des phénomènes statistiques étant seule déterminée. De la manière de voir adoptée ici, au contraire, le point matériel est un réalité essentielle et son mouvement est entièrement déterminé comme étant celui d'une singularité d'amplitude dans une onde qui se propage.*

Niels Bohr reacted to this idea in an unpublished draft for his Como lecture (Bohr 1985, p. 92): *Our point of view is essentially different from that taken by de Broglie in a recent article. This author attempts to reconcile the two apparently contradictory sides of the phenomena by regarding the individual particles or light quanta as singularities in the wave field. It does not seem, however, that any such view resting on the concepts of classical physics is suited to help us over the fundamental difficulties referred to. On the contrary, the dilemma of the nature of light and material particles seems, as far as classical concepts are used, to be unavoidable and to constitute an adequate summary of the analysis of experiments.*

Later, after the work of Bohm in the 1950's, de Broglie returned to his early ideas and came up with his "double solution theory" (Broglie 1987), where he, in addition to the statistical Schrödinger wave, proposed that particles would locally excite a "real" wave field which would simultaneously guide the particle. In de Broglie's lifetime no system was known where his ideas could be tested. This changed suddenly around 2005, when Yves Couder, Emmanuel Fort, and co-workers started looking carefully at something as far from quantum systems as droplets of silicone oil bouncing on the surface of the same fluid (Couder et al. 2005a).

2 Bouncing Droplets, Walkers, and Pilot-Waves

If a droplet of silicone oil is placed on a vertically vibrating bath of the same fluid, it can bounce and surprisingly start "walking" horizontally across the bath (Couder et al. 2005a). If the amplitude of the vibration is large enough, the bouncing droplet will never coalesce with the bath, due to the small lubricating air cushion between the droplet and the bath (Couder et al. 2005b). Increasing the amplitude of the vibration, the system will finally undergo a Faraday instability, where standing surface waves with half of the driving frequency spontaneously appear throughout the bath. Remaining close to but below this instability, one can observe a period-doubling transition, where the bouncing frequency of the droplet and the frequency of the surface waves become equal. In this regime, a symmetry breaking bifurcation can take place, where the droplet starts moving horizontally across the surface with a characteristic speed, an order of magnitude smaller than the phase velocity of the waves. The wave pattern generated by these "walkers" consists of standing waves with the Faraday wavelength, i.e., the wavelength selected by the imminent Faraday instability (Eddi et al. 2011).

Couder and Fort were immediately struck by the similarity between the walkers and de Broglie's pilot waves: the droplet creates waves in the region around it and these waves then propagate the droplet. Consequently, they set out to explore the strength of this analogy, and how closely this system could imitate quantum behaviour. They could together with their co-workers demonstrate the existence of discrete states when they rotated the oscillating bath around a central vertical axis (Fort et al. 2010), and tunnelling across a "barrier", i.e., a subsurface structure that decreases the depth of the fluid layer and thereby increases the threshold for the Faraday instability (Eddi et al. 2009). Using subsurface barriers with slit openings Couder and Fort also imitated the double-slit experiment (Couder and Fort 2006), and they fitted the single-particle statistics, i.e., the number of droplets deflected in a given angle, by a Fraunhofer diffraction and interference curve

$$f(\alpha) = A \left| \frac{\sin\left[\pi(L/\lambda_F)\sin\alpha\right] \cos\left[\pi(d/\lambda_F)\sin\alpha\right]}{\pi(L/\lambda_F)\sin\alpha} \right|, \qquad (1)$$

where L is the width of each slit, d the distance between the slits, λ_F the Faraday wavelength, and A a normalization constant that depends on the total number of counts (Fig. 1). Based on the experimental data and the fit of the Fraunhofer curve, (Couder and Fort 2006, p. 3) concluded on their double-slit experiment with walkers: *The interference fringes are clearly observed and well fitted by this expression. It can be noted that a given droplet is observed to go through one or the other of the slits.*

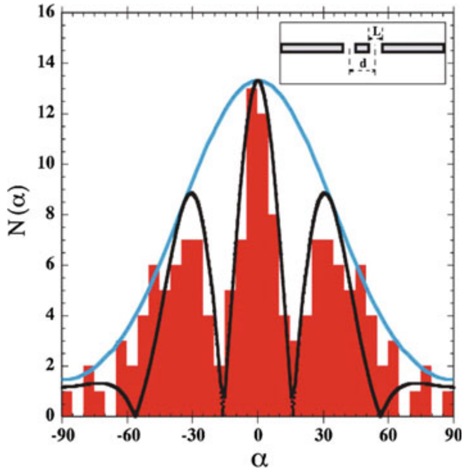

Fig. 1 Single-particle statistics for the droplets in the double-slit experiment with walkers by Couder and Fort. Experiment (*red*), Fraunhofer diffraction and interference curve (1) for a double-slit with $L/\lambda_F = 0.9$ and $d/\lambda_F = 1.7$ (*black*), and Fraunhofer diffraction curve for a comparable single-slit experiment (*blue*). Reproduced with permission from (Couder and Fort 2006)

However its associated wave passes through both slits and the interference of the resulting waves is responsible for the trajectory of the walker.

However, as pointed out in (Andersen et al. 2015), the 75 recorded droplet passages constitute a small data set with on average only approximately two counts for each of the 36 bins in the histogram (Fig. 1). The size of the data set was "doubled" by left-right symmetrization of the histogram, but that did not increase the "true" size of the data set, since the additional 75 data points were not independent. Also, the single-slit diffraction result is not backed by the experiment, since data for a comparable single-slit with the same L/λ_F value are not given by Couder and Fort (2006). Replacing the Fraunhofer curve (1) by a Gaussian distribution with zero mean value and thus deleting all the interference maxima and minima actually gives a fit that is

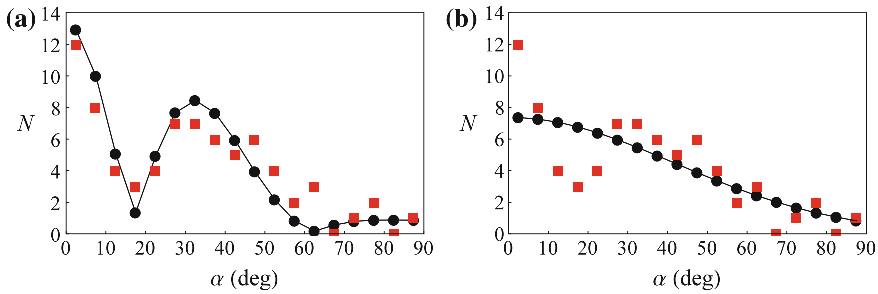

Fig. 2 **a** Fraunhofer diffraction and interference fit (black line and circles) and **b** Gaussian distribution fit (black line and circles) of the data (*red squares*) from the double-slit experiment by Couder and Fort extracted from (Couder and Fort 2006, Fig. 3)

just as good (Fig. 2). Since we do not have access to the non-symmetrised data, we fit only one half of the histogram, where there are 75 data points distributed over 18 bins. A standard approach to determine the goodness-of-fit is to use Pearson's chi-square $\chi_P^2 = \sum_{i=1}^{n_b} (O_i - E_i)^2/E_i$, where n_b is the number of bins, O_i the observed bin counts, and E_i the modelled bin counts (Baker and Cousins 1984). To assess the quality of the fit we determine the reduced chi-square, i.e., the ratio between χ_P^2 and the number of degrees of freedom $\nu = n_b - n_c$, where n_c denotes the number of constraints, i.e., the number of parameters in the fit that are extracted from the data. A good fit should have a value of χ_P^2/ν close to unity (Bevington and Robinson 2003; Taylor 1997). For the Fraunhofer fit (1) with $L/\lambda_F = 0.9$ and $d/\lambda_F = 1.7$ as chosen by (Couder and Fort 2006), we have $\nu = 17$, since the total number of counts is known. In the Gaussian case we have $\nu = 16$, since both the total number of counts and the standard deviation of the distribution are determined from the data. We find $\chi_P^2/\nu = 3.3$ for the fit to the Fraunhofer curve and $\chi_P^2/\nu = 0.9$ for the Gaussian fit with standard deviation equal to 42 deg. The use of Pearson's chi-square for goodness-of-fit tests is well-established for large data sets, but it may not be appropriate here due to the small number of counts. An alternative method is to use the multinomial maximum likelihood estimator $\chi_{\lambda,m}^2 = 2\sum_{i=1}^{n_b} O_i \ln\left(O_i/E_i\right)$

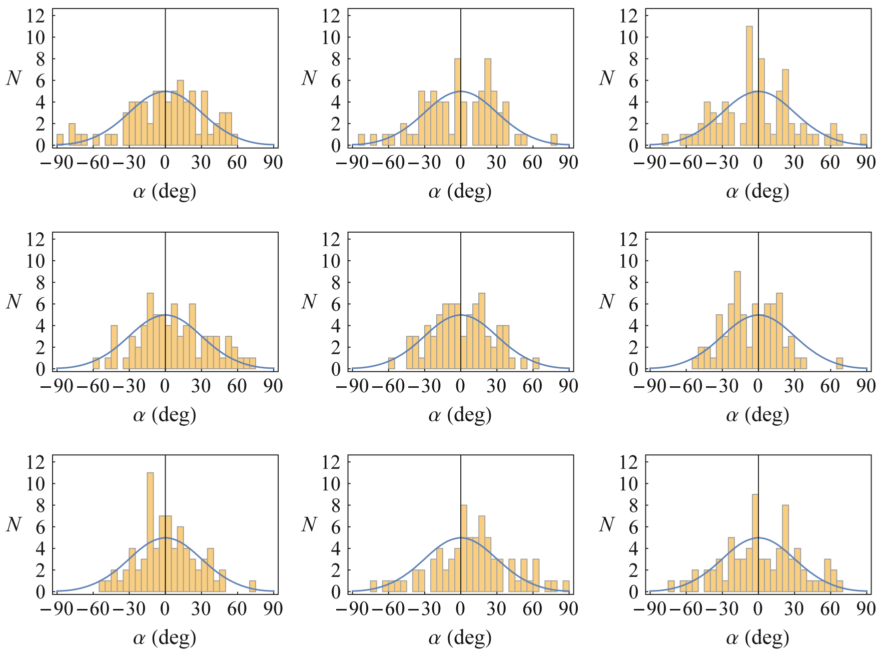

Fig. 3 Histograms (*yellow*) for an ensemble of nine independent data sets each consisting of 75 random angles drawn from a Gaussian distribution (*blue*) with zero mean value and standard deviation 30 deg. The data are binned with the same bin width as the one used by Couder and Fort for their double-slit experiment (Couder and Fort 2006, Fig. 3)

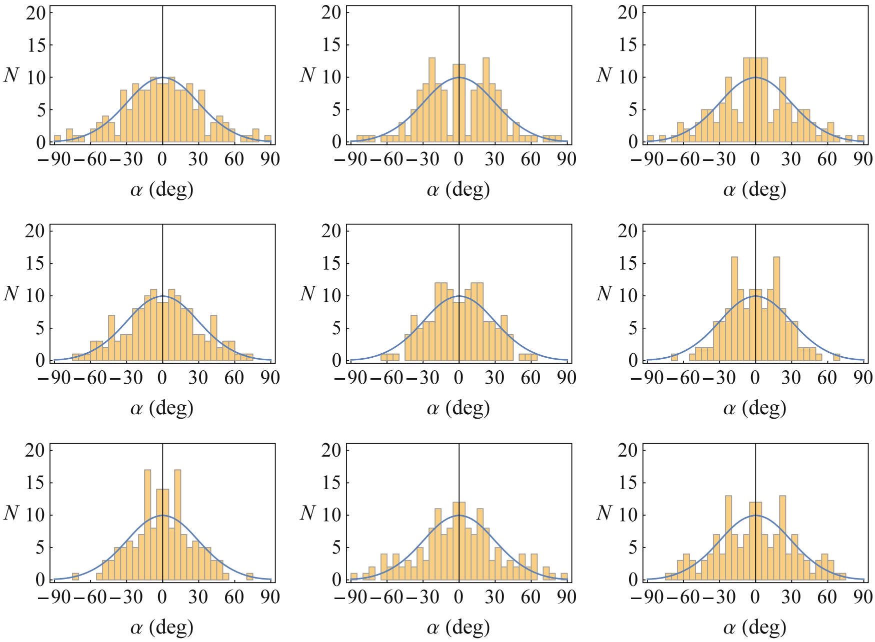

Fig. 4 Histograms (*yellow*) based on the same nine data sets as in Fig. 3 each consisting of 75 random angles drawn from a Gaussian distribution (*blue*) with zero mean value and standard deviation 30 deg, but now left-right symmetrized, thus doubling the number of data points in each data set to 150 as done by Couder and Fort for the data in their double-slit experiment (Couder and Fort 2006, Fig. 3)

when computing the reduced chi-square (Baker and Cousins 1984). Notice that in the limit of a large number of counts $\chi^2_{\lambda,m}$ degenerates to χ^2_P provided that we have $\sum_{i=1}^{n_b} O_i = \sum_{i=1}^{n_b} E_i$. We find $\chi^2_{\lambda,m}/\nu = 1.3$ for the fit to the Fraunhofer curve and $\chi^2_{\lambda,m}/\nu = 1.1$ for the Gaussian fit with standard deviation equal to 42 deg. Given the small data set, Monte Carlo simulations should ideally be used to obtain precise assessments of the goodness-of-fit for the two models (Baker and Cousins 1984).

The deceiving appearance of histograms based on small data sets can be further illustrated by ensembles of data drawn at random from a known distribution. In Fig. 3, 75 angles have been drawn from a Gaussian distribution and binned in a histogram with the same bin width as the one used by Couder and Fort for the data in their double-slit experiment (Couder and Fort 2006). This is shown for nine independent data sets and leads in some cases to histograms with strong left-right asymmetry.[1] In Fig. 4 we have left-right symmetrized the nine data sets, and we see that, in, say, one out of three cases, one might superficially find evidence for interference maxima and minima—simply due to the small number of counts.

[1]Note that some counts may fall outside the interval covered by the histogram, and that the number of counts in the histogram may therefore be less than 75 in some cases.

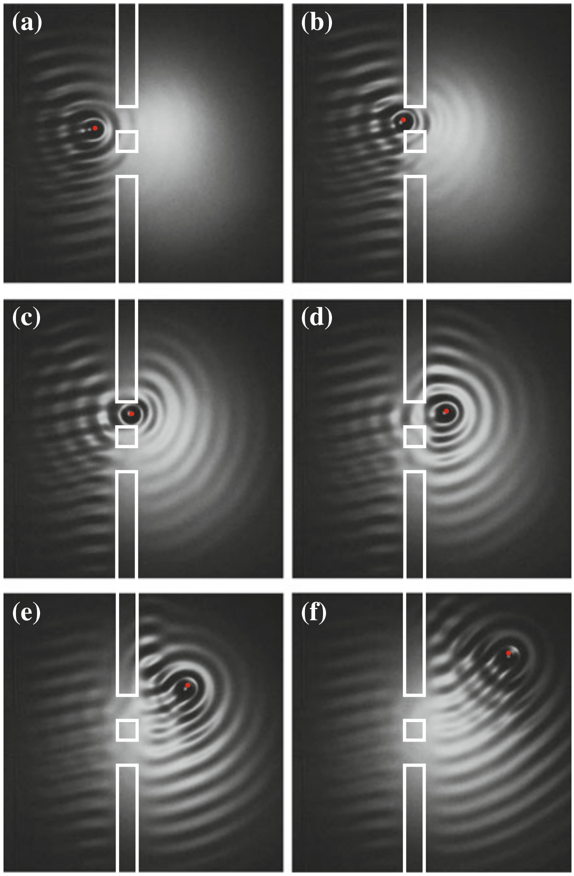

Fig. 5 Example of the wave field during slit passage in the double-slit experiment with walkers carried out by our colleagues and us. The droplet is highlighted in *red* and the subsurface barrier has been accentuated in *white*. Reproduced with permission from (Andersen et al. 2015)

In Fig. 5 we show the wave field and the slit passage of a droplet in the double-slit experiment carried out by our colleagues and us (Andersen et al. 2015) and made with parameters similar to those in the experiment by Couder and Fort (2006). The experimental data presented in (Andersen et al. 2015) shows no evidence for interference in the single-particle statistics and indeed the wide range of passage times through the slit and the weakness of the wave passing through the slit that is not chosen by the droplet cast strong doubt on the interference claimed by Couder and Fort (2006).

3 The Double-Slit Experiment and Its Interpretation

The quantum mechanical description of the double-slit experiment is deceptively simple. One simply has to determine the wave function Ψ by solving the time-dependent Schrödinger equation

$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\right)\Psi(\mathbf{r}, t) \;=\; 0, \tag{2}$$

where m is the mass of the quantum particle and $\hbar$ Planck's constant. The Schrödinger equation is a linear differential equation with properties similar to many other dispersive wave equations. In Fig. 6 we show a two-dimensional numerical solution of Eq. (2) with appropriate boundary conditions on barriers and walls for a quantum particle moving in towards a barrier with two slits. In quantum mechanics, one cannot specify both position and velocity simultaneously. The incoming quantum particle is thus represented by a wave packet. The height of the potential barriers were twice the mean energy of the wave packet. As one can see, this wave packet behaves very much like a water wave: when it reaches the two slits, part of it is reflected and the rest goes through the slits. After the slits, interference takes place and breaks the two wave packets emerging from the slits into a series of small wave packets around the interference maxima at the location of the "measuring" screen.

The strangeness arises when one tries to interpret this result in the light of the fact that the double-slit experiment with, e.g., electrons, does not lead to the formation of many small new particles. An electron is always detected as a localized particle with the same mass and charge. Thus one is forced to accept that the solution of the Schrödinger equation only leads to a statistical prediction, the probabil-

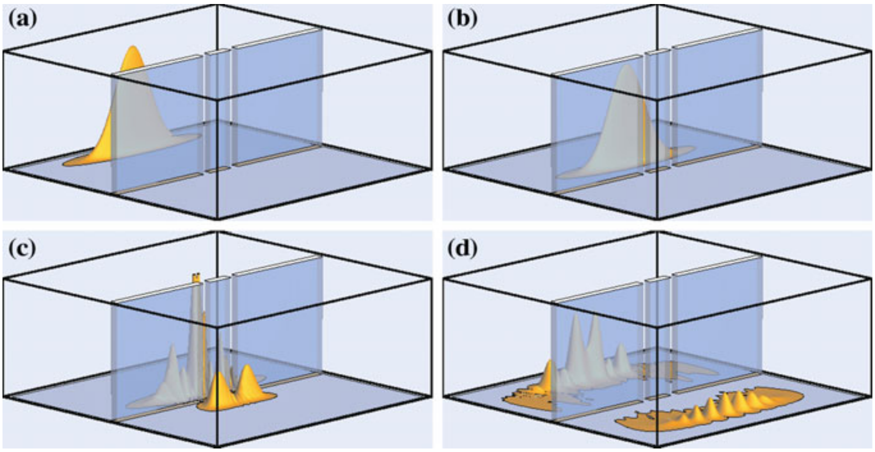

Fig. 6 Time evolution of the probability density in the double-slit experiment in quantum mechanics. Numerical solution of the time-dependent Schrödinger equation with an initial wave packet, which splits on passing the slits. Reproduced with permission from (Andersen et al. 2015)

(a) (b) (c) (d) (e)

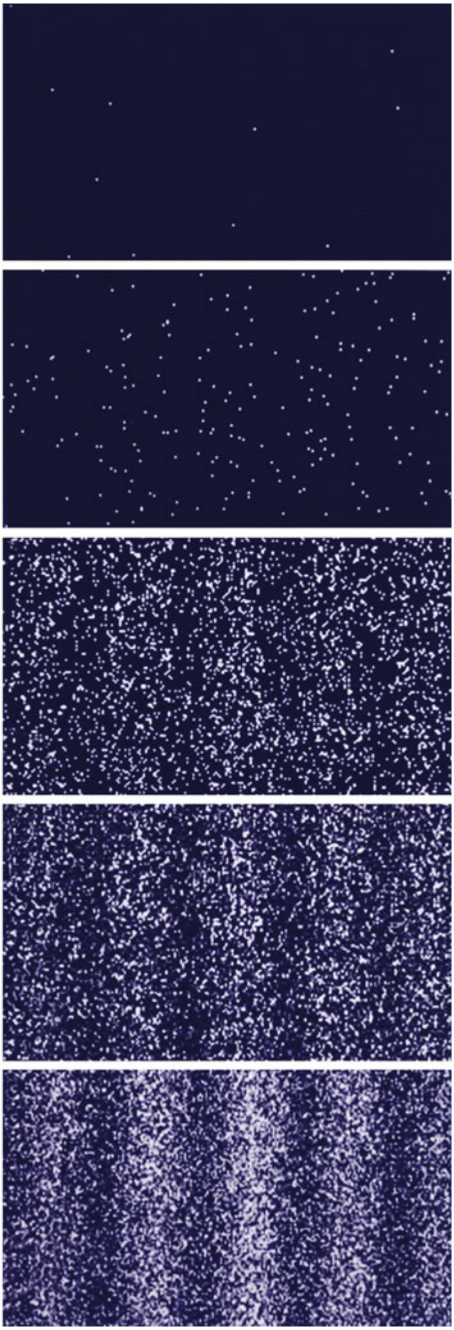

Fig. 7 Build-up of interference pattern in the quantum mechanical double-slit experiment with electrons performed by Tonomura and co-workers (1989). Numbers of electrons are 10 (**a**), 200 (**b**), 6000 (**c**), 40000 (**d**), and 140000 (**e**). Reproduced with permission from (Tonomura 2005). Copyright (2005) National Academy of Sciences, USA

ity of a certain outcome. This is illustrated in Fig. 7 taken from the experiment by Tonomura (1989, 2005), where an electron microscope was used to create a double-slit experiment with one electron passing through at a time. Each bright spot (electron) apparently arrives at a random position on the screen, but after some time an interference pattern with clear maxima and minima emerges.

These strange correlations are at the heart of quantum mechanics and they are closely related to the existence of "entanglement", where two particles remain correlated despite their lack of precise orbits. Such states were introduced theoretically by Einstein et al. (1935) to point out the "incompleteness" of quantum mechanics. In his response, Bohr (1935) made an analogy to the double-slit experiment in a situation, where one tries to measure the path taken by the particle by allowing the "diaphragm" containing the two slits to recoil - thus creating a "two-particle" system consisting of the particle and the diaphragm.

4 Quantum Mechanics with "Real" Particles

The walking droplets have been successfully modelled in situations where no boundaries or barriers need to be taken into account. The current status is forcefully reviewed in (Bush 2015a, b). In this way one can model, e.g., the discrete Landau-like circular orbits found when the oscillating bath is rotated around a central vertical axis (Fort et al. 2010) or the hydrogen-like states found in a central force-field created by adding magnets (Perrard et al. 2014a). In the double-slit experiment, however, we cannot neglect boundaries and barriers, so in (Andersen et al. 2015), we have taken a very different approach, further separated from the experiments. Following the ideas of de Broglie we have tried to write down a simple model explicitly containing particles, which might, in an average sense, contain standard quantum mechanics. As our wave equation we choose

$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\right)\Psi(\mathbf{r},t) \;=\; A(\mathbf{r},t)\,\delta[\mathbf{r}-\mathbf{R}(t)], \tag{3}$$

where a particle is present at position $\mathbf{R}(t)$, which creates an inhomogeneity in the form of a δ-function with amplitude A at the position of the particle. The choice of the Schrödinger equation as the homogeneous wave equation is mostly a matter of convenience: it should be a non-dissipative, dispersive equation. Of course, the experiment *is* damped (and forced), but this is probably not a feature to emulate, if we want a fundamental equation underlying quantum mechanics. The dispersive nature is dictated by observation: we know from de Broglie's association of particle and wave properties, that the wavelength λ of a quantum particle should be related to its momentum p as:

$$\lambda = \frac{h}{p} \tag{4}$$

and thus waves with different wavelengths should travel with different speeds.

Now, the wave equation (3) introduces a particle position, and thus we need another equation to determine the particle dynamics. A natural choice is the "guiding" equation used by de Broglie and Bohm, relating the velocity of the particle to the phase $\boldsymbol{\Phi}$ of the complex wave field Ψ

$$\dot{\mathbf{R}}(t) \;=\; \frac{\hbar}{m} \nabla \boldsymbol{\Phi} \bigg|_{\mathbf{r}=\mathbf{R}(t)} . \tag{5}$$

The dynamical equations (3) and (5) constitute two strongly non-linear coupled equations, which are obviously very hard to solve. In (Andersen et al. 2015) it is shown that one can solve for a free particle moving on a straight line, and also that, for a particle executing a classical bound orbit, e.g., in the harmonic oscillator, one obtains discrete states satisfying a condition like the Bohr-Sommerfeld quantisation rule of the old quantum mechanics. For this latter result, it is, however, important that the equations obey Galilean covariance, which dictates the choice of the amplitude $A(\mathbf{r}, t)$.

We should note that, although the above dynamical equations are inspired by de Broglie's work, they are not to be found there. In particular, we have not found an equation in his work, like our Eq. (3), relating the field to the particle position. So this is entirely our own responsibility. We should also stress the strong difference to Bohmian mechanics (Bohm 1952a, b), where the standard, homogeneous Schrödinger equation (2) is retained without any reference to the particle position. In Bohm's theory the particle is moved by the wave, but the wave is not created by the particle.

5 A Modified Double-Slit Experiment

To directly solve Eqs. (3) and (5) for the double-slit experiment seem forbiddingly hard—even numerically. What we can do instead is to give a simple argument, that shows why this type of dynamics can never reproduce the quantum behaviour in all its strangeness. To do that, we modify the double-slit experiment slightly, so as to enhance the strange quantum behaviour. Before the barrier with the two slits we insert between the slits a long "splitter plate" perpendicular to the barrier (Fig. 8). Thereby we effectively "split the world" before the particle reaches the two slits, and thus force it to "choose its path" already when it reaches the up-stream end of the splitter plate.

In quantum mechanics such a splitter plate presents no more problems than it would for water waves. The wave simply moves around the plate, retaining the symmetry between the slits and the interference maxima and minima are basically unchanged (Fig. 9). This is not so for the walking droplets or for our "quantum" particles described by Eqs. (3) and (5). Here, when a particle reaches the splitter plate, its path must be deflected either toward one or the other of the slits. Let us say that it moves

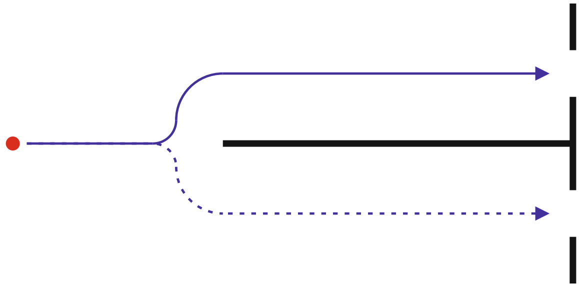

Fig. 8 The modified double-slit experiment with a splitter place inserted before the slits. Reproduced with permission from (Andersen et al. 2015)

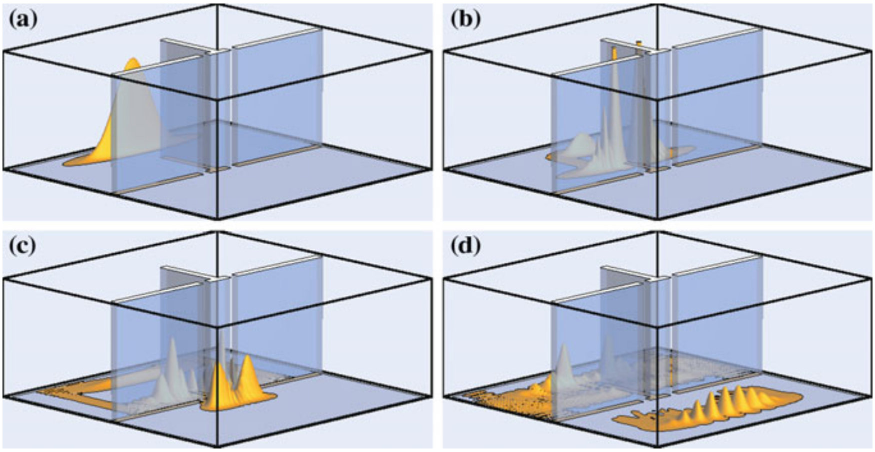

Fig. 9 Time evolution of the probability density in the modified double-slit experiment with a splitter plate in quantum mechanics. Reproduced with permission from (Andersen et al. 2015)

toward the upper slit as shown in Fig. 8. On this path the wave field resembling that of a free particle (perhaps with the addition of an image field to give the right boundary condition on the plate) will move with the speed of the particle. The lower path, however, is in a region where there is no particle and thus no forcing. Thus the initial wave field will decay there, due to its dispersive nature: the initial, localized wave field will have components moving with different velocities, which implies that the wave field will spread and decay. Thus, if the plate is long, and the time to move along it correspondingly large, the particle that finally emerges from the upper slit will see a very weak and dispersed field from the other slit, which cannot change its direction significantly. In fact part of the wave field from the lower slit might emerge only after the particle has hit the measuring screen.

Thus, even though a walking droplet can of course interfere with its own wave going through the other slit, the effect will be far from the quantum mechanical version. In quantum mechanics, Feynman's path integral formulation implies that all possible paths leading, say, from the starting point where the wave packet is introduced (the red dot in Fig. 8) to a point on the screen contribute according to their "action", irrespective of whether a particle has actually chosen that path or not. This equality of paths is broken in the experiment and in our extended quantum theory, since one of the paths is singled out due to the presence of the "real" particle. In the experiment, such a splitter plate would effectively destroy the contribution from the "other" slit if it is long enough. Here "long enough" means that the time it takes the particle to move the length of the splitter plate should be much longer than the *memory time* that determines for how long the wave field created by a droplet will persist (Eddi et al. 2011).

6 Discussion

In conclusion we see that the very fact that the particles' orbits remain well-defined while they generate the "pilot-waves" around them leads to non-quantum behaviour. Both Couder and Fort (2012) and Bush (2010) note that the uncertainty principle obviously does not work here: we can see the orbits of droplets because the photons that we use to observe them impart negligible momentum compared to that of the droplet. However, they envisage that a quantum like uncertainty might reappear if one could only observe the droplets by, say, making them collide with similar droplets in the spirit of Heisenberg's famous γ-ray microscope. However, as stressed by Bohr in the draft for the Como lecture mentioned in the beginning (Bohr 1985), uncertainty is not a question of the limitation of specific measurements. It is an intrinsic property of the particle due to wave mechanics, meaning that it does not even make sense to think of quantum particles in a "realistic" way, i.e., one where they are attributed a well-defined space-time trajectory. Thus, in the aforementioned draft, after introducing Heisenberg's recent uncertainty relations, Bohr continues (p. 93): *It must be remembered, however, that the uncertainty in question is not a simple consequence of a discontinuous change of energy and momentum say during an interaction between radiation and material particles employed in measuring the space-time coordinates of the individuals. According to the above considerations the question is rather that of the impossibility of defining rigorously such a change, when the space-time coordination of the individuals is also considered.*

That deterministic wave-driven particle systems like walking droplets are not capable of reproducing the strangeness of the quantum mechanical double-slit experiment does not render such systems unworthy of attention. Walking droplets are spectacular fluid dynamical systems in their own right, and, e.g., the interaction of walkers with subsurface obstacles and boundaries form fascinating and at present basically unexplored sets of research problems. Wave-driven particle systems may help us in pinpointing the fundamental differences between quantum mechanics and classical physics and form an instructive classical analogy for some quantum phenomena such as orbital quantisation and possibly for many-particle systems for which the analogy is largely unexplored.

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